CS6790: Geometry and Photometry based Computer Vision

Assignment # 1
Image Rectification

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Contents

1 Introduction

Shape is distorted under perspective imaging [1]. Parallel lines in real world may or may not appear to be parallel in an image. In this assignment, we try to recover the original image with correct geometric shape of the objects in the picture by computing an inverse transformation using different techniques. For the given assignment, all the six images are resized into 600×400 size and converted into `jpg` format using OpenCV[1] for faster computation of the transformation matrices. The resized images are used for all the questions. Throughout the report, we focus on two images as shown in Figure **??** and their rectification using different methods. All the images and results are provided in the end of the report. The codes are can be found here.

2 Question 1

Let the inhomogeneous coordinates of a pair of matching points be $\mathbf{x} = (x, y)^\top$ and $\mathbf{x}' = (x', y')^\top$ and the homography matrix be denoted by $\mathbf{H}$. Let $\mathbf{H}$ be as follows:

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \tag{1}$$

If we consider $\mathbf{x}^{*\prime} = \mathbf{H}\mathbf{x}^*$, where $\mathbf{x}^* = (x, y, 1)^\top$ and $\mathbf{x}^{*\prime}$ are in homogeneous coordinates, we get:

$$x' = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + h_{33}} \tag{2}$$

$$y' = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + h_{33}} \tag{3}$$

Out of the nine unknown parameters in $\mathbf{H}$ matrix, we need to know eight as the in homogeneous coordinates are scale-independent. Hence, we need to find mapping between *four* points to calculate the $\mathbf{H}$ matrix. Then, the $\mathbf{H^{-1}}$ matrix is calculated to get the image in the original plane. The pixels that have coordinates outside the specified size of the image are dropped. The rectified image has "holes" in them which is corrected using a custom interpolation function. If the intensity value of a pixel is zero, then it is assigned the maximum intensity value of the neighbours. This interpolation function is preferred over existing ones as it produces a sharper image as compared to the image generated using off-the-shelf interpolation functions. The rectified images are presented in Figure 1.

(a) Image 1

(b) Image 2

Figure 1: Rectified images using four points

3 Question 2

In the second approach, we intend to rectify the image up to similarity by first doing an affine rectification and then computing the dual conic to the using two perpendicular lines.

[1] https://opencv.org/

3.1 Affine Rectification

For doing affine rectification, we compute the image of the line at infinity $\mathbf{l}_\infty$. We choose two pairs of lines $\mathbf{l}$ and $\mathbf{l}'$ in the image that would be parallel in the real world and finding their corresponding point of intersection $\mathbf{x} = \mathbf{l} \times \mathbf{l}'$. Once the two points of intersection $\mathbf{x}$ and $\mathbf{x}'$ are known, then the line passing through them i.e., $\mathbf{l}^* = \mathbf{x} \times \mathbf{x}'$ is calculated. A suitable homography matrix $\mathbf{H}$ under which $\mathbf{l}^* = (l_1, l_2, l_3)^\top$ will map to $\mathbf{l}_\infty$ is given in Equation 4. The affinely rectified images are presented in Figure 2.

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_1 & l_2 & l_3 \end{bmatrix} \tag{4}$$

(a) Image 1

(b) Image 2

Figure 2: Affinely rectified images from projectively distorted images

3.2 Similarity Rectification

Once the affinely rectified image is obtained, we identify lines $\mathbf{l}', \mathbf{m}'$ which are images of $\mathbf{l}, \mathbf{m}$ that are perpendicular in the real world. So, we have $\mathbf{l}'^\top \mathbf{C}^*_\infty{}' \mathbf{m}' = 0$ because $\mathbf{l}^\top \mathbf{C}^*_\infty \mathbf{m} = 0$ (condition of orthogonality). In this case, $\mathbf{C}^*_\infty{}'$ is given in Equation 5.

$$\mathbf{C}^*_\infty{}' = \begin{bmatrix} \mathtt{KK}^\top & \mathbf{0} \\ \mathbf{0}^\top & 0 \end{bmatrix} \tag{5}$$

where K is a 2×2 matrix. So, the number of unknowns is 3 which can be solved by considering two pairs of orthogonal lines. The images rectified up to a similarity is given in Figure 3. We can see that in this case the images obtained are less sharp and distinct as compared to a direct transform using four points which is expected as the second approach is a two step process as opposed to the first one. In the second approach, there is an intermediate step of interpolation which is the primary reason behind the difference in the obtained results.

