

Module 5 Project — Using Linear Programming Models to Maximize Profits

Sourabh D. Khot (ID 002754952)

College of Professional Studies, Northeastern University

ALY 6050 Introduction to Enterprise Analytics CRN 81180 Spring 2022

Professor Azadeh Mobasher

Jun 26, 2022

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Introduction

A hardware company plans to expand into a new area and supply its four main products to the nearby local dealers. As an operation research analyst, I will be assisting the company in determining inventory levels for each product for maximum net profit.

Data on manufacturing and transformation cost per unit is known, and the selling price of each product is fixed. The company has set aside a monthly budget for this expansion and plans to rent a warehouse. The number of shelves in the warehouse, their dimensions, and the dimensions of the product packages are known. To promote band products and penetrate into a new area, the marketing department has provided specific product portfolio mix requirements that need to be accommodated.

Using this information, I will recommend the optimum monthly inventory level for each product to make maximum profit. I will also perform sensitivity analysis to suggest changes in the model parameters, such as selling price and budget allocation.

Analysis

I will build a linear programming model in Microsoft Excel and use Solver Excel add-in with the Simplex LP method to compute the optimum solution for maximum profit. I will extensively analyze the sensitivity report to suggest further courses of action and increase profit.

1. Mathematical formulation

It is assumed that in a month, a single purchasing order is placed at the beginning of the month using the monthly purchasing budget for the required inventory of units of the four products. Hence, the area occupied by a single order should not be more than the total shelf area of the warehouse. Shelves of the warehouse are assumed to be connected, and only the total warehouse area (number of shelves*area per shelf) and area of each product are used to constrain the maximum number of products that may be stored in the warehouse. Also, it is assumed that there is sufficient demand, and every inventory bought is sold in the same month.

To implement linear programming, inventory units per month are allowed to be non-integers, and the fractional part may be assumed to be average, contributing to subsequent months. E.g., if the monthly inventory is 4.5, then nine products should be purchased in two months. Similarly, although water products are packed in a case of five, the decision variable for inventory count is defined for an individual water pump, and the related metrics such as cost price and area per unit are normalized as per individual pump.

Definitions:

$$Profit = Selling\ price - Cost\ price$$

$$x_1 = Units\ per\ month\ of\ Pressure\ Washer$$

$$x_2 = Units\ per\ month\ of\ Gokart$$

$$x_3 = Units\ per\ month\ of\ Generator$$

$$x_4 = Units\ per\ month\ of\ Water\ pump\ (single)$$

Maximize:

$$z = (499.99 - 330)x_1 + (729.99 - 370)x_2 + (700.99 - 410)x_3 + (269.99 - 127)x_4 \text{ or}$$

$$\text{Maximum:} \quad z = 169.99x_1 + 359.99x_2 + 290.99x_3 + 142.99x_4$$

Subject to constraints:

$$\text{Budget:} \quad 330x_1 + 370x_2 + 410x_3 + 127x_4 \leq 170,000$$

$$\text{Shelf Space (sq. ft.):} \quad 5 \times 5x_1 + 8 \times 5x_2 + 5 \times 5x_3 + \frac{5 \times 5}{4 \times 5}x_4 \leq 82 \times 30 \times 5$$

$$\text{Promotion 1:} \quad \frac{x_1 + x_2}{x_1 + x_2 + x_3 + x_4} \geq 30\% \text{ or}$$

$$-0.7x_1 + -0.7x_2 + 0.3x_3 + 0.3x_4 \leq 0$$

$$\text{Promotion 2:} \quad x_3 \geq 2x_2 \text{ or}$$

$$-x_3 + 2x_4 \leq 0$$

$$\text{Non-negativity:} \quad x_1, x_2, x_3, x_4 \geq 0$$

In the Excel model, the non-negativity constraints are not explicitly formulated and are taken care of by selecting the corresponding option in the Solver dialog box.

2. Linear programming formulation setup

The linear programming model is implemented in Excel, as shown in table 1.

