Northern University of Bussiness and Technology Khulna

NUBTK $(^w^)$

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Contest (1)

template.cpp 13 lines

```cpp
#include <bits/stdc++.h>
using namespace std;

#ifdef LOCAL
#include "debug.h"
#else
#define dbg(...)
#endif

int32_t main() {
  cin.tie(0)->sync_with_stdio(0);
  cin.exceptions(cin.failbit);
}
```

debug.h 16 lines

```cpp
template <typename A, typename B>
ostream &operator<<(ostream &os, const pair<A, B> &p) {
  return os << '(' << p.first << ", " << p.second << ')';
}
template <typename C, typename T = typename enable_if<!is_same<
    ↪C, string>::value, typename C::value_type>::type>
ostream &operator<<(ostream &os, const C &v) {
  os << '{';
  string sep;
  for (const T &x : v) os << sep << x, sep = ", ";
  return os << '}';
}
void dbg_out() { cerr << " |\n"; }
template<typename Head, typename... Tail> void dbg_out(Head H,
    ↪Tail... T) {
  cerr << " | " << H; dbg_out(T...);
}
#define dbg(...) cerr << "[" << #__VA_ARGS__ << "] =", dbg_out(
    ↪__VA_ARGS__)
```

gen.h 2 lines

```cpp
#define uid(l, r) uniform_int_distribution<int>(l, r)(rng)
mt19937 rng(chrono::steady_clock::now().time_since_epoch().
    ↪count());
```

.bashrc 5 lines

```
# write every function in single line
alias clr="printf '\33c'"
co() { g++ -std=c++23 -O2 -Wall -Wextra
 -Wshadow -Wconversion -o $1 $1.cpp; }
run() { co $1 && ./$1; }
```

hash.sh 3 lines

```
# Hash file ignoring whitespace and comments. Verifies that
# code was correctly typed. Usage: 'sh hash.sh < A.cpp'
cpp -dD -P -fpreprocessed|tr -d '[:space:]'|md5sum|cut -c-6
```

stress.sh 20 lines

```
#!/usr/bin/env bash

for ((testNum=0;testNum<$4;testNum++))
do
    "./$3".out > input
    "./$2".out < input > outSlow
    "./$1".out < input > outWrong
    if !(cmp -s "outWrong" "outSlow")
    then
        echo "Error found!"
        echo "Input:"
        cat input
        echo "Wrong Output:"
        cat outWrong
        echo "Slow Output:"
        cat outSlow
        exit
    fi
done
echo Passed $4 tests
```

troubleshoot.txt 75 lines

```
General:
* Write down most of your thoughts, even if you're not sure
whether they're useful.
* Give your variables (and files) meaningful names.
* Stay organized and don't leave papers all over the place!
* You should know what your code is doing ...

Pre-submit:
* Write a few simple test cases if sample is not enough.
* Are time limits close? If so, generate max cases.
* Is the memory usage fine?
* Could anything overflow?
* Remove debug output.
* Make sure to submit the right file.

Wrong answer:
* Print your solution! Print debug output as well.
* Read the full problem statement again.
* Have you understood the problem correctly?
* Are you sure your algorithm works?
* Try writing a slow (but correct) solution.
* Can your algorithm handle the whole range of input?
* Did you consider corner cases (ex. n=1)?
* Is your output format correct? (including whitespace)
* Are you clearing all data structures between test cases?
* Any uninitialized variables?
* Any undefined behavior (array out of bounds)?
* Any overflows or NaNs (or shifting ll by >=64 bits)?
* Confusing N and M, i and j, etc.?
* Confusing ++i and i++?
* Return vs continue vs break?
* Are you sure the STL functions you use work as you think?
* Add some assertions, maybe resubmit.
* Create some test cases to run your algorithm on.
* Go through the algorithm for a simple case.
* Go through this list again.
* Explain your algorithm to a teammate.
* Ask the teammate to look at your code.
* Go for a small walk, e.g. to the toilet.
* Rewrite your solution from the start or let a teammate do it.

Geometry:
* Work with ints if possible.
* Correctly account for numbers close to (but not) zero.
    ↪Related:
for functions like acos make sure absolute val of input is not
(slightly) greater than one.
* Correctly deal with vertices that are collinear, concyclic,
coplanar (in 3D), etc.
* Subtracting a point from every other (but not itself)?

Runtime error:
* Have you tested all corner cases locally?
* Any uninitialized variables?
* Are you reading or writing outside the range of any vector?
* Any assertions that might fail?
* Any possible division by 0? (mod 0 for example)
* Any possible infinite recursion?
* Invalidated pointers or iterators?
* Are you using too much memory?
* Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:
* Do you have any possible infinite loops?
* What's your complexity? Large TL does not mean that something
simple (like NlogN) isn't intended.
* Are you copying a lot of unnecessary data? (References)
* Avoid vector, map. (use arrays/unordered_map)
* How big is the input and output? (consider FastIO)
* What do your teammates think about your algorithm?
* Calling count() on multiset?

Memory limit exceeded:
* What is the max amount of memory your algorithm should need?
* Are you clearing all data structures between test cases?
* If using pointers try BumpAllocator.
```

Mathematics (2)

2.1 Equations

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The extremum is given by $x = -b/2a$.

$$\begin{matrix} ax + by = e \\ cx + dy = f \end{matrix} \Rightarrow \begin{matrix} x = \frac{ed - bf}{ad - bc} \\ y = \frac{af - ec}{ad - bc} \end{matrix}$$

In general, given an equation $Ax = b$, the solution to a variable x_i is given by

$$x_i = \frac{\det A_i'}{\det A}$$

where A_i' is A with the i'th column replaced by b.

2.2 Recurrences

If $a_n = c_1 a_{n-1} + \cdots + c_k a_{n-k}$, and $r_1, \ldots, r_k$ are distinct roots of $x^k - c_1 x^{k-1} - \cdots - c_k$, there are $d_1, \ldots, d_k$ s.t.

$$a_n = d_1 r_1^n + \cdots + d_k r_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1 n + d_2) r^n$.

2.3 Trigonometry

$$\sin(v+w) = \sin v \cos w + \cos v \sin w$$
$$\cos(v+w) = \cos v \cos w - \sin v \sin w$$

$$\tan(v+w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$
$$\sin v + \sin w = 2 \sin \frac{v+w}{2} \cos \frac{v-w}{2}$$
$$\cos v + \cos w = 2 \cos \frac{v+w}{2} \cos \frac{v-w}{2}$$

$$(V+W)\tan(v-w)/2 = (V-W)\tan(v+w)/2$$

where V, W are lengths of sides opposite angles v, w.

$$a \cos x + b \sin x = r \cos(x - \phi)$$
$$a \sin x + b \cos x = r \sin(x + \phi)$$

where $r = \sqrt{a^2 + b^2}, \phi = \operatorname{atan2}(b, a)$.

