Optimization Assignment

Name:Soundarya Naru Email : narusoundarya2002@gmail.com.com

1 Problem

If the sum of the lengths of the hypotenuse and a side of a right-angled triangle is given,show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$.

2 Solution

Let the hypotenuse of the right triangle be x and the height by y.

Hence its base is $\sqrt{x^2 - y^2}$
Hence its area $= (\sqrt{x^2 - y^2})y/2$
But it is given $x + y = p$
substituting this in the area,we get

$$Area = \sqrt{((p-y)^2 - y^2)}y/2 \tag{1}$$

squaring on both sides,we get

$$A = p^2y^2/4 - py^3/2 \tag{2}$$

For Maximum or minimum area

$$\frac{dy}{dA} = 0 \tag{3}$$

Here the area of the triangle is maximum when

$$x = 2p/3, y = p/3 \tag{4}$$

$$cos\theta = y/x \tag{5}$$

$$cos\theta = \frac{p/3}{2p/3} \tag{6}$$

$$cos\theta = 1/2 \tag{7}$$

$$\theta = \frac{\pi}{3}(or)60^\circ \tag{8}$$

Hence,the area is maximum if the angle between the hypotenuse and side is 60°.

3 Execution

Verify the above proofs in the following code.

```
https://github.com/soundaryanaru/FWC-assignments/blob/main/Optimization/Advance/code/op2.py
```