HW4

1. $\because x(t)$ is real & odd, $\quad W_0 = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}$

$\therefore a_k$ is imaginary & odd,

$\therefore a_0 = 0$

$$a_k = \frac{1}{8}\int_0^8 x(t)\cdot e^{-j(2\pi/8)kt}\,dt$$

$$= \frac{1}{8}\left[\int_0^4 e^{-j(2\pi/8)kt}\,dt - \int_4^8 e^{-j(2\pi/8)kt}\,dt\right]$$

$$= \frac{1}{8}\left[\frac{1}{-\frac{1}{4}\pi jk}\left(e^{-j\pi k}-1\right) - \frac{1}{-\frac{1}{4}\pi jk}\left(1-e^{-j\pi k}\right)\right]$$

$$= \frac{1}{j\pi k}\left[1-e^{-j\pi k}\right] = \begin{cases} 0 & , k = 0, \pm 2, \pm 4 \dots \\ \frac{2}{j\pi k} & , k = \pm 1, \pm 3, \pm 5 \dots \end{cases}$$

已知：$y(t) = \sum_{k=-\infty}^{\infty} a_k H(jkW_0) e^{jkW_0 t}$

當 $k = 2n \quad (n \in \mathbb{Z})$, $a_k H(jkW_0) e^{jkW_0 t} = 0$

當 $k = 2n+1 \quad (n \in \mathbb{Z})$, $\because H(j(2n+1)W_0) = \frac{\sin(4(2n+1)W_0)}{(2n+1)W_0}$

$$= \frac{\sin((2n+1)\pi)}{(2n+1)\pi/4} = 0$$

$\therefore a_k H(jkW_0) e^{jkW_0 t} = 0$

則 $y(t) = \sum_{k=-\infty}^{\infty} a_k H(jkW_0) e^{jkW_0 t} = 0$ #

2. $H(e^{jw}) = \sum_{n=-\infty}^{\infty} h(n)\, e^{-jwn}$ $h(n) = \begin{cases} 1 & 0 \le n \le 2 \\ -1 & -2 \le n \le -1 \\ 0 & \text{o.w} \end{cases}$

$$= h(-2)e^{j2w} + h(-1)e^{jw} + h(0) + h(1)e^{-jw} + h(2)e^{-j2w}$$

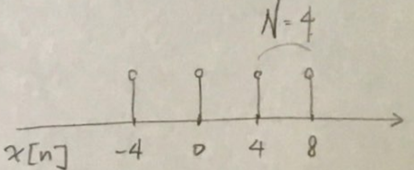
$$= (-1)e^{j2w} + (-1)e^{jw} + 1 + 1e^{-jw} + 1e^{-j2w}$$

$$= 1 + (-e^{jw} + e^{-jw}) + (-e^{j2w} + e^{-j2w})$$

$$= 1 - 2j\left(\frac{e^{jw} - e^{-jw}}{2j}\right) - 2j\left(\frac{e^{j2w} - e^{-j2w}}{2j}\right)$$

$$= 1 - 2j\sin w - 2j\sin(2w)$$

$\because x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$ $\therefore N = 4$



分析 a_k 係數 for $x[n]$, $a_k = \frac{1}{4}\sum_{n=\langle 4\rangle} \sum_{m=-\infty}^{\infty} \delta[n-4m]\cdot e^{-jk(\frac{2\pi}{4})n}$

$$a_0 = \frac{1}{4},\ a_1 = \frac{1}{4},\ a_2 = \frac{1}{4},\ a_3 = \frac{1}{4}$$

from (3.131) $y[n] = \sum_{k=\langle N\rangle} \underbrace{a_k H(e^{j\frac{2\pi}{N}k})}_{\text{Fourier series coefficients } \hat{a}_k \text{ of } y[n]} e^{jk(\frac{2\pi}{N})n}$

$$\hat{a}_0 = a_0 \cdot H(e^{j\frac{2\pi}{N}0}) = \frac{1}{4}(1) = \frac{1}{4}$$

$$\hat{a}_1 = a_1 H(e^{j\frac{2\pi}{N}1}) = \frac{1}{4}(1 - 2j\underbrace{\sin\tfrac{2\pi}{4}}_{1} - 2j\underbrace{\sin\tfrac{4\pi}{4}}_{0}) = \frac{1}{4}(1-2j)$$

$$\hat{a}_2 = a_2 H(e^{j\frac{2\pi}{N}2}) = \frac{1}{4}(1 - 2j\underbrace{\sin\pi}_{0} - 2j\underbrace{\sin 2\pi}_{0}) = \frac{1}{4}$$

$$\hat{a}_3 = a_3 H(e^{j\frac{2\pi}{N}3}) = \frac{1}{4}(1 - 2j\underbrace{\sin\tfrac{3}{2}\pi}_{-1} - 2j\underbrace{\sin 3\pi}_{0}) = \frac{1}{4}(1+2j)$$