2.4 Geometry

2.4.1 Triangles

Side lengths: a, b, c

Semiperimeter: $p = \dfrac{a+b+c}{2}$

Area: $A = \sqrt{p(p-a)(p-b)(p-c)}$

Circumradius: $R = \dfrac{abc}{4A}$

Inradius: $r = \dfrac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two):

$$s_a = \sqrt{bc\left[1 - \left(\frac{a}{b+c}\right)^2\right]}$$

Law of sines: $\dfrac{\sin\alpha}{a} = \dfrac{\sin\beta}{b} = \dfrac{\sin\gamma}{c} = \dfrac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos\alpha$

Law of tangents: $\dfrac{a+b}{a-b} = \dfrac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}}$

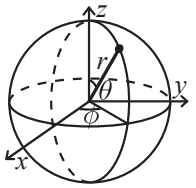
2.4.2 Quadrilaterals

With side lengths a, b, c, d, diagonals e, f, diagonals angle θ, area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin\theta = F \tan\theta = \sqrt{4e^2 f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is $180°$, $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

2.4.3 Spherical coordinates



$$x = r\sin\theta\cos\phi \qquad r = \sqrt{x^2+y^2+z^2}$$
$$y = r\sin\theta\sin\phi \qquad \theta = \operatorname{acos}(z/\sqrt{x^2+y^2+z^2})$$
$$z = r\cos\theta \qquad \phi = \operatorname{atan2}(y, x)$$

2.5 Derivatives/Integrals

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\tan x = 1 + \tan^2 x \qquad \frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$
$$\int \tan ax = -\frac{\ln|\cos ax|}{a} \qquad \int x\sin ax = \frac{\sin ax - ax\cos ax}{a^2}$$
$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2}\operatorname{erf}(x) \qquad \int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1)$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

2.6 Sums

$$c^a + c^{a+1} + \cdots + c^b = \frac{c^{b+1} - c^a}{c-1}, c \neq 1$$

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$
$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(2n+1)(n+1)}{6}$$
$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$
$$1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

2.7 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots, (-\infty < x < \infty)$$
$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots, (-1 < x \leq 1)$$
$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots, (-1 \leq x \leq 1)$$
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots, (-\infty < x < \infty)$$
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots, (-\infty < x < \infty)$$

2.8 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x. It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x x p_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y,

$$V(aX + bY) = a^2 V(X) + b^2 V(Y).$$

2.8.1 Discrete distributions

Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\operatorname{Bin}(n, p)$, $n = 1, 2, \ldots$, $0 \leq p \leq 1$.

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu = np,\ \sigma^2 = np(1-p)$$

$\operatorname{Bin}(n, p)$ is approximately $\operatorname{Po}(np)$ for small p.

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each which yields success with probability p is $\text{Fs}(p)$, $0 \le p \le 1$.

$$p(k) = p(1-p)^{k-1},\ k = 1, 2, \ldots$$

$$\mu = \frac{1}{p},\ \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\text{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda}\frac{\lambda^k}{k!}, k = 0, 1, 2, \ldots$$

$$\mu = \lambda,\ \sigma^2 = \lambda$$

2.8.2 Continuous distributions

Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $\text{U}(a, b)$, $a < b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2},\ \sigma^2 = \frac{(b-a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is $\text{Exp}(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \ge 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda},\ \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu, \sigma^2)$, $\sigma > 0$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

2.9 Markov chains

A *Markov chain* is a discrete random process with the property that the next state depends only on the current state. Let $X_1, X_2, \ldots$ be a sequence of random variables generated by the Markov process. Then there is a transition matrix $\mathbf{P} = (p_{ij})$, with $p_{ij} = \Pr(X_n = i | X_{n-1} = j)$, and $\mathbf{p}^{(n)} = \mathbf{P}^n\mathbf{p}^{(0)}$ is the probability distribution for X_n (i.e., $p_i^{(n)} = \Pr(X_n = i)$), where $\mathbf{p}^{(0)}$ is the initial distribution.

Stationary distribution

π is a stationary distribution if $\pi = \pi\mathbf{P}$. If the Markov chain is *irreducible* (it is possible to get to any state from any state), then $\pi_i = \frac{1}{\mathbb{E}(T_i)}$ where $\mathbb{E}(T_i)$ is the expected time between two visits in state i. π_j/π_i is the expected number of visits in state j between two visits in state i.

For a connected, undirected and non-bipartite graph, where the transition probability is uniform among all neighbors, π_i is proportional to node i's degree.

Ergodicity

A Markov chain is *ergodic* if the asymptotic distribution is independent of the initial distribution. A finite Markov chain is ergodic iff it is irreducible and *aperiodic* (i.e., the gcd of cycle lengths is 1). $\lim_{k\to\infty}\mathbf{P}^k = \mathbf{1}\pi$.

Data Structures (3)

3.1 STL

MapComparator.h

Description: example of function object (functor) for map or set
Usage: `set<int,cmp> s; map<int,int,cmp> m;`

5bfa6c, 1 lines

```
struct cmp{bool operator()(int l,int r)const{return l>r;}};
```

HashMap.h

Description: Hash map with similar API as unordered_map. Initial capacity must be a power of 2 if provided.
Usage: `gp_hash_table<int, int, chash> h({},{},{},{},{1<<16});`
Memory: ~1.5x unordered map
Time: ~3x faster than unordered map

<ext/pb_ds/assoc_container.hpp> b6e07b, 6 lines

```
using namespace __gnu_pbds;
struct chash {
  const uint64_t C = 4e18*acos(0)+71;
  const int RANDOM = chrono::steady_clock::now().
    ↪time_since_epoch().count();
  int64_t operator()(int64_t x) const { return
    ↪__builtin_bswap64((x^RANDOM)*C); }
};
```

OrderStatisticTree.h

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. Change `null_type` to get a map.
Time: $\mathcal{O}(\log N)$

<ext/pb_ds/assoc_container.hpp> 257533, 12 lines

```
using namespace __gnu_pbds;
template<class T> using Tree = tree<T, null_type, less<T>,
  rb_tree_tag, tree_order_statistics_node_update>;
#define ook order_of_key // position of the given element
#define fbo find_by_order // return an iterator

int atMost(Tree<pair<int, int>>& T, int r) {
  return T.order_of_key({r,MOD});
}
int getSum(Tree<pair<int, int>>& T, int l, int r) {
  return atMost(T,r)-atMost(T,l-1);
}
```

3.2 1D Range Queries

SparseTable.h

Description: Static 1D range (min/max/gcd/lcm/or/and) query. If TL is an issue, use arrays instead of vectors and store values instead of indices.
Memory: $\mathcal{O}(N \log N)$
Time: $\mathcal{O}(1)$

471d4b, 19 lines

```
template<class T> struct SparseTable {
  int N, LOG;
  vector<vector<T>> jmp;
  T cmb(T a, T b) { return min(a, b); }
  SparseTable(const vector<T>& v) {
    N = v.size(); LOG = __lg(N);
    jmp.resize(N, vector<T>(LOG + 1));
    for (int i = 0; i < N; ++i) jmp[i][0] = v[i];
    for (int j = 1; j <= LOG; ++j) {
      for (int i = 0; i + (1<<j) <= N; ++i) {
        jmp[i][j] = cmb(jmp[i][j-1], jmp[i+(1<<(j-1))][j-1]);
      }
    }
  }
  T query(int l, int r) {
    int d = __lg(r-l+1);
    return cmb(jmp[l][d], jmp[r-(1<<d)+1][d]);
  }
};
```

FenwickTree.h

Description: Computes partial sums a[0] + a[1] + ... + a[pos - 1], and updates single elements a[i], taking the difference between the old and new value. 0-Indexed.
Time: Both operations are $\mathcal{O}(\log N)$.

