

Improvements in Training of Neural Networks

Vineeth N Balasubramanian

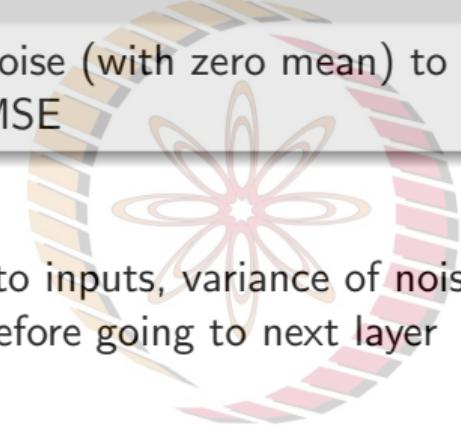
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Review

Exercise

- Show that adding Gaussian noise (with zero mean) to input is equivalent to L_2 weight decay when loss function is MSE
- When we add Gaussian noise to inputs, variance of noise is amplified by squared weight before going to next layer

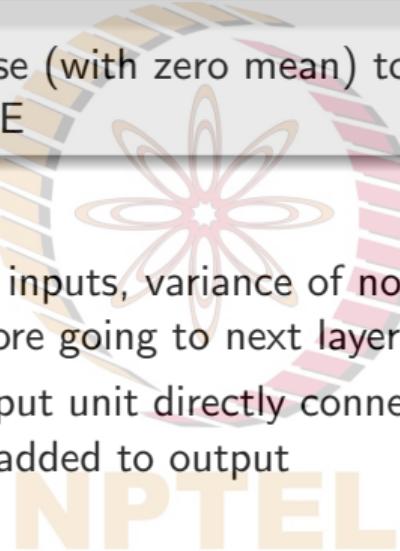


NPTEL

Review

Exercise

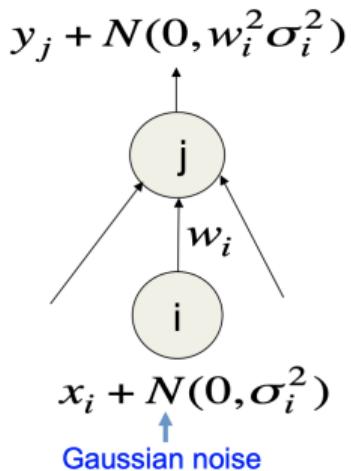
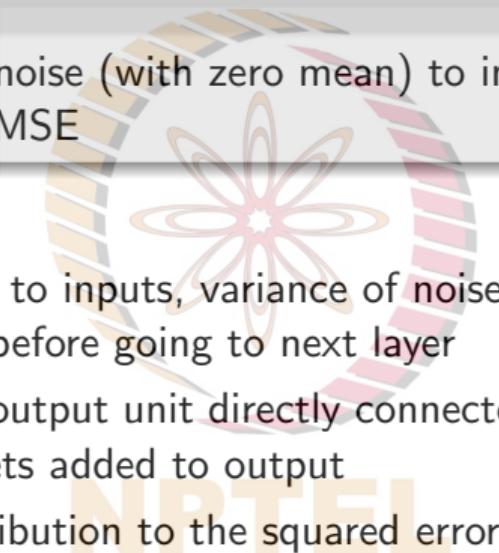
- Show that adding Gaussian noise (with zero mean) to input is equivalent to L_2 weight decay when loss function is MSE
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- In a simple net with a linear output unit directly connected to inputs, the amplified noise gets added to output



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Exercise

- Show that adding Gaussian noise (with zero mean) to input is equivalent to L_2 weight decay when loss function is MSE
- When we add Gaussian noise to inputs, variance of noise is amplified by squared weight before going to next layer
- In a simple net with a linear output unit directly connected to inputs, the amplified noise gets added to output
- This makes an additive contribution to the squared error, i.e. minimizing squared error tends to minimize squared weights when inputs are noisy!



Review

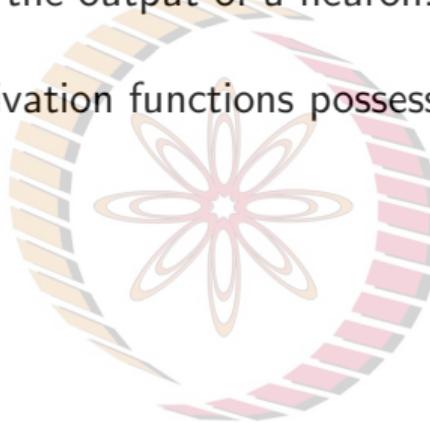
Mathematically, consider one input with added noise. We would have:

$y_{\text{noisy}} = \sum_i w_i x_i + \sum_i w_i \epsilon_i$, where ϵ_i is sampled from $N(0, \sigma_i^2)$. Given expected output t , we have:

$$\begin{aligned}\mathbb{E}[(y_{\text{noisy}} - t)^2] &= \mathbb{E}\left[\left(y + \sum_i w_i \epsilon_i - t\right)^2\right] = \mathbb{E}\left[\left((y - t) + \sum_i w_i \epsilon_i\right)^2\right] \\ &= (y - t)^2 + \mathbb{E}\left[2(y - t) \sum_i w_i \epsilon_i\right] + \mathbb{E}\left[\left(\sum_i w_i \epsilon_i\right)^2\right] \\ &= (y - t)^2 + \mathbb{E}\left[\left(\sum_i w_i \epsilon_i\right)^2\right] \\ &\quad \left(\because \epsilon_i \text{ is independent of } \epsilon_j, \text{ and } \epsilon_i \text{ is independent of } (y - t) \right) \\ &= (y - t)^2 + \sum_i w_i^2 \sigma_i^2 \implies \text{Penalty on L2-norm of weights!}\end{aligned}$$

Activation Functions

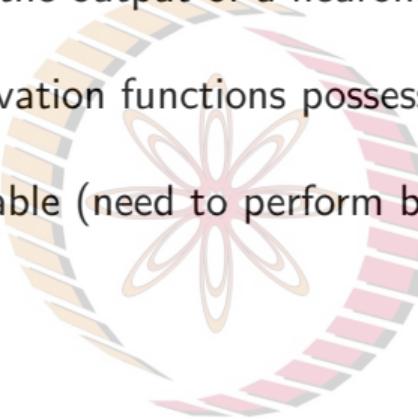
- Non-linear function applied to the output of a neuron.
- What characteristics must activation functions possess?



