Torque Vectoring for Rear Axle using Adaptive Sliding Mode Control

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***Abstract*—Active chassis control systems have been developed and applied increasingly in the automotive industry to improve vehicle global safety and comfort from normal to critical driving situations. These systems like Electronic Stability Program (ESP), Vehicle Dynamic Control (VDC), Direct Yaw moment Control (DYC), Traction Control System (TCS) play the most important role therein but they are all braked based systems. The weakness of these s is to cause energy loss during acceleration, whereas Torque Vectoring System could improve the driving stability without deceleration during cornering or acceleration. An additionally corrective torque based on desired yaw moment is computed from reference yaw rate and sideslip angle and then applied to the left and the right rear wheels. In this paper, a Torque Vectoring Controller on the basis of an Adaptive Sliding Mode Control (SMC) in which a combined sliding surface derived from the error of actual and reference signals of both yaw rate and body sideslip angle and an adaptive gain control law are proposed. The proposed approach is verified by the co-simulations of Matlab®/Simulink® and Carsim®. Simulation results demonstrate the effectiveness of the system and the overall enhancement in vehicle's stability and drivability.**

***Keyword*—Vehicle dynamics, torque vectoring, sliding mode, robust control, tracking control, yaw rate tracking - side slip angle tracking.**

I. INTRODUCTION

Advanced vehicle dynamics controls are being increasingly researched, developed and applied to upgrade handling, driving performance and manoeuvrability for achieving the global stability of vehicles for the past few decades in the automotive industry, such as Active Front Steering (AFS), Active Rear Steering (ARS), Four Wheel Steering (4WS), Electronic Stability Program (ESP), Vehicle Dynamic Control (VDC), Traction Control System (TCS), Direct Yaw moment Control (DYC), Active Differential, Torque Vectoring System... Among them, brake-based control systems like ESP, TCS, VDC, and DYC are very useful and play a vital role in many driving scenarios. However they could impair the longitudinal performance of vehicle and cause loss of energy.

There are many innovations in vehicle yaw control using torque vectoring developed by many automotive manufacturers and researchers. In a series of literatures about torque vectoring of Mitsubishi Motors Corporation found in [3], [4] [6] for expanding the vehicle dynamics limit in many maneuvers as desired during acceleration, deceleration and steady-state driving, the left-right torque vectoring is especially focused to design. In [5] the author modeled torque-biasing devices of a four-wheel-drive system. The proposed driveline system is based on nominal front-wheel-drive operation with on-demand transfer of torque to the rear. The torque biasing components of the system are an electronically controlled center coupler and a rear electronically controlled limited slip differential.

The SMC based control algorithm for the electronically controlled 4WD system to improve traction ability based on Sliding Mode Control strategy to control the front wheels torque to minimize the difference between front and rear wheel speed and the engine torque to reduce the slip of each wheel and to improve the fuel economy could be found in [12]. The results show the promising effectiveness of the proposed approach but the detailed procedure test in simulations is not specified. A MIMO sliding mode system for a global chassis control of torque vectoring between left-right rear wheels and active rear steering was presented in [8]. This controller works well and shows the promising results in a low speed procedure test. Some active differentials with controlled left/right or front/rear torque distribution to the wheels for enhancing the traction control and yaw rate control performances without being intrusive to the driver were presented in [9].

This paper presents the modeling of torque vectoring system for rear axle of a Sport Utility Vehicle (SUV) with all-wheel drive. The remaining paper is organized as follows: A vehicle model for controller design and a reference model, generated from desired yaw rate and vehicle body slip are introduced in section II. It is continued with section 3, where a controller algorithm based on adaptive Sliding Mode Control to regulate torques for left and right rear wheels is presented. The co-simulations results of Matlab/Simulink® and Carsim® showing the improved handling and stability of vehicle under double lane change procedure with the changes of cornering stiffness in various operating conditions are explained in section 4. The last section is the conclusion and future work of this study.

