Model for infections caught by random testing in slums

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1 Introduction

We are addressing the transmission of COVID-19 in densely populated settlements in developing countries, where social distancing is not viable due to crowded living conditions and economic constraints on the residents.
We propose a solution developed through extensive research and by interviewing key stakeholders. 10-15 families in a settlement can be strengthened together as a single cohesive 'unit' of disease transmission. More details about the creation of units, the economic and sociological impacts are included in our final writeup doc.
In this document, we will look at how sampling of a small number of residents for testing will be an efficient way to allocate the limited resources. We will use simple combinatorics to demonstrate how a small sampling from residents can help detect presence of COVID-19 in a unit.

2 Effectiveness of random sampling

Let us assume that residents are equally likely to be chosen for testing. Let n be the number of residents of a unit, x be the number of infected residents of the unit. Let's chose α people for testing. Now, the probability of catching an infection by this testing is,

$$p = 1 - \frac{C(n-x,\alpha)}{C(n,\alpha)}$$

(While implementing this solution, x will be an unknown number between 0 and n. However, it is reasonable to assume same levels of infection in the unit as in the general populace. Another approach is to use the numbers predicted by a mathematical model, like the one we propose towards the end of this paper.)
Please refer to the following figures to get an intuitive understanding of this model.

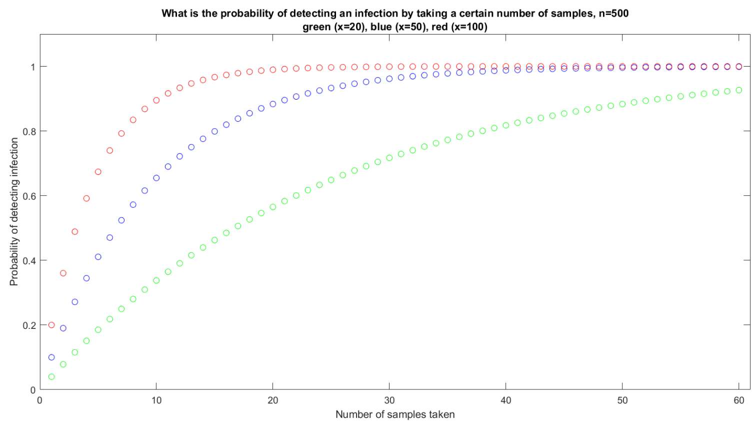

Figure 1: Probability of detecting an infection as a function of samples taken from a unit of population 500. Different colors represent different number of infected people in the population.

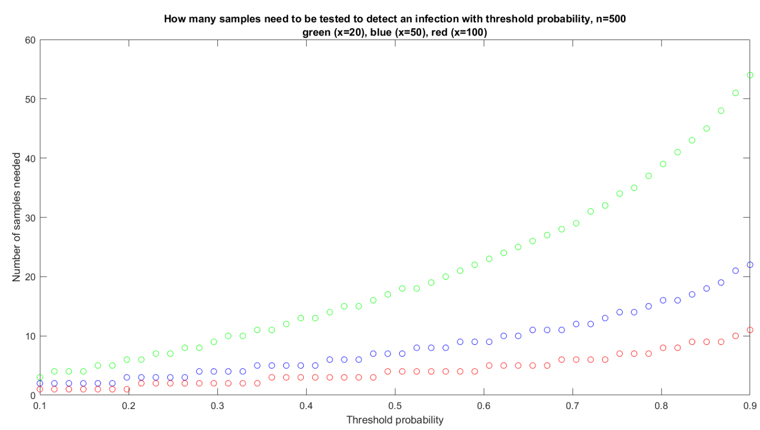

Figure 2: Least number of samples that need to be taken from a from a unit of population 500 as a function of threshold probabilities of detecting an infection. Different colors represent different number of infected people in the population.

3 Model of infection growth

As previously mentioned, while implementing this solution, x will be an unknown number between 0 and n. We outline a mathematical model which can be used to model spread of infection through a unit where a fraction of the residents still have to go out of the unit for work.
Let us consider the following variables for our model.

n	Total number of people inside one unit.
x	Total number of infected people inside one unit.
g	Growth rate of infection inside the unit. (proportional to density)
f	Fraction of people from the unit that need to go out of the unit for work
I	Infection rate in general populace (outside the unit)
t	time
α	Number of people tested.

How quickly does x change with respect to time?

$$\frac{dx}{dt} = gx + fnI, \qquad x|_{t=0} = x_0$$

Here, the term fnI corresponds to the number of people who work outside the unit and catch infection. The solution for this model is

$$x = \left(x_0 + \frac{fnI}{g}\right) e^{gt} - \frac{fnI}{g}$$

More work will be done to hone this model by the author after the end of the challenge.

Contact

Please contact the author (Prajakta Bedekar, pbedekar@uh.edu) with any questions, concerns or suggestions regarding this document.