Northern University of Bussiness and Technology Khulna

NUBTK $(\hat{}w\hat{})$

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Contest (1)

Template.cpp — f321ac, 13 lines

```cpp
#include <bits/stdc++.h>
using namespace std;

#ifdef LOCAL
#include "debug.h"
#else
#define dbg(...)
#endif

int32_t main() {
  cin.tie(0)->sync_with_stdio(0);
  cin.exceptions(cin.failbit);
}
```

DebugShort.h — 16 lines

```cpp
template <typename A, typename B>
ostream &operator<<(ostream &os, const pair<A, B> &p) {
  return os << '(' << p.first << ", " << p.second << ')';
} // pair
template <typename C, typename T = typename enable_if<!is_same<
   ↪C, string>::value, typename C::value_type>::type>
ostream &operator<<(ostream &os, const C &v) {
  os << '{';
  string sep;
  for (const T &x : v) os << sep << x, sep = ", ";
  return os << '}';
} // ds
void dbg_out() { cerr << "|\n"; }
template<typename H, typename... T> void dbg_out(H h, T... t) {
  cerr << " | "<< h; dbg_out(t...);
} // basic
#define dbg(...) cerr << __LINE__<< ":[" << #__VA_ARGS__ << "]
   ↪=", dbg_out(__VA_ARGS__)
```

keybindings.json — 23 lines

```
[
  {
    "key": "alt+b", // build
    "command": "workbench.action.terminal.sendSequence",
    "args": {
      "text": "./compile.sh ${fileBasenameNoExtension} \n"
    }
  },
  {
    "key": "alt+d", // file input/output
    "command": "workbench.action.terminal.sendSequence",
    "args": {
      "text": "./compile.sh ${fileBasenameNoExtension} && ./${
         ↪fileBasenameNoExtension} < input.txt > output.txt\n"
    }
  },
  {
    "key": "alt+d", // manual input
    "command": "workbench.action.terminal.sendSequence",
    "args": {
      "text": "./compile.sh ${fileBasenameNoExtension} && ./${
         ↪fileBasenameNoExtension}\n"
    }
  }
]
```

gen.h — 2 lines

```cpp
#define uid(l, r) uniform_int_distribution<int>(l, r)(rng)
mt19937 rng(chrono::steady_clock::now().time_since_epoch().
   ↪count());
```

.bashrc — 7 lines

```
# write every function in single line
alias clr="printf '\33c'"
co() { g++ -std=c++23 -O2 -Wall -Wextra
 -Wshadow -Wconversion -fsanitize=address
 -fsanitize=undefined -fsanitize=signed-integer-overflow
 -D_GLIBCXX_DEBUG -o $1 $1.cpp; }
run() { co $1 && ./$1; }
```

hash.sh — 3 lines

```
# Hash file ignoring whitespace and comments. Verifies that
# code was correctly typed. Usage: 'sh hash.sh < A.cpp'
cpp -dD -P -fpreprocessed|tr -d '[:space:]'|md5sum|cut -c-6
```

stress.sh — 20 lines

```
#!/usr/bin/env bash

for ((testNum=0;testNum<$4;testNum++))
do
    "./$3".out > input
    "./$2".out < input > outSlow
    "./$1".out < input > outWrong
    if !(cmp -s "outWrong" "outSlow")
    then
        echo "Error found!"
        echo "Input:"
        cat input
        echo "Wrong Output:"
        cat outWrong
        echo "Slow Output:"
        cat outSlow
        exit
    fi
done
echo Passed $4 tests
```

troubleshoot.txt — 75 lines

```
General:
* Write down most of your thoughts, even if you're not sure
whether they're useful.
* Give your variables (and files) meaningful names.
* Stay organized and don't leave papers all over the place!
* You should know what your code is doing ...

Pre-submit:
* Write a few simple test cases if sample is not enough.
* Are time limits close? If so, generate max cases.
* Is the memory usage fine?
* Could anything overflow?
* Remove debug output.
* Make sure to submit the right file.

Wrong answer:
* Print your solution! Print debug output as well.
* Read the full problem statement again.
* Have you understood the problem correctly?
* Are you sure your algorithm works?
* Try writing a slow (but correct) solution.
* Can your algorithm handle the whole range of input?
* Did you consider corner cases (ex. n=1)?
* Is your output format correct? (including whitespace)
* Are you clearing all data structures between test cases?
* Any uninitialized variables?
* Any undefined behavior (array out of bounds)?
* Any overflows or NaNs (or shifting ll by >=64 bits)?
* Confusing N and M, i and j, etc.?
* Confusing ++i and i++?
* Return vs continue vs break?
* Are you sure the STL functions you use work as you think?
* Add some assertions, maybe resubmit.
* Create some test cases to run your algorithm on.
* Go through the algorithm for a simple case.
* Go through this list again.
* Explain your algorithm to a teammate.
* Ask the teammate to look at your code.
* Go for a small walk, e.g. to the toilet.
* Rewrite your solution from the start or let a teammate do it.

Geometry:
* Work with ints if possible.
* Correctly account for numbers close to (but not) zero.
   ↪Related:
for functions like acos make sure absolute val of input is not
(slightly) greater than one.
* Correctly deal with vertices that are collinear, concyclic,
coplanar (in 3D), etc.
* Subtracting a point from every other (but not itself)?

Runtime error:
* Have you tested all corner cases locally?
* Any uninitialized variables?
* Are you reading or writing outside the range of any vector?
* Any assertions that might fail?
* Any possible division by 0? (mod 0 for example)
* Any possible infinite recursion?
* Invalidated pointers or iterators?
* Are you using too much memory?
* Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:
* Do you have any possible infinite loops?
```

```
* What's your complexity? Large TL does not mean that something
simple (like NlogN) isn't intended.
* Are you copying a lot of unnecessary data? (References)
* Avoid vector, map. (use arrays/unordered_map)
* How big is the input and output? (consider FastIO)
* What do your teammates think about your algorithm?
* Calling count() on multiset?

Memory limit exceeded:
* What is the max amount of memory your algorithm should need?
* Are you clearing all data structures between test cases?
* If using pointers try BumpAllocator.
```

Mathematics (2)

2.1 Equations

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The extremum is given by $x = -b/2a$.

$$\begin{matrix} ax + by = e \\ cx + dy = f \end{matrix} \Rightarrow \begin{matrix} x = \dfrac{ed - bf}{ad - bc} \\ y = \dfrac{af - ec}{ad - bc} \end{matrix}$$

In general, given an equation $Ax = b$, the solution to a variable x_i is given by

$$x_i = \frac{\det A_i'}{\det A}$$

where A_i' is A with the i'th column replaced by b.

2.2 Recurrences

If $a_n = c_1 a_{n-1} + \cdots + c_k a_{n-k}$, and $r_1, \ldots, r_k$ are distinct roots of $x^k - c_1 x^{k-1} - \cdots - c_k$, there are $d_1, \ldots, d_k$ s.t.

$$a_n = d_1 r_1^n + \cdots + d_k r_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1 n + d_2) r^n$.

2.3 Trigonometry

$$\sin(v + w) = \sin v \cos w + \cos v \sin w$$
$$\cos(v + w) = \cos v \cos w - \sin v \sin w$$

$$\tan(v + w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$
$$\sin v + \sin w = 2 \sin \frac{v + w}{2} \cos \frac{v - w}{2}$$
$$\cos v + \cos w = 2 \cos \frac{v + w}{2} \cos \frac{v - w}{2}$$
$$(V + W) \tan(v - w)/2 = (V - W) \tan(v + w)/2$$

where V, W are lengths of sides opposite angles v, w.

$$a \cos x + b \sin x = r \cos(x - \phi)$$
$$a \sin x + b \cos x = r \sin(x + \phi)$$

where $r = \sqrt{a^2 + b^2}, \phi = \text{atan2}(b, a)$.

