ME228-S1 - Assignment 9

Department of Mechanical Engineering, IIT Bombay

Spring 2024

Due Date: 9:30 AM, May 03, 2024, Marks 20

Assignment Date: 9:30 AM, April 15, 2024

Objective and Instructions

1. The objective of this assignment is to learn about auto-differentiation using Autograd library and how to use it for solving an ODE.
2. This needs to be performed using Jupyter Notebook or Google Colab Notebook or only.
3. Submit Jupyter Notebook, Jupyter Notebook pdf to Moodle.

Start with Python Notebook "Autograd_Tutorial _Alankar" posted on Moodle. This tutorial shows how to solve an Ordinary Differential Equation (ODE) using Autograd library. The ODE is $\frac{dy}{dx} = -2xy; y(0) = 1$. A neural network with a single hidden layer is setup in which there are 5 neurons. x is the input to the neural network and y is the output. Solution of an ODE based on such single hidden layer is based on *Universal Approximation Theorem* according to which any continuous and differentiable function can be approximated by a single large layer of neurons. The loss function is $(\frac{dy}{dx} + 2xy)^2 + (y_0 - 1)^2$. Note that the loss function essentially enforces that ODE and initial conditions are satisfied. You can get more familiar with this concept by running the notebook for a different number of neurons in the hidden layer, different number of grid points for x and for a different optimizer.

Q 1. **[5 points]** Run the problem for three cases i.e. 5, 10 and 15 neurons in the hidden layer and compare the solutions. As a final answer to this problem, you should show the graph as it is currently given in the notebook, including the solutions for all three cases. Use proper labels and legends on the graphs to identify which solution is from which setup.

Q 2. **[15 points]** Solve the equation $\frac{dy}{dx} = 2x^3 - exp(-x); y(0) = 1$ by making suitable changes in the notebook. You can use any number of neurons in the hidden layer as it suits. You may also need to use a larger number of x inputs.

−end−