Quantity Type	Quantity	Product				Total	Comment	
		Pressure Washer	Go-kart	Generator	One Water Pump			
Decision variable	Inventory unit/month	x_1	x_2	x_3	x_4	4.00		
Objective	Monthly profit	\$ 169.99	\$ 359.99	\$ 290.99	\$ 142.99	\$ 963.96	Objective function to maximize	
(Computed)	Profit/unit	\$ 169.99	\$ 359.99	\$ 290.99	\$ 142.99		Profit = Selling price - Cost Price	
Parameter	Selling price	\$ 499.99	\$ 729.99	\$ 700.99	\$ 269.99			
Parameter	Cost price	\$ 330.00	\$ 370.00	\$ 410.00	\$ 127.00		e water pump = cost of case / unit	
Constraints						LHS	RHS	Slack
Parameter	Budget	\$ 330.00	\$ 370.00	\$ 410.00	\$ 127.00	\$ 1,237.00	\$ 170,000.00	\$168,763.00
Parameter	Warehouse area in sq. ft.	25.00	40.00	25.00	1.25	91.25	12300.00	\$ 12,208.75
(Computed)	Washers + Karts \geq 30% of total	-0.70	-0.70	0.30	0.30	-0.80	0.00	0.80
(Computed)	Generators \geq 2 Pumps	0.00	0.00	-1.00	2.00	1.00	0.00	-1.00
(Included in Solver)								

Table 1. Linear Programming Model implemented in Excel (with dummy decision variables)

3. Problem solution and sensitivity report

The linear programming problem is solved in Excel using the above mathematical formulation and model. The solution is populated in the model shown in table 2, and the generated sensitivity report is in table 3.

Quantity Type	Quantity	Pressure Washer	Go-kart	Generator	One Water Pump	Total	Comment
		x ₁	x ₂	x ₃	x ₄		
Decision variable	Inventory unit/month	0.00	155.18	237.77	118.88	511.83	
Objective	Monthly profit	\$ -	\$ 55,862.91	\$ 69,188.48	\$ 16,999.31	\$ 142,050.70	Objective function to maximize
(Computed)	Profit/unit	\$ 169.99	\$ 359.99	\$ 290.99	\$ 142.99		Profit = Selling price - Cost Price
Parameter	Selling price	\$ 499.99	\$ 729.99	\$ 700.99	\$ 269.99		
Parameter	Cost price	\$ 330.00	\$ 370.00	\$ 410.00	\$ 127.00		e water pump = cost of case / unit
Constraints						LHS	RHS Slack
Parameter	Budget	\$ 330.00	\$ 370.00	\$ 410.00	\$ 127.00	\$ 170,000.00 ≤	\$ 170,000.00 \$ -
Parameter	Warehouse area in sq. ft.	25.00	40.00	25.00	1.25	12300.00 ≤	12300.00 \$ -
(Computed)	Washers + Karts ≥ 30% of total	-0.70	-0.70	0.30	0.30	-1.63 ≤	0.00 1.63
(Computed)	Generators ≥ 2 Pumps (Included in Solver)	0.00	0.00	-1.00	2.00	0.00 ≤	0.00 0.00

Current allocation: 30.32%
Current x₃/x₄ ratio: 2.00

Table 2. Solved Linear Programming Problem using Solver

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$E\$5	Washer_units	-	(110.07)	169.99	110.07	1E+30
\$F\$5	Gokart_units	155.18	-	359.99	205.84	76.74
\$G\$5	Generator_units	237.77	-	290.99	98.20	131.87
\$H\$5	Pump_units	118.88	-	142.99	196.41	89.12

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$I\$13	Budget LHS	170,000.00	0.56	170,000.00	428.80	56,225.00
\$I\$14	Warehouse area in sq. ft. LHS	12,300.00	3.84	12,300.00	6,078.38	30.95
\$I\$15	Washers + Karts ≥ 30% of total LHS	(1.63)	-	-	1E+30	1.63
\$I\$16	Generators ≥ 2 Pumps LHS	-	33.68	-	974.12	27.92

Table 3. Sensitivity Report of the solved problem

4. Optimal inventory level and monthly profit

The optimum inventory level solution for each product is given in table 4. To maximize the profit given the constraints, there should be zero pressure washer inventory, Go-karts should be ordered at 155.18 per month, generators at 237.77 per month, and water pump units at 118.88 per month (i.e., 23.78 water pump cases per month). Maintaining this inventory level, we will make a maximum monthly profit of \$142.05K.