II. VEHICLE MODEL

A. Vehicle model for controller design

For the representation of the vehicle handling dynamics with rear torque vectoring, a planar motion vehicle model is adopted as shown in Fig. 1. The linearized 3 DOF model includes lateral motion, longitudinal motion and yaw motion about vertical axis. In this model, the dynamics of wheels at

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rear axle are also employed to describe the additional input torque vectoring between the left and right rear wheels.

The schematic of vehicle model is shown in Figure 1.

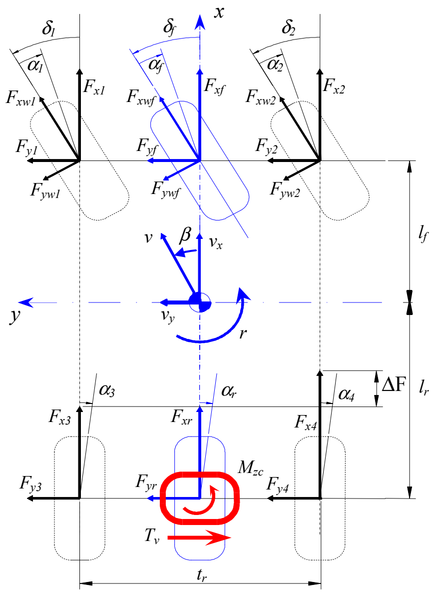

Figure 1: The linearized 3DOF model

The planar motions are described as follows:

$$\begin{cases} m(\dot{v}_x - rv_y) = \sum F_x \\ mv_x(\dot{\beta},+r) = \sum F_y \\ I_{zz}.\dot{r} = \sum M_z + M_{z\_corective} \end{cases} \quad (1)$$

Wheel rotational dynamics at rear axle:

$$\begin{cases} I_w\dot{\omega}_3 = T_3 - R_w F_{xw3} \\ I_w\dot{\omega}_4 = T_4 - R_w F_{xw4} \end{cases} \quad (2)$$

The ideal torque vectoring differential at rear axle is considered 'ideal' if it is unrestricted in both the magnitude and direction of torque transfer it can apply between the left and right wheels. Such a differential allows the wheel torques to be expressed as:

$$T_3 = \frac{T_{in}}{2} + T_v,\ T_4 = \frac{T_{in}}{2} - T_v \quad (3)$$

where the torque transfer T_v can be of any desired sign or magnitude

$$\begin{aligned} M_{z\_corective} &= (F_{x3} - F_{x4}).\frac{t_r}{2} = \Delta F.\frac{t_r}{2} = \frac{\Delta T}{R_w}.\frac{t_r}{2} \\ &= \frac{2T_V}{R_w}.\frac{t_r}{2} = \frac{T_V.t_r}{R_w} \end{aligned} \quad (4)$$

where r is yaw rate, v_y is lateral velocity, δ_f is front steering angle, m is the vehicle mass, I_{zz} is the yaw moment of inertia around the vertical axis, l_f/l_r are the distances between CG to front/rear axles, $M_{z\text{-}corrective}$ is the desired yaw moment generated by torque vectoring differential, F_{xi}/F_{yi} are longitudinal forces and lateral forces in vehicle body fixed coordinate, F_{xwi}, F_{ywi} are the corresponding tire forces in the local tire coordinate, T_{in} is the torque from center transfer case to rear torque vectoring differential, T_v is the torque transferred between left and right rear wheel, index $1,2,3,4,f,r$ stand for front left, front right, rear left, rear right, front, rear wheels respectively, R_w is the dynamic radius of wheel, t_r is the wheel track of the rear axle.

