

Exerciese

A.6

Determine the rank of the following matrices

```
A = [3, 0, 1, 3;  
     2, 0, 3, 2;  
     0, 2, -8, 1;  
     -2, 1, 2, 1];  
  
rank_A = rank(A);  
disp(['Rank of the A is ', num2str(rank_A)]);
```

Rank of the A is 4

A.20

check if the following systems of equations are consistent. If they are, calculate their general solutions

$$x_1 + x_2 + x_3 + x_4 = 10$$

$$-x_1 + x_2 - x_3 + x_4 = 2$$

$$2x_1 - 3x_2 + 2x_3 - 2x_4 = -6$$

```
A = [1, 1, 1, 1; -1, 1, -1, 1; 2, -3, 2, -2];  
b = [10; 2; -6];  
  
Ab = [A, b];  
rank_Ab = rank(Ab);  
disp(['Rank of the Ab is ', num2str(rank_Ab)]);
```

Rank of the Ab is 3

```
% Perform Gaussian elimination to row echelon form  
rref_Ab = rref(Ab);  
  
if rank(A) == rank(Ab)  
    disp('The system is consistent.');    disp('General solution:');  
    % Extract the reduced row echelon form of A and b  
    rref_A = rref_Ab(:, 1:end-1);  
    rref_b = rref_Ab(:, end);  
  
    % Find the particular solution  
    particular_solution = rref_A \ rref_b;
```

```

% Display the particular solution
disp(particular_solution);

% Find the null space (homogeneous solution)
null_space = null(A, 'r');

% Display the null space (homogeneous solution)
disp('Null space (homogeneous solution):');
disp(null_space);

% Display the general solution
disp('General solution:');
disp(particular_solution + null_space);
else
    disp('The system is inconsistent.');
```

The system is consistent.

General solution:

4
2
0
4

Null space (homogeneous solution):

-1
0
1
0

General solution:

3
2
1
4

A.29

check the linear independence of the foollowig set of values

$$a^{(1)} = [1, 2, 3, 4, 5]^T$$

$$a^{(2)} = [-2, 1, 0, 1, -1]^T$$

$$a^{(3)} = [4, 0, -3, 2, 1]^T$$

```

a1 = [1, 2, 3, 4, 5]';
a2 = [-2, 1, 0, 1, -1]';
a3 = [4, 0, -3, 2, 1]';
```

```

A = [a1, a2, a3];
rank_A = rank(A);
```

```

% check if vector are linearly indepedent
if rank_A == size(A, 2)
```

```

disp('The vectos are linear independent')
else
disp('The vectors are linearly independent')

end

```

The vectos are linear independent

A.32

Find eigenvalues for the following matrices

```

A = [1, 1, 0;
     1, 4, 0;
     0, 0, 5];

```

```
e = eig(A);
```

```
disp(e);
```

```

0.6972
4.3028
5.0000

```

Taylor Series Expansion

1. Write down the taylor series expansion of the function about x_0 .
2. Plot the original function and taylor series expansion near x_0 .
3. Neglect terms of order three or higher

$$f(x) = x_1 e^{-x_1} + x_2 + 1, x_0 = [1, 0]^T$$

taylor series expansion for one-dimensional variables

$$\text{cf) } f(x^*) = \sum_{n=0}^{\infty} \frac{f^n(x^*)}{n!} (x - x^*)^n$$

```

clear;

syms x1 x2;

% original function
f1 = @(x1, x2) x1 .* exp(-x1) + x2 + 1;

% taylor series expansion near x0
f2 = @(x1, x2) 2 + ...
    ([exp(-x2); -x1 .* exp(-x2) + 1].' * [x1 - 1; x2]) + ...

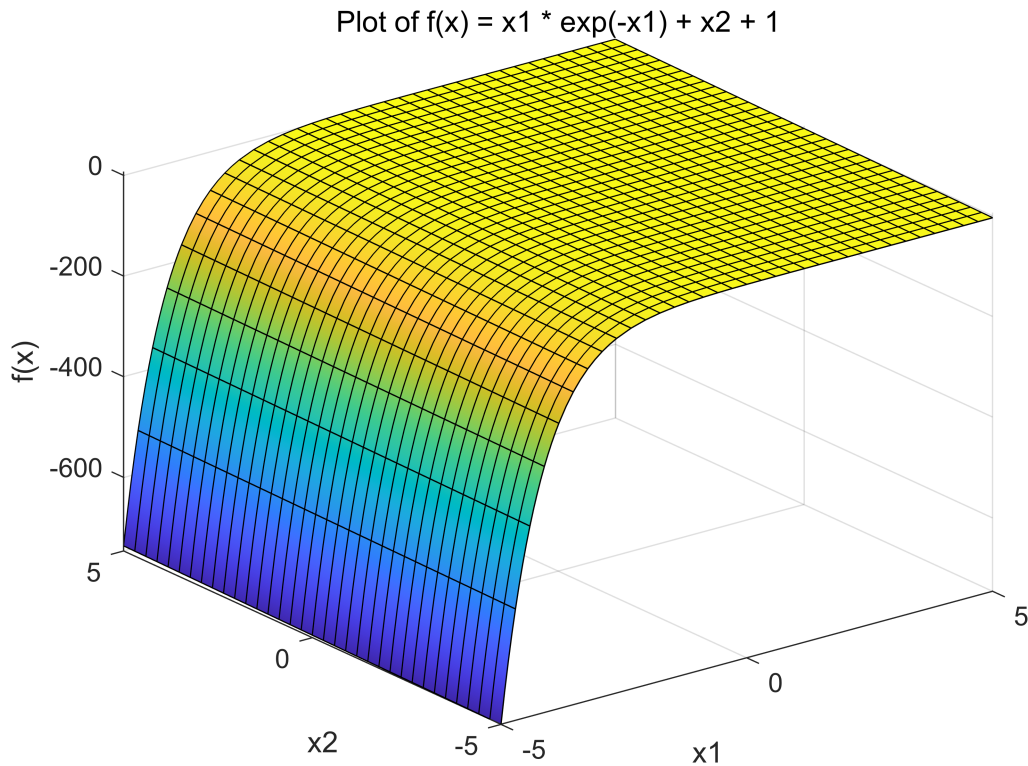
```

```

(1/2 .* [x1 - 1; x2].' * [0, -exp(-x2); -exp(-x2), x1 .* exp(-
x2)] * [x1 - 1; x2]);

% plot
figure(1);
fsurf(f1, [-5, 5, -5, 5]);
xlabel('x1');
ylabel('x2');
zlabel('f(x)');
title('Plot of f(x) = x1 * exp(-x1) + x2 + 1');

