CS6790: Geometry and Photometry based Computer Vision

Assignment # 2
Camera Calibration

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Contents

List of Figures

1 Introduction

Camera calibration refers to the estimation of lens parameters and image sensors of a video camera. As mentioned in [1], the image $\mathbf{x}$ of a point $\mathbf{X}$ in three dimensional world coordinate can be found by Equation 1.

$$\mathbf{x} = \mathbf{P}\mathbf{X} \tag{1}$$

where $\mathbf{P}$ is the camera projection matrix. The camera projection matrix can be decomposed as $\mathbf{P} = \mathtt{K}[\mathtt{R}|\mathbf{t}]$ where $\mathtt{R}$ and $\mathbf{t}$ are the rotation and translation respectively of the camera centre with respect to the world coordinate system and $\mathtt{K}$ is the internal camera calibration matrix. The relation between the image of the absolute conic,ω and $\mathtt{K}$ is given as $\omega^{-1} = \mathtt{K}\mathtt{K}^\top$. In this assignment, we aim to compute the internal calibration matrices using two different methods as discussed in the following subsections. All the codes for this assignment can be found here.

1.1 Perpendicularity Relations between Vanishing Points

The angle between two rays with directions $\mathbf{d_1}, \mathbf{d_2}$ and corresponding images as $\mathbf{x_1}, \mathbf{x_2}$ is given by the following Equation 2.

$$cos\ \theta = \frac{\mathbf{x_1}^\top \omega \mathbf{x_2}}{\sqrt{\mathbf{x_1}^\top \omega \mathbf{x_1}}\sqrt{\mathbf{x_2}^\top \omega \mathbf{x_2}}} \tag{2}$$

where ω is the image of absolute conic. If we assume $\mathbf{v_1}, \mathbf{v_2}$ are the vanishing points corresponding to orthogonal directions, then we have $\mathbf{v_1}^\top \omega \mathbf{v_2} = 0$.

1.2 Metric Plane imaged with Known Homography

Suppose we know the homography $\mathbf{H} = [\mathbf{h_1}\ \mathbf{h_2}\ \mathbf{h_3}]$ that maps the actual points on plane π to its imaged points. The circular points of the plane π can be mapped to the points $\mathbf{y} = \mathtt{H}(1, \pm i, 0)$. These imaged points lie on ω, the image of absolute conic. So, they satisfy $\mathbf{y}^\top \omega \mathbf{y} = 0$. From this relation, we get the following conditions mentioned in Equation 3.

$$\mathbf{h_1}^\top \omega \mathbf{h_1} = \mathbf{h_2}^\top \omega \mathbf{h_2}, \quad \mathbf{h_1}^\top \omega \mathbf{h_2} = 0 \tag{3}$$

Once ω is calculated, $\mathtt{K}$ matrix can also be computed. Though Cholesky factorization can be a way of calculating $\mathtt{K}$ from $\omega^{-1} = \mathtt{K}\mathtt{K}^\top$, $\mathtt{K}$ is computed in the method described below. This method is followed because Cholesky factorization is NOT unique. Assume $\mathtt{K}$ as mentioned in Equation 4.

$$\mathtt{K} = \begin{bmatrix} f_x & s & u_x \\ 0 & f_y & u_y \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

Then we have $\omega^{-1} = \mathtt{K}\mathtt{K}^\top$ as described in Equation 5.

$$\omega^{-1} = \mathtt{K}\mathtt{K}^\top = \begin{bmatrix} f_x^2 + s^2 + u_x^2 & s f_y + u_x u_y & u_x \\ s f_y + u_x u_y & f_y^2 + u_y^2 & u_y \\ u_x & u_y & 1 \end{bmatrix} \tag{5}$$

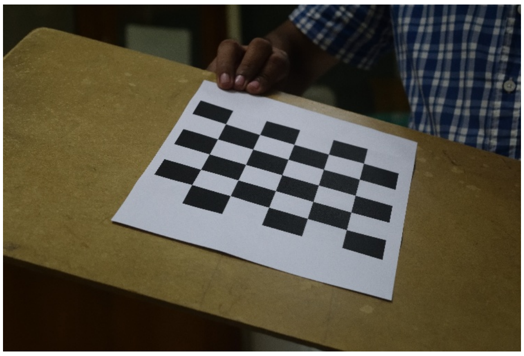
Once, ω is found, ω^{-1} is calculated and the five parameters of $\mathtt{K}$ matrix can be calculated. If we consider square pixels and zero skew, then we have to compute three params i.e. f, u_x and u_y.

