EE2703: Applied Programming Lab

Assignment # 8

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Contents

1 Introduction

In mathematics, the discrete Fourier transform (DFT) converts a finite sequence of equally-spaced samples of a function into a same-length sequence of equally-spaced samples of the discrete-time Fourier transform (DTFT), which is a complex-valued function of frequency. The interval at which the DTFT is sampled is the reciprocal of the duration of the input sequence. An inverse DFT is a Fourier series, using the DTFT samples as coefficients of complex sinusoids at the corresponding DTFT frequencies. It has the same sample-values as the original input sequence. The DFT is therefore said to be a frequency domain representation of the original input sequence. If the original sequence spans all the non-zero values of a function, its DTFT is continuous (and periodic), and the DFT provides discrete samples of one cycle. If the original sequence is one cycle of a periodic function, the DFT provides all the non-zero values of one DTFT cycle. [1]

$$Analysis\ Equation\ : X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn} \tag{1}$$

$$Synthesis\ Equation\ : x[n] = \frac{1}{N} \sum_{n=0}^{N-1} X[k] W_N^{-kn} \tag{2}$$

where $W_N = e^{j2\pi/N}$

2 Problem 1

Import the following libraries.

```
import numpy as np
import matplotlib.pyplot as plt
```

2.1 $f(t) = sin(5t)$

2.1.1 Magnitude Response

```
x = np.linspace(0,2*np.pi,129);x = x[:-1]
y = np.sin(5*x)
Y = np.fft.fft(y)
Y = np.fft.fftshift(Y)/128.0
w = np.linspace(-64,64,129);w = w[:-1]
print(np.angle(Y,deg = False))
plt.figure(figsize=(16,8))
plt.grid()
plt.plot(w,abs(Y))
plt.xlim([-10,10])
plt.title(r'Magnitude of FFT of $sin(5t)$')
plt.xlabel(r'$k\rightarrow$',size = 16)
plt.ylabel(r'$|Y|\rightarrow$',size = 16)
plt.show()
```

2.1.2 Phase Response

```
plt.figure(figsize=(16,8))
plt.grid()
plt.plot(w,np.angle(Y,deg=False),'+',markersize = 10)
ii = np.where(abs(Y)>=0.001)
plt.plot(w[ii],np.angle(Y[ii]),'ro',markersize = 10)
plt.xlim([-10,10])
plt.title(r'Phase of DFT of $sin(5t)$')
```

[1] https://en.wikipedia.org/wiki/Discrete_Fourier_transform

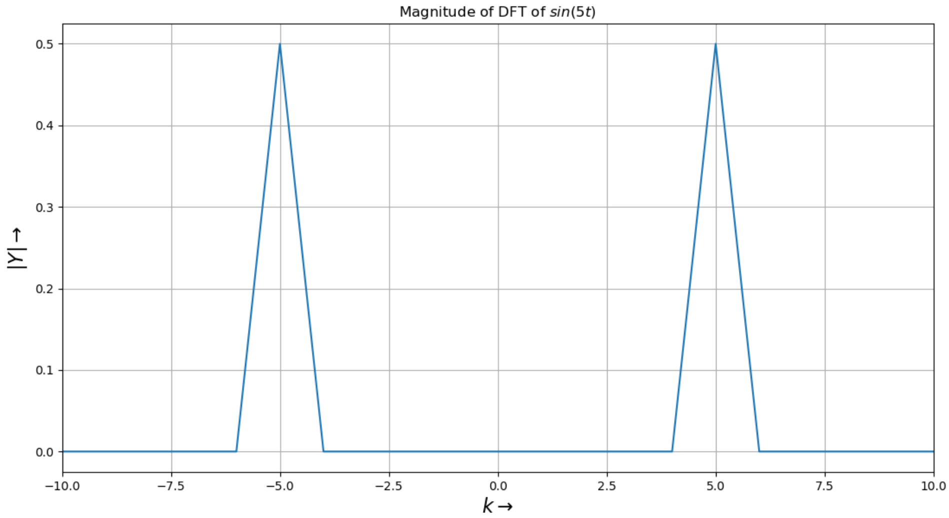

Figure 1: Magnitude of DFT of $sin(5t)$

```
plt.xlabel(r'$k\rightarrow$',size = 16)
plt.ylabel(r'$\angle Y\rightarrow$',size = 16)
plt.show()
```

