Signals & Systems HW5

1. (a) No. Since $n \in \mathbb{Z}$, we can't find $N_0 \in \mathbb{N}$ s.t. $x_2[n] = x_2[n+N_0]$ $\forall n \in \mathbb{Z}$

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(b) $X_1(e^{jw}) = \sum_{n=-\infty}^{\infty} x_1[n]e^{-jwn} = 1 + e^{-jw} + e^{-2jw} + e^{-3jw} + e^{-4jw}$

$X_2(e^{jw}) = 2\pi \sum_{l=-\infty}^{\infty} \delta(w-2-2\pi l)$ (by Table 5.2 with $w_0 = 2$)

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(c) $X_3(e^{jw}) = X_1(e^{jw})X_2(e^{jw}) = 2\pi(1+e^{-jw}+e^{-2jw}+e^{-3jw}+e^{-4jw}) \sum_{l=-\infty}^{\infty} \delta(w-2-2\pi l)$

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2. (a) $H_1(e^{jw}) = \sum_{n=-\infty}^{\infty} h_1[n]e^{-jwn} = \sum_{n=0}^{\infty} (\frac{1}{2})^n e^{-jwn} = \dfrac{1}{1-\frac{1}{2}e^{-jw}}$

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(b) $H_2(e^{jw}) = H(e^{jw}) - H_1(e^{jw}) = \dfrac{-12+5e^{-jw}}{(4-e^{-jw})(3-e^{-jw})} - \dfrac{1}{1-\frac{1}{2}e^{-jw}}$

$= \dfrac{-2}{1-\frac{1}{4}e^{-jw}} + \dfrac{1}{1-\frac{1}{3}e^{-jw}} - \dfrac{1}{1-\frac{1}{2}e^{-jw}}$

$h_2[n] = -2(\frac{1}{4})^n u[n] + (\frac{1}{3})^n u[n] - (\frac{1}{2})^n u[n]$

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3. (a) $H(j\omega) = \dfrac{Y(j\omega)}{X(j\omega)} = \dfrac{-\omega^2+3j\omega+2}{-\omega^2+6j\omega+9} = 1 + \dfrac{-3}{(j\omega+3)} + \dfrac{2}{(j\omega+3)^2}$

$\Rightarrow h(t) = \delta(t) - 3e^{-3t}u(t) + 2te^{-3t}u(t)$ (by Table 4.2)

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(b)

$G(j\omega) = \dfrac{1}{H(j\omega)} = \dfrac{-\omega^2+6j\omega+9}{-\omega^2+3j\omega+2} = 1 + \dfrac{-1}{(j\omega+2)} + \dfrac{4}{(j\omega+1)}$

$\Rightarrow g(t) = \delta(t) - e^{-2t}u(t) + 4e^{-t}u(t)$ (by Table 4.2)

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4. (a) $H(e^{j\omega}) = \dfrac{Y(e^{j\omega})}{X(e^{j\omega})} = \dfrac{1}{1-\frac{1}{4}e^{-j\omega}-\frac{1}{4}e^{-2j\omega}} = \dfrac{1}{\left(1+\frac{\sqrt{17}-1}{8}e^{-j\omega}\right)\left(1-\frac{\sqrt{17}+1}{8}e^{-j\omega}\right)}$

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(b) $H(e^{j\omega}) = \dfrac{8}{17+\sqrt{17}} \cdot \dfrac{1}{\left(1+\frac{\sqrt{17}-1}{8}e^{-j\omega}\right)} + \dfrac{9+\sqrt{17}}{17+\sqrt{17}} \cdot \dfrac{1}{\left(1-\frac{\sqrt{17}+1}{8}e^{-j\omega}\right)}$

$\Rightarrow h[n] = \dfrac{8}{17+\sqrt{17}}\left(-\dfrac{\sqrt{17}-1}{8}\right)^n u[n] + \dfrac{9+\sqrt{17}}{17+\sqrt{17}}\left(\dfrac{\sqrt{17}+1}{8}\right)^n u[n]$

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