Fusion of Tire Lateral Force Estimation and Sliding Mode Control for Torque Vectoring

Jinmin Kim1), Hyunseup Jo1) and Sang Won Yoon2)\*

1)Department of Automotive Engineering (Automotive – Computer Convergence), Hanyang University, Seoul 04763, Korea

2)Department of Electrical and Computer Engineering, Seoul National University, Seoul 08826, Korea

(Received date ; Revised date ; Accepted date ) \* Please leave blank

**ABSTRACT−**Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here.Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. Type the abstract here. [Capital letter at the beginning of each sentence, put a period at the end, Please write in 100 ~ 200 words, Times New Roman, 9pt]

**KEY WORDS** : Type key words here, Type key words here, Type key words here, Type key words here

nomenclature

*Vx* : longitudinal velocity, m/s

*Vy* : lateral velocity, m/s

γ : yaw rate, rad/s

*ax* : longitudinal acceleration, m/s2

*ay* : lateral acceleration, m/s2

*δ* : steering angle, rad

*Td* : wheel driving torque, N·m

*Tb* : wheel braking torque, N·m

|  |
| --- |
| subscripts  *FL, FR, RL, RR*: front left, front right, rear left, rear right |
| \* *Corresponding author*. e-mail: [swyoon@snu.ac.kr](mailto:swyoon@snu.ac.kr) |

*r* : effective radius of tire, m

*m* : vehicle mass, kg

*Iz* : moment of inertia about z axis, kg·m2

*lF* : distance from front axle to the center of gravity, m

*lR* : distance from rear axle to the center of gravity, m

*L* : wheel base length, m

*t* : half of track width, m

*h* : height from ground to the center of gravity, m

*Cα* : cornering stiffness, N/rad

*σ* : relaxation length, m

*μ* : road friction coefficient, -

1. INTRODUCTION

The manuscript elements have been formatted for you through the “styles” capability of the software. To use the styles, select the text you wish to apply a style to, then,

2. Estimation of Lateral tire force

In this section, we describe the lateral force estimation process using an Adaptive Extended Kalman Filter (AEKF) and introduce an offline optimization approach for modifying corenering stiffness. Lateral forces are influenced by various factors, including slip angle, road conditions, vertical load on the tire, and tire’s cornering stiffness. The estimation process consists of three main parts: (1) the vehicle lateral dynamics model under the three degrees of freedom (3-DoF), (2) vertical force calculation, and (3) a brief explanation of Dugoff’s tire model, which is widely used for its simplicity. However, Dugoff’s tire model assumes that cornering stiffness as a constant value, this can lead to inaccurate results in overall estimation as the slip angle increases.

To address this, an offline optimization approach is utilized to modifiy cornering stiffness, allowing it to more accruately represent the changing conditions as the slip angle increases. This is achieved through an axle distribution-based lateral force calculation method. The detailed FWVM is illustrated in Figure 1, and the equations of vehicle dynamics are formulated as follows:

|  |  |
| --- | --- |
|  | (1) |
|  | (2) |
|  | (3) |

where  are the longitudinal velocity, lateral velocity, yaw rate, front left wheel steering angle, front right wheel steering angle, vehicle mass, moment of inertia about yaw axis, distance from front axle to the center of gravity (CG), distance from rear axle to the CG, half of track width and aerodynamic drag resistance, respectively. Tire forces,  and  (*i* denotes the axle position) represent the longitudinal and lateral forces with the subscript .



Figure 1. Representation of a four-wheel vehicle model

Lateral forces on the tire generated by the interaction with the road surface are primarily due to the presence of a slip angle. Therefore, calculating the slip angle is critical for determining lateral tire forces; this can be calculated as described in Eq. (4).

|  |  |
| --- | --- |
|  | (4) |

where  denotes the front left wheel steering angle and front right wheel steering angle.