2.4 Geometry

2.4.1 Triangles

Side lengths: a, b, c

Semiperimeter: $p = \dfrac{a + b + c}{2}$

Area: $A = \sqrt{p(p - a)(p - b)(p - c)}$

Circumradius: $R = \dfrac{abc}{4A}$

Inradius: $r = \dfrac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two): $s_a = \sqrt{bc\left[1 - \left(\dfrac{a}{b + c}\right)^2\right]}$

Law of sines: $\dfrac{\sin \alpha}{a} = \dfrac{\sin \beta}{b} = \dfrac{\sin \gamma}{c} = \dfrac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc \cos \alpha$

Law of tangents: $\dfrac{a + b}{a - b} = \dfrac{\tan \frac{\alpha + \beta}{2}}{\tan \frac{\alpha - \beta}{2}}$

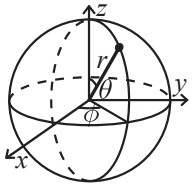
2.4.2 Quadrilaterals

With side lengths a, b, c, d, diagonals e, f, diagonals angle θ, area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2 f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is $180°$, $ef = ac + bd$, and $A = \sqrt{(p - a)(p - b)(p - c)(p - d)}$.

2.4.3 Spherical coordinates



$$\begin{matrix} x = r \sin \theta \cos \phi \\ y = r \sin \theta \sin \phi \\ z = r \cos \theta \end{matrix} \qquad \begin{matrix} r = \sqrt{x^2 + y^2 + z^2} \\ \theta = \text{acos}(z/\sqrt{x^2 + y^2 + z^2}) \\ \phi = \text{atan2}(y, x) \end{matrix}$$

2.5 Derivatives/Integrals

$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1 - x^2}} \qquad \frac{d}{dx} \arccos x = -\frac{1}{\sqrt{1 - x^2}}$$
$$\frac{d}{dx} \tan x = 1 + \tan^2 x \qquad \frac{d}{dx} \arctan x = \frac{1}{1 + x^2}$$
$$\int \tan ax = -\frac{\ln|\cos ax|}{a} \qquad \int x \sin ax = \frac{\sin ax - ax \cos ax}{a^2}$$
$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2} \text{erf}(x) \qquad \int x e^{ax} dx = \frac{e^{ax}}{a^2}(ax - 1)$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

2.6 Sums

$$c^a + c^{a+1} + \cdots + c^b = \frac{c^{b+1} - c^a}{c - 1}, c \neq 1$$

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$
$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(2n+1)(n+1)}{6}$$
$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$
$$1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

2.7 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots, (-\infty < x < \infty)$$

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots, (-1 < x \leq 1)$$

$$\sqrt{1 + x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots, (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots, (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots, (-\infty < x < \infty)$$

2.8 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x. It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x x p_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y,

$$V(aX + bY) = a^2V(X) + b^2V(Y).$$

2.8.1 Discrete distributions

Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\mathrm{Bin}(n, p)$, $n = 1, 2, \ldots, 0 \le p \le 1$.

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu = np,\ \sigma^2 = np(1-p)$$

$\mathrm{Bin}(n, p)$ is approximately $\mathrm{Po}(np)$ for small p.

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each which yields success with probability p is $\mathrm{Fs}(p)$, $0 \le p \le 1$.

$$p(k) = p(1-p)^{k-1},\ k = 1, 2, \ldots$$

$$\mu = \frac{1}{p},\ \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\mathrm{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda}\frac{\lambda^k}{k!}, k = 0, 1, 2, \ldots$$

$$\mu = \lambda,\ \sigma^2 = \lambda$$

2.8.2 Continuous distributions

Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $\mathrm{U}(a, b)$, $a < b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2},\ \sigma^2 = \frac{(b-a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is $\mathrm{Exp}(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \ge 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda},\ \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu, \sigma^2)$, $\sigma > 0$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

2.9 Markov chains

A *Markov chain* is a discrete random process with the property that the next state depends only on the current state. Let $X_1, X_2, \ldots$ be a sequence of random variables generated by the Markov process. Then there is a transition matrix $\mathbf{P} = (p_{ij})$, with $p_{ij} = \Pr(X_n = i | X_{n-1} = j)$, and $\mathbf{p}^{(n)} = \mathbf{P}^n\mathbf{p}^{(0)}$ is the probability distribution for X_n (i.e., $p_i^{(n)} = \Pr(X_n = i)$), where $\mathbf{p}^{(0)}$ is the initial distribution.

Stationary distribution

π is a stationary distribution if $\pi = \pi\mathbf{P}$. If the Markov chain is *irreducible* (it is possible to get to any state from any state), then $\pi_i = \frac{1}{\mathbb{E}(T_i)}$ where $\mathbb{E}(T_i)$ is the expected time between two visits in state i. π_j/π_i is the expected number of visits in state j between two visits in state i.

For a connected, undirected and non-bipartite graph, where the transition probability is uniform among all neighbors, π_i is proportional to node i's degree.

Ergodicity

A Markov chain is *ergodic* if the asymptotic distribution is independent of the initial distribution. A finite Markov chain is ergodic iff it is irreducible and *aperiodic* (i.e., the gcd of cycle lengths is 1). $\lim_{k\to\infty} \mathbf{P}^k = \mathbf{1}\pi$.

Data Structures (3)

3.1 STL

MapComparator.h

Description: example of function object (functor) for map or set
Usage: `set<int,cmp> s; map<int,int,cmp> m;`

5bfa6c, 1 lines

```
struct cmp{bool operator()(int l,int r)const{return l>r;}};
```

HashMap.h

Description: Hash map with similar API as unordered_map. Initial capacity must be a power of 2 if provided.
Usage: `gp_hash_table<int, int, chash> h({},{},{},{},{1<<16});`
Memory: ~1.5x unordered map
Time: ~3x faster than unordered map

<ext/pb_ds/assoc_container.hpp> b6e07b, 6 lines

```
using namespace __gnu_pbds;
struct chash {
  const uint64_t C = 4e18*acos(0)+71;
  const int RANDOM = chrono::steady_clock::now().
    ↪time_since_epoch().count();
  int64_t operator()(int64_t x) const { return
    ↪__builtin_bswap64((x^RANDOM)*C); }
};
```

OrderStatisticTree.h

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. Change `null_type` to get a map.
Time: $\mathcal{O}(\log N)$

<ext/pb_ds/assoc_container.hpp> bd4c35, 16 lines

```
using namespace __gnu_pbds;
template<class T> using IndexedSet = tree<T, null_type, less<T
  ↪>,
  rb_tree_tag, tree_order_statistics_node_update>;
#define ook order_of_key // position of the given element
#define fbo find_by_order // return an iterator
// Number of element <= k
int atMost(IndexedSet<pair<int, int>>& T, int k) {
  return T.order_of_key({k, INT_MAX});
}
// Number of element grater >= k
int atLeast(IndexedSet<pair<int, int>>& T, int k) {
  return T.size() - T.order_of_key({k, INT_MIN});
}
int getSum(Tree<pair<int, int>>& T, int l, int r) {
  return atMost(T,r)-atMost(T,l-1);
}
```

CoordCompress.h

Description: coordinate compression, get index of x by counting number of element less than x
Time: $\mathcal{O}(N \log N)$

14ee69, 7 lines

```
template<class T> void compress(vector<T>& v) {
  ranges::sort(v);
  v.erase(unique(v.begin(), v.end()), v.end());
}
template<class T> int get(vector<T>& v, T x) {
  return ranges::lower_bound(v, x) - begin(v);
```

```
}
```

3.2 1D Range Queries

SparseTable.h

Description: Static 1D range (min/max/gcd/lcm/or/and) query. If TL is an issue, use arrays instead of vectors and store values instead of indices.
Memory: $\mathcal{O}(N \log N)$
Time: $\mathcal{O}(1)$

471d4b, 19 lines

```
template<class T> struct SparseTable {
  int N, LOG;
  vector<vector<T>> jmp;
  T cmb(T a, T b) { return min(a, b); }
  SparseTable(const vector<T>& v) {
    N = v.size(); LOG = __lg(N);
    jmp.resize(N, vector<T>(LOG + 1));
    for (int i = 0; i < N; ++i) jmp[i][0] = v[i];
    for (int j = 1; j <= LOG; ++j) {
      for (int i = 0; i + (1<<j) <= N; ++i) {
        jmp[i][j] = cmb(jmp[i][j-1], jmp[i+(1<<(j-1))][j-1]);
      }
    }
  }
  T query(int l, int r) {
    int d = __lg(r-l+1);
    return cmb(jmp[l][d], jmp[r-(1<<d)+1][d]);
  }
};
```

FenwickTree.h

Description: Computes partial sums a[0] + a[1] + ... + a[pos - 1], and updates single elements a[i], taking the difference between the old and new value. 0-Indexed.
Time: Both operations are $\mathcal{O}(\log N)$.

