Homework 4: Signal Processing

Version: 2020 Fall

1. (20%) Consider a continuous-time LTI system whose frequency response is

$$H(j\omega)=\int_{+\infty}^{\infty} h(t)e^{-j\omega t}dt=\frac{\sin(4\omega)}{\omega}$$

If the input to this system is a periodic signal

$$x(t)=\begin{cases}1, & 0<=t<4\\ -1, & 4<=t<8\end{cases}$$

with period $T=8$, determine the corresponding system output $y(t)$.

2. (20%) Consider a discrete-time LTI system with impulse response is

$$h[n]=\begin{cases}1, & 0<=n<=2\\ -1, & -2<=n<=-1\\ 0, & otherwise\end{cases}$$

Given that the input to this system is

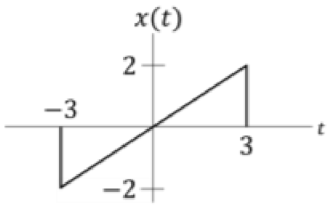
$$x[n]=\sum_{k=-\infty}^{+\infty}\delta[n-4k]$$

determine the Fourier series coefficients ak of the output $y[n]$.

3. Let the Fourier transform of the signal $y(t)$ as the following form

$$Y(j\omega)=\frac{2\sin\omega}{\omega}e^{-j2\omega}$$

and $X(j\omega)$ denotes the Fourier transform of the following signal $x(t)$



(a) (10%) Find $y(t)$ and plot $y(t)$ vs. t.
(b) (10%) Find $X(j\omega)$.
(c) (10%) Evaluate $\int_{-\infty}^{\infty} X(j\omega)Y(j\omega)d\omega$.

4. The output $y(t)$ of a causal LTI system is related to the input $x(t)$ by the equation

$$\frac{dy(t)}{dt}+10y(t)=\int_{-\infty}^{\infty} x(\tau)z(t-\tau)d\tau-x(t)$$

where $z(t)=e^{-t}u(t)+3\delta(t)$

(a) (15%) Find the frequency response $H(j\omega)$ of this system.
($H(j\omega)=\frac{Y(j\omega)}{X(j\omega)}$,where $X(j\omega)$ and $Y(j\omega)$ are the Fourier transform of $x(t)$ and $y(t)$ respectively.)
(b) (15%) Determine the impulse response $h(t)$ of the system in time domain