MTHM508 Summative Assessment 1

Professor Daniel Williamson

Instructions

Please use Rmarkdown to render your solutions as a pdf. Examples of Rmarkdown documents rendered to PDF are posted in ELE. You must display all code that was critical to an answer (in particular for question 5) and should allow R to compute your samplers upon rendering (so please do not explicitly type the solution to a calculation based on random sampler, and allow R to print your answer when the PDF is built). **5 marks** are awarded for submitting a pdf rendered with Rmarkdown and no other format will be awarded these marks. Solutions not rendered in Rmarkdown may also receive lower marks if the alternative format renders assessing the sampler in question 5 difficult.

Coursework Set: 28th October 2022

Coursework Due: 11th November 2022 by noon

Problem

Lucy has had 65 "yes" responses to baby Percy's Birthday party invitation. She is planning the catering and there is a sickness bug going around that means it is likely a number of people will cancel. Suppose that she judges that the probability of any given guest dropping out due to illness is $1-\theta$ (so probability of attending is θ).

1. Describe the exchangeability judgement that Lucy is making. (**4 marks**)
2. Suppose that you make the same exchangeability judgement as Lucy yourself. Derive and carefully justify your Beta prior distribution for θ, $\pi(\theta)$. You must either

- a. Use the quantile method seen in lectures and fit your prior with the MATCH tool.
- b. Use a "pseudo data" argument to select the Beta parameters (expressing current knowledge as if updating a uniform prior with "pseudo observations" as discussed in lectures), or
- c. Use the prior predictive distribution to justify your choice. (**16 marks**)

3. Lucy attends 5 children's birthday parties in the run up to Percy's and she judges that each one has the same probability of an attendee dropping out due to sickness. The number of expected attendees at each was 10, 50, 35, 25 and 40. There were 2, 8, 12, 6 and 8 absences respectively. What is your posterior distribution for θ (which specific Beta distribution is it)? *For this part you may derive the posterior by hand or use a formula you have found (with proper citation). The marks are awarded for the correct a, b parameters not the derivation.* (**15 marks**)
4. Plot your prior and posterior on the same figure and give critical comment on the influence of the data on your prior beliefs. (**10 marks**)

5. *Using only uniform random numbers*, estimate the posterior predictive probability that more than 50 people attend Percy's birthday party using Monte Carlo, ensuring that the error on your estimate is bounded above by 0.01. You must:

- Write down the probability you require as the appropriate integral.
- State clearly how your uniform samples are converted to samples from the right distributions.
- Only use sampling algorithms shown during the course and when more than one sampler is available, you must choose the most efficient.
- Report the Monte Carlo Error of your estimate.
- Include the code from your sampler in your report.
- Whilst you may not use any inbuilt R samplers, you may use their inbuilt density/distribution functions (so `rbinom` cannot be used but `dbinom` and `pbinom` can be).

(**50 marks**)

For Q5, 5% of the marks will be awarded for writing the mathematics correctly. Choosing the correct samplers (method) will be worth 20%, computing a faithful Monte Carlo estimate of your integral (even if it itself is not the right integral, or your distribution sampling has errors) will be worth 20%, The Monte Carlo error estimate being correct and within the bound is worth 20% and your sampling code will be worth 35% (specifically correctly coding up the different algorithms for converting uniform numbers). This distribution of marks is provisional.

(**Total: 100 marks**)