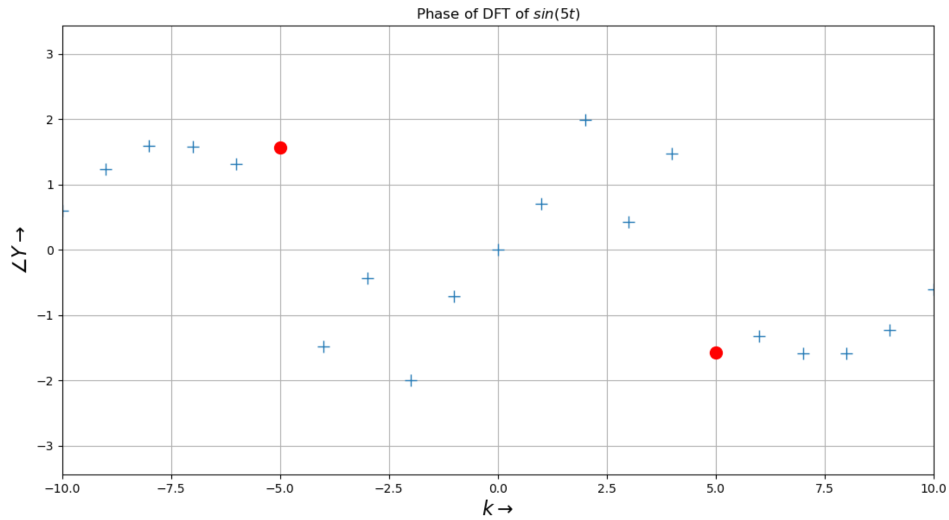

Figure 2: Phase of DFT of $sin(5t)$

2.2 $f(t) = (1 + 0.1cos(t))cos(10t)$

2.2.1 Magnitude Response

```
t = np.linspace(-4*np.pi,4*np.pi,513);t = t[:-1]
y = np.cos(10*t)*(1+0.1*np.cos(t))
Y = np.fft.fft(y)
Y = np.fft.fftshift(Y)/512.0
w = np.linspace(-64,64,513);w = w[:-1]
```

```
print(np.angle(Y,deg = False))
plt.figure(figsize=(16,8))
plt.grid()
plt.plot(w,abs(Y))
plt.xlim([-15,15])
plt.title(r'Magnitude of DFT of $(1+0.1cos(t))cos(10t)$')
plt.xlabel(r'$k\rightarrow$',size = 16)
plt.ylabel(r'$|Y|\rightarrow$',size = 16)
plt.show()
```