4 Question 3

Let the transform of dual conic $\mathbf{C}^*_\infty$ be given by Equation 6.

$$\mathbf{C}^*_\infty{}' = \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix} \tag{6}$$

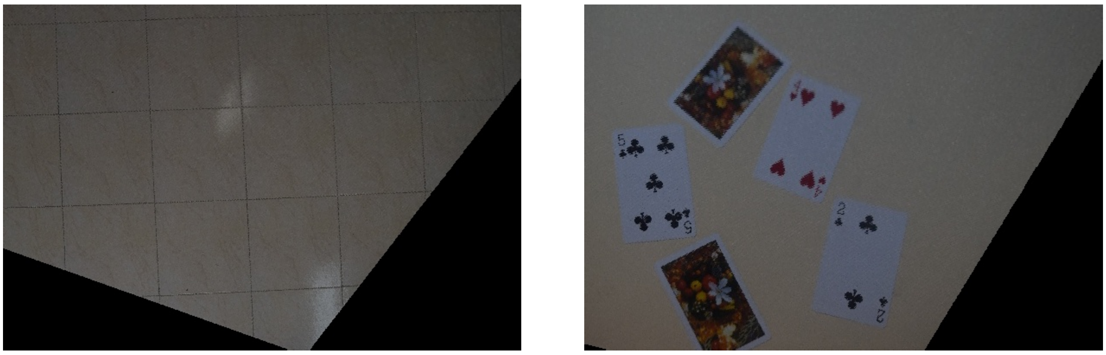
In this question, we rectify the images up to similarity using *five* pairs of perpendicular lines. Let $\mathbf{l}$ and $\mathbf{m}$ be images of orthogonal lines in real world, then using $\mathbf{l}^\top \mathbf{C}^*_\infty \mathbf{m} = 0$, we get a single constraint. From five such pairs, we get a 5×6 dimensional matrix A. We need to calculate $x = (a, b, c, d, e, f)^\top$, which belongs to the null space of A. We have, $A = USV^\top$ by singular value decomposition (SVD). As proved in the class, $x = (a, b, c, d, e, f)^\top$ is the last column of V. Once $\mathbf{C}^*_\infty{}'$ is found, by SVD,

(a) Image 1 (b) Image 2

Figure 3: Images rectified upto a similarity in a two step process

we have $\mathbf{C}_\infty^*{}' = UDU^\top$ as the matrix is symmetric. As $\mathbf{C}_\infty^*{}' = H\mathbf{C}_\infty^* H^\top$, $H = UD^{1/2}$ and the right corner of D matrix is set to 1 to make H non-singular. The images rectified using this procedure are shown in Figure 4.

When compared to the images seen in Figure 1 and Figure 3, we find that the results are better than Question 2, but slightly worse as compared to the results in Question 1. This could be attributed to the reason that while both are single step process, the method used in Question 1 is a more direct method using point-to-point correspondences, while the second method uses orthogonal lines for calculating the homography matrix. However, in certain cases like the fifth and sixth image shown in Figure 8, we can see that this method is very sensitive to the choice of pairs of perpendicular lines and can break down in the presence of noise.



(a) Image 1 (b) Image 2

Figure 4: Images rectified upto a similarity in a single step process using five pairs of perpendicular lines

5 Question 4

In this question, it is asked to do an affine rectification of the image and then a similarity rectification using a transformed circle and its points of intersection with $\mathbf{l}_\infty$. The transformed circle is an ellipse in an affinely rectified image. Like Equation 6, the $\mathbf{C}$ has *five* unknowns (upto scale). So, we need five points on the ellipse to find its equation. The four corners of the transformed square can be certainly used as four points on the ellipse. However, the fifth point can not be determined in the affinely rectified image. So, the equation of the ellipse can not be uniquely determined.