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Activation Functions

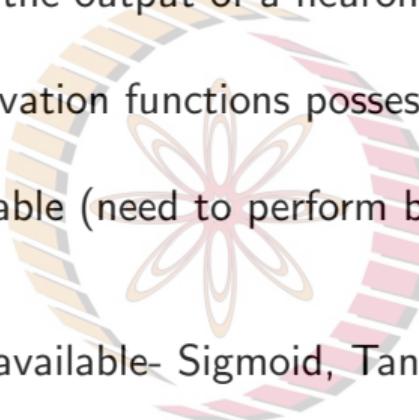
- Non-linear function applied to the output of a neuron.
- What characteristics must activation functions possess?
- Must be continuous, differentiable (need to perform backpropagation), non-decreasing and easy to compute.



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Activation Functions

- Non-linear function applied to the output of a neuron.
- What characteristics must activation functions possess?
- Must be continuous, differentiable (need to perform backpropagation), non-decreasing and easy to compute.
- Common activation functions available- Sigmoid, Tanh, ReLU, etc.
 - Which one to choose?
- What effect does a chosen activation function have on training?

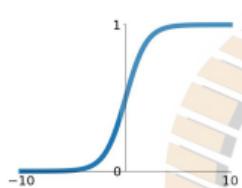


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Activation Functions

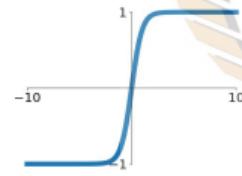
Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



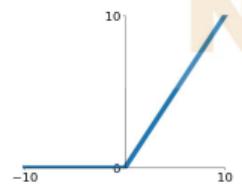
tanh

$$\tanh(x)$$



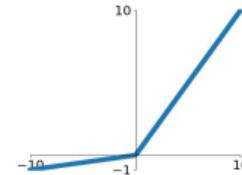
ReLU

$$\max(0, x)$$



Leaky ReLU

$$\max(0.1x, x)$$

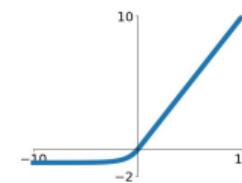


Maxout

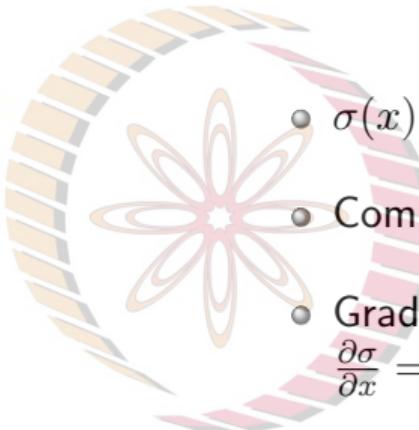
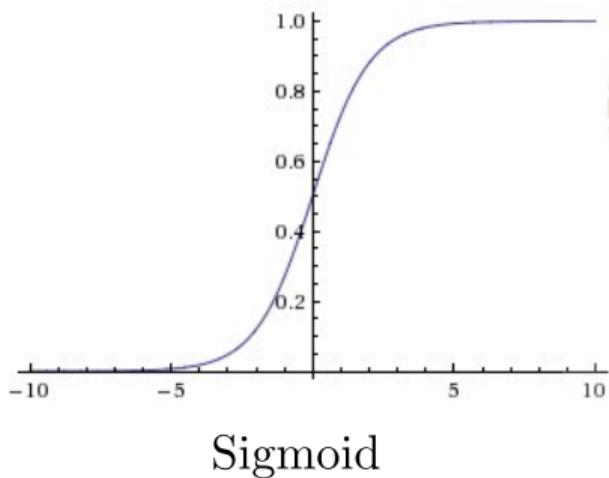
$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

NPTEL ELU

$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



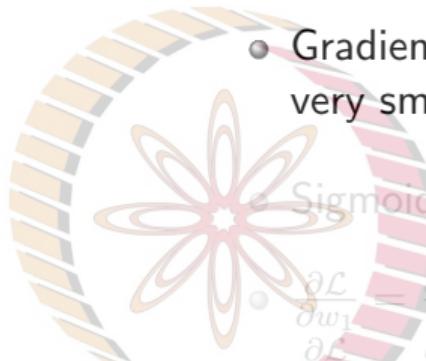
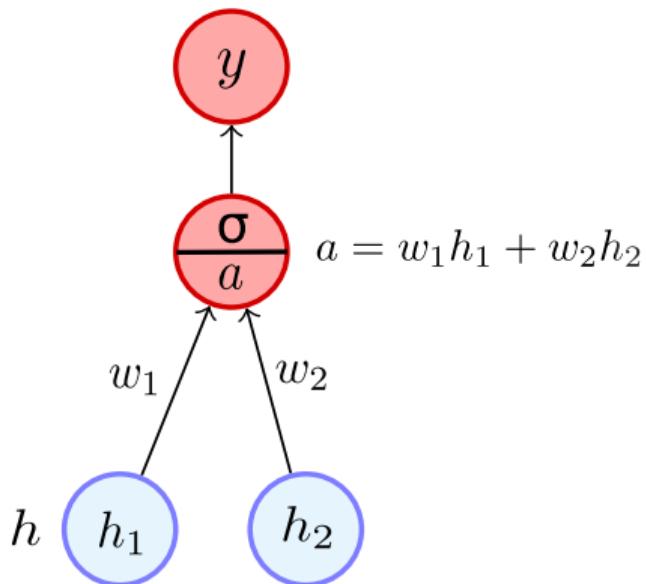
Activation Functions: Sigmoid



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- $\sigma(x) = \frac{1}{1+e^{-x}}$
- Compresses all inputs to range $[0, 1]$
- Gradient of sigmoid function is:
$$\frac{\partial \sigma}{\partial x} = \sigma(x)(1 - \sigma(x))$$
- Sigmoid activations are rarely used in vanilla NNs today, and more often in specific architectures. Why?