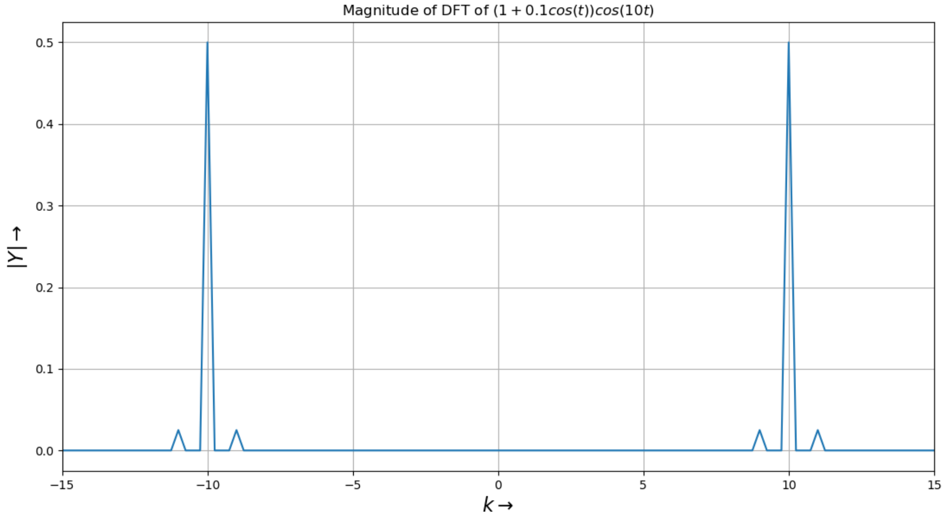

Figure 3: Magnitude of DFT of $(1 + 0.1cos(t))cos(10t)$

2.2.2 Phase Response

```
plt.figure(figsize=(16,8))
plt.grid()
plt.plot(w,np.angle(Y,deg=False),'+',markersize = 5)
ii = np.where(abs(Y)>=0.001)
plt.plot(w[ii],np.angle(Y[ii]),'ro',markersize = 5)
plt.xlim([-15,15])
plt.title(r'Phase of DFT of $(1+0.1cos(t))cos(10t)$')
plt.xlabel(r'$k\rightarrow$',size = 16)
plt.ylabel(r'$\angle Y\rightarrow$',size = 16)
plt.show()
```

3 Problem 2

3.1 $f(t) = sin^3(t)$

3.1.1 Magnitude Response

```
t = np.linspace(-4*np.pi,4*np.pi,513);t = t[:-1]
y = (np.sin(t))**3
Y = np.fft.fft(y)
Y = np.fft.fftshift(Y)/512.0
w = np.linspace(-64,64,513);w = w[:-1]
```

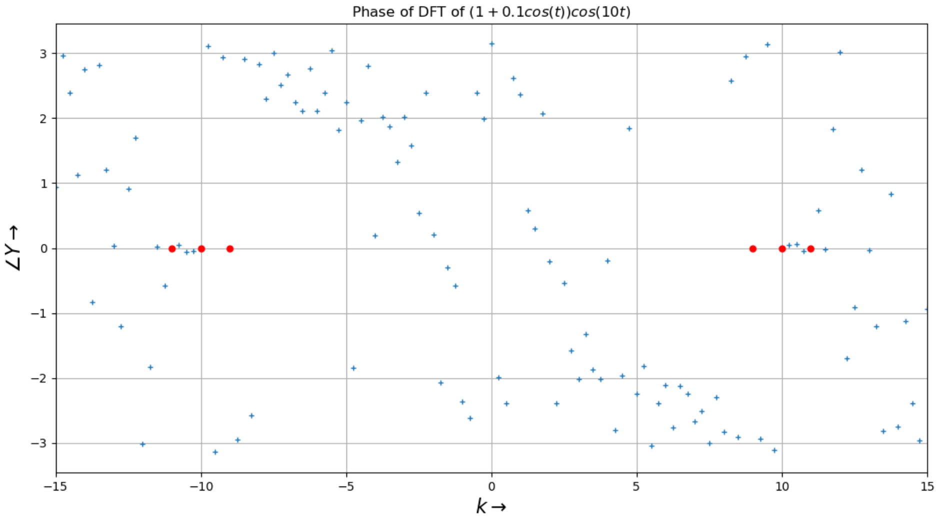

Figure 4: Phase of DFT of $(1 + 0.1cos(t))cos(10t)$

```
plt.figure(figsize=(16,8))
plt.grid()
plt.plot(w,abs(Y))
plt.xlim([-15,15])
plt.title(r'Magnitude of DFT of $sin^3(t)$')
plt.xlabel(r'$k\rightarrow$',size = 16)
plt.ylabel(r'$|Y|\rightarrow$',size = 16)
plt.show()
```