122df3, 23 lines

```
template<class T> struct FenwickTree {
  int N; vector<T> bit;
  FenwickTree(int _N) { N = _N; bit.resize(N); }
  void add(int p, T x) {
    for (++p; p <= N; p += p&-p) bit[p-1] += x;
  }
  T sum(int l, int r) { return sum(r+1)-sum(l); }
  T sum(int r) {
    T res = 0;
    for(; r; r -= r&-r) res += bit[r-1];
    return res;
  }
  int lower_bound(T sum) {
    if (sum <= 0) return -1;
    int pos = 0;
    for (int pw = 1<<25; pw; pw >>= 1) {
      int npos = pos+pw;
      if (npos <= N && bit[npos-1] < sum)
        pos = npos, sum -= bit[pos-1];
    }
    return pos;
  }
};
```

SegmentTree.h

Description: 1D point update and range query where `cmb` is any associative operation. `seg[1]==query(0,N-1)`.
Time: $\mathcal{O}(\log N)$

406185, 37 lines

```
template<class T> struct SegmentTree {
  const T ID{};
  T cmb(T a, T b) { return a + b; }
  int N = 1; vector<T> seg;
  SegmentTree(int _N) {
    while (N < _N) N *= 2;
    seg.assign(2*N, ID);
  }
  void update(int p, T val) { // set val at position p
    seg[p += N] = val;
    for (p /= 2; p > 0; p /= 2) {
      seg[p] = cmb(seg[2*p], seg[2*p+1]);
    }
  }
  T query(int l, int r) { // zero-indexed, inclusive
    T lf = ID, rf = ID;
    for (l += N, r += N + 1; l < r; l /= 2, r /= 2) {
      if (l & 1) lf = cmb(lf, seg[l++]);
      if (r & 1) rf = cmb(seg[--r], rf);
    }
    return cmb(lf, rf);
  }
  // int first_at_least(int lo, int val, int ind, int l, int r)
  //   -> { // if seg stores max across range
  //   if (r < lo || val > seg[ind]) return -1;
  //   if (l == r) return l;
  //   int m = (l+r)/2;
  //   int res = first_at_least(lo,val,2*ind,l,m); if (res !=
  //   -1) return res;
  //   return first_at_least(lo,val,2*ind+1,m+1,r);
  // }
  //   int k_th_one(int k, int ind, int l, int r) {
  //   if (l == r) return l;
  //   int m = (l + r) / 2;
  //   int lc = 2*ind, rc = 2*ind+1;
  //   if (seg[lc] >= k) return k_th_one(k, lc, l, m);
  //   else return k_th_one(k-seg[lc], rc, m+1, r);
  // }
};
```

LazySegmentTree.h

Description: 1D range increment and sum query.
Usage: `LazySeg<int64_t, 1<<20> T; T.update(l, r, val);`
Time: $\mathcal{O}(\log N)$

e0742f, 42 lines

```
template<class T, int SZ>struct LazySeg {
  // SZ must be power of 2
  static_assert(__builtin_popcount(SZ) == 1);
  const T ID{};
  T cmb(T a, T b) { return a + b; }
  T seg[2*SZ], lazy[2*SZ];
  LazySeg() {
    for (int i = 0; i < 2*SZ; ++i) seg[i] = lazy[i] = ID;
  }
  // modify values for current node
  void push(int ind, int L, int R) {
    seg[ind] += (R-L+1) * lazy[ind]; // dependent on operation
    if (L != R) for (int i = 0; i < 2; ++i) lazy[2*ind+i] +=
      lazy[ind];
    lazy[ind] = 0;
  }
  void pull(int ind) {
    seg[ind] = cmb(seg[2*ind], seg[2*ind+1]);
  }
  void build() {
    for (int i = SZ-1; i >= 1; --i) pull(i);
  }
  void update(int lo, int hi, T inc, int ind = 1, int L = 0,
    int R = SZ - 1) {
    push(ind, L, R);
    if (hi < L || R < lo) return;
    if (lo <= L && R <= hi) {
      lazy[ind] = inc;
      push(ind, L, R);
      return;
    }
    int M = (L + R) / 2;
    update(lo, hi, inc, 2*ind, L, M);
    update(lo, hi, inc, 2*ind+1, M+1, R);
    pull(ind);
  }
  T query(int lo, int hi, int ind = 1, int L = 0, int R = SZ -
    1) {
    push(ind, L, R);
    if (lo > R || L > hi) return ID;
    if (lo <= L && R <= hi) return seg[ind];
    int M = (L + R) / 2;
    return cmb(query(lo, hi, 2*ind, L, M), query(lo, hi, 2*ind
      +1, M+1, R));
  }
};
```

3.3 2D Range Queries

PrefixSum2D.h

Description: calculates rectangle sums in constant time

65b070, 17 lines

```
template<typename T> struct PrefixSum2D {
  vector<vector<T>> sum;
  PrefixSum2D(const vector<vector<T>> &v) {
    int n = v.size(), m = v[0].size();
    sum.assign(n+1, vector<T>(m+1, T{}));
    for (int i = 0; i < n; ++i) {
      for (int j = 0; j < m; ++j) {
        sum[i+1][j+1] = v[i][j]
        - sum[i][j] + sum[i+1][j] + sum[i][j+1];
      }
    }
  }
  T query(int x1, int y1, int x2, int y2) {
    return sum[x2][y2] + sum[x1-1][y1-1]
    - sum[x1-1][y2] - sum[x2][y1-1];
  }
};
```

Number Theory (4)

4.1 Modular Arithmetic

ModIntShort.h

Description: Modular arithmetic. Assumes MOD is prime.
Usage: `mint a = MOD+5; inv(a); // 400000003`

7c1e94, 30 lines

```
struct mint {
  int v;
  static const int MOD = 1E9 + 7;
  explicit operator int() const { return v; }
  mint() : v(0) {}
  mint(int64_t _v) : v(int(_v % MOD)) { v += (v < 0) * MOD; }
  mint &operator+=(mint o) {
    if ((v += o.v) >= MOD) v -= MOD;
    return *this;
  }
  mint &operator-=(mint o) {
    if ((v -= o.v) < 0) v += MOD;
    return *this;
  }
  mint &operator*=(mint o) {
    v = int((int64_t)v * o.v % MOD);
    return *this;
  }
  friend mint pow(mint a, int64_t p) {
    assert(p >= 0);
```

```
    return p==0? 1:pow(a*a, p/2)*(p&1? a:1);
  }
  friend mint inv(mint a) {
    assert(a.v != 0);
    return pow(a, MOD - 2);
  }
  friend mint operator+(mint a, mint b) { return a += b; }
  friend mint operator-(mint a, mint b) { return a -= b; }
  friend mint operator*(mint a, mint b) { return a *= b; }
};
```

ModFactInt.h

Description: Combinations modulo a prime MOD. Assumes $2 \le N \le MOD$.