Activation Functions: Sigmoid



- Gradient of sigmoid neuron at saturation is very small \Rightarrow training is slow

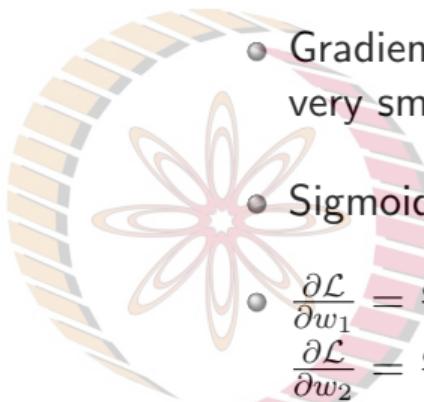
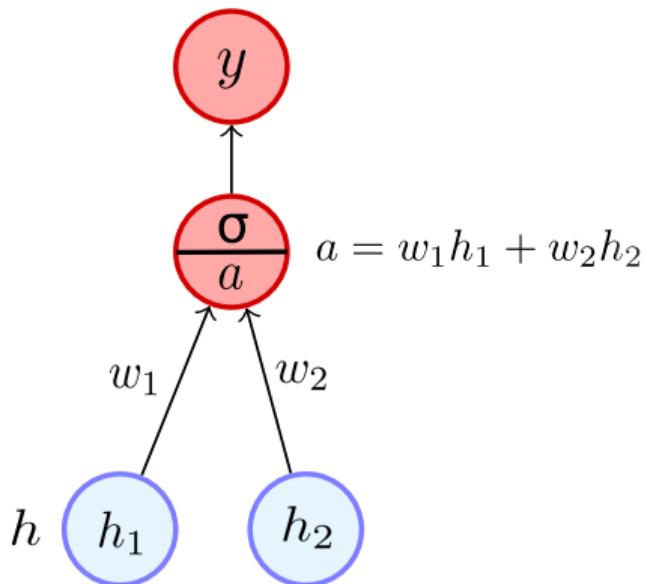
Sigmoid is not zero-centered

$$\frac{\partial \mathcal{L}}{\partial w_1} = \frac{\partial \mathcal{L}}{\partial y} \frac{\partial y}{\partial \sigma} \frac{\partial \sigma}{\partial a} h_1$$
$$\frac{\partial \mathcal{L}}{\partial w_2} = \frac{\partial \mathcal{L}}{\partial y} \frac{\partial y}{\partial \sigma} \frac{\partial \sigma}{\partial a} h_2$$

If h_1 and h_2 are outputs of sigmoid neurons, they are positive. All gradients at a layer are either positive or negative.

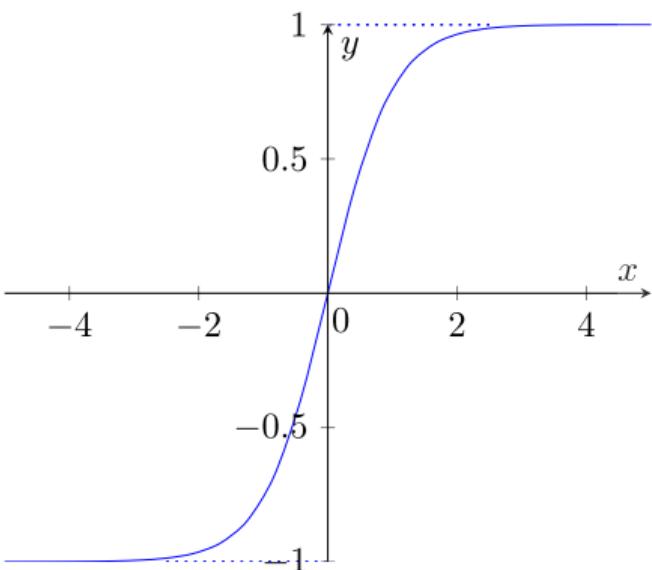
- Computationally expensive, need to compute exponential function

Activation Functions: Sigmoid

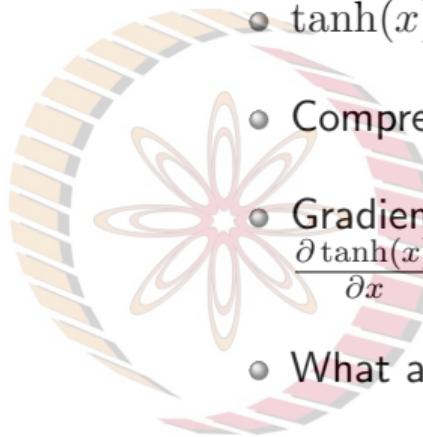


- Gradient of sigmoid neuron at saturation is very small \Rightarrow training is slow
- Sigmoid is not zero-centered
- $\frac{\partial \mathcal{L}}{\partial w_1} = \frac{\partial \mathcal{L}}{\partial y} \frac{\partial y}{\partial \sigma} \frac{\partial \sigma}{\partial a} h_1$
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If h_1 and h_2 are outputs of sigmoid neurons, they are positive. All gradients at a layer are either positive or negative.
- Computationally expensive, need to compute exponential function

Activation Functions: Tanh



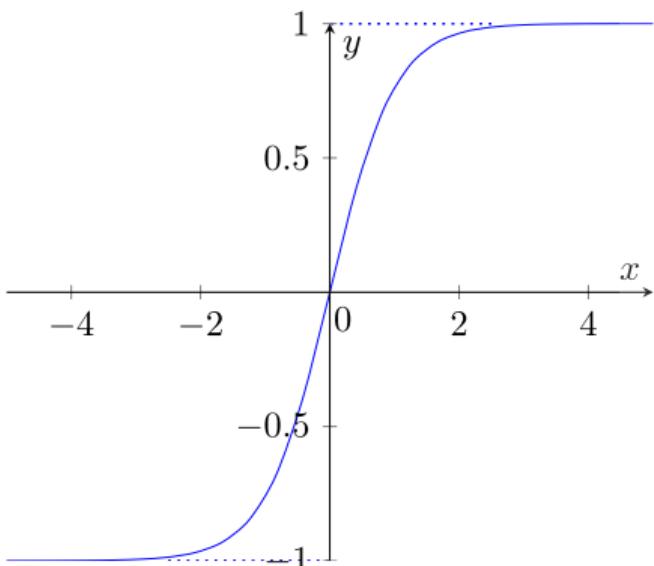
$$f(x) = \tanh(x)$$



- $\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$
- Compresses all inputs to range $[-1, 1]$
- Gradient of tanh function is:
$$\frac{\partial \tanh(x)}{\partial x} = (1 - \tanh^2(x))$$
- What advantage does it have over sigmoid?