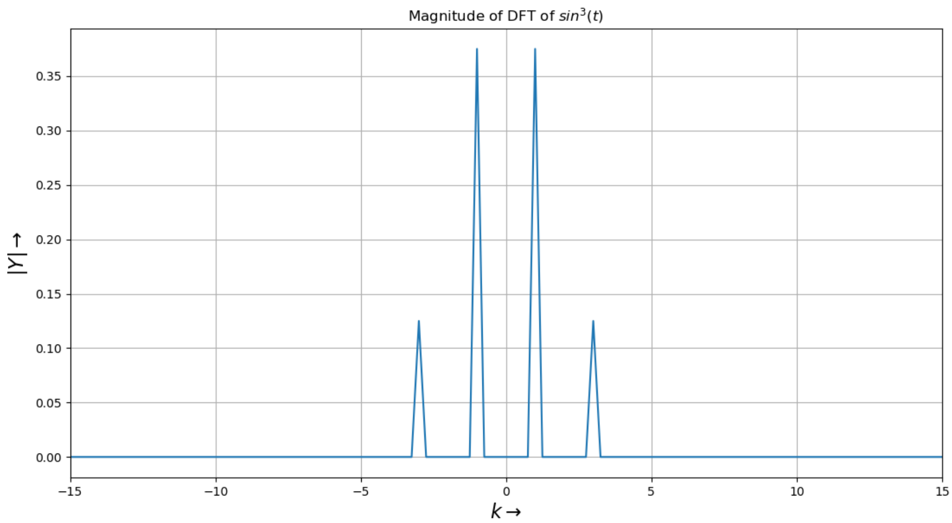

Figure 5: Magnitude of DFT of $sin^3(t)$

3.1.2 Phase Response

```
plt.figure(figsize=(16,8))
plt.grid()
```

```
plt.plot(w,np.angle(Y,deg=False),'+',markersize = 5)
ii = np.where(abs(Y)>=0.001)
plt.plot(w[ii],np.angle(Y[ii]),'ro',markersize = 5)
plt.xlim([-15,15])
plt.title(r'Phase of DFT of $sin^3(t)$')
plt.xlabel(r'$k\rightarrow$',size = 16)
plt.ylabel(r'$\angle Y\rightarrow$',size = 16)
plt.show()
```

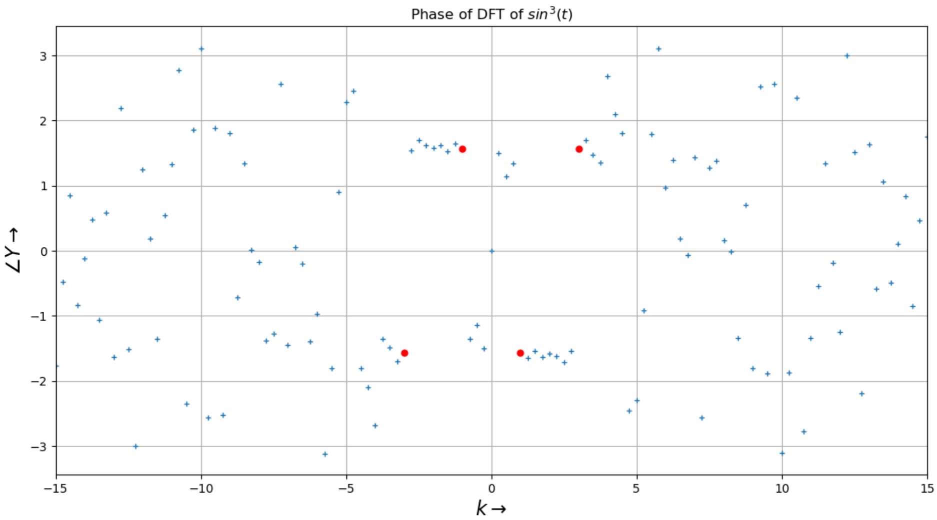

Figure 6: Phase of DFT of $sin^3(t)$

3.2 $f(t) = cos^3(t)$

3.2.1 Magnitude Response

```
t = np.linspace(-4*np.pi,4*np.pi,513);t = t[:-1]
y = (np.cos(t))**3
Y = np.fft.fft(y)
Y = np.fft.fftshift(Y)/512.0
w = np.linspace(-64,64,513);w = w[:-1]
plt.figure(figsize=(16,8))
plt.grid()
plt.plot(w,abs(Y))
plt.xlim([-15,15])
plt.title(r'Magnitude of DFT of $cos^3(t)$')
plt.xlabel(r'$k\rightarrow$',size = 16)
plt.ylabel(r'$|Y|\rightarrow$',size = 16)
plt.show()
```