Usage: `F.gen(100), F.C(6, 4); // 15`

Time: $\mathcal{O}(N)$

232d42, 22 lines

```
struct {
  vector<int> inv, fac, ifac;
  void gen(int N) {
    inv.resize(N); fac.resize(N); ifac.resize(N);
    inv[1] = fac[0] = ifac[0] = 1;
    for (int i = 2; i < N; i++){
      inv[i] = int(MOD-(int64_t)MOD/i*inv[MOD%i]%MOD);
    }
    for (int i = 1; i < N; i++) {
      fac[i] = int((int64_t)fac[i-1]*i%MOD);
      ifac[i] = int((int64_t)ifac[i-1]*inv[i]%MOD);
    }
  }
  int P(int n, int r) { // nPr
    if (n < r || r < 0) return 0;
    return (int64_t)fac[n]*ifac[n-r]%MOD;
  }
  int C(int n, int r) { // nCr
    if (n < r || r < 0) return 0;
    return (int64_t)fac[n]*ifac[r]%MOD*ifac[n-r]%MOD;
  }
} F;
```

ModMulLL.h

Description: Multiply two 64-bit integers mod another if 128-bit is not available. modMul is equivalent to `(ul)(__int128(a)*b%mod)`. Works for $0 \le a, b < mod < 2^{63}$. NOTE : Only works if system supports 80-bit floating point.

d22eee, 10 lines

```
using ul = uint64_t;
ul modMul(ul a, ul b, const ul mod){
  return __int128(a) * __int128(b) % mod;
}
ul modPow(ul a, ul b, const ul mod) {
  if (b == 0) return 1;
  ul res = modPow(a, b/2, mod);
  res = modMul(res, res, mod);
  return b&1 ? modMul(res, a, mod) : res;
}
```

4.2 Primality

4.2.1 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

4.2.2 Divisors

$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

Dirichlet Convolution: Given a function $f(x)$, let

$(f * g)(x) = \sum_{d|x} g(d) f(x/d)$.

If the partial sums $s_{f*g}(n), s_g(n)$ can be computed in $O(1)$ and $s_f(1 \dots n^{2/3})$ can be computed in $O\left(n^{2/3}\right)$ then all $s_f\left(\frac{n}{d}\right)$ can as well. Use

$$s_{f*g}(n) = \sum_{d=1}^{n} g(d) s_f(n/d).$$

If $f(x) = \mu(x)$ then $g(x) = 1, (f * g)(x) = (x == 1)$, and $s_f(n) = 1 - \sum_{i=2}^{n} s_f(n/i)$.

If $f(x) = \phi(x)$ then $g(x) = 1, (f * g)(x) = x$, and $s_f(n) = \frac{n(n+1)}{2} - \sum_{i=2}^{n} s_f(n/i)$.

Sieve.h

Description: Tests primality up to N. Runs faster if only odd indices are stored.

Time: $\mathcal{O}(N \log \log N)$ or $\mathcal{O}(N)$

5a9b1c, 20 lines

```
template<int N> struct Sieve {
  bitset<N> is_prime; vector<int> primes;
  Sieve() {
    is_prime.set(); is_prime[0] = is_prime[1] = 0;
    for (int i = 4; i < N; i += 2) is_prime[i] = 0;
    for (int i = 3; i*i < N; i += 2) if (is_prime[i])
      for (int j = i*i; j < N; j += i*2) is_prime[j] = 0;
    for (int i = 0; i < N; i++) if (is_prime[i]) primes.
      ↪push_back(i);
  }
  // array<int, N> spf{} // smallest prime that divides
  // Sieve() { // above is faster
  //  for (int i = 2; i < N; i++) {
  //    if (spf[i] == 0) spf[i] = i, primes.push_back(i);
  //    for (int p: primes) {
  //      if (p > spf[i] || i*p >= N) break;
  //      spf[i*p] = p;
  //    }
  //  }
  // }
};
```

PrimeCnt.h

Description: Counts number of primes up to N. Can also count sum of primes.

Time: $\mathcal{O}\left(N^{3/4}/\log N\right)$, 60ms for $N = 10^{11}$, 2.5s for $N = 10^{13}$

c04e96, 20 lines

```
ll count_primes(ll N) { // count_primes(1e13) == 346065536839
  if (N <= 1) return 0;
  int sq = (int)sqrt(N);
  vl big_ans((sq+1)/2), small_ans(sq+1);
  FOR(i,1,sq+1) small_ans[i] = (i-1)/2;
  F0R(i,sz(big_ans)) big_ans[i] = (N/(2*i+1)-1)/2;
  vb skip(sq+1); int prime_cnt = 0;
  for (int p = 3; p <= sq; p += 2) if (!skip[p]) { // primes
    for (int j = p; j <= sq; j += 2*p) skip[j] = 1;
    F0R(j,min((ll)sz(big_ans),(N/p/p+1)/2)) {
      ll prod = (ll)(2*j+1)*p;
      big_ans[j] -= (prod > sq ? small_ans[(double)N/prod]
            : big_ans[prod/2])-prime_cnt;
    }
    for (int j = sq, q = sq/p; q >= p; --q) for (;j >= q*p;--j)
      small_ans[j] -= small_ans[q]-prime_cnt;
    ++prime_cnt;
  }
  return big_ans[0]+1;
}
```

MillerRabin.h

Description: Deterministic primality test, works up to 2^{64}. For larger numbers, extend A randomly.

"ModMulLL.h" f5b0fe, 11 lines

```
bool prime(ul n) { // not ll!
  if (n < 2 || n % 6 % 4 != 1) return n - 2 < 2;
  ul s = __builtin_ctzll(n - 1), d = n >> s;
  ul A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022};
  for (auto a : A) {
    ul p = modPow(a, d, n), i = s;
    while (p != 1 && p != n-1 && a % n && i--) p = modMul(p, p,
      ↪ n);
    if (p != n - 1 && i != s) return false;
  }
  return true;
}
```

FactorBasic.h

Description: Factors integers.

Time: $\mathcal{O}\left(\sqrt{N}\right)$

8b0341, 53 lines

```
inline namespace factorBasic {
template<class T> vector<pair<T, int>> factor(T x) {
  vector<pair<T, int>> primes;
  for (T i = 2; i*i <= x; ++i) if (x % i == 0) {
    int t = 0;
    while (x % i == 0) x /= i, t++;
    primes.push_back({i, t});
  }
  if (x > 1) primes.push_back({x, 1});
  return primes;
}

/* Note:
 * number of operations needed s.t.
 *          phi(phi(...phi(n)...))=1
 * is O(log n).
 * Euler's theorem: a^{\phi(p)}\equiv 1 (mod p), gcd(a,p)=1
 */
template<class T> T phi(T x) {
  for (auto& [p, exp]: factor(x)) x -= x/p;
  return x;
}

template<class T> void tour(vector<pair<T, int>> &v,
vector<T> &V, int ind, T cur) {
  if (ind == (int)size(v)) V.push_back(cur);
  else {
    T mul = 1;
    for (int i = 0; i <= v[ind].second; i++) {
```

```
      tour(v, V, ind + 1, cur * mul);
      mul *= v[ind].first;
    }
  }
}

template<class T> vector<T> getDiv(T x) {
  auto v = factor(x);
  vector<T> V;
  tour(v, V, 0, (T)1);
  ranges::sort(V);
  return V;
}

template<class T> vector<T> getDiv(T x) {
  vector<T> V;
  for (T i = 1; i*i <= x; ++i) if (x % i == 0) {
    V.push_back(i);
    if (x / i != i) V.push_back(x / i);
  }
  ranges::sort(V);
  return V;
}
}
```

FactorFast.h

Description: Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).