122df3, 23 lines

```
template<class T> struct FenwickTree {
  int N; vector<T> bit;
  FenwickTree(int _N) { N = _N; bit.resize(N); }
  void add(int p, T x) {
    for (++p; p <= N; p += p&-p) bit[p-1] += x;
  }
  T sum(int l, int r) { return sum(r+1)-sum(l); }
  T sum(int r) {
    T res = 0;
    for(; r; r -= r&-r) res += bit[r-1];
    return res;
  }
  int lower_bound(T sum) {
    if (sum <= 0) return -1;
    int pos = 0;
    for (int pw = 1<<25; pw; pw >>= 1) {
      int npos = pos+pw;
      if (npos <= N && bit[npos-1] < sum)
        pos = npos, sum -= bit[pos-1];
    }
    return pos;
  }
};
```

SegmentTree.h
Description: 1D point update and range query where cmb is any associative operation. seg[1]==query(0,N-1).
Time: $\mathcal{O}(\log N)$
749f3e, 29 lines

```
template<class T> struct SegTree {
  const T ID{};
  T cmb(T a, T b) { return a + b; }
  int n; vector<T> seg;
  SegTree(int _n) {
    for (n = 1; n < _n; ) n *= 2;
    seg.assign(2*n, ID);
  }
  void pull(int p) { seg[p] = cmb(seg[2*p], seg[2*p+1]); }
  void update(int p, T val) { // set val at position p
    seg[p += n] = val;
    for (p /= 2; p; p /= 2) pull(p);
  }
  T query(int l, int r) { // zero-indexed, inclusive
    T ra = ID, rb = ID;
    for (l += n, r += n + 1; l < r; l /= 2, r /= 2) {
      if (l & 1) ra = cmb(ra, seg[l++]);
      if (r & 1) rb = cmb(seg[--r], rb);
    }
    return cmb(ra, rb);
  }
  // int first_at_least(int lo, int val, int ind, int l, int r)
     ↪ { // if seg stores max across range
  //   if (r < lo || val > seg[ind]) return -1;
  //   if (l == r) return l;
  //   int m = (l+r)/2;
  //   int res = first_at_least(lo,val,2*ind,l,m); if (res !=
     ↪-1) return res;
  //   return first_at_least(lo,val,2*ind+1,m+1,r);
  // }
};
```

LazySegmentTree.h
Description: 1D range increment and sum query.
Usage: LazySeg<int64_t, 1<<20> T; T.update(l, r, val);
Time: $\mathcal{O}(\log N)$
e0742f, 42 lines

```
template<class T, int SZ>struct LazySeg {
  // SZ must be power of 2
  static_assert(__builtin_popcount(SZ) == 1);
  const T ID{};
  T cmb(T a, T b) { return a + b; }
  T seg[2*SZ], lazy[2*SZ];
  LazySeg() {
    for (int i = 0; i < 2*SZ; ++i) seg[i] = lazy[i] = ID;
  }
  // modify values for current node
  void push(int ind, int L, int R) {
    seg[ind] += (R-L+1) * lazy[ind]; // dependent on operation
    if (L != R) for (int i = 0; i < 2; ++i) lazy[2*ind+i] +=
       ↪lazy[ind];
    lazy[ind] = 0;
  }
  void pull(int ind) {
    seg[ind] = cmb(seg[2*ind], seg[2*ind+1]);
  }
  void build() {
    for (int i = SZ-1; i >= 1; --i) pull(i);
  }
  void update(int lo, int hi, T inc, int ind = 1, int L = 0,
     ↪int R = SZ - 1) {
    push(ind, L, R);
    if (hi < L || R < lo) return;
    if (lo <= L && R <= hi) {
      lazy[ind] = inc;
      push(ind, L, R);
      return;
    }
    int M = (L + R) / 2;
    update(lo, hi, inc, 2*ind, L, M);
    update(lo, hi, inc, 2*ind+1, M+1, R);
    pull(ind);
  }
  T query(int lo, int hi, int ind = 1, int L = 0, int R = SZ -
     ↪1) {
    push(ind, L, R);
    if (lo > R || L > hi) return ID;
    if (lo <= L && R <= hi) return seg[ind];
    int M = (L + R) / 2;
    return cmb(query(lo, hi, 2*ind, L, M), query(lo, hi, 2*ind
       ↪+1, M+1, R));
  }
};
```

3.3 2D Range Queries

PrefixSum2D.h
Description: calculates rectangle sums in constant time
65b070, 17 lines

```
template<typename T> struct PrefixSum2D {
  vector<vector<T>> sum;
  PrefixSum2D(const vector<vector<T>> &v) {
    int n = v.size(), m = v[0].size();
    sum.assign(n+1, vector<T>(m+1, T{}));
    for (int i = 0; i < n; ++i) {
      for (int j = 0; j < m; ++j) {
        sum[i+1][j+1] = v[i][j]
        - sum[i][j] + sum[i+1][j] + sum[i][j+1];
      }
    }
  }
  T query(int x1, int y1, int x2, int y2) {
    return sum[x2][y2] + sum[x1-1][y1-1]
    - sum[x1-1][y2] - sum[x2][y1-1];
  }
};
```

Number Theory (4)

4.1 Modular Arithmetic

Modular arithmetic operations are defined as follows:
Addition: $(a + b) \bmod m$
Subtraction: $((a - b) \bmod m + m) \bmod m$ (ensures non-negative result)
Multiplication: $(a \cdot b) \bmod m$
Division: $a/b \bmod m$ is defined as $a \cdot b^{-1} \bmod m$ if $\gcd(b, m) = 1$, otherwise it is impossible.
Modular inverse: $b \cdot b^{-1} \equiv 1 \pmod m$

ModIntShort.h
Description: Modular arithmetic. Assumes MOD is prime.
Usage: mint a = MOD+5; inv(a); // 400000003
7c1e94, 30 lines

```
struct mint {
  int v;
  static const int MOD = 1E9 + 7;
  explicit operator int() const { return v; }
  mint() : v(0) {}
  mint(int64_t _v) : v(int(_v % MOD)) { v += (v < 0) * MOD; }
  mint &operator+=(mint o) {
    if ((v += o.v) >= MOD) v -= MOD;
    return *this;
  }
  mint &operator-=(mint o) {
    if ((v -= o.v) < 0) v += MOD;
    return *this;
  }
  mint &operator*=(mint o) {
    v = int((int64_t)v * o.v % MOD);
    return *this;
  }
  friend mint pow(mint a, int64_t p) {
    assert(p >= 0);
    return p==0? 1:pow(a*a, p/2)*(p&1? a:1);
  }
  friend mint inv(mint a) {
    assert(a.v != 0);
    return pow(a, MOD - 2);
  }
  friend mint operator+(mint a, mint b) { return a += b; }
  friend mint operator-(mint a, mint b) { return a -= b; }
  friend mint operator*(mint a, mint b) { return a *= b; }
};
```

ModMulLL.h
Description: Multiply two 64-bit integers mod another if 128-bit is not available. modMul is equivalent to (ul)(__int128(a)*b%mod). Works for $0 \le a, b < mod < 2^{63}$. NOTE : Only works if system supports 80-bit floating point.
d22eee, 10 lines

```
using ul = uint64_t;
ul modMul(ul a, ul b, const ul mod){
  return __int128(a) * __int128(b) % mod;
}
ul modPow(ul a, ul b, const ul mod) {
  if (b == 0) return 1;
  ul res = modPow(a, b/2, mod);
  res = modMul(res, res, mod);
  return b&1 ? modMul(res, a, mod) : res;
}
```

FastMod.h
Description: Barrett reduction computes $a\%b$ about 4 times faster than usual where $b > 1$ is constant but not known at compile time. Division by b is replaced by multiplication by m and shifting right 64 bits.
ce43ec, 9 lines

```
using ul = uint64_t;
struct FastMod {
  ul b, m;
  FastMod(ul b) : b(b), m(-1ULL / b) {}
  ul reduce(ul a) {
    ul q = (ul)((__uint128_t(m) * a) >> 64), r = a - q*b;
    return r - (r>=b) * b;
  }
};
```

4.2 Primality

4.2.1 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

4.2.2 Divisors

$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

Dirichlet Convolution: Given a function $f(x)$, let

$(f * g)(x) = \sum_{d|x} g(d) f(x/d)$.