3.2.2 Phase Response

```
plt.figure(figsize=(16,8))
plt.grid()
plt.plot(w,np.angle(Y,deg=False),'+',markersize = 5)
ii = np.where(abs(Y)>=0.001)
plt.plot(w[ii],np.angle(Y[ii]),'ro',markersize = 5)
```

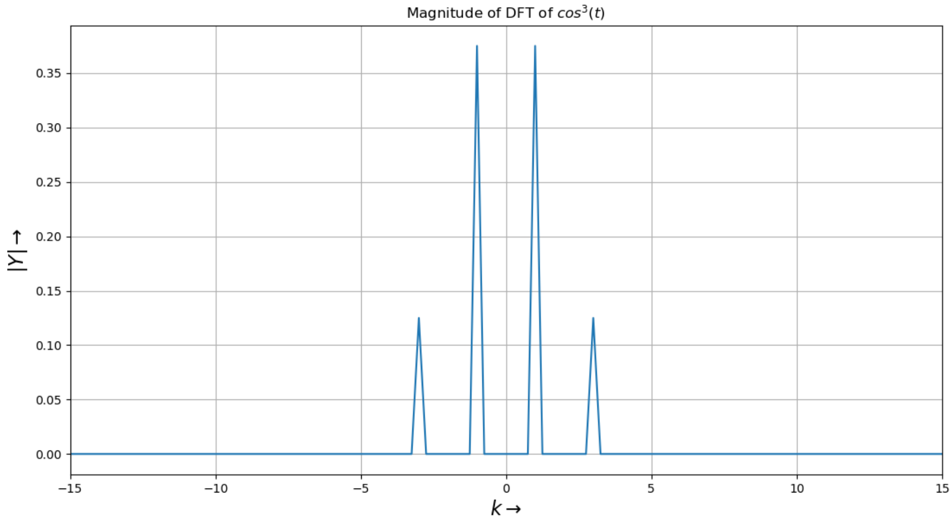

Figure 7: Magnitude of DFT of $cos^3(t)$

```
plt.xlim([-15,15])
plt.title(r'Phase of DFT of $cos^3(t)$')
plt.xlabel(r'$k\rightarrow$',size = 16)
plt.ylabel(r'$\angle Y\rightarrow$',size = 16)
plt.show()
```