2 Computation of ω and $\mathtt{K}$

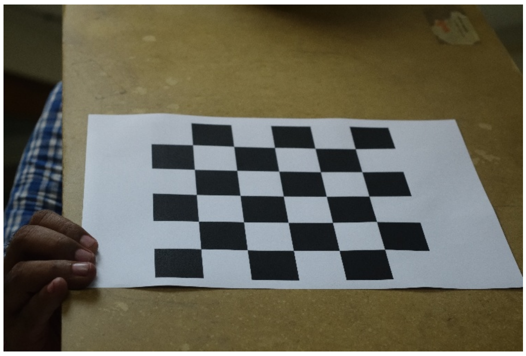
The image of absolute conic, ω and the corresponding internal camera calibration matrix are calculated for the images discussed in the subsequent subsections.

2.1 Checkerboard Pattern

We have images of a checkerboard pattern taken in five different orientations using a camera with same internal camera calibration matrix. Two orientations are shown in Figure 1



(a) Orientation A



(b) Orientation B

Figure 1: Checkerboard pattern in two different orientations

2.1.1 Method I

The image of the absolute conic and corresponding full $\mathtt{K}$ matrix is computed using orthogonal vanishing points. The values are mentioned in Equation 6 and 7.

$$\omega = \begin{bmatrix} 1.867\times10^{-8} & -1.057\times10^{-9} & -6.908\times10^{-5} \\ -1.057\times10^{-9} & 3.650\times10^{-8} & -1.388\times10^{-4} \\ -6.908\times10^{-5} & -1.388\times10^{-4} & 1 \end{bmatrix} \tag{6}$$

$$\mathtt{K} = \begin{bmatrix} 3152.013 & 127.730 & 3921.068 \\ 0 & 2256.201 & 3915.882 \\ 0 & 0 & 1 \end{bmatrix} \tag{7}$$

If we consider zero skew condition and square pixels, then we require only three out of the five orientations of the pattern. The *first, fourth and fifth* orientation is chosen for this purpose. The corresponding ω and $\mathtt{K}$ matrices are given in Equation 8 and 9.

$$\omega = \begin{bmatrix} 3.131\times10^{-9} & 0 & -4.465\times10^{-5} \\ 0 & 3.131\times10^{-9} & -1.591\times10^{-5} \\ -4.465\times10^{-5} & -1.591\times10^{-5} & 1 \end{bmatrix} \tag{8}$$

$$\mathtt{K} = \begin{bmatrix} 9496.286 & 0 & 14258.225 \\ 0 & 9496.286 & 5081.464 \\ 0 & 0 & 1 \end{bmatrix} \tag{9}$$

2.1.2 Method II

In the case of a full $\mathtt{K}$ matrix, we need three homographies to calculate ω according to Equation 2. The calculated ω and $\mathtt{K}$ matrix is given in Equation 10 and 11.

$$\omega = \begin{bmatrix} 5.289\times10^{-9} & 8.209\times10^{-10} & -5.061\times10^{-5} \\ 8.209\times10^{-10} & 5.869\times10^{-9} & -2.112\times10^{-5} \\ -5.061\times10^{-5} & -2.112\times10^{-5} & 1 \end{bmatrix} \tag{10}$$

$$\mathtt{K} = \begin{bmatrix} 9575.242 & -1426.333 & 9210.238 \\ 0 & 9190.464 & 2310.571 \\ 0 & 0 & 1 \end{bmatrix} \tag{11}$$

In case of zero skew condition, we require only two homographies as we need to determine only three parameters. The ω and $\mathtt{K}$ matrices are mentioned in Equation 12 and 13.

$$\omega = \begin{bmatrix} 5.207\times10^{-9} & 0 & -5.318\times10^{-5} \\ 0 & 5.207\times10^{-9} & -7.175\times10^{-6} \\ -5.318\times10^{-5} & -7.175\times10^{-6} & 1 \end{bmatrix} \tag{12}$$

$$
\mathtt{K} = \begin{bmatrix} 9263.346 & 0 & 10214.593 \\ 0 & 9263.346 & 1378.055 \\ 0 & 0 & 1 \end{bmatrix} \tag{13}
$$

2.2 Three Squares

In this question, we use a simple camera calibration device to calculate the internal camera calibration matrix, as shown in Figure 2. At first, the image of absolute conic and full $\mathtt{K}$ matrix is calculated and the values are mentioned in Equation 14 and 15 using three homographies. It is to be noted that five orthogonal relations between vanishing points can not be determined from the image.