If the partial sums $s_{f*g}(n), s_g(n)$ can be computed in $O(1)$ and $s_f(1 \dots n^{2/3})$ can be computed in $O\left(n^{2/3}\right)$ then all $s_f\left(\frac{n}{d}\right)$ can as well. Use

$$s_{f*g}(n) = \sum_{d=1}^{n} g(d) s_f(n/d).$$

If $f(x) = \mu(x)$ then $g(x) = 1, (f * g)(x) = (x == 1)$, and $s_f(n) = 1 - \sum_{i=2}^{n} s_f(n/i)$.

If $f(x) = \phi(x)$ then $g(x) = 1, (f * g)(x) = x$, and $s_f(n) = \frac{n(n+1)}{2} - \sum_{i=2}^{n} s_f(n/i)$.

Sieve.h

Description: Tests primality up to SZ. Runs faster if only odd indices are stored.

Time: $\mathcal{O}(SZ \log \log SZ)$ or $\mathcal{O}(SZ)$ 935b91, 22 lines

```
template<int SZ> struct Sieve {
  bitset<SZ> is_prime; vector<int> primes;
  Sieve() {
    is_prime.set(); is_prime[0] = is_prime[1] = 0;
    for (int i = 4; i < SZ; i += 2) is_prime[i] = 0;
    for (int i = 3; i*i < SZ; i += 2) if (is_prime[i])
      for (int j = i*i; j < SZ; j += i*2) is_prime[j] = 0;
    for (int i = 0; i < SZ; i++) {
      if (is_prime[i]) primes.push_back(i);
    }
  }
  // array<int, SZ> spf{} // smallest prime that divides
  // Sieve() { // above is faster
  //  for (int i = 2; i < SZ; i++) {
  //    if (spf[i] == 0) spf[i] = i, primes.push_back(i);
  //    for (int p: primes) {
  //      if (p > spf[i] || i*p >= SZ) break;
  //      spf[i*p] = p;
  //    }
  //  }
  // }
};
```

FactorBasic.h

Description: Factors integers.

Time: $\mathcal{O}\left(\sqrt{N}\right)$ 91f34a, 38 lines

```
inline namespace factorBasic {
template<class T> vector<pair<T,int>> factor(T x) {
  vector<pair<T,int>> primes;
  for (T i = 2; i*i <= x; ++i) if (x % i == 0) {
    int t = 0;
    while (x % i == 0) x /= i, t++;
    primes.pb({i,t});
  }
  if (x > 1) primes.push_back({x,1});
  return primes;
}
/* Note:
 * number of operations needed s.t.
 *          phi(phi(...phi(n)...))=1
 * is O(log n).
 * Euler's theorem: a^{\phi(p)}\equiv 1 (mod p), gcd(a,p)=1
 */
ll phi(ll x) {
  each(a,factor(x)) x -= x/a.f;
  return x;
}
template<class T> void tour(vector<pair<T,int>>& v,
  vector<T>& V, int ind, T cur) {
    if (ind == sz(v)) V.pb(cur);
    else {
      T mul = 1;
      F0R(i,v[ind].s+1) {
        tour(v,V,ind+1,cur*mul);
        mul *= v[ind].f;
      }
    }
  }
template<class T> vector<T> getDivi(T x) {
  auto v = factor(x);
  vector<T> V; tour(v,V,0,(T)1); sort(all(V));
  return V;
}
}
```

PrimeCnt.h

Description: Counts number of primes up to N. Can also count sum of primes.

Time: $\mathcal{O}\left(N^{3/4}/\log N\right)$, 60ms for $N = 10^{11}$, 2.5s for $N = 10^{13}$ c04e96, 20 lines

```
ll count_primes(ll N) { // count_primes(1e13) == 346065536839
  if (N <= 1) return 0;
  int sq = (int)sqrt(N);
  vl big_ans((sq+1)/2), small_ans(sq+1);
  FOR(i,1,sq+1) small_ans[i] = (i-1)/2;
  F0R(i,sz(big_ans)) big_ans[i] = (N/(2*i+1)-1)/2;
  vb skip(sq+1); int prime_cnt = 0;
  for (int p = 3; p <= sq; p += 2) if (!skip[p]) { // primes
    for (int j = p; j <= sq; j += 2*p) skip[j] = 1;
    F0R(j,min((ll)sz(big_ans),(N/p/p+1)/2)) {
      ll prod = (ll)(2*j+1)*p;
      big_ans[j] -= (prod > sq ? small_ans[(double)N/prod]
            : big_ans[prod/2])-prime_cnt;
    }
    for (int j = sq, q = sq/p; q >= p; --q) for (;j >= q*p;--j)
      small_ans[j] -= small_ans[q]-prime_cnt;
    ++prime_cnt;
  }
  return big_ans[0]+1;
}
```

MillerRabin.h

Description: Deterministic primality test, works up to 2^{64}. For larger numbers, extend A randomly.

"ModMulLL.h" ef770a, 12 lines

```
bool prime(ul n) {
  if (n < 2 || n % 6 % 4 != 1) return n - 2 < 2;
  ul s = __builtin_ctzll(n-1), d = n >> s;
  ul A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022};
  for (auto &a : A) {
    ul p = modPow(a, d, n), i = s;
    while (p != 1 && p != n-1 && a % n && i--)
      p = modMul(p, p, n);
    if (p != n-1 && i != s) return false;
  }
  return true;
}
```

FactorFast.h

Description: Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).

Time: $\mathcal{O}\left(N^{1/4}\right)$, less for numbers with small factors

"MillerRabin.h", "ModMulLL.h" 99cf33, 16 lines

```
ul pollard(ul n) { // return some nontrivial factor of n
  auto f = [n](ul x) { return modMul(x, x, n) + 1; };
  ul x = 0, y = 0, t = 30, prd = 2, i = 1, q;
  while (t++ % 40 || gcd(prd, n) == 1) {
    if (x == y) x = ++i, y = f(x);
    if ((q = modMul(prd, max(x,y)-min(x,y), n))) prd = q;
    x = f(x), y = f(f(y));
  }
  return gcd(prd, n);
}
void factor_rec(ul n, map<ul,int>& cnt) {
  if (n == 1) return;
  if (prime(n)) { ++cnt[n]; return; }
  ul u = pollard(n);
  factor_rec(u,cnt), factor_rec(n/u,cnt);
}
```

4.3 Euclidean Algorithm

4.3.1 Bézout's identity

For $a \neq, b \neq 0$, then $d = \gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

Euclid.h

Description: Generalized Euclidean algorithm. `euclid` and `invGeneral` work for $A, B < 2^{62}$.