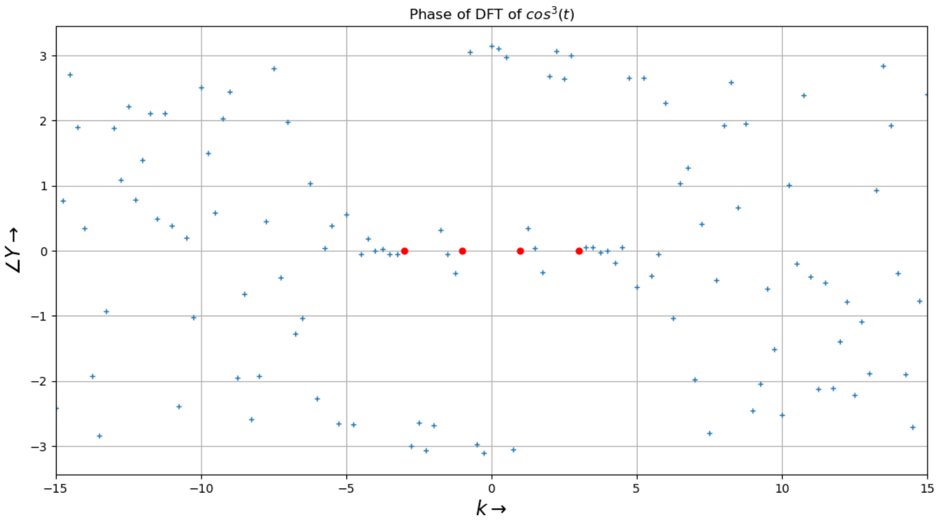

Figure 8: Phase of DFT of $cos^3(t)$

3.3 Analysis

$$cos(3t) = 4cos^3(t) - 3cos(t) \tag{3}$$

$$\implies cos^3(t) = \frac{cos(3t) + 3cos(t)}{4} \tag{4}$$

$$\implies cos^3(t) = 0.125(e^{3jt} + e^{-3jt}) + 0.375(e^{jt} + e^{-jt}) \tag{5}$$

So,we get the spikes at the locations where we expect i.e ± 1 and ± 3 of desired magnitude in the DFT.

4 Problem 3

4.1 Magnitude Response

```
t = np.linspace(-4*np.pi,4*np.pi,513);t = t[:-1]
y = np.cos(20*t+5*np.cos(t))
Y = np.fft.fft(y)
Y = np.fft.fftshift(Y)/512.0
w = np.linspace(-64,64,513);w = w[:-1]
plt.figure(figsize=(16,8))
plt.grid()
plt.plot(w,abs(Y))
plt.xlim([-40,40])
plt.title(r'Magnitude of DFT of $cos(20t+5cos(t))$')
plt.xlabel(r'$k\rightarrow$',size = 16)
plt.ylabel(r'$|Y|\rightarrow$',size = 16)
plt.show()
```

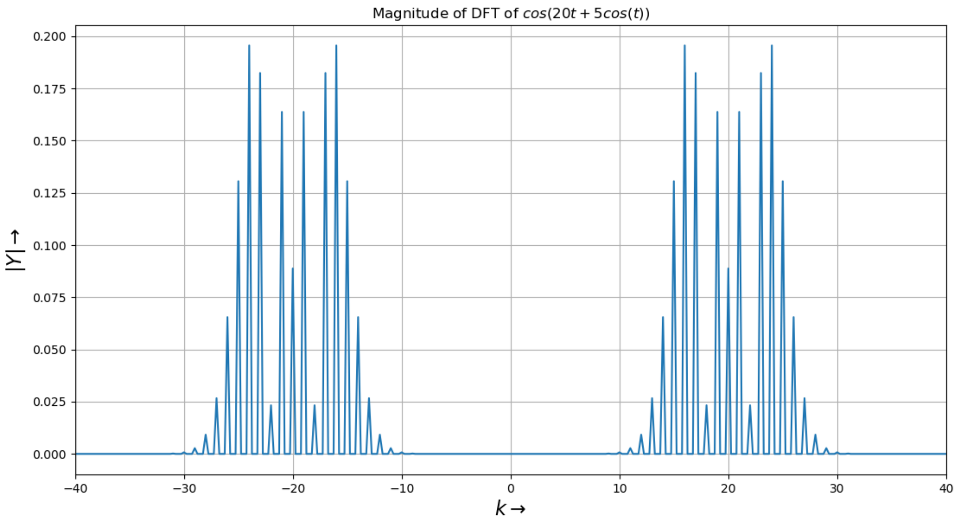

Figure 9: Magnitude of DFT of $cos(20t + 5cos(t))$

4.2 Phase Response

```
plt.figure(figsize=(16,8))
plt.grid()
plt.plot(w,np.angle(Y,deg=False),'+',markersize = 5)
ii = np.where(abs(Y)>=0.001)
plt.plot(w[ii],np.angle(Y[ii]),'ro',markersize = 5)
plt.xlim([-40,40])
plt.title(r'Phase of DFT of $cos(20t+5cos(t))$')
plt.xlabel(r'$k\rightarrow$',size = 16)
plt.ylabel(r'$\angle Y\rightarrow$',size = 16)
plt.show()
```

