PMATH 347: Homework #1

Due: Wednesday, May 14th at 11:59 on Crowdmark.

1. Let $\text{Fun}(X, X)$ be the set of functions from $X \to X$, and let $\circ$ denote composition.
   (a) Show that $\circ$ is associative and that the identity function Id_X is an identity for $\circ$.
   (b) Show that f is left invertible if and only if f is injective, and that f is right invertible if and only if f is surjective.
   (c) Show that if X is infinite, then there are elements of $\text{Fun}(X, X)$ which are left invertible but not invertible, and elements which are right invertible but not invertible.
   (d) Show that if X is infinite, then left (resp. right) inverses in $\text{Fun}(X, X)$ are not necessarily unique.
2. (a) Let M be a set, and let $\boxtimes$ be an associative binary operation on M with identity e. Let
   $$G = \{g \in M : g \text{ is invertible with respect to } \boxtimes\}.$$
   Let $\tilde{\boxtimes} = \boxtimes|_{G \times G}$ be the restriction of $\boxtimes$ to $G \times G$. Show that $(G, \tilde{\boxtimes})$ is a group.
   (Hint: don't forget to show that $\tilde{\boxtimes}$ is a binary operation on G.)
   (b) Conclude that the set S_X from class is a group for all sets X.
3. Let $\circ$ denote the binary operation
   $$M_n\mathbb{C} \times M_n\mathbb{C} \to M_n\mathbb{C} : (A, B) \mapsto \frac{(AB + BA)}{2},$$
   where $M_n\mathbb{C}$ is the vector space of $n \times n$ matrices over $\mathbb{C}$.
   (a) Show that $\circ$ is commutative, and has an identity.
   (b) Show that for every $n \geq 2$, $\circ$ is not associative.
4. Show that if G is a group, and $g^m = e$ for $g \in G$ and $m \geq 1$, then $|g|$ divides m.
5. Find the order of all elements of S_5.
   (Make sure to completely justify your answer for this and the next question.)
6. Find the order of all elements of D_8.