# The Study of Game Theory on Graph

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### 1 Introduction

Given each vertex  $v_i$  on the graph X has its *benefit*  $p_i(v_i)$ , an *input*  $a \in v_i$  could convey  $signal \ x_a$  to  $v_i$  by distribution of strategies  $p_n \in v_n$ .

I.g., it could be optimised by DL for the given set of inputs  $a_n \in A$  and its entries  $v \in V$ , to know the tendency in A or V; like the network of financial system for the outer stimuli like the official discount rate. The second example is for topography model of psychology by Freud, in more minute vertices, to model the unconsciousness. The third example is ecosystem in general against any change outside.

In metaphor, if the pinball has *Freud's pleasure principle* for the gross benefit  $p^*$  as the score board on it, with the players as the environment to fit in, by learning the distribution of the springs dynamically; the pinball could have say it owns the mind to grow against the coming players, by *conflict* among the springs set aside.

By this topography, i.g., conflict like ambivalent interpretation becomes possible like the image of a woman "beautiful" on topos  $v_1$  while "dangerous" on topos  $v_2$  with deciding how to output in the gross benefit  $p^*$ .

## 2 On PyTorch

As the easier case than AI, we firstly model the ideal financial market.

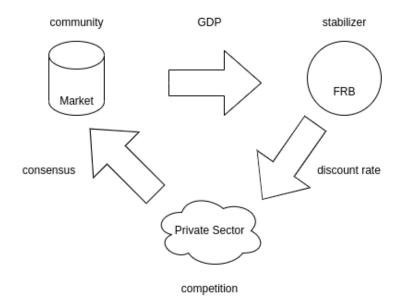


Figure 1: The ideal financial market

which could be a metaphor to the Freudian metanl model.

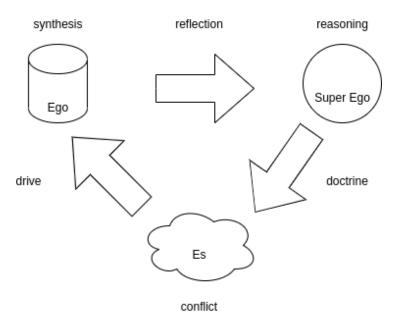


Figure 2: The Freudian Topography

The weight matrix W be on the cloud of private competitors;

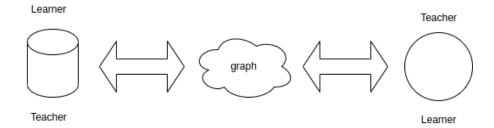


Figure 3: Ping-pong Model

## 3 In Theory

Path-dependency best response function

$$B_i(a_{-i}) = \{a_i \in A_i : u_i(a_i, a_{-i}) \ge u_i(a_{i'}, a_{-i}), \forall a_{i'} \in A_i\}$$

Cournot's oligopoly Game

$$\pi_i(q_1,\cdots,q_n)=q_iP(q_1+\cdots+q_n)-C_i(q_i)$$

The War of Attrition

$$-t_{i} \qquad if \quad t_{i} < t_{j}$$

$$u_{i}(t_{1}, t_{2}) = \frac{1}{2}v_{i} - t_{i} \qquad if \quad t_{i} = t_{j}$$

$$v_{i} - t_{j} \qquad if \quad t_{i} > t_{j}$$

$$(1)$$

Accident Game

$$-a_i - \rho(a_1, a_2)L(a_1, a_2)$$
  
 $-a_2 - (1 - \rho(a_1, a_2)L(a_1, a_2))$ 

## References

[1] An Introduction to Game Theory, Martin J. Osborne, 2000.