Homework 5: Signal Processing

Version: 2020 Fall

1. Let $x1[n]$ and $x2[n]$ be two discrete-time signals defined as

$$x_1[n] = \begin{cases} 1, & 0 <= n <= 4 \\ 0, & otherwise \end{cases}$$

and $x_2[n] = e^{j2n}$

(a) (10%) Is $x_2[n]$ a periodic signal? If yes, what is its fundamental period? If no, please show why?

(b) (20%) Find the discrete-time Fourier transforms $X_1(e^{j\omega})$ and $X_1(e^{j\omega})$ for $x_1[n]$ and $x_2[n]$, respectively.

(c) (10%) Let $x_3[n]$=$x_1[n] * x_2[n]$, where $*$ is denoted as convolution. Then find the discrete-time Fourier transform $X_3(e^{j\omega})$ for $x_3[n]$.

2. An LTI system with impulse response $h_1[n] = (\frac{1}{2})^n u[n]$ is connected in parallel with another causal LTI system with impulse response $h_2[n]$. However, we just know that the equivalent discrete-time Fourier transform of the total resulting parallel interconnection $(h_1[n] / h_2[n])$ is

$$H(e^{j\omega}) = \frac{-12 + 5e^{-j\omega}}{12 - 7e^{-j\omega} + e^{-j2\omega}}$$

(a) (10%) Find the discrete-time Fourier transform $H_1(e^{j\omega})$ for $h_1[n]$.

(b) (20%) Determine $h_2[n]$ and its discrete-time Fourier transform $H_2(e^{j\omega})$.

3. Consider a continuous-time LTI system $h(t)$ whose input $x(t)$ and output $y(t)$ are related through the following differential equation representation:

$$\frac{d^2 y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 9y(t) = \frac{d^2 x(t)}{dt^2} + 3\frac{dx(t)}{dt} + 2x(t)$$

(a) (10%) Determine the impulse response, $h(t)$, of the system.

(b) (5%) Determine the impulse response, $g(t)$, of the inverse system.

4. Consider a causal and stable LTI system $h[n]$ whose input $x[n]$ and output $y[n]$ are related through the following second-order differential equation representation:

$$y[n] - \frac{1}{4}y[n-1] - \frac{1}{4}y[n-2] = x[n]$$

(a) (10%) Determine the frequency response $H(e^{j\omega})$ for the system $h[n]$.

(b) (5%) Determine the impulse response $h[n]$ of the system.

The End of Homework