Time: $\mathcal{O}(\log AB)$ 8bab06, 13 lines

```
// For A,B>=0, finds (x,y) s.t.
pair<int64_t, int64_t> euclid(int64_t A, int64_t B) {
  // Ax+By=gcd(A,B), |Ax|,|By|<=AB/gcd(A,B)
  if (!B) return {1, 0};
  pair<int64_t, int64_t> p = euclid(B, A%B);
  return {p.second, p.first - A/B*p.second};
}
// find x in [0,B) such that Ax=1 mod B
int64_t invGeneral(int64_t A, int64_t B) {
  pair<int64_t, int64_t> p = euclid(A, B);
  assert(p.first*A + p.second*B == 1);
  return p.first + (p.first<0)*B;
} // must have gcd(A,B)=1
```

4.4 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \ b = k \cdot (2mn), \ c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

4.5 Lifting the Exponent

For $n > 0$, p prime, and ints x, y s.t. $p \nmid x, y$ and $p|x-y$:

- $p \neq 2$ or $p = 2, 4|x-y \implies v_p(x^n - y^n) = v_p(x-y) + v_p(n)$.
- $p = 2, 2|n \implies v_2(x^n - y^n) = v_2((x^2)^{n/2} - (y^2)^{n/2})$.

Combinatorial (5)

5.1 Permutations

5.1.1 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800

n	11	12	13	14	15	16	17
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14

n	20	25	30	40	50	100	150	171
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX

5.1.2 Cycles

Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n)\frac{x^n}{n!} = \exp\left(\sum_{n\in S}\frac{x^n}{n}\right)$$

5.1.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1)+D(n-2)) = nD(n-1)+(-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

5.1.4 Burnside's lemma

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|}\sum_{g\in G}|X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n}\sum_{k=0}^{n-1} f(\gcd(n,k)) = \frac{1}{n}\sum_{k|n} f(k)\phi(n/k).$$

5.2 Partitions and subsets

5.2.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1,\ p(n) = \sum_{k\in\mathbb{Z}\setminus\{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	∼2e5	∼2e8

5.2.2 Lucas' Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + ... + n_1 p + n_0$ and $m = m_k p^k + ... + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k}\binom{n_i}{m_i} \pmod p$.

5.3 General purpose numbers

5.3.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t - 1}$ (FFT-able).
$B[0, \ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} i^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k}B_k(n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty}\frac{B_k}{k!}f^{(k-1)}(m)$$
$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

5.3.2 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ c(0,0) = 1$$
$$\textstyle\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\ldots(x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

5.3.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$
$$E(n,0) = E(n,n-1) = 1$$
$$E(n,k) = \sum_{j=0}^{k}(-1)^j\binom{n+1}{j}(k+1-j)^n$$

5.3.4 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$
$$S(n,1) = S(n,n) = 1$$
$$S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n$$

5.3.5 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod p$$

5.3.6 Labeled unrooted trees

on n vertices: n^{n-2}
on k existing trees of size n_i: $n_1 n_2 \cdots n_k n^{k-2}$
with degrees d_i: $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$

5.3.7 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2}C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

5.4 Young Tableaux

Let a **Young diagram** have shape $\lambda = (\lambda_1 \geq \cdots \geq \lambda_k)$, where λ_i equals the number of cells in the i-th (left-justified) row from the top. A **Young tableau** of shape λ is a filling of the $n = \sum \lambda_i$ cells with a permutation of $1 \ldots n$ such that each row and column is increasing.

Hook-Length Formula: For the cell in position (i,j), let $h_\lambda(i,j) = |\{(I,J)|i \leq I, j \leq J, (I = i \text{ or } J = j)\}|$. The number of Young tableaux of shape λ is equal to $f^\lambda = \frac{n!}{\prod h_\lambda(i,j)}$.

Schensted's Algorithm: converts a permutation σ of length n into a pair of Young Tableaux $(S(\sigma), T(\sigma))$ of the same shape. When inserting $x = \sigma_i$,

1. Add x to the first row of S by inserting x in place of the largest y with $x < y$. If y doesn't exist, push x to the end of the row, set the value of T at that position to be i, and stop.

2. Add y to the second row using the same rule, keep repeating as necessary.

All pairs $(S(\sigma), T(\sigma))$ of the same shape correspond to a unique σ, so $n! = \sum (f^\lambda)^2$. Also, $S(\sigma^R) = S(\sigma)^T$.

Let $d_k(\sigma), a_k(\sigma)$ be the lengths of the longest subseqs which are a union of k decreasing/ascending subseqs, respectively. Then $a_k(\sigma) = \sum_{i=1}^{k} \lambda_i, d_k(\sigma) = \sum_{i=1}^{k} \lambda_i^*$, where λ_i^* is size of the i-th column.

Numerical (6)