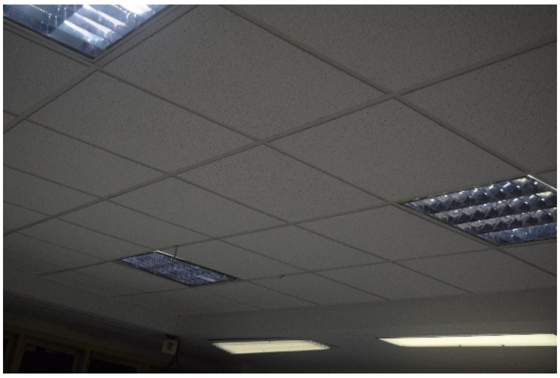
6 Conclusion

In this assignment, we tried different methods for doing a similarity rectification on a projectively distorted image. From our observations, we see that the first method gives the best and stable results in all the images. This is expected as the first method is essentially a Direct Linear Transformation (DLT), that is widely used in practice in computer vision related tasks. The second method, though is a two-step process is robust to noise unlike the third process that breaks down when there is slight errors in selecting the perpendicular lines.

References

[1] Richard Hartley and Andrew Zisserman. *Multiple view geometry in computer vision.* Cambridge university press, 2003.

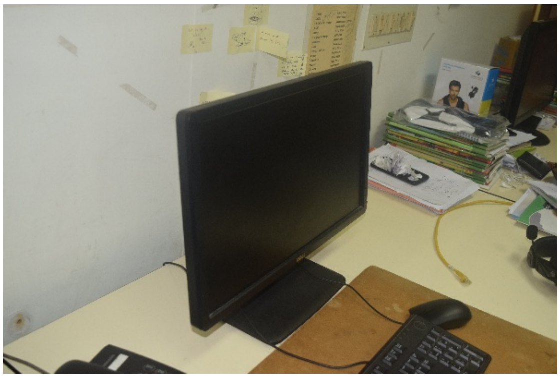
(a) First image



(b) Second image

(c) Third image

(d) Fourth image

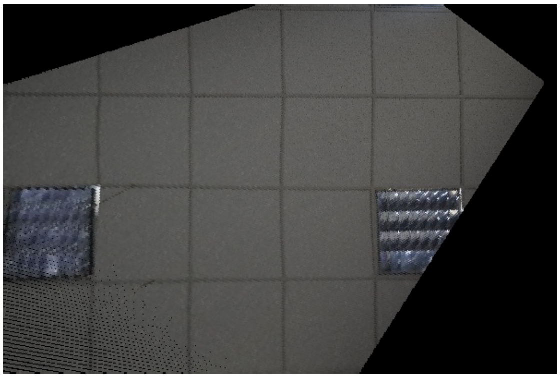


(e) Fifth image

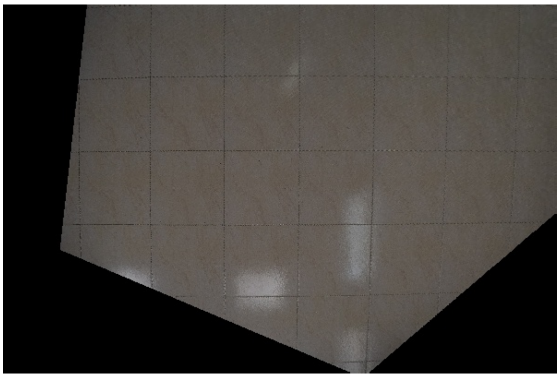
(f) Sixth image

Figure 5: Original images with projective distortion

(a) First image



(b) Second image



(c) Third image

(d) Fourth image

(e) Fifth image

(f) Sixth image

Figure 6: Rectified images using four points

(a) First image

(b) Second image

(c) Third image

(d) Fourth image

(e) Fifth image

(f) Sixth image

Figure 7: Images rectified upto a similarity in a two step process

(a) First image (b) Second image

(c) Third image (d) Fourth image

(e) Fifth image (f) Sixth image

Figure 8: Images rectified upto a similarity in a single step process using five pairs of perpendicular lines