```



```

figure(2);
fsurf(f2, [-5, 5, -5, 5]);

```

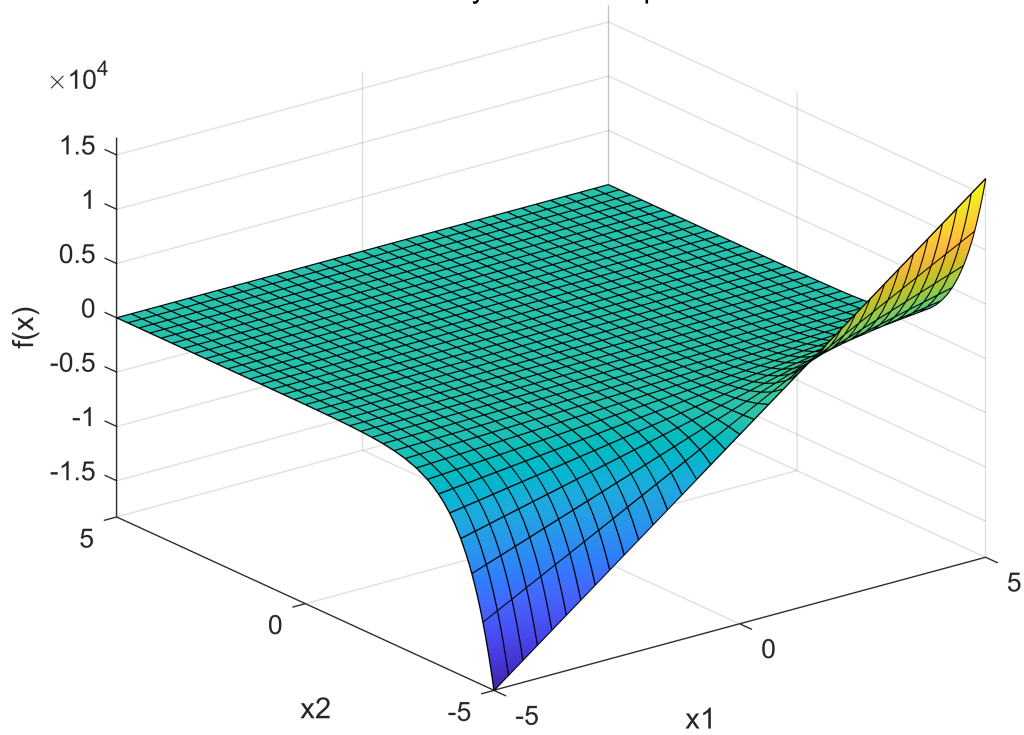
경고: 배열 입력값에 대해 함수가 예기치 않게 동작하고 있습니다. 성능을 개선하려면 크기와 형태가 입력 인수와 동일한 출력값을 반환하도록 함수를 적절히 벡터화하십시오.

```

xlabel('x1');
ylabel('x2');
zlabel('f(x)');
title('Plot of taylor series expansion');

```

Plot of taylor series expansion



```
result1 = f1(1, 0);  
result2 = f2(1, 0);  
disp(['Original value: ', num2str(result1)]);
```

Original value: 1.3679

```
disp(['Taylor expansion value: ', num2str(result2)]);
```

Taylor expansion value: 2