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# A Dynamic Model for Tire/Road Friction Estimation under Combined Longitudinal/Lateral Slip Situation

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## Abstract

A new dynamic tire model for estimating the longitudinal/lateral road-tire friction force was derived in this paper. The model was based on the previous Dugoff tire model, in consideration of its drawback that it does not reflect the actual change trend that the tire friction force decreases with the increment of wheel slip ratio when it enters into the nonlinear region. The Dugoff model was modified by fitting a series of tire force data and compared with the commonly used Magic Formula model. This new dynamic friction model is able to capture accurately the transient behavior of the friction force observed during pure longitudinal wheel slip, lateral sideslip and combined slip situation. Simulation has been done under different situations, while the results validate the accuracy of the new tire friction model in predicting tire/road friction force during transient vehicle motion.

## Introduction

It's obvious that the contact area between the tire and road surface acts as an interface of the vehicle and driving environment. The forces generated in this interface not only govern the motion states of the driving vehicle, but also determine their certain performances under extreme situations. On the other hand, the tire-road forces and moments are also the media through which transferred the drivers' intention and maneuverability operation. Generated from the friction mechanism between the tire tread and road surface texture, the tire-road force shaped the characteristics of a highly nonlinear function related to many factors such as vehicle motion states, wheel slip situation, wheel load, tire stiffness, and road condition etc., and also limited by the maximum friction potentials of this interface. It's the tire-road forces that affect the automotive dynamics and kinematics responses, and determine the vehicular stability and maneuverability under critical driving condition. So many active safety systems like ABS, TCS and ESP, regulate the tire forces or torques adapt to vehicle states and driving environment situations, and make full

use of road friction capability to the most possible extent. Since friction is the major mechanism for generating forces on the vehicle, it is extremely important to have an accurate characterization of the magnitude (and direction) of the friction force generated at the tire-road interface [1]. However, it's always impractical to measure the tire-road forces directly for dynamics control systems equipped on production vehicle, due to the inconvenient application and high costs of expensive sensors. Therefore the problem of modeling and predicting tire-road friction has become an area of intense research in automotive community in the past decades. Moreover, tire friction models are also indispensable for accurate studying the friction force characteristics when designing vehicle dynamics control systems.

Abundant research has been carried on exploring the dynamic phenomenon and transient properties of the tire-road forces, and various tire friction models were proposed, include the theoretical and empirical models. Trying to look for the explanations for tire-road friction characteristics with experimental data and fit their changing rules by mathematical equations, the empirical tire model has good utility in developing vehicle control systems. The normalized tire-road friction force (or called road friction coefficient)  $\mu$  was described as a nonlinear function of wheel slip ratio, wheel load and road conditions. Apart from the famous "Magic Formula (MF)" model given by Pacejka that expressed the friction coefficient as the composite trigonometric function of wheel slip, the exponential friction model was also used to illustrate the features of friction coefficient with the change of tire slip, such as Burckhardt's linear-exponential composite model and Bian's dual-exponential model [2-3]. Different from the above models, Kiencke and Daiss put forward a quadratic function to express the change trend of friction coefficient, in consideration the low precision by fitting the  $\mu$ -S curve with two linear functions [4]. A simplified trigonometric function model based on MF was proposed by Bian to describe the longitudinal tire-road friction coefficient [5]. Most of these models were acceptable in capturing the characteristics of longitudinal road friction  $\mu$ -S curve with a good

precision, but they were incapable of illustrating the dynamic characters of tire-road friction in lateral or combined longitudinal/lateral wheel slip situations.

Besides the above models that described the properties of road friction coefficient, other tire models were mainly focused on expressing the longitudinal or lateral tire forces as complicated functions of vehicle and tire dynamics parameters. The Dugoff model was one that was broadly used [6]. Engaged in describing the longitudinal and lateral tire friction forces with a series of variables such as longitudinal wheel slip ratio, tire slip angle and tire cornering stiffness etc., this model had the advantages of simple equations, low computation load, and lateral/longitudinal multi driving condition adaption. The most prominent advantage of Dugoff model was that it adopts the variable of maximum road friction  $\mu_{max}$ , which made it preponderant in road friction potentials and road condition estimation. However, Dugoff model is incapable of describing the dynamic characters of tire forces with the change of wheel slip in many conditions when compared with many other accurate models like Magic Formula, which limited its utility. Other scholars did some research to enhance the accuracy of Dugoff model, but their case study determined the results can only be used in special driving situation [7].

