HW 3 solution

1. (a) $T=2,\ \omega_0=\pi$

$$a_0 = \frac{1}{2}\int_{-1}^{1} t\,dt = 0$$

$$a_k = \frac{1}{2}\int_{-1}^{1} t e^{-jk\pi t}\,dt = \frac{1}{2}\left[\frac{e^{-jk\pi t}\cdot t}{-jk\pi}\Bigg|_{-1}^{1} - \int_{-1}^{1}\frac{e^{-jk\pi t}}{-jk\pi}\,dt\right]$$

$$= \frac{1}{2}\left[\frac{e^{jk\pi}+e^{-jk\pi}}{-jk\pi} - \cancel{\frac{e^{jk\pi}-e^{-jk\pi}}{k^2\pi^2}}^{\,0}\right]$$

$\because k \in \mathbb{Z}$

$$= \frac{j\cos k\pi}{k\pi} = \frac{j(-1)^k}{k\pi},\quad k\neq 0$$

(b) $T=4,\ \omega_0=\frac{\pi}{2}$

$$k=0,\quad a_0 = \frac{1}{4}\int_0^2 \sin\pi t\,dt = \frac{1}{4}\cdot\frac{-\cos\pi t}{\pi}\Bigg|_0^2 = 0$$

$$k\neq 0,\quad a_k = \frac{1}{4}\int_0^2 \sin\pi t\, e^{-jk\frac{\pi}{2}t}\,dt$$

$$= \frac{1}{4}\int_0^2 \frac{e^{j\pi t}-e^{-j\pi t}}{2j} e^{-jk\frac{\pi}{2}t}\,dt$$

$$= \frac{1}{8j}\left(\frac{e^{j(\pi-\frac{k}{2}\pi)t}}{j(\pi-\frac{k}{2}\pi)}\Bigg|_0^2 + \frac{e^{-j(\pi+\frac{k}{2}\pi)t}}{j(\pi+\frac{k}{2}\pi)}\Bigg|_0^2\right)$$

$$= \frac{1}{8}\left(1-(-1)^k\right)\frac{2}{\pi-\frac{k^2}{4}\pi} = \frac{1-(-1)^k}{\pi(4-k^2)},\quad k \text{ can't be } \pm 2$$

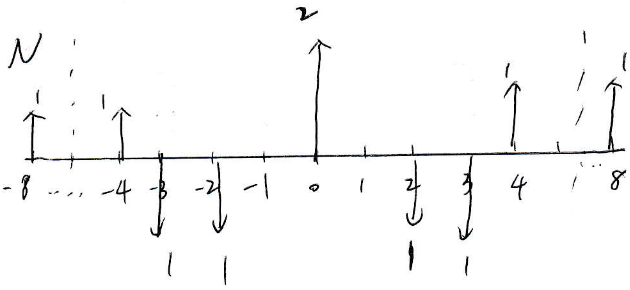
$$\therefore\ k=2,\quad a_k = \frac{1}{8j}\left(\frac{-j\pi}{-\frac{1}{2}j\pi}+0\right) = -\frac{1}{4}j$$

$$k=-2,\quad a_k = \frac{1}{8j}\left(0+\frac{-j\pi}{\frac{1}{2}j\pi}\right) = \frac{1}{4}j$$

$$\Rightarrow a_k = \begin{cases} \dfrac{1-(-1)^k}{\pi(4-k^2)}, & k\neq\pm 2 \\ -\frac{1}{4}j, & k=2 \\ \frac{1}{4}j, & k=-2 \end{cases}$$

, (k=0 can be included)

2. (a) graph to find N:



$N=12,\ w_0=\frac{\pi}{6}$

$$a_k=\frac{1}{12}\sum_{n=-5}^{6}x[n]e^{-jk\frac{\pi}{6}n}=\frac{1}{12}\left(e^{-j(-4)\frac{\pi}{6}k}-e^{-j(-3)\frac{\pi}{6}k}-e^{-j(-2)\frac{\pi}{6}k}+2-e^{-j(2)\frac{\pi}{6}k}-e^{-j(3)\frac{\pi}{6}k}+e^{-j(4)\frac{\pi}{6}k}\right)$$

$$=\frac{1}{6}\cos\left(\frac{2\pi}{3}k\right)-\frac{1}{6}\cos\left(\frac{\pi}{2}k\right)-\frac{1}{6}\cos\left(\frac{\pi}{3}k\right)+\frac{1}{6}$$

(b) from ex. 3.12

$N=N_1,\ W_0=\frac{2\pi}{N}$

$$\Rightarrow a_k=\begin{cases}\frac{2N_1+1}{N}, & k=0,\pm N,\pm 2N,\dots\\ \frac{1}{N}\cdot\frac{\sin\left[2\pi k\left(\frac{N_1+\frac{1}{2}}{N}\right)\right]}{\sin(\pi k/N)}, & k\neq 0,\pm N,\pm 2N,\dots\end{cases}$$

3. (a) $y[n]=x[n]\cos\left(\frac{8\pi n}{N}\right)$

$$=\frac{x[n]}{2}\left(e^{j(4)\frac{2\pi}{N}n}+e^{j(-4)\frac{2\pi}{N}n}\right)$$

from table 3.2 frequency shift property

$$a_k'=\frac{1}{2}\left(a_{k+4}+a_{k-4}\right)$$

(b) $y[n]=\begin{cases}2x[n], & n\in\text{even}\\ 0, & n\in\text{odd}\end{cases}$ (period N is an even number)

$$\Rightarrow y[n]=x[n]+(-1)^n x[n]$$

$$=x[n]+e^{j\left(\frac{N}{2}\right)\frac{2\pi}{N}n}\cdot x[n]$$

$$\xleftrightarrow{F.S.}\ b_k=a_k+a_{k-\frac{N}{2}}$$

4. from fact 1 & 2

$$a_0 = \frac{1}{6} \sum_{n=-2}^{3} x[n] = \frac{1}{2}$$

from fact 3

$$\sum_{n=0}^{5} (-1)^{n+6} x[n] = \sum_{n=0}^{5} e^{j(3)\frac{2\pi}{6}(n+6)} \cdot x[n] = \sum_{n=0}^{5} e^{j(3)\frac{2\pi}{6}n} x[n] = 1 = 6a_3$$

$$\therefore a_3 = \frac{1}{6}$$

with minimum power $\therefore a_1 = a_2 = a_4 = a_5 = 0$

$$\Rightarrow x[n] = \frac{1}{2} + \frac{1}{6} e^{j(3)\frac{2\pi}{6}n} = \frac{1}{2} + \frac{1}{6}(-1)^n$$