Homework 6: Signal Processing

Version: 2020 Fall

1. Consider a DT signal x[n] with DT Fourier transform (DTFT) $X(e^{j\omega})$ given by Figure 1.

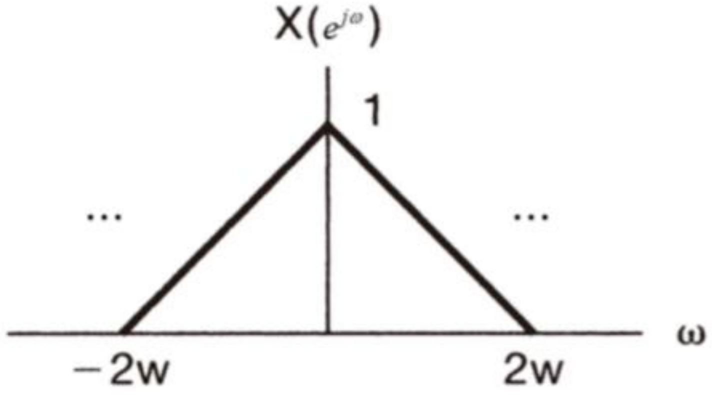


Figure 1

where $w = \pi / 2$.

Next, we create a DT signal g[n] given by

$$g[n] = x_{(4)}[n] = \begin{cases} x[n/4], & n = 0, \pm 4, \pm 8, \pm 12, \ldots \\ 0, & otherwise \end{cases}$$

(a) (10%) Find and plot the magnitude and phase response of the DTFT $G(e^{j\omega})$ of g[n].

(b) (15%) We pass the signal g[n] through an ideal lowpass filter with cutoff frequency $\pi / 4$ and passband gain=1 to produce a signal p[n]. Find and plot the magnitude and phase responses of the DTFT $P(e^{j\omega})$ of p[n].

(c) (15%) The DT signal p[n] is employed to generate an amplitude modulated(AM) signal given by $y[n] = p[n]\cos(3\pi n / 4)$ .Find and plot the magnitude and phase responses of the DTFT $Y(e^{j\omega})$ of y[n].

2.

(a) (15%) Consider that a continuous-time(CT) linear time-invariant(LTI) system h(t) with input x(t) and the output y(t) related by the following linear constant-coefficient differential equation(LCCDE)

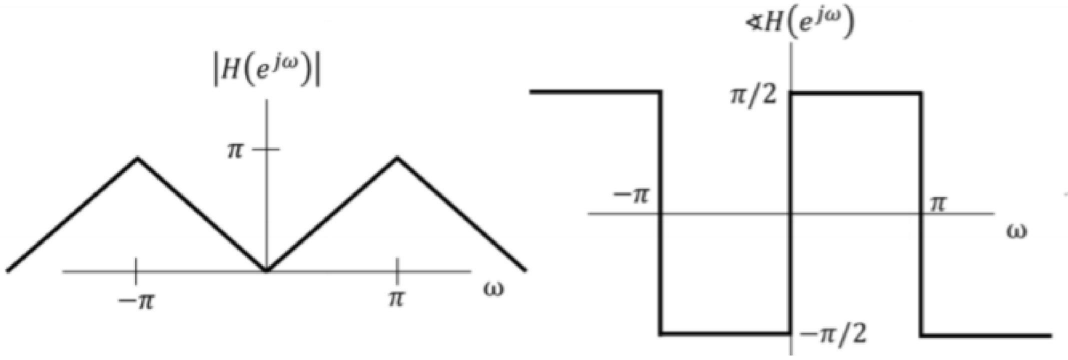
$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

Find the group delay of the CT LTI system. Justify your answer.

(b) (20%) Consider a causal finite impulse response(FIR) filter with real-valued impulse response h[n]=0 for n$\geq N$. Let N be odd and h[n] be symmetric about (N-1)/2. Find the group delay of the FIR filter. Justify your answer.

3.

The system frequency response $H(e^{j\omega})$ of a discrete-time differentiator is shown in below:



(25%) Determine the output signal y[n] as a function of ω_0, θ if the system input x[n] is

$$x[n] = \cos[\omega_0 n + \theta]$$

The End of Homework