The relationships between F_{xi}, F_{yi} and F_{xwi}, F_{ywi}

$$\begin{bmatrix} F_{xi} \\ F_{yi} \end{bmatrix} = \begin{bmatrix} \cos\delta_i & -\sin\delta_i \\ \sin\delta_i & \cos\delta_i \end{bmatrix} \begin{bmatrix} F_{xwi} \\ F_{ywi} \end{bmatrix} \quad (5)$$

Under the assumptions: vehicle drives on a horizontal plane, the longitudinal velocity is constant, vehicle has stiff suspensions, the aerodynamic resistance and the wind lateral thrust are not considered, no rear wheel steering angle, front steering angle and vehicle side slip angle are small enough to linearize their trigonometrical functions. The linearized model around the operating condition can be obtained in state-space representation as follows:

$$\dot{X}(t) = f(X, \delta_f, t) + BU(t) \quad (6)$$

where $f(X, \delta_f, t) = AX + b_f\delta_f$

$$X = \begin{bmatrix} \beta \\ r \end{bmatrix}\ A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\ b_f = \begin{bmatrix} b_{f1} \\ b_{f2} \end{bmatrix}\ B = \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix}\ U = [T_v] \quad (7)$$

$$a_{11} = -\frac{(C_f + C_r)}{mv_x} \quad a_{12} = \frac{(l_r C_r - l_f C_f)}{mv_x^2} - 1$$

$$a_{21} = \frac{l_r C_r - l_f C_f}{I_z} \quad a_{22} = \frac{-(l_f^2 C_f + l_r^2 C_r)}{I_z v_x}$$

$$b_{f1} = \frac{C_f}{mv_x} \quad b_{f2} = \frac{l_f C_f}{I_z} \quad b_{11} = 0 \quad b_{21} = \frac{t_r}{R_w}\frac{1}{I_{zz}}$$

where X is the state vector, U is the input vector, C_f/C_r are the cornering stiffness of front/rear tyres.

From the point of view of robustness of the proposed adaptive sliding mode controller in which the system uncertainties, the variation of tire cornering stiffness for front and rear tires and the effect of low friction condition as disturbance, are taken into account. In this model we can assume that the uncertainties in cornering stiffness of tyres are:

$$\begin{aligned} C_f &= \overline{C}_f + \Delta C_f \\ C_r &= \overline{C}_r + \Delta C_r \end{aligned} \quad (8)$$

where: $\overline{C}_f / \overline{C}_r$ are nominal cornering stiffness of front and rear tyre, ΔC_f, ΔC_r are uncertainty values of $\overline{C}_f / \overline{C}_r$. With these perturbations, the uncertainty function becomes:

$$\Delta f(X, \delta_f, t) = \Delta A.X + \Delta b.\delta_f \quad (9)$$

The final model for controller design is described as:

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} + \begin{bmatrix} b_{f1} \\ b_{f2} \end{bmatrix} \delta_f + B.u(t) + \begin{bmatrix} \Delta f_1(X, \delta_f, t) \\ \Delta f_2(X, \delta_f, t \end{bmatrix} \quad (10)$$

This system can be also presented as follows:

$$\dot{X} = \begin{bmatrix} f_1(X, \delta_f, t) + \Delta f_1(X, \delta_f, t) \\ f_2(X, \delta_f, t) + \Delta f_2(X, \delta_f, t) \end{bmatrix} + \begin{bmatrix} 0 \\ b_{21} \end{bmatrix} u(t) \quad (11)$$

This model is adopted to design the adaptive SMC controller for torque vectoring system in this paper.

B. Reference Model

This reference model generates the desired responses for actual vehicle model states to track them. For this study, the reference model developed by Masao Nagai in [2] is employed. This model represents vehicle model driving in the linear dynamics range that generates an ideal response as the input. The outputs of reference model are the desired yaw rate and zero side slip angle of vehicle body, based on the driver's demanded steering angle and vehicle's actual states from the full vehicle model in Carsim®.

