Homework 1: Signal Processing

Version: 2020 Fall

1. Below $x(t)$, $x[n]$ are the input and $y(t)$, $y[n]$ are the output of continuous-time and discrete-time systems respectively. Determine if the system is:
   (1) memoryless,
   (2) time-invariant,
   (3) linear,
   (4) causal,
   (5) stable.
   Justify your answer.

   (a) (10%) $y(t) = \int_{-\infty}^{2t} x(\tau)d\tau$

   (b) (10%) $y(t) = \cos^2(2t)x(t)$

   (c) (10%) $y[t] = x[n-2] - 2x[n-6]$

   (d) (10%) $y[n] = \begin{cases} x[n-1], & n >= 1 \\ 0, & n = 0 \\ x[n], & n <= -1 \end{cases}$

2. Determine whether the continuous-time signal $x(t)$ or discrete-time signal $x[n]$ is periodic or not.
   If yes, please determine its fundamental period; if not, please explain why.

   (a) (10%) $x(t) = \cos(w_n t)\cos(\sin(w_m t))$

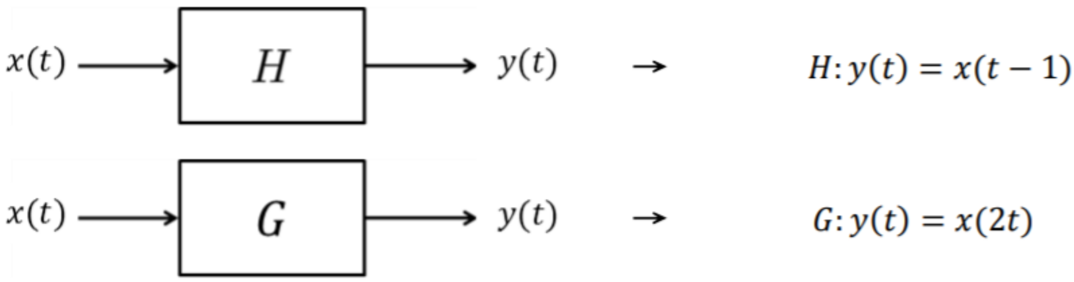
   (b) (10%) $x[n] = \cos(\pi n) + \cos(2\pi\sqrt{3}n)$

3. Below $x(t)$, $x[n]$ are the input and $y(t)$, $y[n]$ are the output of continuous-time and discrete-time systems respectively. Determine if the system is invertible. If yes, please find out the inverse system; if not, please explain why.

   (a) (10%) $y(t) = \int_{-\infty}^{t} e^{-(t-\tau)}x(\tau)d\tau$

   (b) (5%) $y[n] = x[n]x[n-2]$

4. Consider the following systems:



(a) Determine the output $y(t)$ if inputting $x(t)$ into the system H -1 , which is the inverse of H. (5%)

(b) Determine the output $y(t)$ if inputting $x(t)$ into the system G -1 , which is the inverse of G. (5%)

(c) Consider the system in the following Figure 1. Moreover, F is equivalent to the cascaded interconnection of H and G. Find the output $w(t)$ if inputting $x(t)$ into the system *F.* (i.e., in terms of x(t) or its shift/scaled versions). (5%)

(d) Consider the system in the following Figure 1. Find the output $z(t)$ if inputting $x(t)$ into the system F^{-1}, which is the inverse of F. (i.e., in terms of $x(t)$ or its shift/scaled versions). And, draw it in block diagram form in terms of H^{-1}, G^{-1} between $x(t)$ and $z(t)$. (10%)

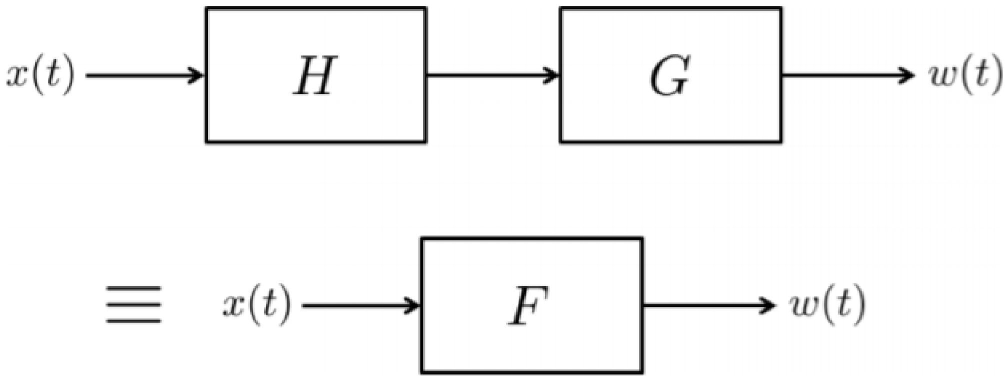


Figure 1.

The End of Homework