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Activation Functions: Tanh

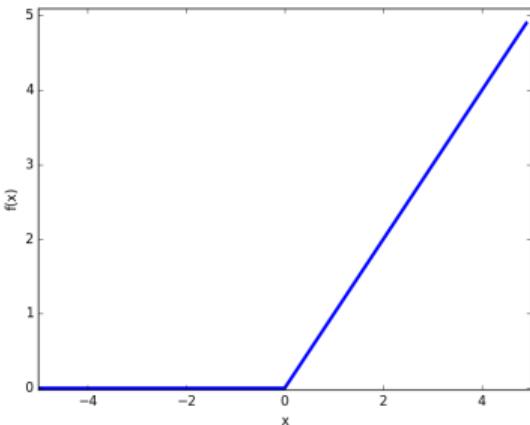


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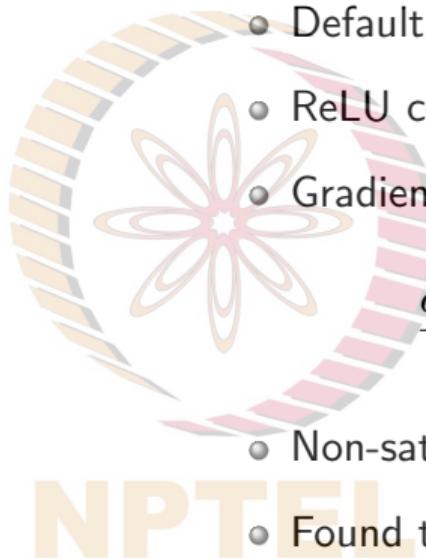


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$$\frac{\partial \tanh(x)}{\partial x} = (1 - \tanh^2(x))$$
- What advantage does it have over sigmoid?
It is zero-centered
- However, gradient of tanh at saturation also vanishes
- Also computationally expensive due to the exponential

Activation Functions: Rectified Linear Unit (ReLU)

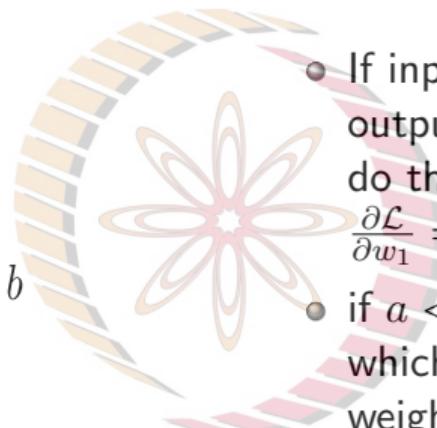
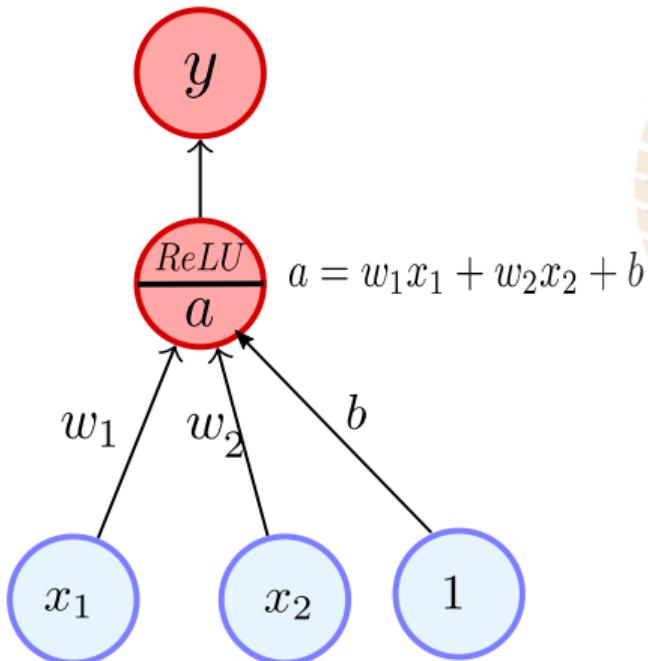


$$f(x) = \max(0, x)$$



- Default activation used today
 - ReLU computes $f(x) = \max(0, x)$
 - Gradient of ReLU is:
- $$\frac{\partial \text{ReLU}(x)}{\partial x} = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x > 0 \end{cases}$$
- Non-saturating for positive inputs
 - Found to accelerate convergence of SGD compared to sigmoid/tanh functions (by a factor of 6 in AlexNet)
 - Computationally inexpensive

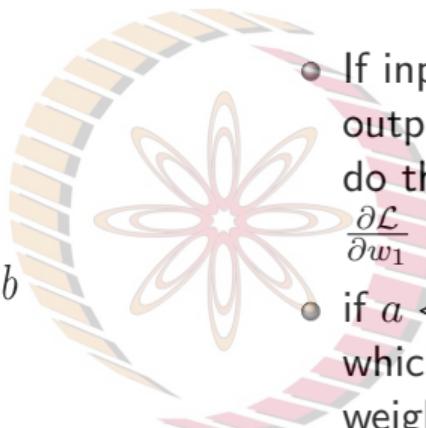
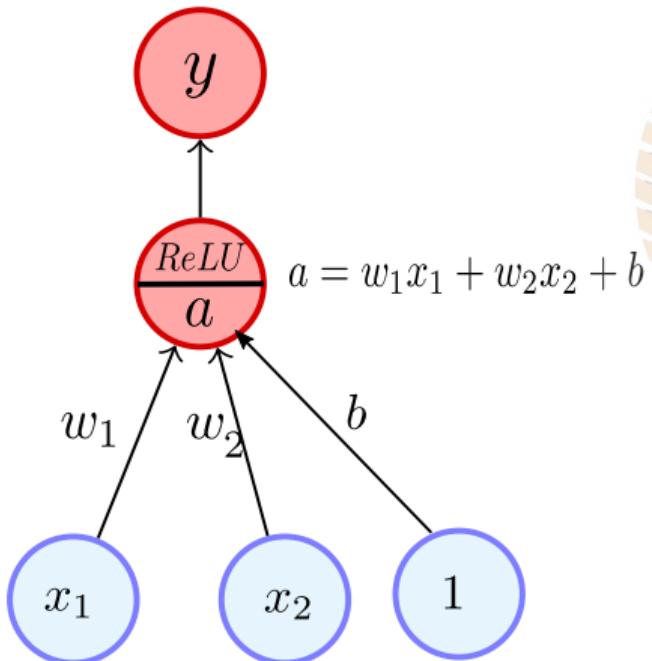
The Dying ReLU Problem



- If input to ReLU neuron is negative, output is zero (“**dead neuron**”). How do the gradients flow?
$$\frac{\partial \mathcal{L}}{\partial w_1} = \frac{\partial \mathcal{L}}{\partial y} \frac{\partial y}{\partial \text{ReLU}} \frac{\partial \text{ReLU}}{\partial a} x_1$$
- if $a < 0$, gradient term $\frac{\partial \text{ReLU}}{\partial a}$ is zero, which makes whole gradient zero. Thus, weights w_1 and w_2 do not get updated

