Deep Learning for Computer Vision

VAEs and Disentanglement

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What is Disentanglement?

- Isolating sources of variation in observational data
  - E.g. separating underlying concepts of "**Big Red Apple**": size (*big*), color (*red*) and shape (*apple*)

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- Can we isolate these factors using some representation learning method?
- Why do we need this?

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- Isolating sources of variation in observational data
  - E.g. separating underlying concepts of "**Big Red Apple**": size (*big*), color (*red*) and shape (*apple*)
- Can we isolate these factors using some representation learning method?
- Why do we need this? Useful to generate new images that are not in observed dataset
  - E.g. Generate an image corresponding to "**Small Black Apple**" using a model that was trained on "*Small Black Grapes*" and "*Big Red Apples*"

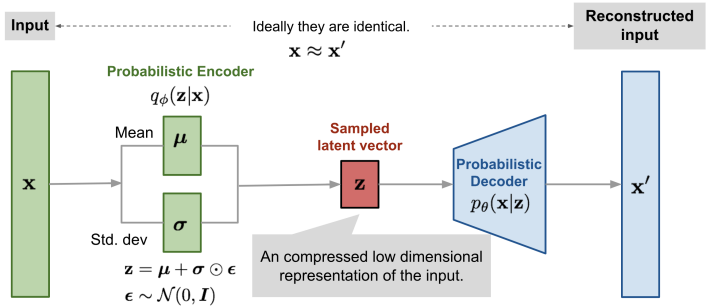
Disentanglement: Example

Images generated when latents (dimensions encoding generative factors) corresponding to gender are changed; more control when latents are disentangled

Credit: Chen et al, Isolating Sources of Disentanglement in Variational Autoencoders, NeurIPS 2018

Disentanglement: Why VAEs?

Recall VAEs:



VAEs learn latent variables which can be used to generate data; if these latent variables are disentangled, allows controlled generation of images

Credit: *Lilian Weng*

β-VAE[1]

- A variant of VAE which allows disentanglement

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- A variant of VAE which allows disentanglement
- Recall **VAE loss**: $L_{\text{VAE}} = -\log p_\theta(\mathbf{x}) + D_{\text{KL}}(q_\phi(\mathbf{z}|\mathbf{x})\|p_\theta(\mathbf{z}|\mathbf{x}))$

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- Another way of writing the VAE objective:

$$\max_{\phi,\theta} \mathbb{E}_{\mathbf{x}\sim\mathcal{D}}[\mathbb{E}_{\mathbf{z}\sim q_\phi(\mathbf{z}|\mathbf{x})} \log p_\theta(\mathbf{x}|\mathbf{z})]$$
$$\text{subject to } D_{\text{KL}}(q_\phi(\mathbf{z}|\mathbf{x})\|p_\theta(\mathbf{z})) < \delta$$

Maximize probability of generating real data, while keeping distance between real and approximate posterior distributions small (under a small constant δ)

[1] Higgins et al, beta-VAE: Learning Basic Visual Concepts with a Constrained Variational Framework, ICLR 2017

β-VAE

- VAE maximization objective can then be rewritten as a Lagrangian with a Lagrangian multiplier β under KKT conditions (similar to SVM):

$$\begin{aligned}
&\mathbb{E}_{\mathbf{z}\sim q_\phi(\mathbf{z}|\mathbf{x})} \log p_\theta(\mathbf{x}|\mathbf{z}) - \beta(D_{\mathsf{KL}}(q_\phi(\mathbf{z}|\mathbf{x})\|p_\theta(\mathbf{z})) - \delta) \\
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- β-**VAE** loss hence given by:

$$L_{\mathsf{BETA}}(\phi, \beta) = -\mathbb{E}_{\mathbf{z}\sim q_\phi(\mathbf{z}|\mathbf{x})} \log p_\theta(\mathbf{x}|\mathbf{z}) + \beta D_{\mathsf{KL}}(q_\phi(\mathbf{z}|\mathbf{x})\|p_\theta(\mathbf{z}))$$

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- When $\beta > 1 \rightarrow$ stronger constraint on latent bottleneck, follow generative process and thus encourage **disentanglement**

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- When $\beta > 1 \rightarrow$ stronger constraint on latent bottleneck, follow generative process and thus encourage **disentanglement**
- Could limit representation capacity of $\mathbf{z}$, creating a trade-off between reconstruction quality and extent of disentanglement

Credit: Lilian Weng

β-TCVAE[2]

- **Disadvantage of β-VAE:** Trade-off between disentanglement and reconstruction capability. How can we get both?

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- KL-divergence term can be decomposed as:

$$D_{\mathsf{KL}}(q_\phi(\mathbf{z}|\mathbf{x})\|p_\theta(\mathbf{z})) = \underbrace{I_q(\mathbf{z},\mathbf{n})}_{\text{index-code mutual information (MI)}} + \underbrace{D_{\mathsf{KL}}(q_\phi(\mathbf{z})\|p_\theta(\mathbf{z}))}_{\text{marginal KL to prior}}$$

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- **Marginal KL to prior** more important to learn disentangled representations; reducing **MI** might be causing poor reconstruction. What to do?

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- Further decompose marginal KL:

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- Weight $\beta > 1$ to disentangle without affecting reconstruction

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- Why not simply use MI? Why MI gap? Homework! *(Hint: Read metric section in Chen et al, Isolating Sources of Disentanglement in Variational Autoencoders, NeurIPS 2018)*

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DCI Metric[a]

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- Create a **relative importance matrix** R such that $R_{ij} = |W_{ij}|$

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DCI Metric: Disentanglement

- Degree to which a representation disentangles underlying factors of variation
- Disentanglement score of i^{th} latent: $D_i = \big(1 - H(P_i)\big)$ where H is entropy and $P_{ij} = \frac{R_{ij}}{\sum_k R_{ik}}$, importance of $\mathbf{z}_i$ to predict $\mathbf{g}_j$
- Total disentanglement score: $D = \sum_i \rho_i D_i$ where $\rho_i = \frac{\sum_j R_{ij}}{\sum_{ij} R_{ij}}$, relative latent importance used to normalize the score

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DCI Metric: Completeness

- Degree to which each underlying generative factor is captured by a single latent variable
- For each generative factor $\mathbf{g}_j$, $C_j = (1H(P_j))$ where the distribution $P_{\cdot j}$ is as above
- If a single latent variable contributes to $\mathbf{g}_j$'s prediction, score is 1 (**complete**); if all latent variables equally contribute to $\mathbf{g}_j$'s prediction, score is 0 (**maximally overcomplete**)

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- Useful for natural tasks which require knowledge of important attributes of data; e.g. for classification task, representation should capture information about object of interest

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- Note that I value depends on capacity of model f_i also

Homework

Readings

- Lilian Weng, From Autoencoders to Beta-VAE
- Prashnna Gyawali, Disentanglement with VAEs: A Review
- (Optional) Papers on respective slides

Questions

- Why is MI Gap and not MI used as a metric for disentanglement?

References

[1] Ricky TQ Chen et al. "Isolating sources of disentanglement in variational autoencoders". In: *Advances in Neural Information Processing Systems*. 2018, pp. 2610–2620.

[2] Cian Eastwood and Christopher KI Williams. "A framework for the quantitative evaluation of disentangled representations". In: *International Conference on Learning Representations*. 2018.

[3] Irina Higgins et al. "beta-VAE: Learning Basic Visual Concepts with a Constrained Variational Framework". In: *ICLR*. 2017.

[4] Diederik P Kingma and Max Welling. *Auto-Encoding Variational Bayes*. 2013. arXiv: 1312.6114 [stat.ML].