In this paper, we developed a dynamic friction model based on modification of the Dugoff model that can be used to describe the tire/road interaction forces under combined longitudinal and lateral wheel slip situation. The proposed model not only inherited the advantages of Dugoff model, but also improved the precision of capturing the dynamic characters of tire forces with the change of wheel slip ratio and tire slip angle, which guaranteed its validity in estimation of the friction potentials between tire/road interface under different driving conditions. Hence, in contrast to many other static models, our model is appropriate for dynamics analyzing under any motion situations as well as for vehicle control law design. The paper was organized in the following five parts. Section 1 gave a review on tire models on describing the friction forces between tire/road interface. The analysis of the Dugoff model and our modified tire model is proposed in section 2 and 3 respectively. In section 4 the simulation of tire forces under various situation by using the proposed model is performed and results are compared with the data from Magic Formula model. Summary and conclusions are provided in section 5.

## Analysis on Dugoff Tire Model

With a simple form, Dugoff tire model can calculate the longitudinal and lateral tire force under pure longitudinal slip, pure side slip and associated longitudinal-side slip situation. So it was used by lots of researchers to reach better real-time performance.

Dugoff tire model is described as

$$F_x = C_s \frac{S}{1+S} \cdot f(\lambda) \quad (1)$$

$$F_y = C_\alpha \cdot \frac{\tan(\alpha)}{1+S} \cdot f(\lambda) \quad (2)$$

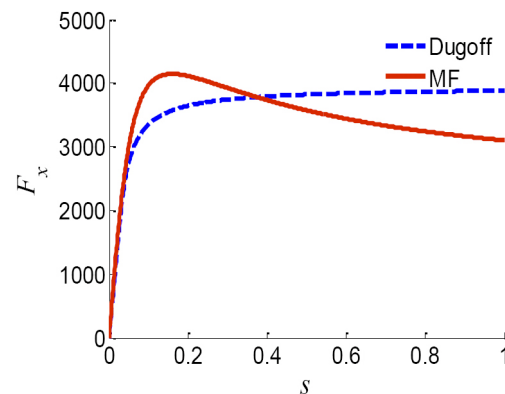
In above equations the variables are used as follows:  $F_x$  and  $F_y$  are the longitudinal and lateral forces of tires.  $C_s$  and  $C_\alpha$  are the longitudinal and lateral stiffness of tires.  $S$  is the longitudinal slip ratio of tires and  $\alpha$  is the tire sideslip angle.  $\lambda$  and the function  $f(\lambda)$  are described as

$$\lambda = \frac{\mu_{max} F_z (1+S)}{2\sqrt{(C_s S)^2 + (C_\alpha \tan \alpha)^2}} \quad (3)$$

$$f(\lambda) = \begin{cases} (2-\lambda)\lambda, & \lambda < 1 \\ 1, & \lambda \geq 1 \end{cases} \quad (4)$$

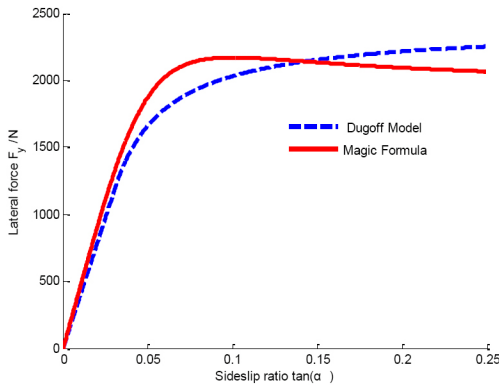
Among the above formula  $F_z$  is the vertical load force of tires;  $\mu_{max}$  is the maximum friction coefficient.

Although fit for expressing the change trend of the friction forces generated from the tire and road interface, the precision of Dugoff model decreases a lot in condition of large wheel slip ratio or large sideslip angle when compared with Magic Formula, which is commonly considered as the accurate model for road friction illustration. The comparison of the longitudinal and lateral tire forces calculated by Dugoff and MF tire model is shown in Figure 1. It indicates that: there is no peak point in Dugoff model; and the difference of these two models is larger with large slip ratios. The Dugoff model must be modified because the model precision is very important when the slip ratio is large, in such a situation a vehicle is easy to lose stability.