6.1 Arithmetic

BigInt.h

Description: Big Integer

1cbd2c, 289 lines

```
// base and base_digits must be consistent
const int base = 1e9, base_digits = 9;
struct bigint { // value == 0 is represented by empty z
  vi z; // digits
  int sign; // sign == 1 <==> value >= 0
  bigint() : sign(1) {} // sign == -1 <==> value < 0
  bigint(ll v) { *this = v; }
  bigint &operator=(ll v) {
    sign = v < 0 ? -1 : 1; v *= sign; // make v positive
    z.clear(); for (;v;v/=base) z.pb(v%base);
    return *this;
  }
  bigint(const str &s) { read(s); } // add char by char

  bigint &operator+=(const bigint &other) {
    //dbg("ADDING",*this,other,sign,other.sign);
    if (sign == other.sign) {
      for (int i = 0, carry = 0; i < sz(other.z) || carry; ++i)
        ↪ {
        if (i == sz(z)) z.pb(0);
        z[i] += carry+(i<sz(other.z)?other.z[i]:0);
        carry = z[i] >= base; if (carry) z[i] -= base;
      }
    } else if (other != 0 /* prevent infinite loop */) *this -=
      ↪ -other;
    return *this;
  }
  friend bigint operator+(bigint a, const bigint &b) { return a
    ↪ += b; }
  bigint &operator-=(const bigint &other) {
    if (sign == other.sign) {
      if ((sign == 1 && *this >= other) || (sign == -1 && *this
        ↪ <= other)) {
        for (int i = 0, carry = 0; i < sz(other.z) || carry; ++
          ↪i) {
          z[i] -= carry+(i<sz(other.z)?other.z[i]:0);
          carry = z[i]<0; if (carry) z[i] += base;
        }
        trim();
      } else { // result will change sign
        *this = other-*this;
        this->sign = -this->sign;
      }
    } else *this += -other;
    return *this;
  }
  friend bigint operator-(bigint a, const bigint &b) { return a
    ↪ -= b; }

  bigint &operator*=(int v) { // oops make sure not to multiply
    ↪ by ll ...
    if (v < 0) sign = -sign, v = -v;
    for (int i = 0, carry = 0; i < sz(z) || carry; ++i) {
      if (i == sz(z)) z.pb(0);
      ll cur = (ll)z[i]*v+carry;
      carry = cur/base; z[i] = cur%base;
    }
    trim(); return *this;
  }
  bigint operator*(int v) const { return bigint(*this) *= v; }
  friend pair<bigint, bigint> divmod(const bigint &a1, const
    ↪bigint &b1) {
    int norm = base/(b1.z.bk+1);
    bigint a = a1.abs()*norm, b = b1.abs()*norm, q, r; // make
      ↪last element of b big
    q.z.rsz(sz(a.z));
    R0F(i,sz(a.z)) {
      r *= base; r += a.z[i];
      int s1 = sz(b.z) < sz(r.z) ? r.z[sz(b.z)] : 0;
      int s2 = sz(b.z)-1 < sz(r.z) ? r.z[sz(b.z)-1] : 0;
      int d = ((ll)s1*base+s2)/b.z.bk; // best approximation
      r -= b*d; while (r < 0) r += b, --d;
      q.z[i] = d;
    }
    q.sign = a1.sign*b1.sign; r.sign = a1.sign;
    q.trim(); r.trim(); return {q,r/norm};
  }
  friend bigint sqrt(const bigint &a1) {
    bigint a = a1; while (!sz(a.z) || sz(a.z)&1) a.z.pb(0);
    int n = sz(a.z), firstDigit = ::sqrt((db)a.z[n-1]*base+a.z[
      ↪n-2]);
    int norm = base/(firstDigit+1); a *= norm; a *= norm;
    while (!sz(a.z) || sz(a.z)&1) a.z.pb(0);
    bigint r = (ll)a.z[n-1]*base+a.z[n-2];
    firstDigit = (int)::sqrt((db)a.z[n-1]*base+a.z[n-2]);
    int q = firstDigit; bigint res;
    R0F(j,n/2) {
      for (;; --q) {
        bigint r1 = (r-(res*2*base+q)*q)*base*base +
              (j>0?(ll)a.z[2*j-1]*base+a.z[2*j-2]:0);
        if (r1 >= 0) { r = r1; break; }
      }
      res *= base; res += q; // add a bit to sqrt
      if (j > 0) {
        int d1 = sz(res.z)+2 < sz(r.z) ? r.z[sz(res.z)+2] : 0;
          ↪// always 0/1?
        int d2 = sz(res.z)+1 < sz(r.z) ? r.z[sz(res.z)+1] : 0;
        int d3 = sz(res.z) < sz(r.z) ? r.z[sz(res.z)] : 0;
        q = ((ll) d1*base*base+(ll)d2*base+d3)/(firstDigit*2);
      }
    }
    res.trim(); return res/norm;
  }
  bigint operator/(const bigint &v) const { return divmod(*this
    ↪, v).f; }
  bigint operator%(const bigint &v) const { return divmod(*this
    ↪, v).s; }
  bigint &operator/=(int v) {
    if (v < 0) sign = -sign, v = -v;
    for (int i = sz(z)-1, rem = 0; i >= 0; --i) {
      ll cur = z[i]+rem*(ll)base;
      z[i] = cur/v; rem = cur%v;
    }
    trim(); return *this;
  }
  bigint operator/(int v) const { return bigint(*this) /= v; }
  int operator%(int v) const {
    if (v < 0) v = -v;
    int m = 0; R0F(i,sz(z)) m = (z[i]+m*(ll)base)%v;
    return m*sign; }
  bigint &operator*=(const bigint &v) { return *this = *this*v;
    ↪ }
  bigint &operator/=(const bigint &v) { return *this = *this/v;
    ↪ }

  bool operator<(const bigint &v) const {
    if (sign != v.sign) return sign < v.sign;
    if (sz(z) != sz(v.z)) return sz(z)*sign < sz(v.z) * v.sign;
    R0F(i,sz(z)) if (z[i] != v.z[i]) return z[i]*sign < v.z[i]*
      ↪sign;
    return 0; // equal
  }
  bool operator>(const bigint &v) const { return v < *this; }
  bool operator<=(const bigint &v) const { return !(v < *this);
    ↪ }
  bool operator>=(const bigint &v) const { return !(*this < v);
    ↪ }
  bool operator==(const bigint &v) const { return !(*this < v)
    ↪&& !(v < *this); }
  bool operator!=(const bigint &v) const { return *this < v ||
    ↪v < *this; }
  void trim() {
    while (sz(z) && z.bk == 0) z.pop_back();
    if (!sz(z)) sign = 1; // don't output -0
  }
  bool isZero() const { return !sz(z); }
  friend bigint operator-(bigint v) {
    if (sz(v.z)) v.sign = -v.sign;
    return v; }
  bigint abs() const { return sign == 1 ? *this : -*this; }
  ll longValue() const {
    ll res = 0; R0F(i,sz(z)) res = res*base+z[i];
    return res*sign; }
  friend bigint gcd(const bigint &a, const bigint &b) {
    return b.isZero() ? a : gcd(b, a % b); } // euclidean algo
  friend bigint lcm(const bigint &a, const bigint &b) {
    return a/gcd(a, b) * b; }

  void read(const str &s) {
    sign = 1; z.clear(); int pos = 0;
    while (pos < sz(s) && (s[pos] == '-' || s[pos] == '+')) {
      if (s[pos] == '-') sign = -sign;
      ++pos; } // account for sign
    for (int i = sz(s)-1; i >= pos; i -= base_digits) {
      int x = 0;
      for (int j = max(pos, i-base_digits+1); j <= i; j++)
        x = x*10+s[j]-'0';
      z.pb(x);
    }
    trim();
  }
  friend istream &operator>>(istream &is, bigint &v) {
    str s; is >> s; v.read(s); return is; }
  friend ostream &operator<<(ostream &os, const bigint &v) {
    if (v.sign == -1) os << '-';
    os << (!sz(v.z) ? 0 : v.z.bk);
    R0F(i,sz(v.z)-1) os << setw(base_digits) << setfill('0') <<
      ↪ v.z[i];
    return os; // pad with zeroes
  }
  static vi convert_base(const vi &a, int old_digits, int
    ↪new_digits) {
    vl p(max(old_digits, new_digits) + 1); // blocks of 10^{old
      ↪} -> 10^{new}
    p[0] = 1; FOR(i,1,sz(p)) p[i] = p[i-1]*10;
    vi res; ll cur = 0; int cur_digits = 0;
    for (int v:a) {
      cur += v*p[cur_digits]; cur_digits += old_digits;
      while (cur_digits >= new_digits) {
```

```
      res.pb(cur%p[new_digits]);
      cur /= p[new_digits]; cur_digits -= new_digits;
    }
  }
  res.pb(cur); while (sz(res) && res.bk == 0) res.pop_back();
  return res;
}
static vl karatMul(const vl &a, const vl &b) { // karatsuba
  int n = sz(a); vl res(2*n);
  if (n <= 32) { // naive multiply
    F0R(i,n) F0R(j,n) res[i+j] += a[i]*b[j];
    return res; }
  int k = n/2;
  vl a1(begin(a),begin(a)+k), a2(k+all(a));
  vl b1(begin(b),begin(b)+k), b2(k+all(b));
  vl a1b1 = karatMul(a1, b1), a2b2 = karatMul(a2, b2);
  F0R(i,k) a2[i] += a1[i], b2[i] += b1[i];
  vl r = karatMul(a2, b2); // three instead of four products
  F0R(i,sz(a1b1)) r[i] -= a1b1[i];
  F0R(i,sz(a2b2)) r[i] -= a2b2[i];
  F0R(i,sz(r)) res[i+k] += r[i];
  F0R(i,sz(a1b1)) res[i] += a1b1[i];
  F0R(i,sz(a2b2)) res[i+n] += a2b2[i];
  return res;
}
bigint operator*(const bigint &v) const {
  if (min(sz(z),sz(v.z)) < 150) return mul_simple(v);
  bigint res; res.sign = sign*v.sign; // should work as long
    ↪as # of digits isn't too large (> LLONG_MAX/10^{12})
  vi a6 = convert_base(this->z, base_digits, 6); // blocks of
    ↪ 10^6 instead of 10^9
  vi b6 = convert_base(v.z, base_digits, 6);
  vl a(all(a6)), b(all(b6));
  while (sz(a) < sz(b)) a.pb(0);
  while (sz(b) < sz(a)) b.pb(0);
  while (sz(a)&(sz(a)-1)) a.pb(0), b.pb(0); // make size
    ↪power of 2
  vl c = karatMul(a, b);
  ll cur = 0; F0R(i,sz(c)) { // process carries
    cur += c[i]; res.z.pb(cur%1000000); cur /= 1000000; }
  res.z = convert_base(res.z,6,base_digits);
  res.trim(); return res;
}
bigint mul_simple(const bigint &v) const {
  bigint res; res.sign = sign*v.sign;
  res.z.rsz(sz(z)+sz(v.z));
  F0R(i,sz(z)) if (z[i]) {
    ll cur = 0; for (int j = 0; j < sz(v.z) || cur; ++j) {
      cur += res.z[i+j]+(ll)z[i]*(j<sz(v.z)?v.z[j]:0);
      res.z[i+j] = cur%base; cur /= base;
    }
  }
  res.trim(); return res;
}
friend str ts(const bigint& v) {
  stringstream ss; ss << v;
  str s; ss >> s; return s; }
};

bigint random_bigint(int n) {
  str s; F0R(i,n) s += rand() % 10 + '0';
  return bigint(s); }

// random tests
void bigintTest() {
  bigint x = bigint("120");
  bigint y = bigint("5");
  cout << x / y << endl;
  F0R(i,1000) {
    int n = rand() % 100 + 1;
    bigint a = random_bigint(n), res = sqrt(a);
    bigint xx = res * res, yy = (res + 1) * (res + 1);
    if (xx > a || yy <= a) {
      dbg("SQRT FAILED",i);
      dbg(a,res);
      break;
    }
    int m = rand() % n + 1;
    bigint b = random_bigint(m)+1;
    res = a/b; xx = res*b, yy = b*(res+1);
    if (xx > a || yy <= a) {
      dbg("DIVISION FAILED",i);
      dbg(a,b,res);
      break;
    }
  }
  {
    bigint a = random_bigint(10000);
    bigint b = random_bigint(2000);
    auto t1 = chrono::high_resolution_clock::now();
    bigint c = a / b;
    auto t2 = chrono::high_resolution_clock::now();
    chrono::duration<double, milli> duration = t2 - t1;
    dbg(duration.count(),"ms");
  }
  bigint a = random_bigint(500000);
  bigint b = random_bigint(500000);
  bigint c1, c2;
  {
    auto t1 = chrono::high_resolution_clock::now();
    c1 = a * b;
    auto t2 = chrono::high_resolution_clock::now();
    chrono::duration<double, milli> duration = t2 - t1;
    dbg(duration.count(),"ms");
  }
  {
    auto t1 = chrono::high_resolution_clock::now();
    c2 = a.mul_simple(b);
    auto t2 = chrono::high_resolution_clock::now();
    chrono::duration<double, milli> duration = t2 - t1;
    dbg(duration.count(),"ms");
  }
  dbg(c1 == c2);
  {

    auto t1 = chrono::high_resolution_clock::now();
    F0R(i,1000000) {
      a = random_bigint(30);
      b = random_bigint(30);
      c2 = a.mul_simple(b);
    }
    auto t2 = chrono::high_resolution_clock::now();
    chrono::duration<double, milli> duration = t2 - t1;
    dbg(duration.count(),"ms");
  }
}
```