The desired model can be described as bellows:

$$x_d = \begin{bmatrix} \beta_d \\ r_d \end{bmatrix} = \begin{bmatrix} 0 \\ \dfrac{k_r}{1+\tau.s} \end{bmatrix} \delta_f \tag{12}$$

where r_d, β_d are the desired yaw rate and vehicle body side slip angle

$$k_r = \frac{v}{l_f + \dfrac{ml_f l_r .v_2}{2C_f l_f L}}, \quad \tau_r = \frac{I_{zz}.v_x}{2C_f l_f L + ml_r v_x^2} \tag{13}$$

The equation (12) can also be expressed as state-space equation as:

$$\dot{x}_d = A_d x_d + B_d \delta_f = \begin{bmatrix} 0 & 0 \\ 0 & -1/\tau_r \end{bmatrix} x_d + \begin{bmatrix} 0 \\ \dfrac{k_r}{\tau_r} \end{bmatrix} \delta_f \tag{14}$$

III. Control Algorithm

In this work, an adaptive Sliding Mode Controller (SMC) is proposed in order to achieve the stability and the robustness of the system. This adaptive SMC uses a combined sliding surface with an adaptive parameter control gain instead of the constant one. The adaptive technique will adapt the unknown upper bound of perturbations to achieve the global asymptotical stability, based on the Lyapunov stability theorem.

The aim of the proposed strategy is to implement a controller having a good yaw rate tracking as well as vehicle body sideslip angle tracking, path keeping and disturbance rejection to improve vehicle dynamics, especially handling performance. The tracking errors for vehicle's yaw rate and body side slip angle from Eq. (10) and Eq. (12) are:

$$\begin{aligned} e_1 &= r - r_d \\ e_2 &= \beta - \beta_d \end{aligned} \tag{15}$$

For the above purposes, the combined sliding surface is employed as:

$$s = e_1 + \lambda e_2 = r - r_d + \lambda(\beta - \beta_d) = r - r_d + \lambda\beta \tag{16}$$

where λ is strictly positive constants

From the yaw and lateral motion equations in Eq. (10)

$$\begin{aligned} \dot{\beta} &= f_1 + \Delta f_1 \\ \dot{r} &= f_2 + \Delta f_2 + b_{21}u(t) \end{aligned} \tag{17}$$

By differentiating the sliding surface we can achieve the attractive equations as bellows:

$$\dot{s} = \dot{e}_1 + \lambda\dot{e}_2 = \dot{r} - \dot{r}_d + \lambda\dot{\beta} = -\hat{K}sign(s) \tag{18}$$

Substitute $\dot{r}$ of Eq. 17 into Eq. 18, we get:

$$f_2 + \Delta f_2 + b_{21}u(t) - \dot{r}_d + \lambda f_1 + \lambda\Delta f_1 = -\hat{K}sign(s) \tag{19}$$

An appropriate control law can, therefore, be as:

$$u(t) = \frac{1}{b_{21}}\left(\dot{r}_d - f_2 - \Delta f_2 - \lambda f_1 - \lambda\Delta f_1 - \hat{K}sign(s)\right) \tag{20}$$

The entire control law is, then, obtained as:

$$u = T_v = \frac{1}{b_{21}} \begin{bmatrix} \dot{r}_d - a_{21}\beta - a_{22}r - b_{f2}\delta_f - \Delta f_2 - \lambda\Delta f \\ -\lambda(a_{11}\beta + a_{12}r + b_{f1}\delta_f)_1 - \hat{K}sign(s) \end{bmatrix} \tag{21}$$

In fact, the system uncertainties like Δf_1, Δf_2 are unknown and it is difficult to determine them. The control input, then is modified as:

$$u(t) = \frac{1}{b_{21}}\left(-f_2 - \lambda f_1 + \dot{r}_d - \hat{K}sign(s)\right) \tag{22}$$

Stability Analysis

The error will go to zero as $s \to 0$.