Time: $\mathcal{O}\left(N^{1/4}\right)$, less for numbers with small factors

"MillerRabin.h" 113e1c, 23 lines

```
ul pollard(ul n) { // return some nontrivial factor of n
  auto f = [n](ul x) { return modMul(x, x, n) + 1; };
  ul x = 0, y = 0, t = 30, prd = 2, i = 1, q;
  while (t++ % 40 || gcd(prd, n) == 1) {
    if (x == y) x = ++i, y = f(x);
    if ((q = modMul(prd, max(x, y)-min(x, y), n))) prd = q;
    x = f(x), y = f(f(y));
  }
  return gcd(prd, n);
}

void factor_rec(ul n, map<ul, int> &cnt) {
  if (n == 1) return;
  if (prime(n)) { ++cnt[n]; return; }
  ul u = pollard(n);
  factor_rec(u, cnt), factor_rec(n/u, cnt);
}

vector<pair<ul, int>> factor(ul n) {
  map<ul, int> cnt;
  factor_rec(n, cnt);
  return vector<pair<ul, int>>(cnt.begin(), cnt.end());
}
```

4.3 Euclidean Algorithm

Euclid.h

Description: Generalized Euclidean algorithm. `euclid` and `invGeneral` work for $A, B < 2^{62}$.

Time: $\mathcal{O}(\log AB)$

8bab06, 13 lines

```
// For A,B>=0, finds (x,y) s.t.
pair<int64_t, int64_t> euclid(int64_t A, int64_t B) {
  // Ax+By=gcd(A,B), |Ax|,|By|<=AB/gcd(A,B)
  if (!B) return {1, 0};
  pair<int64_t, int64_t> p = euclid(B, A%B);
  return {p.second, p.first - A/B*p.second};
}
// find x in [0,B) such that Ax=1 mod B
int64_t invGeneral(int64_t A, int64_t B) {
  pair<int64_t, int64_t> p = euclid(A, B);
  assert(p.first*A + p.second*B == 1);
  return p.first + (p.first<0)*B;
} // must have gcd(A,B)=1
```

4.4 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2),\ \ b = k \cdot (2mn),\ \ c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

4.5 Lifting the Exponent

For $n > 0$, p prime, and ints x, y s.t. $p \nmid x, y$ and $p | x - y$:

- $p \neq 2$ or $p = 2, 4 | x - y \implies v_p(x^n - y^n) = v_p(x - y) + v_p(n)$.
- $p = 2, 2 | n \implies v_2(x^n - y^n) = v_2((x^2)^{n/2} - (y^2)^{n/2})$.

Combinatorial (5)

5.1 Permutations

5.1.1 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
n	11		12		13		14	15	16	17
$n!$	4.0e7		4.8e8		6.2e9		8.7e10	1.3e12	2.1e13	3.6e14
n	20		25		30		40	50	100	150	171
$n!$	2e18		2e25		3e32		8e47	3e64	9e157	6e262	>DBL_MAX

5.1.2 Cycles

Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp\left(\sum_{n \in S} \frac{x^n}{n}\right)$$

5.1.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

5.1.4 Burnside's lemma

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k|n} f(k)\phi(n/k).$$

5.2 Partitions and subsets

5.2.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1,\ p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	∼2e5	∼2e8

5.2.2 Lucas' Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + ... + n_1 p + n_0$ and $m = m_k p^k + ... + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k} \binom{n_i}{m_i} \pmod{p}$.

5.3 General purpose numbers

5.3.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t - 1}$ (FFT-able).
$B[0, \ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} i^m = \frac{1}{m+1} \sum_{k=0}^{m} \binom{m+1}{k} B_k (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty} \frac{B_k}{k!} f^{(k-1)}(m)$$

$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

5.3.2 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n, k) = c(n-1, k-1) + (n-1)c(n-1, k),\ \ c(0, 0) = 1$$
$$\textstyle\sum_{k=0}^{n} c(n, k) x^k = x(x+1)\ldots(x+n-1)$$

$c(8, k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n, 2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

5.3.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k} (-1)^j \binom{n+1}{j} (k+1-j)^n$$

5.3.4 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} j^n$$

5.3.5 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

5.3.6 Labeled unrooted trees

on n vertices: n^{n-2}
on k existing trees of size n_i: $n_1 n_2 \cdots n_k n^{k-2}$
with degrees d_i: $(n-2)!/((d_1-1)! \cdots (d_n-1)!)$

5.3.7 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2} C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

5.4 Young Tableaux

Let a **Young diagram** have shape $\lambda = (\lambda_1 \geq \cdots \geq \lambda_k)$, where λ_i equals the number of cells in the i-th (left-justified) row from the top. A **Young tableau** of shape λ is a filling of the $n = \sum \lambda_i$ cells with a permutation of $1 \ldots n$ such that each row and column is increasing.

Hook-Length Formula: For the cell in position (i,j), let $h_\lambda(i,j) = |\{(I,J) | i \leq I, j \leq J, (I = i \text{ or } J = j)\}|$. The number of Young tableaux of shape λ is equal to $f^\lambda = \frac{n!}{\prod h_\lambda(i,j)}$.

Schensted's Algorithm: converts a permutation σ of length n into a pair of Young Tableaux $(S(\sigma), T(\sigma))$ of the same shape. When inserting $x = \sigma_i$,

1. Add x to the first row of S by inserting x in place of the largest y with $x < y$. If y doesn't exist, push x to the end of the row, set the value of T at that position to be i, and stop.
2. Add y to the second row using the same rule, keep repeating as necessary.

All pairs $(S(\sigma), T(\sigma))$ of the same shape correspond to a unique σ, so $n! = \sum (f^\lambda)^2$. Also, $S(\sigma^R) = S(\sigma)^T$.

Let $d_k(\sigma), a_k(\sigma)$ be the lengths of the longest subseqs which are a union of k decreasing/ascending subseqs, respectively. Then $a_k(\sigma) = \sum_{i=1}^{k} \lambda_i, d_k(\sigma) = \sum_{i=1}^{k} \lambda_i^*$, where λ_i^* is size of the i-th column.

Numerical (6)

6.1 Matrix

Matrix.h
Description: 2D matrix operations.
"ModInt.h" b18e29, 21 lines

```
using T = mi;
using Mat = V<V<T>>; // use array instead if tight TL

Mat makeMat(int r, int c) { return Mat(r,V<T>(c)); }
Mat makeId(int n) {
  Mat m = makeMat(n,n); F0R(i,n) m[i][i] = 1;
  return m;
}
Mat operator*(const Mat& a, const Mat& b) {
  int x = sz(a), y = sz(a[0]), z = sz(b[0]);
  assert(y == sz(b)); Mat c = makeMat(x,z);
  F0R(i,x) F0R(j,y) F0R(k,z) c[i][k] += a[i][j]*b[j][k];
  return c;
}
Mat& operator*=(Mat& a, const Mat& b) { return a = a*b; }
Mat pow(Mat m, ll p) {
  int n = sz(m); assert(n == sz(m[0]) && p >= 0);
  Mat res = makeId(n);
  for (; p; p /= 2, m *= m) if (p&1) res *= m;
  return res;
}
```

Graphs (7)

Erdos-Gallai: $d_1 \geq \cdots \geq d_n$ can be degree sequence of simple graph on n vertices iff their sum is even and $\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k), \forall 1 \leq k \leq n$.