(a). Longitudinal tire force calculated by Dugoff and MF model

Figure 1. Tire force calculated by Dugoff and MF model



(b). Lateral tire force calculated by Dugoff and MF model

Figure 1. (cont.) Tire force calculated by Dugoff and MF model

The reason leading to the error with large slip ratio is that the Dugoff model does not reflect the truth that the tire friction coefficient  $\mu$  decreases when the slip ratio is enough large. For solving this problem, some modification method should be taken.

## The Modified Dugoff Tire Model

It can be found from figure 1 that both the longitudinal and lateral tire forces calculated from original Dugoff model are consistently less than the values calculated by MF model when the vehicle is in the linear region of wheel slip/sideslip ratio. While when the wheel slip/sideslip ratio exceed some values and enter into the nonlinear region, the tire forces calculated by Dugoff model become bigger than those by MF model. On the other hand, the friction force-wheel slip curve of the Dugoff model shows a continuously increasing trend with the increment of wheel slip ratio and its shape has no peak point, which forms the reasons that this model disagrees with the MF model and does not reflect the actual principles of tire road friction forces. Therefore we studied to modify the original Dugoff model according to the following principles. Firstly a magnifying factor  $G$  multiplied the original tire force equations to enhance the magnitude of the computing results, and fit the rules that tire forces decrease with the increment of wheel slip/sideslip ratio in nonlinear region. This will result in an obvious peak point on the friction force-slip ratio curve. Secondly, both the longitudinal and lateral tire stiffness are modified and fitted as the functions of tire vertical load, slip ratio and side slip angle, in order to apply the model to various driving conditions. A newly modified Dugoff model was proposed by fitting the experimental results and it can be used to express the dynamic characteristics of tire-road friction forces under pure longitudinal, pure lateral, and combined longitudinal/lateral wheel slip situation. This model composes of two group equations, which are used to describe the longitudinal and lateral tire forces under various driving conditions respectively. The new model is expressed as the following formula.

For the longitudinal tire force,

$$F_x = C_s \cdot \frac{S}{1+S} \cdot f(\lambda) \cdot G_s \quad (5)$$

$$G_s = (1.15 - 0.75\mu_{\max}) \cdot S^2 - (1.63 - 0.75\mu_{\max}) \cdot S + 1.27 \quad (6)$$

$$\lambda = \frac{\mu_{\max} F_z (1+S)}{2\sqrt{(C_s S)^2 + (C_\alpha \tan \alpha)^2}} \quad (7)$$

$$f(\lambda) = \begin{cases} (2-\lambda)\lambda, & \lambda < 1 \\ 1, & \lambda \geq 1 \end{cases} \quad (8)$$

Another group of equations are used to compute the lateral tire friction forces.

$$F_y = C_\alpha \cdot \frac{\tan \alpha}{1+S} \cdot f(\lambda) \cdot G_\alpha \quad (9)$$

$$G_\alpha = (\mu_{\max} - 1.6) \cdot \tan \alpha + 1.155 \quad (10)$$

$$\lambda = \frac{\mu_{\max} F_z (1+S)}{2\sqrt{(C_s S)^2 + (C_\alpha \tan \alpha)^2}} \quad (11)$$

$$f(\lambda) = \begin{cases} (2-\lambda)\lambda, & \lambda < 1 \\ 1, & \lambda \geq 1 \end{cases} \quad (12)$$

The longitudinal wheel slip ratio in above equations is defined as

$$S = \frac{|r\omega - V|}{\max(r\omega, V)} \quad (13)$$

Where the variables  $r$ ,  $\omega$  and  $V$  is wheel radius, wheel rolling speed and wheel longitudinal velocity respectively.

Figure 2 shows the comparison of our modified Dugoff model with the Magic Formula on calculating the longitudinal tire force with the tire vertical load of 4000N. It can be seen that the newly modified model becomes closer to the MF model along with the whole range of wheel slip ratio, which fits for the changing trends of actual tire forces with an acceptable precision. And most important, the new model has an obvious peak point near the fitted one from MF model, which is a distinct improvement over the original Dugoff model.