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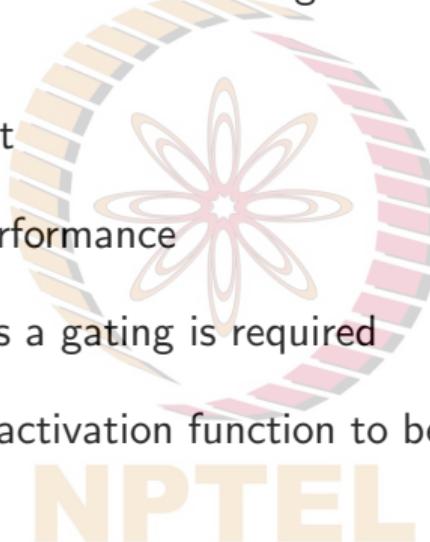
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- Initializing bias to small positive value can help
- Using variants of ReLU (Leaky ReLU, Exponential Linear Unit) can help

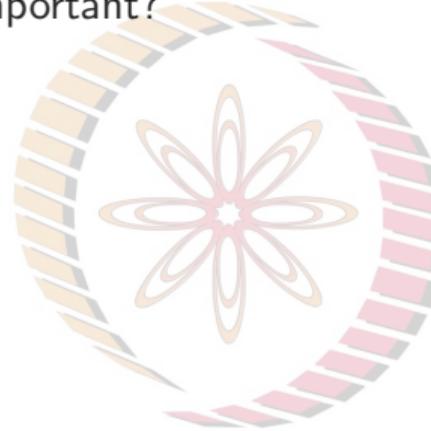
Activation Functions: Which one to choose?

- Use ReLU non-linearity, be careful with learning rates and monitor the fraction of 'dead' units in a network.
- Try Leaky ReLU, ELU, Maxout
- Try tanh, but expect worse performance
- Sigmoid not used much, unless a gating is required
- In general, a good idea for an activation function to be in its linear region for most part of training



Weight Initialization

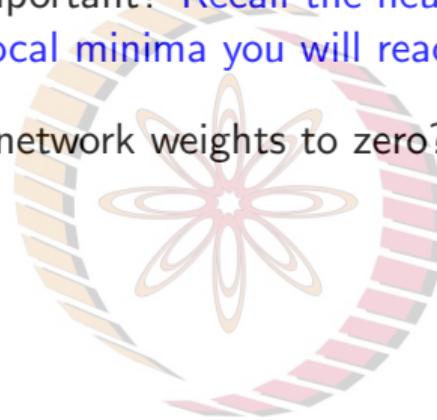
- Why is weight initialization important?



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Weight Initialization

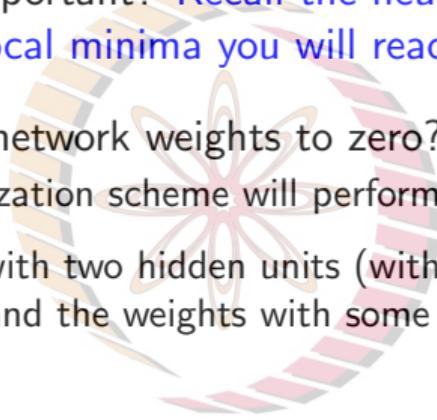
- Why is weight initialization important? Recall the neural network error surface. Where you start is critical to which local minima you will reach
- What happens if we initialize network weights to zero?



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Weight Initialization

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 - In fact, any constant initialization scheme will perform very poorly
 - Consider a neural network with two hidden units (with, say, ReLU activation), and assume we initialize all biases to 0 and the weights with some constant α



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 - If we forward propagate an input (x_1, x_2) in the network, output of both hidden units will be $\text{relu}(\alpha x_1 + \alpha x_2)$!

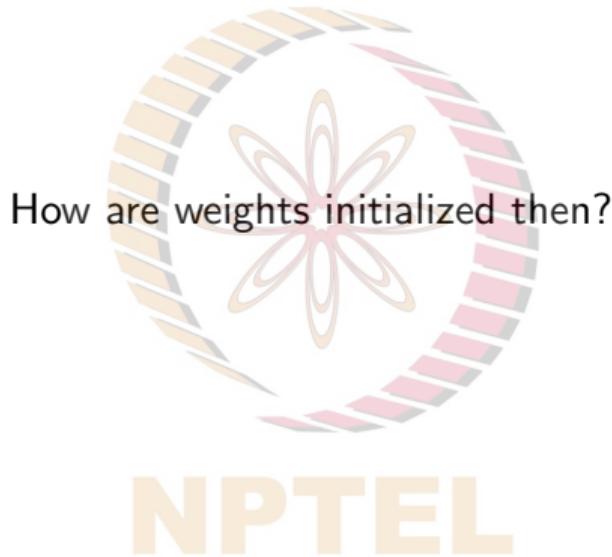
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Weight Initialization

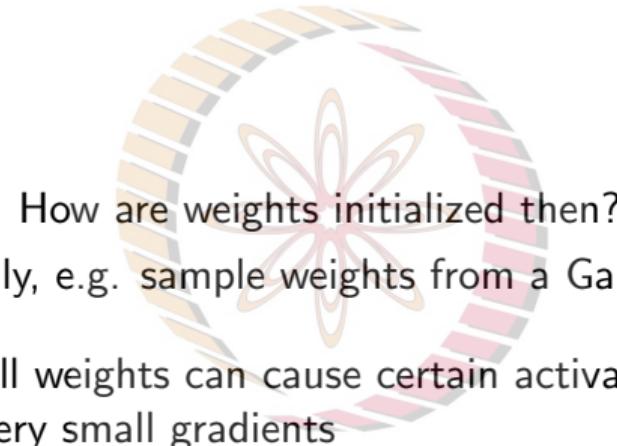
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 - If we forward propagate an input (x_1, x_2) in the network, output of both hidden units will be $\text{relu}(\alpha x_1 + \alpha x_2)!$
 - Both hidden units will have identical influence on cost \implies identical gradients
 - Thus, both neurons evolve symmetrically throughout training, preventing different neurons from learning different things

