Fusion of Tire Lateral Force Estimation and Sliding Mode Control for Torque Vectoring

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**ABSTRACT−**Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here.Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. [Capital letter at the beginning of each sentence, put a period at the end, Please write in 100 ~ 200 words, Times New Roman, 9pt]

**KEY WORDS** : Type key words here, Type key words here, Type key words here, Type key words here

nomenclature

*Vx* : longitudinal velocity, m/s

*Vy* : lateral velocity, m/s

γ : yaw rate, rad/s

*ax* : longitudinal acceleration, m/s2

*ay* : lateral acceleration, m/s2

*δ* : steering angle, rad

*Td* : wheel driving torque, N·m

*Tb* : wheel braking torque, N·m

|  |
| --- |
| subscripts  *FL, FR, RL, RR*: front left, front right, rear left, rear right |
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*r* : effective radius of tire, m

*m* : vehicle mass, kg

*Iz* : moment of inertia about z axis, kg·m2

*lF* : distance from front axle to the center of gravity, m

*lR* : distance from rear axle to the center of gravity, m

*L* : wheel base length, m

*t* : half of track width, m

*h* : height from ground to the center of gravity, m

*Cα* : cornering stiffness, N/rad

*σ* : relaxation length, m

*μ* : road friction coefficient, -

1. INTRODUCTION

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2. Estimation of Lateral tire force

In this section, we describe the lateral force estimation process using an Adaptive Extended Kalman Filter (AEKF) and introduce an offline optimization approach for modified corenering stiffness. The estimation process consists of three main parts: the vehicle lateral dynamics model under the planar motion constraints, vertical force calculation and a brief explanation of Dugoff’s tire model. However, Dugoff’s tire model assumes that cornering stiffness is a constant value, which can lead to inaccurate results in lateral tire force estimation as the slip angle increases.

To address this, we propose the offline optimization approach for modified cornering stiffness using an axle distribution-based lateral force calculation. For the vehicle lateral dynamics model, a three degrees of freedom (3-DoF) model is employed which has been widely used for estimating lateral forces and is critical for understanding vehicle stability and handling. The detailed four-wheel vehicle model is illustrated in Figure 1, and the equations of vehicle lateral dynamics are described as follows:

|  |  |
| --- | --- |
|  | (1) |
|  | (2) |
|  | (3) |

where  are the longitudinal velocity, lateral velocity, yaw rate, front left wheel steering angle, front right wheel steering angle, vehicle mass, moment of inertia about yaw axis, distance from front axle to the center of gravity (CG), distance from rear axle to the CG, half of track width and aerodynamic drag coefficient respectively. For the tire forces,  and  (*i* denotes the axle position) represent the longitudinal and lateral forces with the subscript . On the other hand, the lateral slip angle



Figure 1. 2D representation of a vehicle motion model

is a critical factor in determining lateral tire forces; this can be calculated as described in Eq. (4).

|  |  |
| --- | --- |
|  | (4) |

where  denotes the front left wheel steering angle and front right wheel steering angle. Compared to , can be directly calculated using drivetrain output, specifically through the wheel torque value. In contrast, estimating  is complex because it is influenced by various factors, including the lateral slip angle, road conditions, vertical tire force and tire characteristics such as cornering stiffness.

2.1. Vertical tire force calculation

The lateral tire force is affected by several factors, but the vertical tire force plays a crucial role in its accurate estimation. It is essential to account for  throguth the equations of considering load transfer and acceleration, as theses forces are directly influcenced by the vehicle’s dynamic behavior, including braking, acceleration and cornering. For simplicity, the coupings between pitch and roll dynamics are neglected, the vertical forces can be simplified and calucatled as outlined in Eq. (5). (Doumiati *et al.*, 2012)

|  |  |
| --- | --- |
|  | (5) |

where are the vehicle mass, gravitational acceleration, distance trom ground to CG, half of track width, wheel base length, x- axis acceleration and y-axis acceleration respectively.

2.2 Axle dristribution based lateral force calculation

The linear tire model in assumes that cornering stiffness represents a linear relationship between the lateral force and slip angle when the tire slip angle is small as described in Eq. (6). However, as it increases, the behavior of the tire becomes nonlinear. In the nonlinear regime, the lateral force no longer increases proportionally with the slip angle; instead, it appeoaches a saturation point where additional increases in slip angle yield diminishing in lateral force. Therefore, predicting the lateral tire force using linear models becomes less accurate.

|  |  |
| --- | --- |
|  | (6) |

To address this, a more effective approach is to predict the lateral tire force directly by considering the distribution of vertical load across the tires relative to the total load on certain axle. The equations of axle distribution based lateral force calculation is described in Eq. (7) and it accounts for the influence of vertical load without relying on filtering-based estimation techniques.

|  |  |
| --- | --- |
|  | (7) |
|  | (8) |

Where represents the lateral tire force,  and  are the total lateral forces on the front and rear axles, respectively.

2.3. Optimization for modified cornering stiffness

|  |  |
| --- | --- |
|  | (9) |

However, Eq. (7) still has an issue with accurately estimating lateral force because it does not fully account for the nonlinear relationship between lateral force and slip angle. Instead, Eq. (9) denotes the cornering stiffness has a quadratic function relationship with the vertical load on lateral force. Through the previous studies, the lateral force has a a quadratic relationship with the vertical load. We used it only as a basis to incorporate the effects of vertical load more accurately and modified Eq. (9) by integrating the initial cornering stiffness  to better capture the nonlinear dynamics of tire’s behavior.

|  |  |
| --- | --- |
|  | (10) |

By considering both the slip angle and vertical load, we performed a least squares optimization to minimize the error between the predicted for modified cornering stiffness and determine the coefficient (*a, b*) of modified cornering stiffness, as described in Eq. (10). From the experiment, the optimal value of *a* is 0.006, and *b* is 3.501 respectively.

2.4. Dugoff’s tire model

In this study, we employed Dugoff tire formula because it provides simplicity and is easy to implement. It calculates tire forces based on the slip ratio of longitudinal forces and the slip angle. Assuming longitudinal slip ratio is negeligible, the simplified Dugoff model for lateral force is described in Eq. (11).

|  |  |
| --- | --- |
|  | (11) |

where represent the corenering stiffness of each axle, and  is the road firction coefficient, which we assume to be 1.0 for a high-friction road surface. With this simplified model, the lateral force is generated with a time lag relative to change in slip angle, it causes transient response of the tire. The dynamic model of lateral force is represented as follows:

|  |  |
| --- | --- |
|  | (12) |

Where denotes the relaxation length, and assumed it as constant value with 0.1 in this study.

2.5. Adaptive extended kalman filter

To estimate the lateral force in state-space model, the AEKF is employed to dynamically adjust the process noise. While the process noise is dynamically adjusted, the measurement noise is kept constant. This is because the measurement noise is primarily influenced by the sensor charactericstics, which remain stable under normal operating conditions. AEKF utilizes 8-dimensional state vector and 5-dimensional control vector  as follows:

|  |  |
| --- | --- |
|  | (13) |
|  | (14) |

Here, longitudinal force of each axle is used as a input control vector and calculated as Eq. (14). Where are the wheel driving torque, wheel bracking torque, and effective radius of tire. Meanwhile, the measurement model is described in Eq. (15)

|  |  |
| --- | --- |
|  | (15) |

And the estimator process is described as follows:

|  |  |
| --- | --- |
|  | (16) |

The nonlinear function of are described as follows:

|  |  |
| --- | --- |
|  | (16) |
|  | (17) |

The nonlinear function of are described as follows:

5. CONCLUSION

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REFERENCES

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