2.1. Vertical tire force calculation

The vertical tire force plays a crucial role in accurately estimating lateral forces. It is essential to account for  through that consider load transfer and acceleration, as these are directly influcenced during the vehicle’s dynamic behavior such as cornering, accelerating, and braking.

The coupings between pitch and roll dynamics are neglected in this study, assuming that these have a minimal effect on the overall vertical force calculation. The vertical forces can be simplified and calucatled using the approach outlined in Eq. (5). (Doumiati *et al.*, 2012)

|  |  |
| --- | --- |
|  | (5) |

where are the vehicle mass, gravitational acceleration, distance trom ground to CG, half of track width, wheel base length, longitudinal acceleration and lateral acceleration, respectively.

2.2. Dugoff’s tire model

To represent tire forces, Dugoff’s tire model combines both lateral and longitudinal tire forces. It calculates these forces based on the slip ratio of longitudinal forces and the slip angle for lateral forces. By neglecting longitudinal slip ratio, simplified Dugoff’s tire model for lateral force is described in Eq. (6).

|  |  |
| --- | --- |
|  | (6) |

where represent the corenering stiffness of each axle, and  is the tire-road firction coefficient, assumed to be 1.0 for a high-friction road surface. Meanwhile, the lateral force is generated with a time lag relative to change in slip angle, it causes transient response of the tire. The lateral tire force dynamics is first order and represented as follows:

|  |  |
| --- | --- |
|  | (7) |

Here, denotes the relaxation length, which is assumed to be constant value of 0.1m in this study.

2.3 Axle dristribution based-lateral force

As mentioned before, the Dugoff’s tire model assumes that the lateral tire force is proportional to slip angle. However, this assumption is valid within a limited small range of slip angle. As slip angle increases, the behavior of the tire becomes nonlinear and no longer increases proportionally with the slip angle; instead, it appeoaches a saturation point where additional increases in slip angle yield diminishing in lateral force. Thus, predicting the lateral tire force using linear models becomes less accurate.

On the other hand, an alterative approach is proposed for estimating lateral tire force without relying on sophiscated tire modeling and filtering methods. Instead, they predicts the lateral tire forece directly by focusing on the distribution of vertical load across the tires relative to the total load on certain axle. The equations of the axle distribution based lateral force calculation is described in Eq. (7).

|  |  |
| --- | --- |
|  | (8) |
|  | (9) |

Here,represents the lateral tire force,  and  are the total lateral forces on the front and rear axles, respectively.

2.4. Optimization for modifying cornering stiffness

Previous studies have proposed that load transfer affects cornering stiffness, and this relationship can be presented by a second-order polynominal with respect to vertical force. In this study, the second-order polynominal equation is adjusted by adding a bias term, representing initial cornering stiffness as described in Eq. (10). While the axle dristribution based method does not fully capture the nonlinear relationship between lateral force and slip angle, it remains effective for modifying the cornering stiffness and reflecting nonlinear changes as the slip angle increases.

|  |  |
| --- | --- |
|  | (10) |
|  | (11) |

To further refine cornering stiffness, an optimization problem is then formulated aimed at reflecting the effect of vertical load, as defined in Eq. (12). This involves minimizing the sum of squared error between Eq.(8) and Eq.(11).

|  |  |
| --- | --- |
|  | (11) |

The Levenberg Marquardt method is utilized for this optimization task and the optimal values for the coefficients and as -0.006 and 3.501, respectively.