Credit: <https://www.deeplearning.ai/ai-notes>

Weight Initialization



Weight Initialization



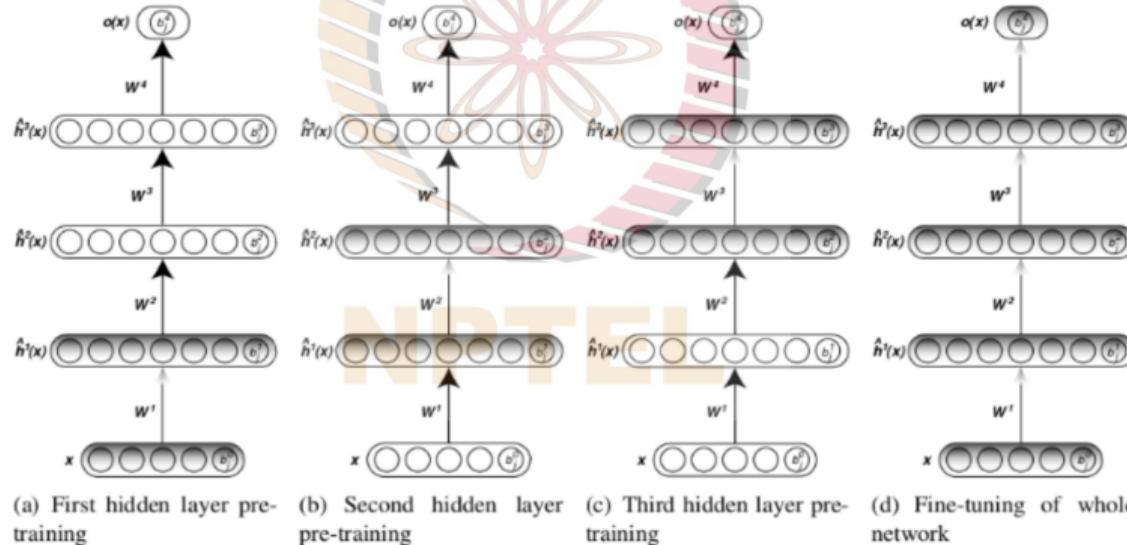
How are weights initialized then?

- Typically chosen randomly, e.g. sample weights from a Gaussian distribution
- Both very large and small weights can cause certain activation functions (sigmoid/tanh) to saturate, leading to very small gradients

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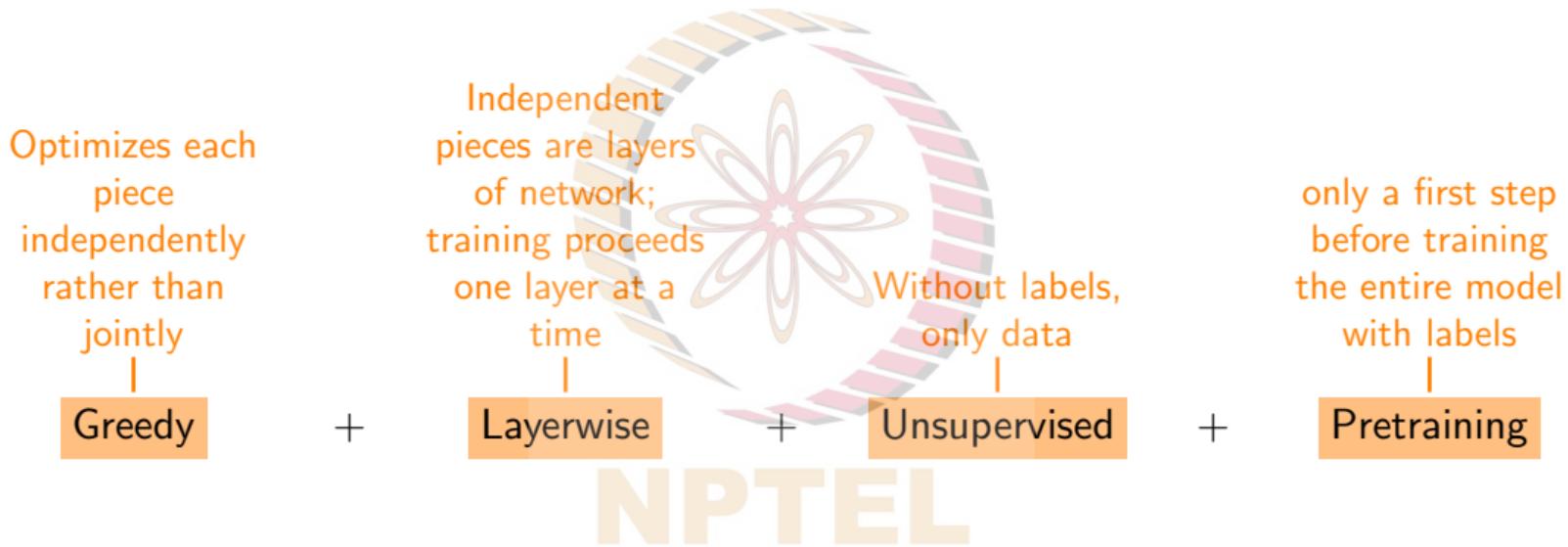
Weight Initialization through Unsupervised Pretraining

- In 2006, Salakhutdinov and Hinton¹ introduced **greedy layerwise unsupervised training**
→ largely considered to be a significant catalyst in initial success of DNNs at that time
(now replaced by better weight initialization methods)



¹Salakhutdinov and Hinton, "Reducing the Dimensionality of Data with Neural Networks", Science, 2006

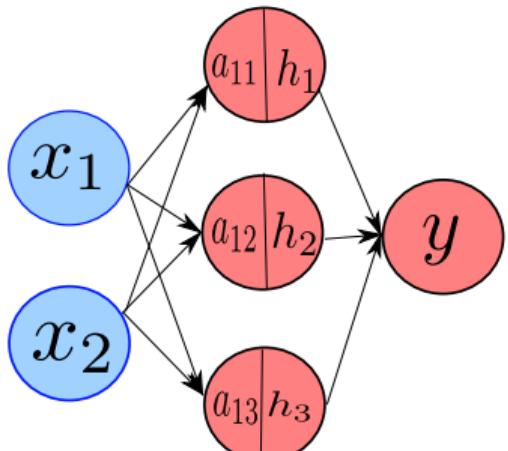
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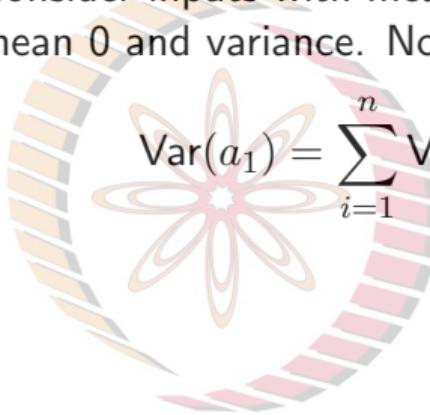
Achieved using networks called autoencoders or Restricted Boltzmann Machines (or equivalent), which we will see later

