卐 Short Revision Notes 卐

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Contents

Think twice code once!

1 Maths

1.1 Game Theory

Games like chess or checkers are partizan type.

1.1.1 What is a Combinatorial Game?

1. There are 2 players.
2. There is a set of possible positions of Game
3. If both players have same options of moving from each position, the game is called impartial; otherwise partizan
4. The players move alternating.
5. The game ends when a postion is reached from which no moves are possible for the player whose turn it is to move. Under **normal play rule**, the last player to move wins. Under **misere play rule** the last player to move loses.
6. The game ends in a finite number of moves no matter how it is played.

P - Previous Player, **N** - Next Player

1. Label every terminal position as P - postion
2. Position which can move to a P position is N position
3. Position whole all moves are to N positoin is P position.

Note: Every Position is either a P or N.

Directed graph G = (X, F), where X is positions (vertices) and F is a function that gives for each $x \in X$ a subset of X, i.e. *followers of* x. If $F(x)$ is empty, x is called a terminal position.

$g(x) = min\{n \geq 0 : n \neq g(y)$ for $y \in F(x)\}$

Positions x for which $g(x)$ is 0 are P postions and all others are N positions.

4.1 The Sum of n Graph Games. Suppose we are given n progressively bounded graphs, $G_1 = (X_1, F_1), G_2 = (X_2, F_2), \ldots, G_n = (X_n, F_n)$. One can combine them into a new graph, $G = (X, F)$, called the **sum** of $G_1, G_2, \ldots, G_n$ and denoted by $G = G_1 + \cdots + G_n$ as follows. The set X of vertices is the Cartesian product, $X = X_1 \times \cdots \times X_n$. This is the set of all n-tuples $(x_1, \ldots, x_n)$ such that $x_i \in X_i$ for all i. For a vertex $x = (x_1, \ldots, x_n) \in X$, the set of followers of x is defined as

$$\begin{aligned} F(x) = F(x_1, \ldots, x_n) = {} & F_1(x_1) \times \{x_2\} \times \cdots \times \{x_n\} \\ & \cup \{x_1\} \times F_2(x_2) \times \cdots \times \{x_n\} \\ & \cup \cdots \\ & \cup \{x_1\} \times \{x_2\} \times \cdots \times F_n(x_n). \end{aligned}$$

Theorem 2. *If g_i is the Sprague-Grundy function of G_i, $i = 1, \ldots, n$, then $G = G_1 + \cdots + G_n$ has Sprague-Grundy function $g(x_1, \ldots, x_n) = g_1(x_1) \oplus \cdots \oplus g_n(x_n)$.*

Thus, if a position is a **N** position, we can cleverly see which position should we go to (which component game move to take) such that we reach **P** position.

2 Graphs

2.1 Tree

Undirected, acyclic, connected, $|V| - 1$ edges.

All edges are bridges, and internal vertices (degree > 1) are articulation points.

SSSP: Simply take the sum of edge weights of that unique path. $O(|V|)$

APSP: Simply do SSSP from all vertices. $O(|V^2|)$

```
void preorder (v) {
  visit (v);
  preorder (left (v));
  preorder (right (v));
}
void inorder (v) {
  inorder (left (v));
  visit (v);
  inorder (right (v));
}
void postorder (v) {
  postorder (left (v));
  postorder (right (v));
  visit (v);
}
```

2.1.1 Important Problems

- UVA 11695 Sol: Problem Desc: Find which edge to remove and add so as to minimise the number of hops to travel between flights.
  Problem Sol: Just link the center of diameters. Brute force which edge to remove.
- UVA 112 Sol, UVA 112 Prob: Just see how I processed the input.
- UVA 10029 Sol, UVA 10029 Prob: Edit steps, (lexicographic sequence of words)