We consider the Lyapunov function candidate to derive stability condition as in [1]:

$$V = \frac{1}{2}s^2 \tag{23}$$

Then the derivative of V becomes:

$$\begin{aligned} \dot{V} &= s\dot{s} = s(\dot{r} - \dot{r}_d + \lambda\dot{\beta}) \\ &= s\left(\Delta f_2 + \lambda\Delta f_1 - \hat{K}sign(s)\right) \\ &= s.sign(s)\left(\Delta f_2.sign(s) + \lambda\Delta f_1(sign(s) - \hat{K}\right) \\ &\le |s|\left(|\Delta f_2| + |\lambda\Delta f_1| - \hat{K}\right) \end{aligned} \tag{24}$$

To reach the stability condition, the time derivative of V must be negative and can be alternatively expressed as:

$$\text{Since } V = \frac{1}{2}s^2 = \frac{1}{2}|s|^2 \ge 0, \text{ then } \dot{V} = |s|.\frac{d|s|}{dt} \tag{25}$$

The sliding condition is

$$|s|.\frac{d|s|}{dt} \le -\eta|s| \quad \text{or} \quad \frac{d|s|}{dt} \le -\eta \tag{26}$$

where η is a positive constant

This inequality implies that $|s| \to 0$ in a finite time and once on the sliding surface, the system trajectory will not leave it.

As mentioned above, the uncertainties in the cornering stiffness of tyres C_f and C_r are ΔC_f and ΔC_r, respectively. If the uncertainties in cornering stiffness of the front and rear tyres are assumed to be equal, $\Delta C_f = \Delta C_r = \Delta C$, the uncertainty term $\Delta_i(X, t, \delta_f)$ will be bounded as:

$$|\Delta f_1| \le \Delta F_{1\max} \text{ and } |\Delta f_2| \le \Delta F_{2\max}$$

Due to the condition (26), the gain control $\hat{K}$ has the lower bound as:

$$\hat{K} \geq \Delta F_{2mac} + \lambda \Delta F_{1\max} + \eta \\ \hat{K} \geq \Delta_{\max} + \eta \tag{27}$$

Where: $\Delta_{\max} = \Delta F_{2\max} + \lambda \Delta F_{1\max}$ (28)

Chattering in Sliding Mode Control

In practice, the control law involves the sgn() function as the switching term. An ideal SMC requires infinitely fast switching around the sliding surface which may lead to chattering in computational implementation. In order to avoid this chattering phenomenon, the sgn() can be replaced by the sat() function as:

$$sat(s/\varepsilon) = \begin{cases} s/\varepsilon & if\ |s| \leq \varepsilon \\ \operatorname{sgn}(s/\varepsilon) & if\ |s| > \varepsilon \end{cases} \tag{29}$$

Adaptive gain control law

To overcome the problem with estimating exactly the uncertainties and disturbances, the final control law with an adaptive sliding gain is proposed by substituting a time-variant sliding parameter K(t) updated by an adaptation law into the control input that is defined as in [14]:

$$\dot{K}(t) = \gamma.s.sat(s/\varepsilon) \tag{30}$$

where γ is a positive constant. We assume that these positive parameters satisfy $\gamma \geq \varepsilon, \gamma \geq 1$. Under the condition (27), the stability of system will be considered again. The candidate function of Lyapunov is defined as:

$$V = \frac{1}{2}s^2 + \frac{1}{2\gamma}\left(K(t) - \hat{K}\right) \tag{31}$$

$$\dot{V} = s\dot{s} + \frac{1}{\gamma}\left(K(t) - \hat{K}\right)\dot{K}(t) \tag{32}$$

$$= s\left(\Delta f - K(t).\gamma.sat(s/\varepsilon)\right) + \frac{1}{\gamma}\left(K(t) - \hat{K}\right)\gamma.s.sat(s/\varepsilon)$$

$$= s.\Delta f - K(t).\gamma.s.sat(s/\varepsilon)$$

$$\leq |\Delta f||s| - K(t).\gamma.s.sat(s/\varepsilon)$$

$$\leq |\Delta f||s| - (\Delta_{\max} + \eta).\gamma.s.sat(s/\varepsilon)$$

$$= |\Delta f||s| - (\Delta_{\max} + \eta).\gamma \begin{cases} |s|/\varepsilon & if\ |s| \leq \varepsilon \\ |s| & if\ |s| > \varepsilon \end{cases}$$

$$\dot{V} \leq -\eta\gamma \begin{cases} |s|/\varepsilon & if\ |s| \leq \varepsilon \\ |s| & if\ |s| > \varepsilon \end{cases} \tag{33}$$

$\dot{V}(t) < 0$. That means the reaching condition is to be able to guarantee. Each term in Eq.(32) is bounded so $\dot{s}(t)$ and K(t) will be bounded as well.