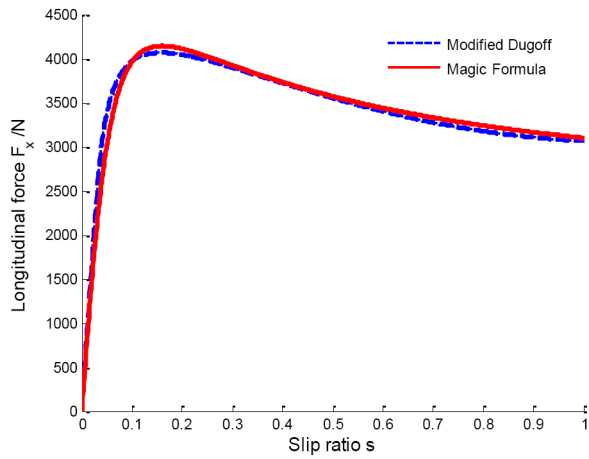


Figure 2. Comparison between Modified Dugoff and MF model on longitudinal tire force calculating

Figure 3, 4, 5, 6, 7, 8 show the comparison results between the modified tire model and Pacejka's MF model for tire force analyzed under various situations. It can be found that our new tire model can compute both the longitudinal and lateral friction forces between tire and road interface simultaneously with the closer accuracy than MF model. At the meantime, the proposed model can capture the characteristics of tire force with the change of wheel vertical load, wheel slip ratio, side slip angle and different road surface conditions, which made it a dynamic tire friction model. However, the modified Dugoff model has simpler equations than those of MF model, which means less computation load and short convergence time when it is used in vehicle dynamics analysis and control engineering.