	Pressure washer	Go-kart	Generator	Water pump (single)	Total
Inventory units	0.00	155.18	237.77	118.88	511.83
Monthly profit	\$ 0.00	\$ 55,862.91	\$ 69,188.48	\$ 16,999.31	\$ 142,050.70

Table 4. Solution of the Linear Programming Problem

From table 2, we can see that the entire budget and warehouse space is used up. There is slack in the proportion of washer+karts, as it is required to be at least 30%, and is 30.32%.

5. Min. selling price of washer for having a non-zero optimal inventory

With the current parameters, no pressure washer is sold for the maximum profit. This could be because the per-unit profit from the pressure washer is not high enough considering the space it occupies in the warehouse, and using that space for other products would generate more net profit. Per-unit profit depends on selling price and cost price. Now, the area occupied and per-unit cost of a fixed product cannot be changed. Hence, the only option is to increase the selling price so that it is worthwhile to sell the space-occupying product.

From the sensitivity report in table 3, we observe that the reduced price for washer units is -110.07, which is the price to be reduced from the coefficient of the decision variable to obtain a non-zero decision variable. Reducing negative means adding mod of the number. Hence, the

coefficient or profit of washer units must be increased by 110.07 for a non-zero optimum inventory level. Since the cost price is constant, the current selling price of \$499.99 must be increased by 110.07 to obtain a non-negative optimal inventory. (To account for rounding off error, we will increase by 110.08). Therefore, the selling price of the pressure washer should be at least \$610.07 for its optimum monthly inventory level to be a non-zero positive number. This was solved in another iteration of the model as in table 5. Note that in this scenario, another product go-kart's optimum inventory level becomes zero!

Quantity Type	Quantity	Product				Total	Comment		
		Pressure Washer	Go-kart	Generator	One Water Pump				
		X ₁	X ₂	X ₃	X ₄				
Decision variable	Inventory unit/month	434.10	0.00	56.49	28.24	518.83			
Objective	Monthly profit	\$121,578.34	\$ -	\$ 16,437.44	\$ 4,038.61	\$ 142,054.38			Objective function to maximize
(Computed)	Profit/unit	\$ 280.07	\$ 359.99	\$ 290.99	\$ 142.99				Profit = Selling price - Cost Price
Parameter	Selling price	\$ 610.07	\$ 729.99	\$ 700.99	\$ 269.99				
Parameter	Cost price	\$ 330.00	\$ 370.00	\$ 410.00	\$ 127.00				e water pump = cost of case / unit
Constraints						LHS	RHS	Slack	
Parameter	Budget	\$ 330.00	\$ 370.00	\$ 410.00	\$ 127.00	\$ 170,000.00	≤ \$ 170,000.00	\$ -	
Parameter	Warehouse area in sq. ft.	25.00	40.00	25.00	1.25	12300.00	≤ 12300.00	\$ -	
(Computed)	Washers + Karts ≥ 30% of total	-0.70	-0.70	0.30	0.30	-278.45	≤ 0.00	278.45	Current allocation: 83.67%
(Computed)	Generators ≥ 2 Pumps (Included in Solver)	0.00	0.00	-1.00	2.00	0.00	≤ 0.00	0.00	Current x ₃ /x ₄ ratio: 2.00

Table 5. Alternate Solved Linear Programming Problem with washer selling price of \$610.07

6. Recommendation of additional budget and its impact on monthly profit

From table 3, we can see that the shadow price for budget LHS is \$0.56. That means for every additional monthly budget of \$1, the objective function of net monthly profit will increase by only 56 cents, i.e. a marginal loss of 44 cents. Hence, I would not recommend any additional monthly budget. (This shadow price is valid within a range of allowable increase of \$428.8) We may focus on the warehouse area, which has a higher shadow price.

7. Recommendation on ideal warehouse size and its impact on monthly profit

Intuitively, a smaller warehouse will hold lesser inventory, causing profits to reduce. In comparison, a bigger warehouse will hold more inventory, increasing profits, assuming we can sell the additional inventory. The procurement cost of an additional sq.ft. of the warehouse is not

given; hence we will assume it is low enough to ignore. In the current optimum solution in table 2, we see that both budget and warehouse space are used up with no slack. We also observe that the product with the highest per-unit profit (Go-kart) takes up the highest space compared to other products. So in the optimum solution, more less profitable products may have been selected because they occupy lesser space. Increasing warehouse space may thus increase the proportion of the most profitable product, reducing the lesser profitable products, with the same budget, thus increasing the net monthly profit.