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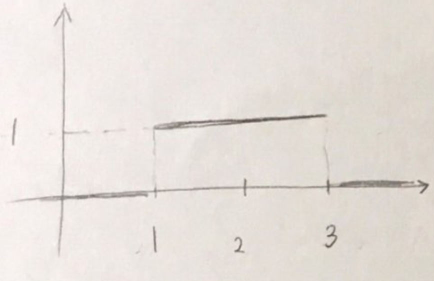
3. (a) $Y(jw) = 2\frac{\sin W}{w} e^{-j2w}$

$\updownarrow$

$y(t) = rect_2(t) * \delta(t-2) = rect_2(t-2)$



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(b) $x(t) = t \cdot rect_6(t) \cdot \frac{2}{3}$ $\quad (\because \; t^n x(t) \overset{\mathcal{F}}{\longleftrightarrow} j^n \frac{d}{dw^n} X(w))$

$= \frac{2}{3} t \cdot rect_6(t) \xrightarrow{\mathcal{F}} \frac{2}{3} j \frac{d}{dw}\left(\frac{2\sin w \cdot 3}{w}\right)$ ...... ①

$X(jw) = \frac{2}{3} 2j \left(\frac{3W\cos(3W) - \sin 3W}{w^2}\right) = \frac{4j}{3W}\left(3\cos(3W) - \frac{\sin(3W)}{W}\right)$

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(c) $\int_{-\infty}^{\infty} X(jw) Y(jw) dw$ $\quad$ < Note> $x^*(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X^*(-jw) e^{jwt} dw$ (4.28)

$= \int_{-\infty}^{\infty} X^*(-jw) \cdot Y(jw) dw = \int_{-\infty}^{\infty} X^*(-jw) \int_{-\infty}^{\infty} y(t) e^{-jwt} dt \, dw$

$= \int_{-\infty}^{\infty} \underline{\int_{-\infty}^{\infty} X^*(-jw) e^{-jwt} dw} \; y(t) dt = 2\pi \int_{-\infty}^{\infty} x^*(-t) y(t) dt$

$= 2\pi \int_{1}^{3} -\frac{2}{3} t \, dt = -\frac{16}{3}\pi$

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4. $$\frac{dy(t)}{dt} + 10y(t) = \int_{-\infty}^{\infty} x(\tau)z(t-\tau)\,d\tau - x(t)$$

$$\rightarrow j\omega Y(j\omega) + 10Y(j\omega) = X(j\omega)Z(j\omega) - X(j\omega) \quad \text{------} \quad ①$$

where $$Z(j\omega) = \int_0^{\infty} e^{-t}e^{-j\omega t}\,dt + 3$$

$$= \frac{1}{j\omega+1} + 3$$

(a) $$Y(j\omega) = X(j\omega)H(j\omega)$$

from ① $$(j\omega+10)Y(j\omega) = \left(\frac{1}{j\omega+1} + 2\right)X(j\omega)$$

$$H(j\omega) = \frac{2+\frac{1}{j\omega+1}}{j\omega+10} = \frac{2}{j\omega+10} + \frac{1}{9}\left(\frac{1}{j\omega+1} - \frac{1}{j\omega+10}\right)$$

$$= \frac{1}{9}\frac{1}{j\omega+1} + \frac{17}{9}\frac{1}{j\omega+10}$$ ※

(b) Note: $$e^{-at}u(t) \xrightarrow{\mathcal{F}} \frac{1}{j\omega+a}$$

$$H(j\omega) \xrightarrow{\mathcal{F}^{-1}} h(t) = \left(\frac{1}{9}e^{-t} + \frac{17}{9}e^{-10t}\right)u(t)$$

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