Figure 2: A simple camera calibration device

$$
\omega = \begin{bmatrix} 5.800 \times 10^{-7} & 6.593 \times 10^{-9} & -2.578 \times 10^{-4} \\ 6.593 \times 10^{-9} & 6.340 \times 10^{-7} & -2.447 \times 10^{-4} \\ -2.578 \times 10^{-4} & -2.447 \times 10^{-4} & 1 \end{bmatrix} \tag{14}
$$

$$
\mathtt{K} = \begin{bmatrix} 1169.399 & -12.714 & 440.191 \\ 0 & 1118.516 & 381.380 \\ 0 & 0 & 1 \end{bmatrix} \tag{15}
$$

We now move to the zero skew regime, in which the required matrices are computed using orthogonal relations between three vanishing points. The calculated values are mentioned in Equation 16 and 17.

$$
\omega = \begin{bmatrix} 5.862 \times 10^{-7} & 0 & -2.526 \times 10^{-4} \\ 0 & 5.862 \times 10^{-7} & -2.955 \times 10^{-4} \\ -2.526 \times 10^{-4} & -2.955 \times 10^{-4} & 1 \end{bmatrix} \tag{16}
$$

$$
\mathtt{K} = \begin{bmatrix} 1125.116 & 0 & 431.034 \\ 0 & 1125.116 & 504.160 \\ 0 & 0 & 1 \end{bmatrix} \tag{17}
$$

Any two squares are randomly chosen out of the three in Figure 2 and the using the two homographies, ω and $\mathtt{K}$ matrices are calculated for zero skew condition and presented in Equation 18 and 19.

$$
\omega = \begin{bmatrix} 6.104 \times 10^{-7} & 0 & -2.476 \times 10^{-4} \\ 0 & 6.104 \times 10^{-7} & -2.328 \times 10^{-4} \\ -2.476 \times 10^{-4} & -2.328 \times 10^{-4} & 1 \end{bmatrix} \tag{18}
$$

$$
\mathtt{K} = \begin{bmatrix} 1152.413 & 0 & 405.675 \\ 0 & 1152.413 & 381.487 \\ 0 & 0 & 1 \end{bmatrix} \tag{19}
$$

2.3 Corner of a Room and Redmi Box

We now compute the required matrices from the images as shown in Figure 3a and 3b. It can be seen that neither five orthogonal relations between vanishing points nor homographies corresponding to imaged planes can be computed for these images. So, the required matrices can be computed only when the zero skew condition is imposed. The three vanishing points are chosen in mutually orthogonal directions which gives us adequate conditions to calculate the required matrices.

(a) Corner of a Room

(b) Redmi Box

Figure 3: Image of a corner of a room and a Redmi box

2.3.1 Corner of a Room

The required matrices for the corner of a room are presented in Equation 20 and 21.

$$
\omega = \begin{bmatrix} 7.599 \times 10^{-9} & 0 & -1.858 \times 10^{-5} \\ 0 & 7.599 \times 10^{-9} & -4.690 \times 10^{-5} \\ -1.858 \times 10^{-5} & -4.690 \times 10^{-5} & 1 \end{bmatrix} \tag{20}
$$

$$
\mathtt{K} = \begin{bmatrix} 9354.474 & 0 & 2445.751 \\ 0 & 9354.474 & 6172.104 \\ 0 & 0 & 1 \end{bmatrix} \tag{21}
$$

2.3.2 Redmi Box

The image of the absolute conic and internal camera calibration matrices for the Redmi Box are presented in Equation 22 and 23.

$$\omega = \begin{bmatrix} 5.803 \times 10^{-9} & 0 & -1.939 \times 10^{-5} \\ 0 & 5.803 \times 10^{-9} & -1.098 \times 10^{-5} \\ -1.939 \times 10^{-5} & -1.098 \times 10^{-5} & 1 \end{bmatrix} \tag{22}$$

$$\mathtt{K} = \begin{bmatrix} 12552.451 & 0 & 3341.706 \\ 0 & 12552.451 & 1893.510 \\ 0 & 0 & 1 \end{bmatrix} \tag{23}$$

3 Inferences and Conclusion

In this assignment, we calculated image of the absolute conic, ω and corresponding internal camera calibration matrix, $\mathtt{K}$ for different images. We can see in Section 2.1 that if we enforce the condition of zero skew in a case when the skew is not actually zero, then we get results that are starkly different from each other.

Ideally, the ω and $\mathtt{K}$ matrices calculated using both the methods (vanishing points and homography method) should be exactly the same. But, as the image sizes are large i.e. 4000×6000, there is some discrepancy in selecting the points for homography calculation which finally results in some error in $\mathtt{K}$ and ω matrix calculation.

References

[1] Richard Hartley and Andrew Zisserman. *Multiple view geometry in computer vision*. Cambridge university press, 2003.