Newer Weight Initialization Methods

- Consider inputs with mean 0 and variance 1, weights with mean 0 and variance. Note that $a_1 = \sum_{i=1}^n w_{i1}x_i$. Then:

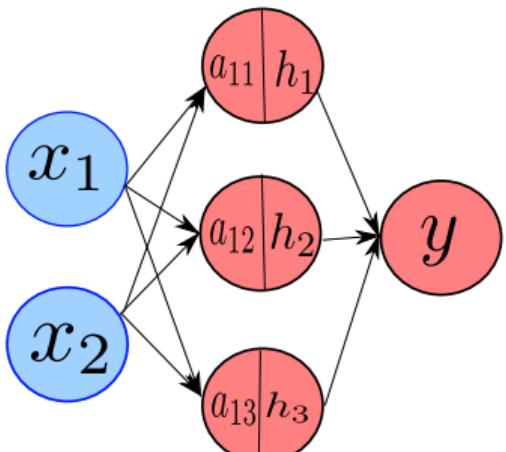


$$\text{Var}(a_i) = n\text{Var}(w)\text{Var}(x)$$



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Newer Weight Initialization Methods



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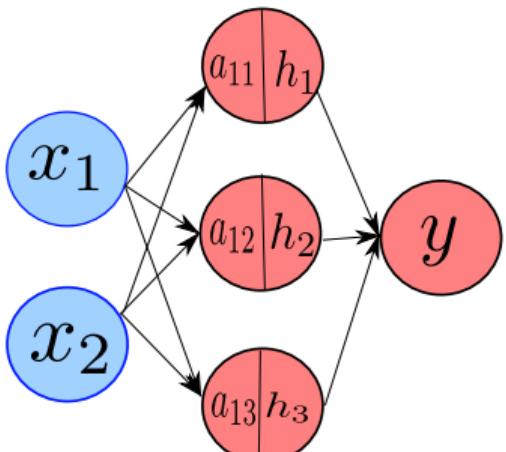
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$$\text{Var}(a_1) = \sum_{i=1}^n \text{Var}(w_{i1}x_i) = n\text{Var}(w)\text{Var}(x)$$

- What happens to variance when we go deeper?

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Newer Weight Initialization Methods



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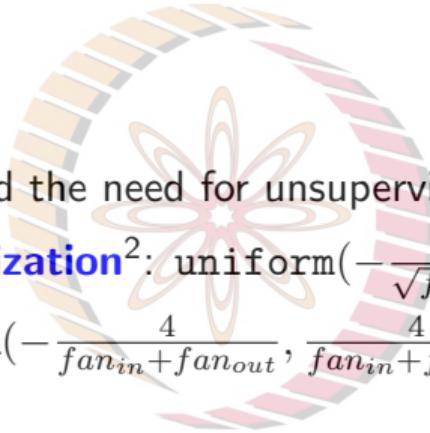
$$\text{Var}(a_1) = \sum_{i=1}^n \text{Var}(w_{i1}x_i) = n\text{Var}(w)\text{Var}(x)$$

- What happens to variance when we go deeper?

$$\text{Var}(a_{ik}) = (n\text{Var}(W))^k\text{Var}(x)$$

- To prevent variance of any layer from blowing up or becoming zero, we need $n\text{Var}(w) = 1$
- For a good initialization, weights can be drawn from standard normal distribution and then scaled by $\frac{1}{\sqrt{n}}$, where n is node's fan-in

Weight Initialization



Most recommended today (removed the need for unsupervised pre-training):

- **Xavier's (or Glorot's) initialization²:** $\text{uniform}\left(-\frac{\sqrt{6}}{\sqrt{fan_{in}+fan_{out}}}, \frac{\sqrt{6}}{\sqrt{fan_{in}+fan_{out}}}\right)$
- **He's initialization³:** $\text{uniform}\left(-\frac{4}{fan_{in}+fan_{out}}, \frac{4}{fan_{in}+fan_{out}}\right)$

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²Glorot and Bengio, "Understanding the difficulty of training deep feedforward neural networks", AISTATS 2010

³He et al, "Delving deep into rectifiers: Surpassing human-level performance on imagenet classification", CVPR 2015

Weight Initialization

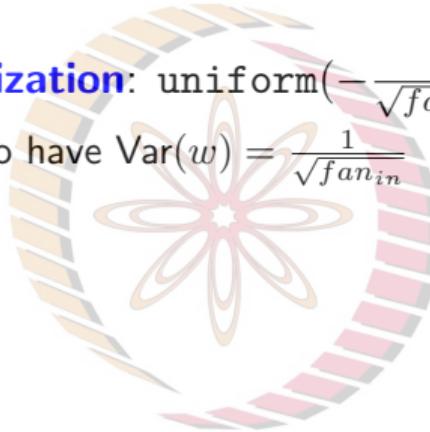
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Weight Initialization

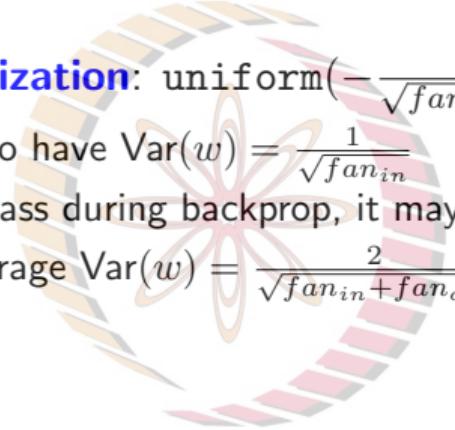
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Weight Initialization

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 - Considering the backward pass during backprop, it may be wise to have $\text{Var}(w) = \frac{1}{\sqrt{fan_{out}}}$ too; hence, go with the average $\text{Var}(w) = \frac{2}{\sqrt{fan_{in}+fan_{out}}}$



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 - If $w \sim \text{Uniform}[-a, a]$, $\text{Var}(w) = \frac{(2a)^2}{12} = \frac{(a)^2}{3}$ (\because variance of uniform distribution in interval $[m, n]$ is $\frac{(n-m)^2}{12}$)

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Weight Initialization

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 - Since we want $fan_{in} \text{Var}(w) = 1 \implies fan_{in} \frac{(a)^2}{3} = 1 \implies a = \frac{\sqrt{3}}{\sqrt{fan_{in}}}$