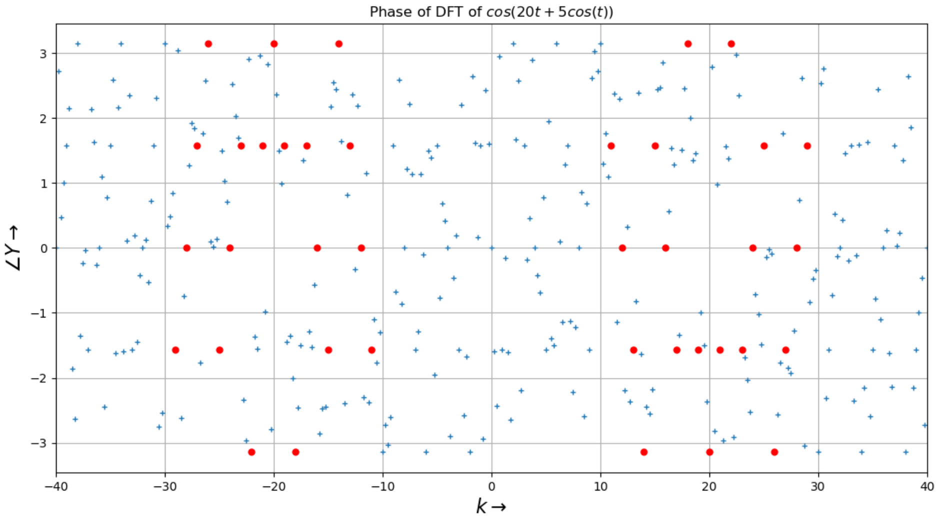

Figure 10: Phase of DFT of $cos(20t + 5cos(t))$

4.3 Inline Question

The spectrum of the function is as expected. The signal has main carrier frequency = 20 rad/s which is the center frequency of the spectrum. The additional spikes centred around +20rad/s and -20rad/s that we see in the spectrum is basically due to the additional cos(5t) part.

5 Problem 4

5.1 Magnitude Response

```
N = 513
t = np.linspace(-32,32,N);t = t[:-1]
y = np.exp(-(t**2)/2.0)
Y = np.fft.fft(y)
Y = np.fft.fftshift(Y)/float(N-1)
w = np.linspace(-64,64,N);w = w[:-1]
y1 =(np.sqrt(2*np.pi))*np.exp(-2*(np.pi**2)*(w**2))/float(N-1)
plt.figure(figsize=(16,8))
plt.grid()
plt.plot(w,abs(Y))
# plt.plot(w,abs(y1),'-.')
plt.xlim([-20,20])
plt.title(r'Magnitude of DFT of $e^{-t^2/2}$')
plt.xlabel(r'$k\rightarrow$',size = 16)
plt.ylabel(r'$|Y|\rightarrow$',size = 16)
plt.show()
```

5.2 Phase Response

```
plt.figure(figsize=(16,8))
plt.grid()
plt.plot(w,np.angle(Y,deg=False),'+',markersize = 5)
ii = np.where(abs(Y)>=0.001)
plt.plot(w[ii],np.angle(Y[ii]),'ro',markersize = 5)
plt.xlim([-20,20])
```