6.2 Matrix

Matrix.h

Description: 2D matrix operations.

"ModInt.h" b18e29, 21 lines

```
using T = mi;
using Mat = V<V<T>>; // use array instead if tight TL

Mat makeMat(int r, int c) { return Mat(r,V<T>(c)); }
Mat makeId(int n) {
  Mat m = makeMat(n,n); F0R(i,n) m[i][i] = 1;
  return m;
}
Mat operator*(const Mat& a, const Mat& b) {
  int x = sz(a), y = sz(a[0]), z = sz(b[0]);
  assert(y == sz(b)); Mat c = makeMat(x,z);
  F0R(i,x) F0R(j,y) F0R(k,z) c[i][k] += a[i][j]*b[j][k];
  return c;
}
Mat& operator*=(Mat& a, const Mat& b) { return a = a*b; }
Mat pow(Mat m, ll p) {
  int n = sz(m); assert(n == sz(m[0]) && p >= 0);
  Mat res = makeId(n);
  for (; p; p /= 2, m *= m) if (p&1) res *= m;
  return res;
}
```

6.3 Polynomials

FFT.h

Description: Multiply polynomials of ints for any modulus $< 2^{31}$. For XOR convolution ignore `m` within `fft`.

Time: $\mathcal{O}(N \log N)$. For $N = 10^6$, conv ~0.13ms, conv_general ~320ms.

"ModInt.h" 19ab20, 39 lines

```
// const int MOD = 998244353;
tcT> void fft(V<T>& A, bool invert = 0) { // NTT
  int n = sz(A); assert((T::mod-1)%n == 0); V<T> B(n);
  for(int b = n/2; b; b /= 2, swap(A,B)) { // w = n/b'th root
    T w = pow(T::rt(),(T::mod-1)/n*b), m = 1;
    for(int i = 0; i < n; i += b*2, m *= w) F0R(j,b) {
      T u = A[i+j], v = A[i+j+b]*m;
      B[i/2+j] = u+v; B[i/2+j+n/2] = u-v;
    }
  }
  if (invert) { reverse(1+all(A));
    T z = inv(T(n)); each(t,A) t *= z; }
} // for NTT-able moduli
tcT> V<T> conv(V<T> A, V<T> B) {
  if (!min(sz(A),sz(B))) return {};
  int s = sz(A)+sz(B)-1, n = 1; for (; n < s; n *= 2);
  A.rsz(n), fft(A); B.rsz(n), fft(B);
  F0R(i,n) A[i] *= B[i];
  fft(A,1); A.rsz(s); return A;
}
template<class M, class T> V<M> mulMod(const V<T>& x, const V<T
  ↪>& y) {
  auto con = [](const V<T>& v) {
    V<M> w(sz(v)); F0R(i,sz(v)) w[i] = (int)v[i];
    return w; };
  return conv(con(x), con(y));
} // arbitrary moduli
tcT> V<T> conv_general(const V<T>& A, const V<T>& B) {
  using m0 = mint<(119<<23)+1,62>; auto c0 = mulMod<m0>(A,B);
  using m1 = mint<(5<<25)+1,  62>; auto c1 = mulMod<m1>(A,B);
  using m2 = mint<(7<<26)+1,  62>; auto c2 = mulMod<m2>(A,B);
  int n = sz(c0); V<T> res(n); m1 r01 = inv(m1(m0::mod));
  m2 r02 = inv(m2(m0::mod)), r12 = inv(m2(m1::mod));
  F0R(i,n) { // a=remainder mod m0::mod, b fixes it mod m1::mod
    int a = c0[i].v, b = ((c1[i]-a)*r01).v,
      c = (((c2[i]-a)*r02-b)*r12).v;
    res[i] = (T(c)*m1::mod+b)*m0::mod+a; // c fixes m2::mod
  }
  return res;
}
```

Graphs (7)

Erdos-Gallai: $d_1 \geq \cdots \geq d_n$ can be degree sequence of simple graph on n vertices iff their sum is even and $\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k), \forall 1 \leq k \leq n$.

