THE MATHEMATICAL CONCEPT OF AN INTEGRAL

A CRISP AND CONCISE INTRODUCTION

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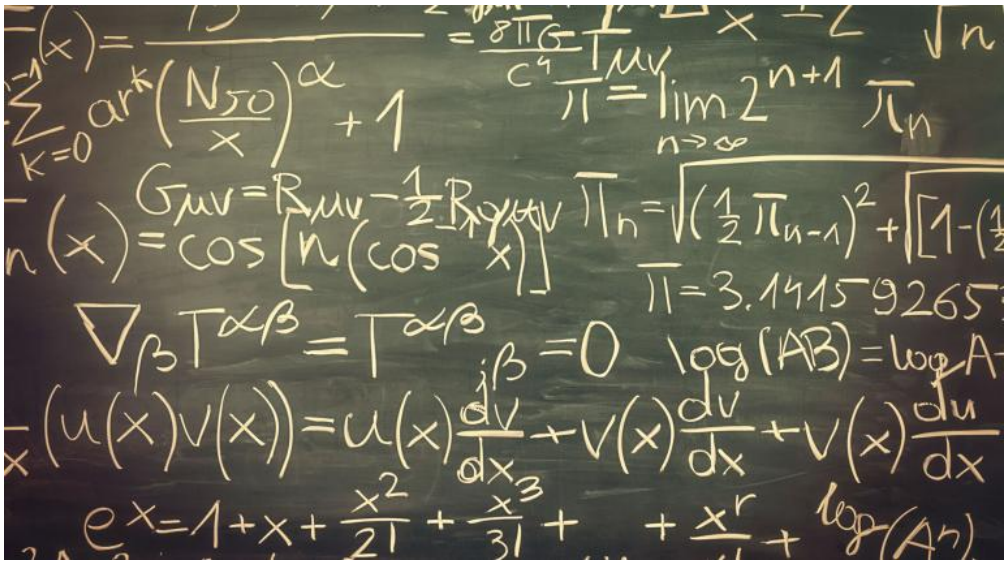

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1 Integral

Given a function $f(x)$, an *anti-derivative* of $f(x)$ is any function $g(x)$ such that $g'(x) = f(x)$. The most general anti-derivative is called the *indefinite integral*.

$$\int f(x)dx = g(x) + c \text{ where c is a constant of integration}$$

$$\begin{aligned} \int f(x)g(x)dx &\neq \int f(x)dx \int g(x)dx \\ \int \frac{f(x)}{g(x)}dx &\neq \frac{\int f(x)dx}{\int g(x)dx} \end{aligned}$$

2 Common Integrals

$$\begin{aligned} \int x^n dx &= \frac{x^{(n+1)}}{(n+1)} + c \\ \int e^x dx &= e^x + c \\ \int a^x dx &= \frac{a^x}{ln\ a} + c \\ \int \frac{1}{x} dx &= ln|x| + c \\ \int cos(x)dx &= sin(x) + c \end{aligned}$$

3 Substitution Technique

$$\int 18x^2 \sqrt[4]{(6x^3 + 5)}dx$$

Let $u = 6x^3 + 5$

$$du = 18x^2 dx$$

$$\int \sqrt[4]{u}\ du$$

4 Integration by Parts

$$[f(x)g(x)]' = f(x)g'(x) + f'(x)g(x)$$

$$f(x)g'(x) = [f(x)g(x)]' - f'(x)g(x)$$

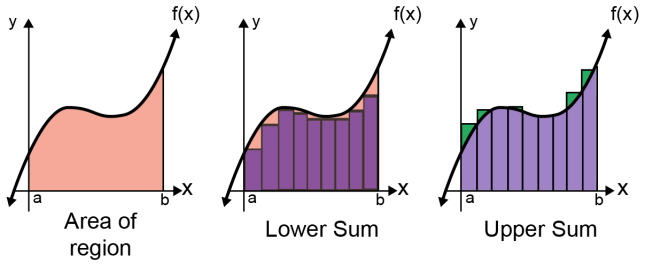
$$\int f(x)g'(x)dx = \int [f(x)g(x)]'dx - \int f'(x)g(x)dx$$

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$$

$$\boxed{\int u(x)v(x)dx = u(x)\int v(x) - \int [u'(x)\int v(x)]dx}$$

Hence, integral of two functions = first function $\times$ integral of second function $-$ integral of (differntiation of first function $\times$ integral of second function).

5 Definite Integral



$$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x = g(x)\Big|_a^b = g(b) - g(a)$$

$f(x_i^*)$ is the value at the middle of the strip Δx

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)dx$$

$\int_a^b f(x)dx = f(c)(b-a)$ where c is in [a,b]

6 Integration Strategies

- Simplify the integrand. E.g., $cos^2(\theta) = \frac{1}{2}(1 + cos(2\theta))$
- Check if simple substitution will work
- If integrand is a rational expression, partial functions may work
- If integrand is polynomial x, $trig$, exp, ln function, integration by parts may work
- If integrand involves $\sqrt{b^2x^2 + a^2}$, trigonometric substitution may work
- If integrand has a quadratic in it, completing the square may work.