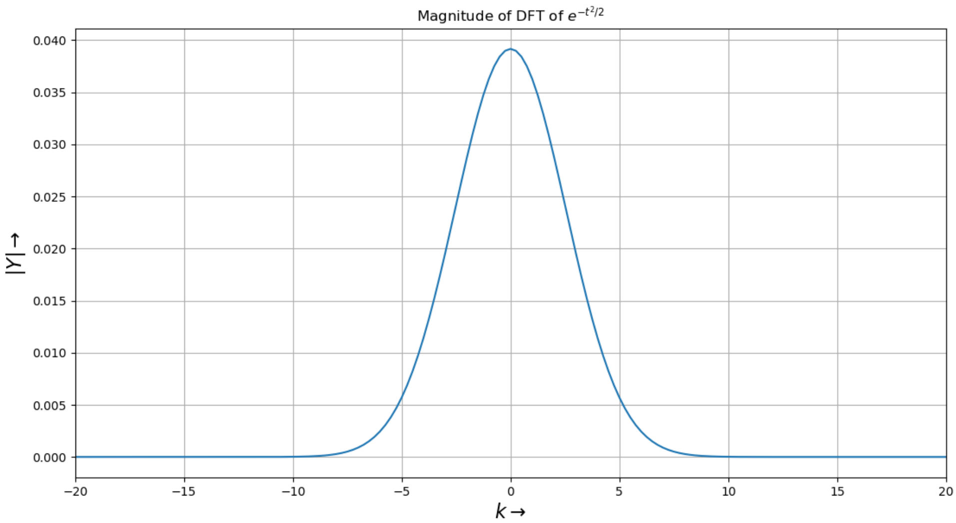

Figure 11: Magnitude of DFT of $e^{-t^2/2}$

```
plt.title(r'Phase of DFT of $e^{-t^2/2}$')
plt.xlabel(r'$k\rightarrow$',size = 16)
plt.ylabel(r'$\angle Y\rightarrow$',size = 16)
plt.show()
```

5.3 Analysis

The DFT of a Gaussian function is an scaled Gaussian. The peak of the output we get depends on the time span over which we are sampling. In this case we took 512 samples over a time span of (-32,32) and got a peak of approximately 0.038.

6 Conclusion

The DFT is the most important discrete transform, used to perform Fourier analysis in many practical applications.In digital signal processing, the function is any quantity or signal that varies over time, such as the pressure of a sound wave, a radio signal, or daily temperature readings, sampled over a finite time interval (often defined by a window function). In image processing, the samples can be the values of pixels along a row or column of a raster image. The DFT is also used to efficiently solve partial differential equations, and to perform other operations such as convolutions or multiplying large integers. [2]In this assignment, we evaluated the DFT of various signals.

[2]https://en.wikipedia.org/wiki/Discrete_Fourier_transform

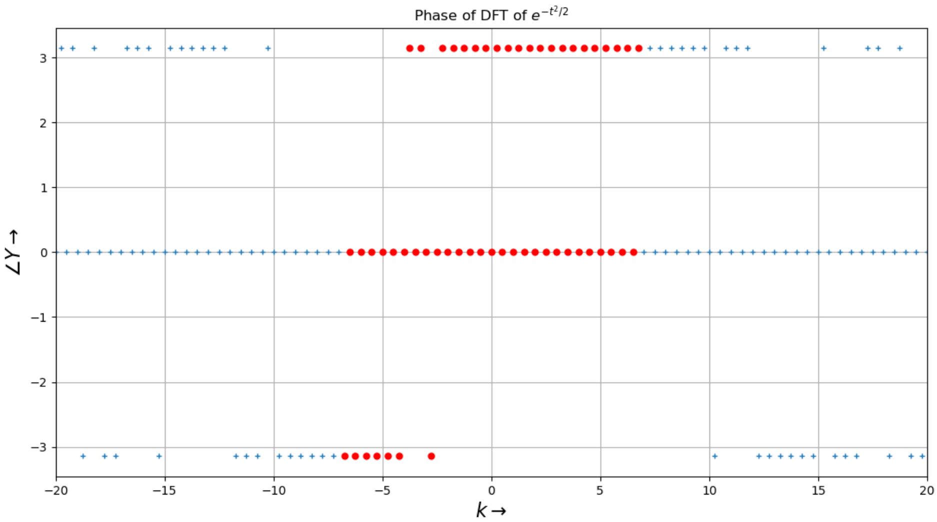

Figure 12: Phase of DFT of $e^{-t^2/2}$