The shadow price of the warehouse area is \$3.84 per sq.ft. That means, per the above understanding, for every additional sq.ft. of warehouse area, there will be an increase of \$3.84 monthly net profit. Thus, I will recommend renting a larger warehouse. As per the sensitivity report, the shadow price of \$3.84 is valid within an allowable range of 6078.38 sq.ft. At the maximum increase (i.e. total area of 18,378.38 sq.ft), the additional contribution would \$23,350.11 (shadow price * additional area). I performed another iteration utilizing this maximum allowable increase with an optimum solution in table 6.

		Product						
Quantity Type	Quantity	Pressure Washer	Go-kart	Generator	One Water Pump	Total	Comment	
		X ₁	X ₂	X ₃	X ₄			
Decision variable	Inventory unit/month	0.00	459.46	0.00	0.00	459.46		
Objective	Monthly profit	\$ -	\$165,400.81	\$ (0.00)	\$ (0.00)	\$ 165,400.81	Objective function to maximize	
(Computed)	Profit/unit	\$ 169.99	\$ 359.99	\$ 290.99	\$ 142.99		Profit = Selling price - Cost Price	
Parameter	Selling price	\$ 499.99	\$ 729.99	\$ 700.99	\$ 269.99			
Parameter	Cost price	\$ 330.00	\$ 370.00	\$ 410.00	\$ 127.00		e water pump = cost of case / unit	
Constraints						LHS	RHS	Slack
Parameter	Budget	\$ 330.00	\$ 370.00	\$ 410.00	\$ 127.00	\$ 170,000.00 ≤	\$ 170,000.00	\$ -
Parameter	Warehouse area in sq. ft.	25.00	40.00	25.00	1.25	18378.38 ≤	18378.38	\$ -
(Computed)	Washers + Karts ≥ 30% of total	-0.70	-0.70	0.30	0.30	-321.62 ≤	0.00	321.62
(Computed)	Generators ≥ 2 Pumps (Included in Solver)	0.00	0.00	-1.00	2.00	0.00 ≤	0.00	0.00

Current allocation: 100.00%

Current x₃/x₄ ratio: 2.00

Current allocation: 100.00%
Current x₃/x₄ ratio: 2.00

Table 6. Alternate Solved Linear Programming Problem with increased area of 18,378 sq.ft.

As initially predicted, the high-profit high-area Go-kart has fully consumed the budget and the area with no slack. There is zero inventory of other products of lower-profit lower-space. The constraint of a minimum 30% allocation to washer+go-kart is still satisfied. The second

constraint of having twice as many pumps as pumps is technically satisfied since both generator and one water pump inventories are zero.

Hence, assuming that cost of the additional area of the warehouse is negligible (because it may already be a part of owned land) and that both generator and pump inventory can be zero, I will recommend having a larger warehouse of a total 18,378 sq.ft. that will lead to an additional monthly profit of \$23,350. The additional area corresponds to additional 40.52 shelves or a total of 122.52 shelves of the same dimensions

Conclusion

For the given parameters and constraints, I designed a linear programming model whose optimum solution was not to order a pressure washer and place an average monthly inventory order of 155.18 go-karts, 237.77 generators, and 118.88 single water pumps (or 23.78 cases of water pumps). These optimum inventories would lead to the maximum profit of \$142,050.70.

To have at least some inventory of pressure washer as part of optimum solution, its selling price should be increased by \$110.08 from currently \$499.99 to \$610.07. However, note that in such a scenario, the optimal inventory of go-kart will become zero.

As per sensitivity analysis, an increase in one dollar of the monthly budget increases the monthly net profit by 56 cents or at a 44% marginal loss; hence I would not recommend increasing the budget. An increase in one square foot of area leads to an increase of \$3.84 in monthly profit (ignoring area cost) since the high-area high-profit go-kart replaces other products keeping the budget same. This is up to a maximum additional increase of 6078.38 sq.ft.

Accordingly, I would recommend a larger warehouse. Assuming the cost of the additional area is negligible and no inventories of generators and pumps are acceptable, I would suggest increasing the area by full allowable range of 6078,38 sq.ft. (40.52 additional shelves) for an increased marginal profit of \$23,350, which would entail selling only go-karts optimally. In terms of totals, the total warehouse area of 18,378 sq.ft. (122.52 total shelves) would lead to a total optimum profit of \$165,400.81.

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