Using equations (21), (29) and (30), the final control law can be written as follows:

$$T_v = \frac{1}{b_{21}}\begin{bmatrix} \dot{r}_d - a_{21}\beta - a_{22}r - b_{f2}\delta_f - \Delta f_2 - \lambda\Delta f_1 \\ \lambda(a_{11}\beta + a_{12}r + b_{f1}\delta_f) - K(t)\gamma.sat(s/\varepsilon) \end{bmatrix} \tag{34}$$

In this paper, the maximum of regulated vectoring torque T_v will be bounded as:

$$-400Nm \leq T_v \leq 400Nm \tag{35}$$

IV. SIMULATION RESULTS

A detailed SUV model in Carsim® is adopted in numerical simulations to analyze some crucial responses of both passive and active vehicle which have dramatic impact on a driving vehicle.

In co-simulations, the so-called closed loop driver model built-in in Carsim® with the preview time parameter 1 second is utilized. The ESC and ABS functions as well as all brake-based systems of vehicle model in Carsim® are deactivated. In numerical simulations, the variations of tire cornering stiffness with ± 20% for front and rear tires were taken into account to prove the insensitivity of system. The simulation results are shown from Figure 3 to Figure 13.

For evaluation purpose, the double lane change manoeuvre on dry road was used to show the performance of the proposed controller in two modes, active vehicle SMC ON and passive vehicle SMC OFF. Additional data for co-simulations can be found in Table 1.

Table 1: Simulation parameters

Parameter	Symbol	Value	Unit
Vehicle mass	m	2257	kg
Distance from CG to front axle	l_f	1.616	m
Distance from CG to rear axle	l_r	1.330	m
Vehicle yaw moment of inertia	I_z	3524.9	kgm^2
Wheel track of the rear axle	t	2.005	m
Dynamic radius of wheel	R_w	0.395	m
Front axle cornering stiffness	C_f	106398	N/rad
Rear axle cornering stiffness	C_r	87577	N/rad

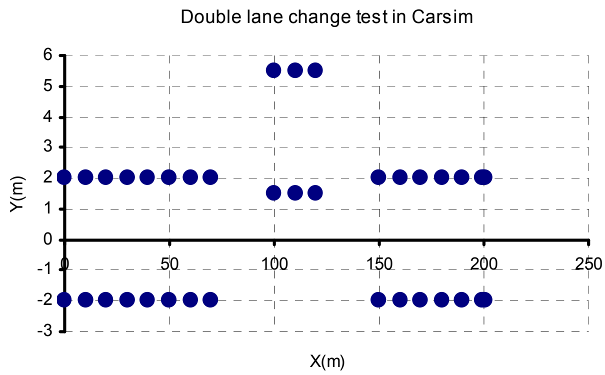

Figure 2: Double lane change test course

The first simulation was implemented at vehicle speed 80km/h, μ=0.85. The results were plotted in Fig. 3 to Fig. 7 as bellows.