7.1 Basics

DFS.h
Description: Basic DFS implementation for undirected graphs
Time: $\mathcal{O}(N+M)$ 14b944, 21 lines

```
struct DFS {
  int N;
  vector<bool> visited;
  vector<vector<int>> adj;
  DFS(int _N) {
    N = _N;
    adj.resize(N);
    visited.resize(N);
  }
  void ae(int u, int v) {
    adj[u].push_back(v);
    adj[v].push_back(u);
  }
  void dfs(int u) {
    visited[u] = true;
    for (auto& v: adj[u]) {
      if (visited[v]) continue;
      dfs(v);
    }
  }
};
```

BFS.h
Description: Performs a standard BFS on an undirected graph.
Time: $\mathcal{O}(N+M)$ a2f9d4, 26 lines

```
struct BFS {
  int SZ;
  vector<bool> vis;
  vector<vector<int>> adj;
  BFS(int _SZ) {
    SZ = _SZ;
    adj.resize(SZ);
    vis.resize(SZ);
  }
  void ae(int u, int v) {
    adj[u].push_back(v);
    adj[v].push_back(u);
  }
  void bfs(int src) {
    vis[src] = 1;
    queue<int> todo; todo.push(src);
    while (!todo.empty()) {
      auto u = todo.front(); todo.pop();
      for (auto& v: adj[u]) {
        if (vis[v]) continue;
        vis[v] = 1;
        todo.push(v);
      }
    }
  }
};
```

Bipartite.h
Description: Checks if an undirected graph is bipartite. Colors nodes with 1/2. If not bipartite, returns false.

Time: $\mathcal{O}(N+M)$ 4f7998, 28 lines

```
struct Bipartite {
  int N;
  vector<int> color;
  vector<vector<int>> adj;
  Bipartite(int _N) {
    N = _N;
    adj.resize(N); color.resize(N);
  }
  void ae(int u, int v) {
    adj[u].push_back(v);
    adj[v].push_back(u);
  }
  bool check() {
    queue<int> todo;
    for (int src = 0; src < N; src++) {
      if (color[src]) continue;
      color[src] = 1; todo.push(src);
      while (!todo.empty()) {
        int u = todo.front(); todo.pop();
        for (auto &v: adj[u]) {
          if (color[v] == color[u]) return false;
          if (!color[v]) color[v] = 3 - color[u], todo.push(v);
        }
      }
    }
    return true;
  }
};
```

TopoSort.h
Description: sorts vertices such that if there exists an edge x->y, then x goes before y 6779f6, 23 lines

```
struct TopoSort {
  int SZ;
  vector<int> in, res;
  vector<vector<int>> adj;
  TopoSort(int _SZ) {
    SZ = _SZ;
    in.resize(SZ);
    adj.resize(SZ);
  }
  void ae(int u, int v) {
    adj[u].push_back(v), ++in[v];
  }
  bool sort() {
    queue<int> todo;
    for (int u = 0; u < SZ; ++u) if (!in[u]) todo.push(u);
    while (!todo.empty()) {
      int u = todo.front(); todo.pop();
      res.push_back(u);
      for (auto &v : adj[u]) if (!(--in[v])) todo.push(v);
    }
    return (int)res.size() == SZ;
  }
};
```

DirectedCycle.h
Description: stack 90d8f6, 29 lines

```
struct DirectedCycle {
  int SZ;
  vector<bool> inStk, vis;
  vector<int> stk, cyc;
  vector<vector<int>> adj;
  DirectedCycle(int _SZ) {
    SZ = _SZ;
    adj.resize(SZ);
    inStk.resize(SZ), vis.resize(SZ);
  }
  void ae(int u, int v) { adj[u].push_back(v); }
  vector<int> gen() {
    for (int u = 0; u < SZ; u++) {
      if (!vis[u] && empty(cyc)) dfs(u);
    }
    return cyc;
  }
  void dfs(int u) {
    stk.push_back(u);
    inStk[u] = vis[u] = 1;
    for (auto& v: adj[u]) {
      if (inStk[v]) cyc = {ranges::find(stk, v), end(stk)};
      else if (!vis[v]) dfs(v);
      if (size(cyc)) return;
    }
    stk.pop_back();
    inStk[u] = 0;
  }
};
```

Dijkstra.h
Description: shortest path
Time: $\mathcal{O}(N \log N + M)$ 5afc43, 27 lines

```
template<class T, bool directed> struct Dijkstra {
  int SZ;
  vector<T> dist;
  vector<vector<pair<int, T>>> adj;
  Dijkstra(int _SZ) {
    SZ = _SZ;
    adj.resize(SZ);
  }
  void ae(int u, int v, T w) {
    adj[u].push_back({v, w});
    if (!directed) adj[v].push_back({u, w});
  }
  void gen(int st) {
    dist.assign(SZ, numeric_limits<T>::max());
    priority_queue<pair<T, int>> PQ;
    auto relax = [&](int v, T d) {
      if (dist[v] <= d) return;
      dist[v] = d;
      PQ.push({-dist[v], v});
    }; relax(st, 0);
    while (!PQ.empty()) {
      auto [d, u] = PQ.top(); PQ.pop(); d = -d;
      if (dist[u] < d) continue;
      for (auto& [v, w]: adj[u]) relax(v, w+d);
    }
  }
};
```

FloydWarshall.h
Description: All-Pairs Shortest Path a2bfcc, 14 lines

```
void floydWarshall(V<vl> &m)
{
  int n = sz(m);
  F0R(i, n)
  ckmin(m[i][i], 0LL);
  F0R(k, n)
  F0R(i, n) F0R(j, n) if (m[i][k] != BIG && m[k][j] != BIG)
  {
    auto newDist = max(m[i][k] + m[k][j], -BIG);
    ckmin(m[i][j], newDist);
  }
  F0R(k, n)
  if (m[k][k] < 0) F0R(i, n) F0R(j, n) if (m[i][k] != BIG && m[
    ↪k][j] != BIG) m[i][j] = -BIG;
}
```

7.2 Trees

TreeDiameter.h
Description: Calculates longest path in tree. The vertex furthest from 0 must be an endpoint of the diameter. 3c22a4, 37 lines

```
struct TreeDiameter {
  int N, diaLen;
  vector<int> dist, dia;
  vector<vector<int>> adj;
  TreeDiameter(int _N) {
    N = _N;
    dia = {0, 0};
    adj.resize(N);
    dist.resize(N);
  }
  void ae(int u, int v) {
    adj[u].push_back(v);
    adj[v].push_back(u);
  }
  void genDist(int R) { dist[R] = 0; dfs(R); }
  void gen() {
    genDist(0);
    for (int i = 0; i < N; ++i) {
      if (dist[i] > dist[dia[0]]) dia[0] = i;
    }
    genDist(dia[0]);
    for (int i = 0; i < N; ++i) {
      if (dist[i] > dist[dia[1]]) dia[1] = i;
    }
    diaLen = dist[dia[1]];
    // int cen = dia[1];
    // for (int i = 0; i < diaLen/2; ++i) cen = par[cen];
    // center = {cen};
    // if (diaLen&1) center.push_back(par[cen]);
  }
  void dfs(int u, int p = -1) {
    for (auto& v: adj[u]) {
      if (v == p) continue;
      dist[v] = dist[u] + 1; dfs(v, u);
    }
  }
};
```

LCAjump.h
Description: Calculates least common ancestor in tree with verts $0 \ldots N-1$ and root R using binary jumping.
Memory: $\mathcal{O}(N \log N)$
Time: $\mathcal{O}(N \log N)$ build, $\mathcal{O}(\log N)$ query 5c0112, 47 lines

```
struct LCA {
  int N, LOG;
  vector<int> depth;
  vector<vector<int>> par, adj;
  LCA(int _N) {
    N = _N; LOG = 1;
    while ((1<<LOG) < N) ++LOG;
    adj.resize(N);
    depth.resize(N);
    par.resize(N, vector<int>(LOG));
  }
  void ae(int u, int v) {
    adj[u].push_back(v);
    adj[v].push_back(u);
  }
  void gen(int R = 0) { par[R][0] = R; dfs(R); }
```

```
  void dfs(int u = 0) {
    for (int j = 1; j < LOG; ++j) {
      par[u][j] = par[par[u][j-1]][j-1];
    }
    for (auto& v: adj[u]) {
      if (v == par[u][0]) continue;
      depth[v] = depth[u]+1;
      par[v][0] = u;
      dfs(v);
    }
  }
  int jump(int x, int d) {
    for (int j = 0; j < LOG; ++j) {
      if ((d>>j) & 1) x = par[x][j];
    }
    return x;
  }
  int lca(int u, int v) {
    if (depth[u] > depth[v]) swap(u, v);
    v = jump(v, depth[v]-depth[u]);
    if (u == v) {return u; }
    for (int j = LOG-1; j >= 0; --j) {
      int U = par[u][j], V = par[v][j];
      if (U != V) u = U, v = V;
    }
    return par[u][0];
  }
  int dist(int u, int v) {
    return depth[u] + depth[v] - 2*depth[lca(u, v)];
  }
};
```

7.3 DFS Algorithms

SCCK.h

Description: Kosaraju's Algorithm, DFS twice to generate strongly connected components in topological order. a, b in same component if both $a \to b$ and $b \to a$ exist.