2.5. Adaptive extended kalman filter

To estimate the lateral force in state-space model, the AEKF is employed to dynamically adjust the process noise. Unlike the process noise, which is adjusted dynamically to reflect changes in the system state transitions, the measurement noise is kept constant and remain relatively stable under normal operating conditions. The AEKF utilizes 8-dimensional state vector , 5-dimensional input control vector , and 5-dimenstional measurement vector  as follows:

|  |  |
| --- | --- |
|  | (13) |
|  | (14) |

 is excluded from the state vector and used as a input control vector calculated by Eq. (14). It is determined by using the wheel driving torque, wheel braking torque, and the effective radius of the tire denoted as,, respectively. The priori state  of AEKF is calculated by integrating over discre time deviation  as described in Eq. (12) ~ (13).

|  |  |
| --- | --- |
|  | (12) |
|  | (13) |

The measurement model is described in Eq. (15)~(16).

|  |  |
| --- | --- |
|  | (15) |
|  | (16) |

where denotes the white gaussian measurement noise and is represented as a nonlinear function of 

The entire estimation process during discrete time deviationis formulated in Eq. (16).

|  |  |
| --- | --- |
|  | (16) |

Here, are the state covariance matrix, system noise covariance, and measurement noise covariance. are the jacobian matrices of the nonlinear function of Compared to conventianl EKF process, AEKF adaptively adjust the system noise covariance matrix , by balancing the weight  between the  and the innovation  term.

5. CONCLUSION

Type the contents of the conclusion here. Type the contents of the conclusion here. Type the contents of the conclusion here. Type the contents of the conclusion here. Type the contents of the conclusion here. Type the contents of the conclusion here. Type the contents of the conclusion here. Type the contents of the conclusion here. Type the contents of the conclusion here. Type the contents of the conclusion here. Type the contents of the conclusion here.

**ACKNOWLEDGEMENT−**This work was supported by Institute of Information & communications Technology Planning & Evaluation (IITP) grant funded by the Korea government (MSIT) (No.2022-0-01053, Development of Network Load Balancing Techniques Based on Multiple Communication/Computing/Storage Resources)

REFERENCES

\* The Harvard System of references is to be used.

\* In the body of the text a paper is to be referred to by the author’s Last Name with the year of publication in parentheses.

Example :

-One author (Last Name, Year)

-Two authors (Last Name and Last Name, Year)

-More than three authors (Last Name *et al*., Year)

Example :

-One author : (Huh, 2000)

-Two authors : (Incropera and DeWitt, 1990)

-More than three authors : (Park *et al*., 2002)

\* References should be listed together at the end of the paper in alphabetical order by author’s Last Name.

-Type in all Author’s name and Paper title

-Type in reference of this paper in the text.

-Transactions of the Korean Society of Automotive Engineers : *Trans. Korean Society of Automotive*

*Engineers*

IJAT : *Int. J. Automotive Technology*

\* Citing a minimum of one citation from the International Journal of Automotive Technology (IJAT) is recommended.

**1. Journals**

All Author(s)(Type Last Name, then first and middle initials). (Year). Paper title(Capital letter at the beginning of sentence). *Journal Name(Italic)* **Volume number(bold)**, **Issue number(bold)**, pp.−pp.

Example :

Huh, H. (2000). Identification of autobody crashworthiness for space- framed vehicle models by finite element limit analysis. *Int. J. Automotive Technology* **1**, **1**, 101−112.

**2. Books**

All Author(s)(Type Last Name, then first and middle initials). (Year). *Book Title(Italic, Capital letter at the beginning of each keyword)*. Edition number. Publisher. The Location of Publisher(City).

Example :

Incropera, F. P. and DeWitt, D. P. (1990*). Fundamentals of Heat and Mass Transfer*. 3rd edn. John Wiley & Sons. New York.

**3. Website**

All Author(s) or Title (Access Year). Detailed Address.

Example :

PermaPure LLC (2006). http://www.permapure.com/

Presentations/Fuel%20Cell%20Humidification%20

presentation.ppt

**4. Reports and User Guide**

All Author/Organization or Title (Year). Title. Organization. Serial number.

Example :

Green, P. (1980). A Computer Simulation of Headlamp Variables and Drivers Sight Distances: Operating Instructions. HSRI Technical Report. UM-HSRI-80-44.

AVL (2006). BOOST Ver. 5.0 User Guide.