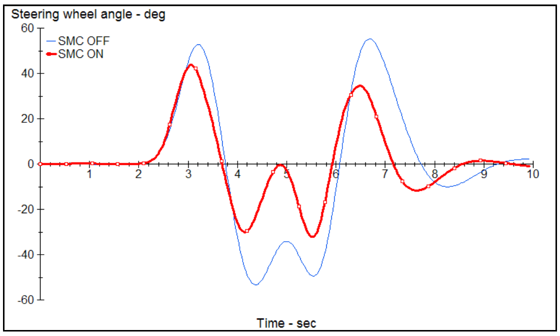

Figure 3: Steering wheel angle

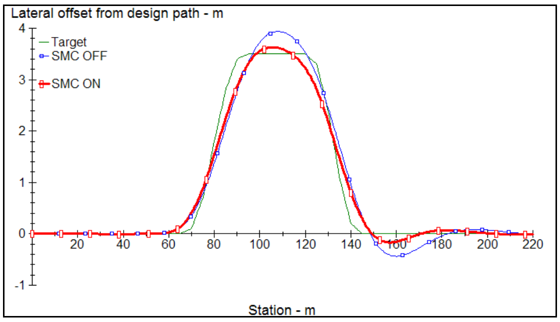

Figure 4: Trajectory of vehicle

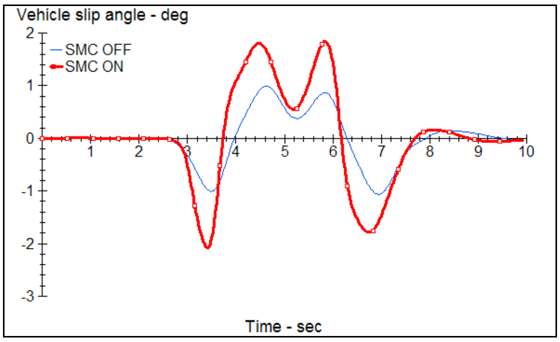

Figure 5: Vehicle body sideslip angle

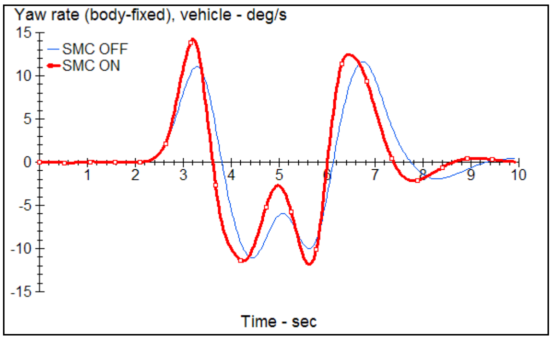

Figure 6: Yaw rate

As shown in Fig. 3 to Fig. 7, the proposed controller helps vehicle to track the target trajectory very well despite a minor overshoot at the end of the path. In the low and mid ranges of lateral acceleration, the normal steering system of passive vehicle (SMC OFF) can help vehicle get through the test. The torque vectoring actions implemented around 2.2s, 3.25s, 6s can keep vehicle tracking well and smoothly.

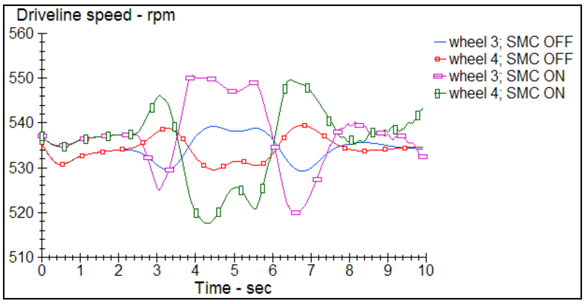

Figure 7: Wheel rotational speed

The next simulation was implemented at vehicle speed 100km/h as plotted in Fig. 8 to Fig. 13.