Time: $\mathcal{O}(N+M)$

8b29a5, 32 lines

```
struct SCC {
  int N;
  vector<bool> vis;
  vector<vector<int>> adj, radj;
  vector<int> todo, comp, comps;
  SCC(int _N) {
    N = _N;
    adj.resize(N), radj.resize(N);
    comp.resize(N, -1), vis.resize(N);
  }
  void ae(int u, int v) {
    adj[u].push_back(v);
    radj[v].push_back(u);
  }
  void dfs(int u) {
    vis[u] = true;
    for (auto &v: adj[u]) if (!vis[v]) dfs(v);
    todo.push_back(u);
  }
  void dfs2(int u, int id) {
    comp[u] = id;
    for (auto &v: radj[u]) if (comp[v] == -1) dfs2(v, id);
  }
  void gen() {
    for (int u = 0; u < N; u++) if (!vis[u]) dfs(u);
    ranges::reverse(todo);
    for (auto &u: todo) if (comp[u] == -1) {
      dfs2(u, u);
      comps.push_back(u);
    }
  }
};
```

TwoSAT.h

Description: Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type $(a||!b)\&\&(!a||c)\&\&(d||!b)\&\&\ldots$ becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions (~x).

Usage: TwoSat ts;
ts.either(0, ~3); // Var 0 is true or var 3 is false
ts.setVal(2); // Var 2 is true
ts.atMostOne({0,~1,2}); // <= 1 of vars 0, ~1 and 2 are true
ts.solve(N); // Returns true iff it is solvable
ts.ans[0..N-1] holds the assigned values to the vars

"SCCK.h" ff0f3d, 31 lines

```
struct TwoSAT {
  int N = 0; vpi edges;
  void init(int _N) { N = _N; }
  int addVar() { return N++; }
  void either(int x, int y) {
    x = max(2*x,-1-2*x), y = max(2*y,-1-2*y);
    edges.eb(x,y); }
  void implies(int x, int y) { either(~x,y); }
  void must(int x) { either(x,x); }
  void atMostOne(const vi& li) {
    if (sz(li) <= 1) return;
    int cur = ~li[0];
    FOR(i,2,sz(li)) {
      int next = addVar();
      either(cur,~li[i]); either(cur,next);
      either(~li[i],next); cur = ~next;
    }
    either(cur,~li[1]);
  }
  vb solve() {
    SCC S; S.init(2*N);
    each(e,edges) S.ae(e.f^1,e.s), S.ae(e.s^1,e.f);
    S.gen(); reverse(all(S.comps)); // reverse topo order
    for (int i = 0; i < 2*N; i += 2)
      if (S.comp[i] == S.comp[i^1]) return {};
    vi tmp(2*N); each(i,S.comps) if (!tmp[i])
      tmp[i] = 1, tmp[S.comp[i^1]] = -1;
    vb ans(N); F0R(i,N) ans[i] = tmp[S.comp[2*i]] == 1;
    return ans;
  }
};
```

7.4 DSU

DSU.h

Description: Disjoint Set Union with path compression and union by size. Add edges and test connectivity. Use for Kruskal's or Boruvka's minimum spanning tree.

Time: $\mathcal{O}(\alpha(N))$

a876ce, 12 lines

```
struct DSU {
  vector<int> e;
  DSU(int N) { e = vector<int>(N, -1); }
  int get(int x) { return e[x] < 0 ? x : e[x] = get(e[x]); }
  bool sameSet(int a, int b) { return get(a) == get(b); }
  int size(int x) { return -e[get(x)]; }
  bool unite(int x, int y) { // union by size
    x = get(x), y = get(y); if (x == y) return false;
    if (e[x] > e[y]) swap(x, y);
    e[x] += e[y]; e[y] = x; return true;
  }
};
```

Kruskal.h

Description: Computes the weight of a Minimum Spanning Tree (MST) using Kruskal's algorithm. Requires DSU.

Time: $\mathcal{O}(M \log M)$, where M = number of edges

"DSU.h" f6c35c, 8 lines

```
template<class T> T Kruskal(int N, vector<array<T, 3>> eg) {
  ranges::sort(eg);
  T ans = 0; DSU D(N);
  for (auto &[w, u, v] : eg) {
    if (D.unite(u, v)) ans += w;
  }
  return ans;
};
```

DSUrb.h

Description: Disjoint Set Union with Rollback

d21534, 25 lines

```
struct DSUrb {
  vector<int> e;
  vector<array<int,4>> mod;
  DSUrb(int n) { e = vector<int>(n,-1); }
  int get(int x) { return e[x] < 0 ? x : get(e[x]); }
  bool sameSet(int a, int b) { return get(a) == get(b); }
  int size(int x) { return -e[get(x)]; }
  bool unite(int x, int y) { // union-by-rank
    x = get(x), y = get(y);
    if (x == y) {
      mod.push_back({-1,-1,-1,-1});
      return 0;
    }
    if (e[x] > e[y]) swap(x,y);
    mod.push_back({x,y,e[x],e[y]});
    e[x] += e[y]; e[y] = x; return 1;
  }
  void rollback() {
    auto a = mod.back(); mod.pop_back();
    if (a[0] != -1) {
      e[a[0]] = a[2];
      e[a[1]] = a[3];
    }
  }
};
```

Geometry (8)

8.1 Primitives

PointShort.h

Description: Use in place of complex<T>.

268ecf, 36 lines

```
using T = db; // or ll
const T EPS = 1e-9; // adjust as needed
using P = pair<T,T>; using vP = V<P>; using Line = pair<P,P>;
int sgn(T a) { return (a>EPS)-(a<-EPS); }
T sq(T a) { return a*a; }

T abs2(P p) { return sq(p.f)+sq(p.s); }
T abs(P p) { return sqrt(abs2(p)); }
T arg(P p) { return atan2(p.s,p.f); }
P conj(P p) { return P(p.f,-p.s); }
P perp(P p) { return P(-p.s,p.f); }
P dir(T ang) { return P(cos(ang),sin(ang)); }

P operator+(P l, P r) { return P(l.f+r.f,l.s+r.s); }
P operator-(P l, P r) { return P(l.f-r.f,l.s-r.s); }
P operator*(P l, T r) { return P(l.f*r,l.s*r); }
P operator/(P l, T r) { return P(l.f/r,l.s/r); }
P operator*(P l, P r) { // complex # multiplication
```

```
  return P(l.f*r.f-l.s*r.s,l.s*r.f+l.f*r.s); }
P operator/(P l, P r) { return l*conj(r)/abs2(r); }

P unit(const P& p) { return p/abs(p); }
T dot(const P& a, const P& b) { return a.f*b.f+a.s*b.s; }
T dot(const P& p, const P& a, const P& b) { return dot(a-p,b-p)
  ↪; }
T cross(const P& a, const P& b) { return a.f*b.s-a.s*b.f; }
T cross(const P& p, const P& a, const P& b) {
  return cross(a-p,b-p); }
P reflect(const P& p, const Line& l) {
  P a = l.f, d = l.s-l.f;
  return a+conj((p-a)/d)*d; }
P foot(const P& p, const Line& l) {
  return (p+reflect(p,l))/(T)2; }
bool onSeg(const P& p, const Line& l) {
  return sgn(cross(l.f,l.s,p)) == 0 && sgn(dot(p,l.f,l.s)) <= 0
    ↪; }
ostream& operator<<(ostream& os, P p) {
  return os << "(" << p.f << "," << p.s << ")"; }
```