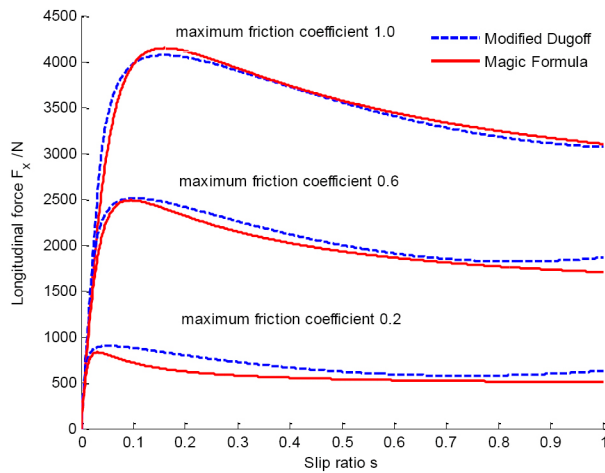


Figure 3. Longitudinal tire force on different road conditions

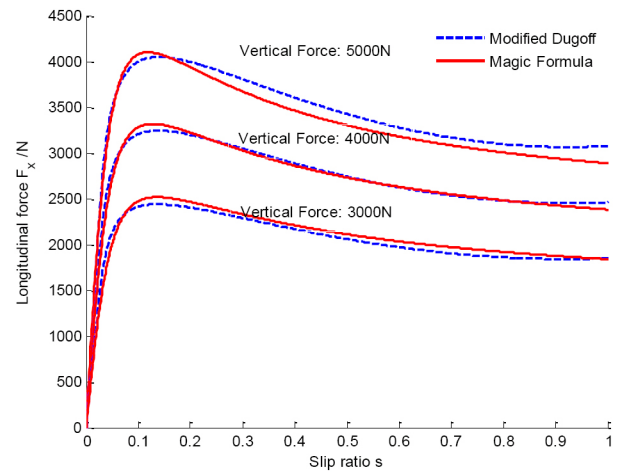


Figure 4. Longitudinal tire force under different vertical wheel load

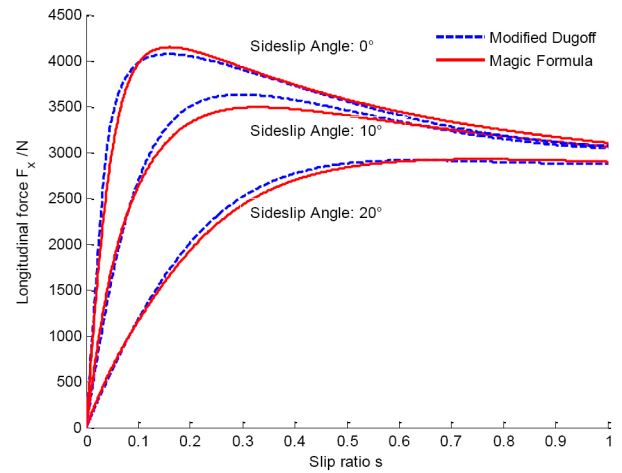


Figure 5. Longitudinal tire force under different side slip angle

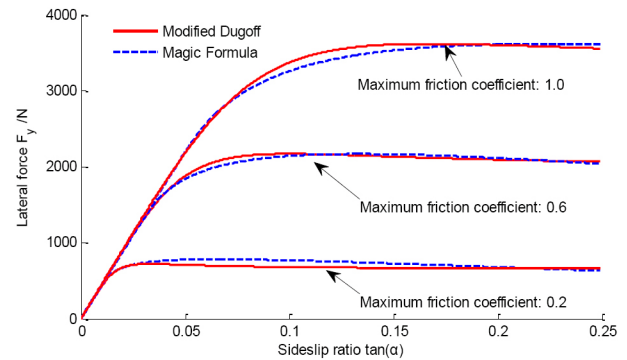


Figure 6. Lateral tire force on different road conditions

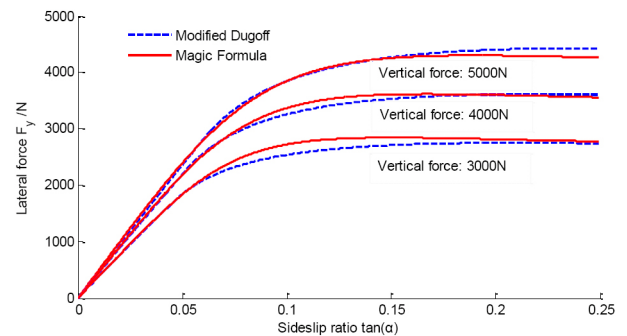


Figure 7. Lateral tire force under different vertical wheel load

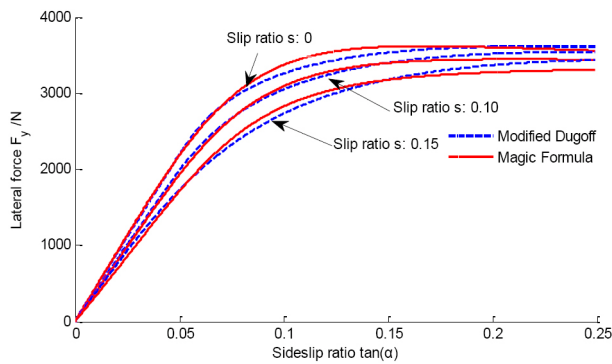


Figure 8. Lateral tire force under different longitudinal wheel slip ratio

## Simulation of Tire Force Estimation in Different Driving Conditions

The vehicle dynamics analyzing model was established by using Matlab/Simulink and CarSim platform, and the simulated tire force estimation was done for acceleration/brake, sinusoidal steering input and double lane change driving conditions. The CarSim model incorporates the Magic Formula tire model to produce tire force results, and the estimated results of longitudinal and lateral tire force by using our modified tire model were compared with those of the CarSim model.

Figure 9 shows the wheel slip ratio when the vehicle was sequentially accelerated and brake on a straight way without any steering input operation. The longitudinal tire friction force estimation results are shown in Figure 10. It was found that the calculated values from proposed tire model were close to the simulation values.

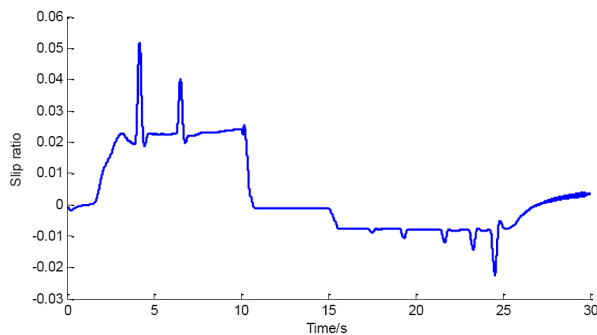


Figure 9. Longitudinal wheel slip ratio

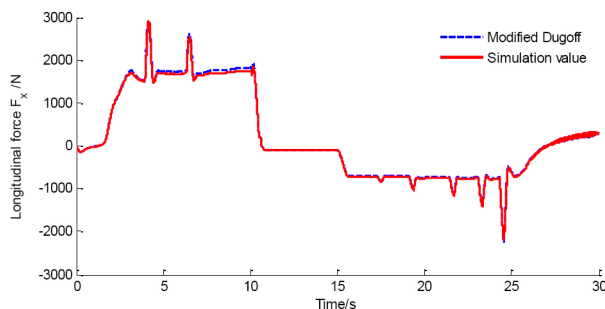


Figure 10. Longitudinal tire force estimation under acceleration and brake situation

The tire sideslip angle under a sinusoidal steering situation is shown in Figure 11, while the lateral tire force estimation results are shown in Figure 12. It was found that the estimated tire forces were consistently close to the simulation values during all the process, except that existed an acceptable error when the value of sideslip angle was in a large region.

