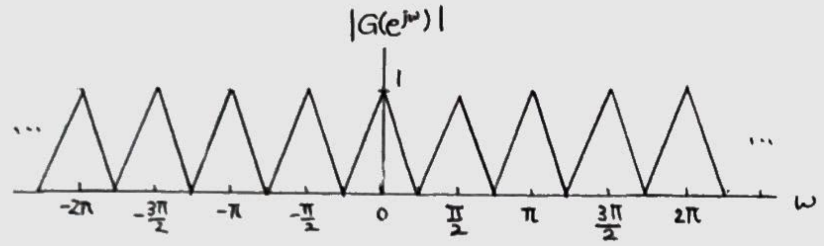
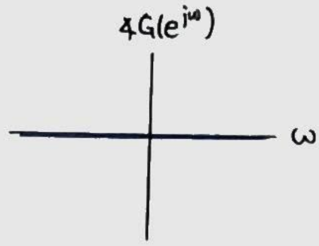
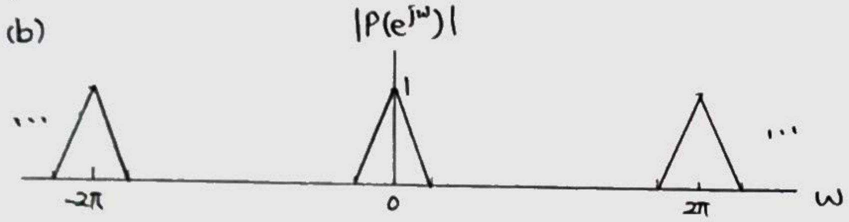
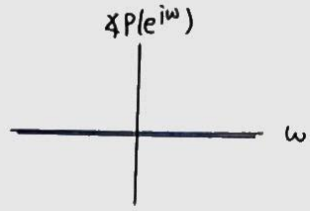
1. (a) $G(e^{j\omega}) = X(e^{j4\omega})$





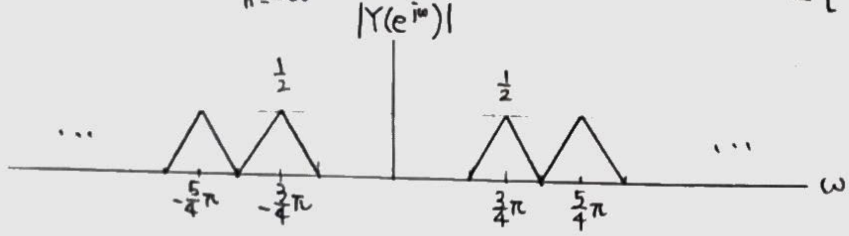
(b)

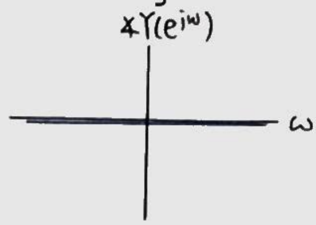




(c) $\cos(\frac{3}{4}\pi n) = \frac{1}{2}(e^{j\frac{3}{4}\pi n} + e^{-j\frac{3}{4}\pi n})$

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} p[n]\cdot\frac{1}{2}(e^{j\frac{3}{4}\pi n} + e^{-j\frac{3}{4}\pi n})e^{-j\omega n} = \frac{1}{2}\left[P(e^{j(\omega-\frac{3}{4}\pi)}) + P(e^{j(\omega+\frac{3}{4}\pi)})\right]$$





2. (a) $H(j\omega) = \dfrac{1}{j\omega+2} \Rightarrow \measuredangle H(j\omega) = -\tan^{-1}(\frac{\omega}{2})$

$$\Rightarrow \tau(\omega) = -\frac{d}{d\omega}\left(\measuredangle H(j\omega)\right) = \underline{\frac{2}{\omega^2+4}} \ \divideontimes$$

(b) $H(e^{j\omega}) = \sum_{n=0}^{N-1} h[n]e^{-j\omega n}$ (causal)

$$= \sum_{n=0}^{(N-1)/2} h[n](e^{-j\omega n} + e^{-j\omega(N-1-n)}) - h[\tfrac{N-1}{2}]e^{-j\omega\frac{N-1}{2}} \quad \text{(symmetry)}$$

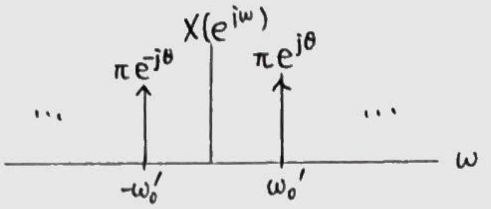
$$= \sum_{n=0}^{(N-1)/2} h[n]\cos\left(\omega(n-\tfrac{N-1}{2})\right)e^{-j\omega\frac{N-1}{2}} - h[\tfrac{N-1}{2}]e^{-j\omega\frac{N-1}{2}}$$

$$\Rightarrow \measuredangle H(e^{j\omega}) = -\omega\frac{N-1}{2} \Rightarrow \tau(\omega) = \underline{\frac{N-1}{2}} \ \divideontimes$$

3. $H(e^{j\omega}) = j\omega$ for $-\pi < \omega < \pi$, let $\omega_0' = \omega_0 + 2\pi k \in [-\pi, \pi]$ where $k \in \mathbb{Z}$

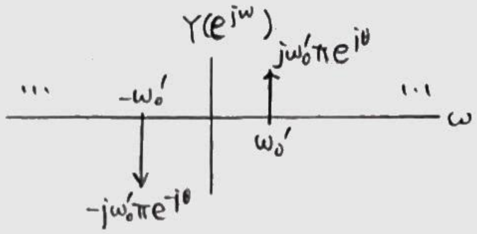
$$x[n] = \cos(\omega_0 n + \theta) = \frac{1}{2}\left(e^{j(\omega_0 n+\theta)} + e^{-j(\omega_0 n+\theta)}\right)$$

$\Rightarrow$



$$Y(e^{j\omega}) = H(e^{j\omega})\, X(e^{j\omega})$$

$\Rightarrow$



$$j\omega_0'\pi e^{j\theta} = \omega_0'\pi e^{j(\theta+\frac{\pi}{2})}$$
$$-j\omega_0'\pi e^{-j\theta} = \omega_0'\pi e^{-j(\theta+\frac{\pi}{2})}$$

$$\Rightarrow y[n] = \omega_0' \cos\left(\omega_0 n + \theta + \frac{\pi}{2}\right)$$

$$= \underline{-\omega_0' \sin(\omega_0 n + \theta)} \quad \divideontimes$$