Weight Initialization

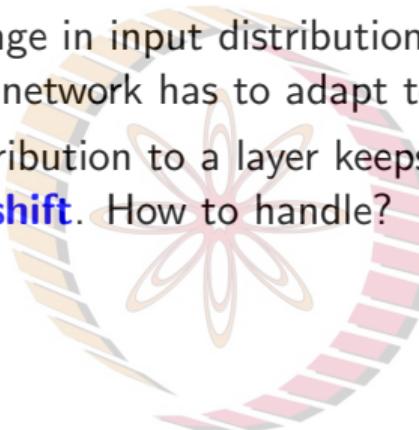
- **Xavier's (or Glorot's) initialization:** $\text{uniform}\left(-\frac{\sqrt{6}}{\sqrt{fan_{in}+fan_{out}}}, \frac{\sqrt{6}}{\sqrt{fan_{in}+fan_{out}}}\right)$
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 - Considering the backward pass during backprop, it may be wise to have $\text{Var}(w) = \frac{1}{\sqrt{fan_{out}}}$ too; hence, go with the average $\text{Var}(w) = \frac{2}{\sqrt{fan_{in}+fan_{out}}}$
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- **He's initialization:** $\text{uniform}\left(-\frac{4}{fan_{in}+fan_{out}}, \frac{4}{fan_{in}+fan_{out}}\right)$. **Homework!**

Batch Normalization

- **Covariate Shift** refers to change in input distribution between training and test scenario. This is a problem because the network has to adapt to the new distribution
- During training, the input distribution to a layer keeps changing over iterations. This is known as **internal covariate shift**. How to handle?



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- During training, the input distribution to a layer keeps changing over iterations. This is known as **internal covariate shift**. How to handle?
- It is known that network training converges faster if its inputs are whitened⁴ i.e linearly transformed to have zero means and unit variances.
- Explicitly ensure each layer's inputs are unit Gaussians (across each dimension), i.e.

$$\hat{x}^{(k)} = \frac{x^{(k)} - \mathbb{E}[x^{(k)}]}{\sqrt{\text{Var}(x^{(k)})}}$$

How do we ensure this?

⁴Lecun et al, Efficient Backpropagation, 1998

Batch Normalization

- The mini-batch! $\mathbb{E}[x^{(k)}]$ and $\text{Var}(x^{(k)})$ are computed empirically from a mini-batch, i.e. ensure that distribution of inputs does not change across batches
- Let's go one step further - introduce $\gamma^{(k)}$ and $\beta^{(k)}$, additional parameters that network learns. This allows network to learn a suitable distribution than use a Gaussian

$$\gamma^{(k)} = \sqrt{\text{Var}(x^{(k)})}$$

$$\beta^{(k)} = \mathbb{E}[x^{(k)}]$$

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Batch Normalization

Input: Values of x over a mini-batch: $\mathcal{B} = \{x_{1\dots m}\}$;

Parameters to be learned: γ, β

Output: $\{y_i = \text{BN}_{\gamma, \beta}(x_i)\}$

$$\mu_{\mathcal{B}} \leftarrow \frac{1}{m} \sum_{i=1}^m x_i \quad // \text{mini-batch mean}$$

$$\sigma_{\mathcal{B}}^2 \leftarrow \frac{1}{m} \sum_{i=1}^m (x_i - \mu_{\mathcal{B}})^2 \quad // \text{mini-batch variance}$$

$$\hat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}} \quad // \text{normalize}$$

$$y_i \leftarrow \gamma \hat{x}_i + \beta \equiv \text{BN}_{\gamma, \beta}(x_i) \quad // \text{scale and shift}$$



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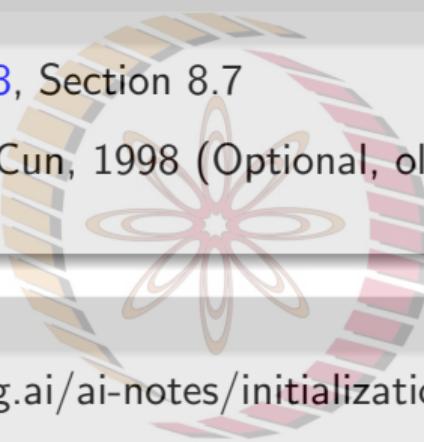
- BN layer is usually inserted before non-linearity (activation)
- Allows higher learning rates. Why? No fear of exploding weights/gradients
- Reduces strong dependence on initialization
- Acts as a form of regularization
- **BatchNorm layer functions differently at test time:** Mean/std are not computed based on test batch; instead, a running average of training mini-batch mean and variance (computed during training) is used.

Ioffe and Szegedy, Batch Normalization: Accelerating Deep Network Training by Reducing Internal Covariate Shift, ICML 2015

Homework

Readings

- Deep Learning book, [Chapter 8](#), Section 8.7
- [Efficient Backprop](#) by Yann LeCun, 1998 (Optional, old article but has interesting insights, worth a read!)

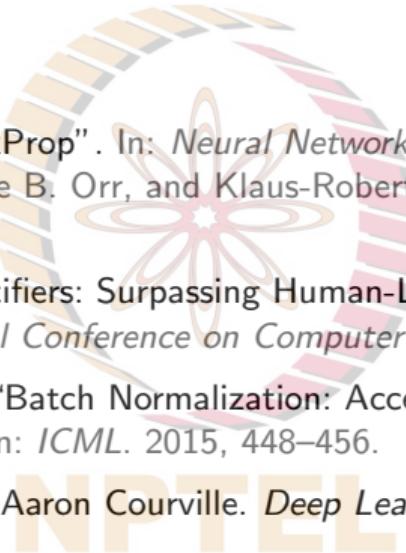


Exercise

- Visit <https://www.deeplearning.ai/ai-notes/initialization>, and try out the animations for weight initialization!
- BN layer is fully differentiable. Consider a neural network with a single hidden layer. Can you show how you would use backpropagation to compute gradients for the batchnorm parameters, γ and β ?
- Derive Kaiming He's initialization.

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References

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- [1] Yann A. LeCun et al. "Efficient BackProp". In: *Neural Networks: Tricks of the Trade: Second Edition*. Ed. by Grégoire Montavon, Geneviève B. Orr, and Klaus-Robert Müller. Springer Berlin Heidelberg, 2012, pp. 9–48.
 - [2] K. He et al. "Delving Deep into Rectifiers: Surpassing Human-Level Performance on ImageNet Classification". In: *IEEE International Conference on Computer Vision (ICCV)*. 2015, pp. 1026–1034.
 - [3] Sergey Ioffe and Christian Szegedy. "Batch Normalization: Accelerating Deep Network Training by Reducing Internal Covariate Shift". In: *ICML*. 2015, 448–456.
 - [4] Ian Goodfellow, Yoshua Bengio, and Aaron Courville. *Deep Learning*. MIT Press, 2016.