Strings (9)

9.1 Light

KMP.h

Description: f[i] is length of the longest proper suffix of the i-th prefix of s that is a prefix of s

Time: $\mathcal{O}(N)$

4538e4, 13 lines

```
vi kmp(str s) {
  int N = sz(s); vi f(N+1); f[0] = -1;
  FOR(i,1,N+1) {
    for (f[i]=f[i-1];f[i]!=-1&&s[f[i]]!=s[i-1];)f[i]=f[f[i]];
    ++f[i]; }
  return f;
}
vi getOc(str a, str b) { // find occurrences of a in b
  vi f = kmp(a+"@"+b), ret;
  FOR(i,sz(a),sz(b)+1) if (f[i+sz(a)+1] == sz(a))
    ret.pb(i-sz(a));
  return ret;
}
```

Z.h

Description: f[i] is the max len such that s.substr(0,len) == s.substr(i,len)

Time: $\mathcal{O}(N)$

566170, 15 lines

```
vi z(str s) {
  int N = sz(s), L = 1, R = 0; s += '#';
  vi ans(N); ans[0] = N;
  FOR(i,1,N) {
    if (i <= R) ans[i] = min(R-i+1,ans[i-L]);
    while (s[i+ans[i]] == s[ans[i]]) ++ans[i];
    if (i+ans[i]-1 > R) L = i, R = i+ans[i]-1;
  }
  return ans;
}
vi getPrefix(str a, str b) { // find prefixes of a in b
  vi t = z(a+b); t = vi(sz(a)+all(t));
  each(u,t) ckmin(u,sz(a));
  return t;
}
```

Various (10)

10.1 Binary search

fstTrue.h

Description: Finds the first value x in [lo, hi] such that predicate f(x) is true.

Usage: int ans = fstTrue()

Time: $\mathcal{O}(\log(hi - lo))$

2654fa, 9 lines

```
template<typename T, typename U> T fstTrue(T lo, T hi, U f) {
  ++hi;
  assert(lo <= hi);
  while (lo < hi) {
    T mid = lo + (hi - lo) / 2;
    f(mid) ? hi = mid : lo = mid + 1;
  }
  return lo;
};
```

lstTrue.h

Description: Finds the last value x in [lo, hi] such that predicate f(x) is true.

Usage: int ans = fstTrue()

Time: $\mathcal{O}(\log(hi - lo))$

8aff85, 9 lines

```
template<typename T, typename U> T lstTrue(T lo, T hi, U f) {
  --lo;
  assert(lo <= hi);
  while (lo < hi) {
    T mid = lo + (hi - lo + 1) / 2;
    f(mid) ? lo = mid : hi = mid - 1;
  }
  return lo;
};
```

realTrue.h

Description: Binary search on continuous (floating-point) domain.

Usage: int ans = realTrue()

Time: $\mathcal{O}(\log(precision))$ — write 100 iterations and have a relax

691e1d, 7 lines

```
template<typename T, typename U>T realTrue(T lo, T hi, U f) {
  for (int i = 0; i < 100; ++i) {
    T mid = 0.5 * (hi + lo);
    f(mid) ? lo = mid : hi = mid;
  }
  return lo;
};
```

10.2 Dynamic programming

When doing DP on intervals:
$a[i][j] = \min_{i<k<j}(a[i][k] + a[k][j]) + f(i,j)$, where the (minimal) optimal k increases with both i and j,

- one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $a[i][j]$ only between $p[i][j-1]$ and $p[i+1][j]$.
- This is known as Knuth DP. Sufficient criteria for this are if $f(b,c) \le f(a,d)$ and $f(a,c) + f(b,d) \le f(a,d) + f(b,c)$ for all $a \le b \le c \le d$.
- Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.

10.3 Debugging tricks

- signal(SIGSEGV, [](int) { _Exit(0); }); converts segfaults into Wrong Answers. Similarly one can catch SIGABRT (assertion failures) and SIGFPE (zero divisions). _GLIBCXX_DEBUG violations generate SIGABRT (or SIGSEGV on gcc 5.4.0 apparently).
- feenableexcept(29); kills the program on NaNs (1), 0-divs (4), infinities (8) and denormals (16).

10.4 Optimization tricks

10.4.1 Bit hacks

- x & -x is the least bit in x.
- for (int x = m; x;) { --x &= m; ... } loops over all subset masks of m (except m itself).
- c = x&-x, r = x+c; (((r^x) >> 2)/c) | r is the next number after x with the same number of bits set.
- F0R(b,k) F0R(i,1<<K) if (i&1<<b) D[i] += D[i^(1<<b)]; computes all sums of subsets.

10.4.2 Pragmas

- #pragma GCC optimize ("Ofast") will make GCC auto-vectorize for loops and optimizes floating points better (assumes associativity and turns off denormals).
- #pragma GCC target ("avx,avx2") can double performance of vectorized code, but causes crashes on old machines. Also consider older #pragma GCC target ("sse4").
- #pragma GCC optimize ("trapv") kills the program on integer overflows (but is really slow).

FastInput.h

Description: Read an integer from stdin. Usage requires your program to pipe in input from file.

Usage: ./a.out < input.txt

Time: About 5x as fast as cin/scanf.

7b3c70, 17 lines

```
inline char gc() { // like getchar()
  static char buf[1 << 16];
  static size_t bc, be;
  if (bc >= be) {
    buf[0] = 0, bc = 0;
    be = fread(buf, 1, sizeof(buf), stdin);
  }
  return buf[bc++]; // returns 0 on EOF
}

int readInt() {
  int a, c;
  while ((a = gc()) < 40);
  if (a == '-') return -readInt();
  while ((c = gc()) >= 48) a = a * 10 + c - 480;
  return a - 48;
}
```

10.5 Miscellaneous

GridDirection.h

Description: Predefined direction for grid-based problems. Includes common movement sets for 2D and 3D grids such as 4-directional, 8-directional, diagonal-only, knight moves, 3D etc.

efed5e, 22 lines

```
// Down, Right, Up, Left
const int dx[4] = {1, 0, -1, 0};
const int dy[4] = {0, 1, 0, -1};
const char dc[4] = {'D', 'R', 'U', 'L'};

// Down, Down-Right, Right, Up-Right, Up, Up-Left, Left, Down-
   ↪Left
const int dx8[8] = {1, 1, 0, -1, -1, -1, 0, 1};
const int dy8[8] = {0, 1, 1, 1, 0, -1, -1, -1};

// Diagonals only
const int dxDiag[4] = {1, 1, -1, -1};
const int dyDiag[4] = {1, -1, 1, -1};

// Knight moves
const int dxK[8] = {2, 2, 1, 1, -1, -1, -2, -2};
const int dyK[8] = {1, -1, 2, -2, 2, -2, 1, -1};

// 3D Grid
const int dx[6] = {1, -1, 0, 0, 0, 0};
const int dy[6] = {0, 0, 1, -1, 0, 0};
const int dz[6] = {0, 0, 0, 0, 1, -1};
```