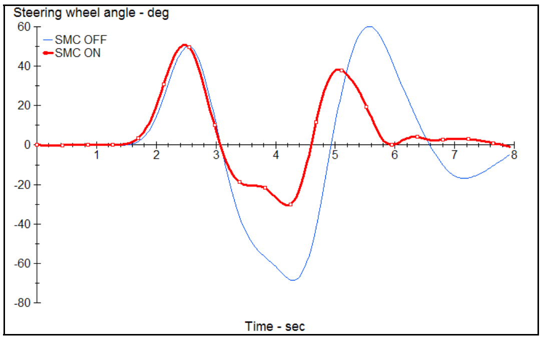

Figure 8: Steering wheel angle

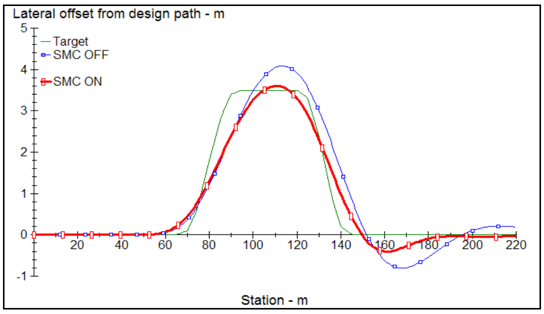

Figure 9: Trajectory of vehicle

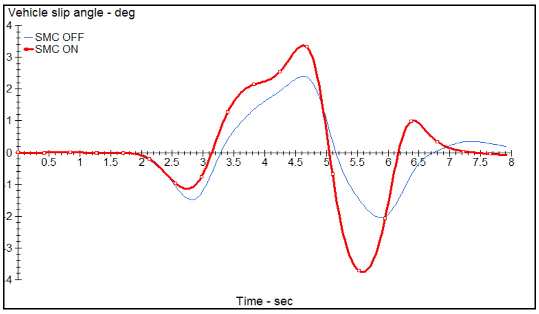

Figure 10: Vehicle body sideslip angle

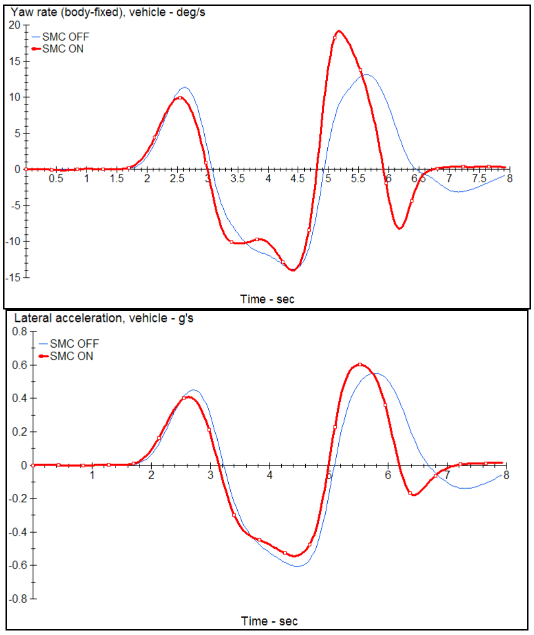

Figure 12: Lateral acceleration

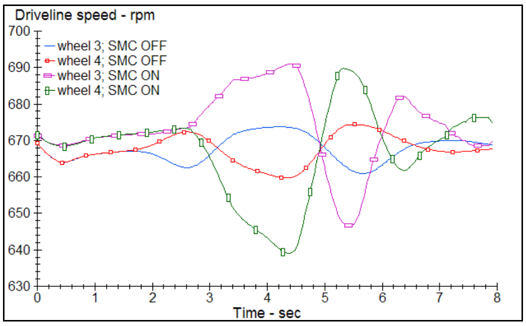

Figure 13: Wheel rotational speed

It is clear that during this manoeuvre, in the mid and high range of lateral acceleration, the vehicle handling performance has been much improved by torque vectoring system and it could contribute to minimising the driver intervention. The vehicle side slip angle is exceeding over 3^o. At high speeds up to 100km/h, however, it is necessary to generate the large lateral forces so that the vehicle could keep path-tracking during cornering. The vehicle body sideslip angle of passive vehicle is less than the active vehicle. As a result, this passive vehicle in this situation could not keep a good path-tracking, while the active vehicle can track the target trajectory effectively.

V. CONCLUSION

In this work, an adaptive sliding mode controller was designed for torque vectoring rear differential at rear axle of a SUV. This proposed approach can achieve the vehicle stability and driveability improvement. The robust stabilization problem was considered with respect to uncertainties in the cornering stiffness of front and rear axle. Further, an adaptive parameter gain control law which is different from conventional sliding mode theory was also proposed to overcome the uncertainties, eliminate chattering and obtain the overall stability and handling performances of vehicle as desired.

Additionally, the control system like this should be integrated with other active chassis control systems in order to achieve the global stability of vehicle and that is the crucial issue for future studies.

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