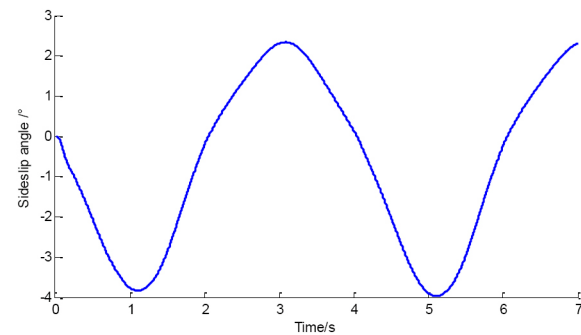


Figure 11. Sideslip angle of tire under the sinusoidal steering situation

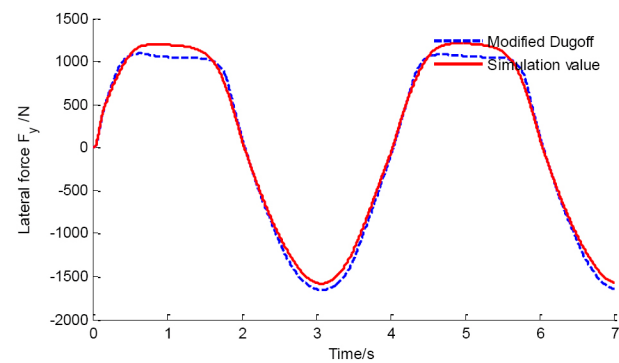


Figure 12. Lateral tire force estimation results under the sinusoidal steering situation

The tire sideslip angle for a double lane change situation is shown in Figure 13, while the lateral tire force estimation results are shown in Figure 14. The estimated tire forces were also consistently close to the simulation values during all the process, except that existed an acceptable error when the value of sideslip angle was in a large region.

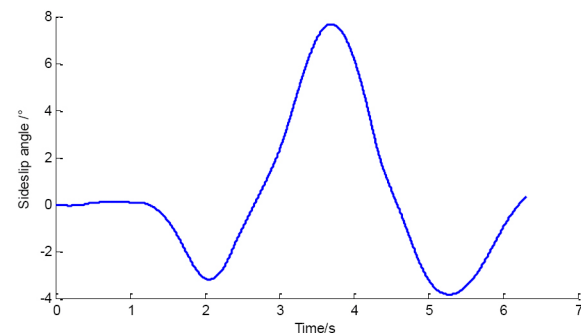


Figure 13. Sideslip angle of tire under the double lane change situation



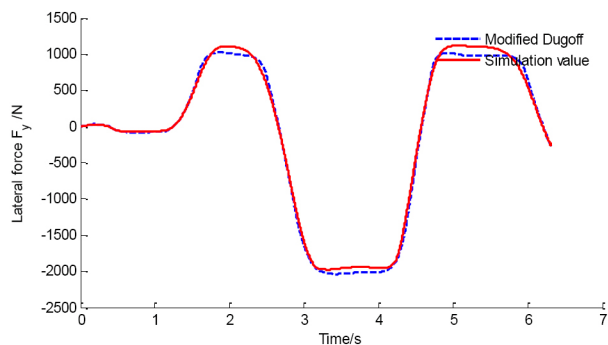


Figure 14. Lateral tire force estimation results under double lane change situation

The simulation and results analysis showed that the proposed tire model can estimate both the longitudinal and lateral tire forces under different driving conditions, which demonstrated its validity in vehicle dynamics analysis.

## Summary/Conclusions

A modified tire model was proposed to estimate the friction force between tire and road interface based on the original Dugoff model. With the simple equations the new model can calculate both the longitudinal and lateral tire friction forces under different wheel slip conditions with results that are comparable to those of commonly used Magic Formula tire model, while the modified tire model needs less computation load than MF. Simulation was done under longitudinal, lateral and combined wheel slip/sideslip situations, and the tire-road forces were consistently estimated quickly and closed to the simulation values, which showed that the proposed model can capture the dynamic characteristics of tire forces along with the change of various tire dynamics states. Therefore, it is considered that this modified Dugoff model is valid for vehicle dynamics analysis and simulation.

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The Engineering Meetings Board has approved this paper for publication. It has successfully completed SAE's peer review process under the supervision of the session organizer. The process requires a minimum of three (3) reviews by industry experts.

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