7.1 Basics

DirectedCycle.h
Description: stack
90d8f6, 29 lines

```
struct DirectedCycle {
  int SZ;
  vector<bool> inStk, vis;
  vector<int> stk, cyc;
  vector<vector<int>> adj;
  DirectedCycle(int _SZ) {
    SZ = _SZ;
    adj.resize(SZ);
    inStk.resize(SZ), vis.resize(SZ);
  }
  void ae(int u, int v) { adj[u].push_back(v); }
  vector<int> gen() {
    for (int u = 0; u < SZ; u++) {
      if (!vis[u] && empty(cyc)) dfs(u);
    }
    return cyc;
  }
  void dfs(int u) {
    stk.push_back(u);
    inStk[u] = vis[u] = 1;
    for (auto& v: adj[u]) {
      if (inStk[v]) cyc = {ranges::find(stk, v), end(stk)};
      else if (!vis[v]) dfs(v);
      if (size(cyc)) return;
    }
    stk.pop_back();
    inStk[u] = 0;
  }
};
```

TopoSort.h
Description: sorts vertices such that if there exists an edge x->y, then x goes before y
6779f6, 23 lines

```
struct TopoSort {
  int SZ;
  vector<int> in, res;
  vector<vector<int>> adj;
  TopoSort(int _SZ) {
    SZ = _SZ;
    in.resize(SZ);
    adj.resize(SZ);
  }
  void ae(int u, int v) {
    adj[u].push_back(v), ++in[v];
  }
  bool sort() {
    queue<int> todo;
    for (int u = 0; u < SZ; ++u) if (!in[u]) todo.push(u);
    while (!todo.empty()) {
      int u = todo.front(); todo.pop();
      res.push_back(u);
      for (auto &v : adj[u]) if (!(--in[v])) todo.push(v);
    }
    return (int)res.size() == SZ;
  }
};
```

Dijkstra.h
Description: shortest path
Time: $\mathcal{O}(N \log N + M)$
5afc43, 27 lines

```
template<class T, bool directed> struct Dijkstra {
  int SZ;
  vector<T> dist;
  vector<vector<pair<int, T>>> adj;
  Dijkstra(int _SZ) {
    SZ = _SZ;
    adj.resize(SZ);
  }
  void ae(int u, int v, T w) {
    adj[u].push_back({v, w});
    if (!directed) adj[v].push_back({u, w});
  }
  void gen(int st) {
    dist.assign(SZ, numeric_limits<T>::max());
    priority_queue<pair<T, int>> PQ;
    auto relax = [&](int v, T d) {
      if (dist[v] <= d) return;
      dist[v] = d;
      PQ.push({-dist[v], v});
    }; relax(st, 0);
    while (!PQ.empty()) {
      auto [d, u] = PQ.top(); PQ.pop(); d = -d;
      if (dist[u] < d) continue;
      for (auto& [v, w]: adj[u]) relax(v, w+d);
    }
  }
};
```

Kruskal.h
Description: Computes the weight of a Minimum Spanning Tree (MST) using Kruskal's algorithm. Requires DSU.
Time: $\mathcal{O}(M \log M)$, where M = number of edges
"DSU.h" f6c35c, 8 lines

```
template<class T> T Kruskal(int N, vector<array<T, 3>> eg) {
  ranges::sort(eg);
  T ans = 0; DSU D(N);
  for (auto &[w, u, v] : eg) {
    if (D.unite(u, v)) ans += w;
  }
  return ans;
};
```

FloydWarshall.h
Description: All-Pairs Shortest Path
a2bfcc, 11 lines

```
void floydWarshall(V<vl>& m) {
    int n = sz(m);
    F0R(i,n) ckmin(m[i][i], 0LL);
    F0R(k,n) F0R(i,n) F0R(j,n)
        if (m[i][k] != BIG && m[k][j] != BIG) {
            auto newDist = max(m[i][k]+m[k][j],-BIG);
            ckmin(m[i][j],newDist);
        }
    F0R(k,n) if (m[k][k] < 0) F0R(i,n) F0R(j,n)
        if (m[i][k] != BIG && m[k][j] != BIG) m[i][j] = -BIG;
}
```

7.2 Trees

TreeDiameter.h
Description: Calculates longest path in tree. The vertex furthest from 0 must be an endpoint of the diameter.
3c22a4, 38 lines

```
struct TreeDiameter {
  int N, diaLen;
  vector<int> dist, dia;
  vector<vector<int>> adj;
  TreeDiameter(int _N) {
    N = _N;
    dia = {0, 0};
    adj.resize(N);
    dist.resize(N);
  }
  void ae(int u, int v) {
    adj[u].push_back(v);
    adj[v].push_back(u);
  }
  void genDist(int R) { dist[R] = 0; dfs(R); }
  void gen() {
    genDist(0);
    for (int i = 0; i < N; ++i) {
      if (dist[i] > dist[dia[0]]) dia[0] = i;
    }
    genDist(dia[0]);
    for (int i = 0; i < N; ++i) {
      if (dist[i] > dist[dia[1]]) dia[1] = i;
    }
    diaLen = dist[dia[1]];
    // center vertex of dia[0] -> dia[1]
    // int cen = dia[1];
    // for (int i = 0; i < diaLen/2; ++i) cen = par[cen];
    // center = {cen};
    // if (diaLen&1) center.push_back(par[cen]);
  }
  void dfs(int u, int p = -1) {
    for (auto& v: adj[u]) {
      if (v == p) continue;
      dist[v] = dist[u] + 1; dfs(v, u);
    }
  }
};
```

LCAjump.h
Description: Calculates least common ancestor in tree with verts $0 \ldots N-1$ and root R using binary jumping.
Memory: $\mathcal{O}(N \log N)$
Time: $\mathcal{O}(N \log N)$ build, $\mathcal{O}(\log N)$ query
5c0112, 47 lines

```
struct LCA {
  int N, LOG;
  vector<int> depth;
  vector<vector<int>> par, adj;
  LCA(int _N) {
    N = _N; LOG = 1;
    while ((1<<LOG) < N) ++LOG;
    adj.resize(N);
    depth.resize(N);
    par.resize(N, vector<int>(LOG));
  }
  void ae(int u, int v) {
    adj[u].push_back(v);
    adj[v].push_back(u);
  }
  void gen(int R = 0) { par[R][0] = R; dfs(R); }
  void dfs(int u = 0) {
    for (int j = 1; j < LOG; ++j) {
      par[u][j] = par[par[u][j-1]][j-1];
    }
    for (auto& v: adj[u]) {
      if (v == par[u][0]) continue;
      depth[v] = depth[u]+1;
      par[v][0] = u;
      dfs(v);
    }
  }
```

```
  int jump(int x, int d) {
    for (int j = 0; j < LOG; ++j) {
      if ((d>>j) & 1) x = par[x][j];
    }
    return x;
  }
  int lca(int u, int v) {
    if (depth[u] > depth[v]) swap(u, v);
    v = jump(v, depth[v]-depth[u]);
    if (u == v) {return u; }
    for (int j = LOG-1; j >= 0; --j) {
      int U = par[u][j], V = par[v][j];
      if (U != V) u = U, v = V;
    }
    return par[u][0];
  }
  int dist(int u, int v) {
    return depth[u] + depth[v] - 2*depth[lca(u, v)];
  }
};
```

7.3 DSU

DSU.h

Description: Disjoint Set Union with path compression and union by size. Add edges and test connectivity. Use for Kruskal's or Boruvka's minimum spanning tree.

Time: $\mathcal{O}(\alpha(N))$

a876ce, 12 lines

```
struct DSU {
  vector<int> e;
  DSU(int N) { e = vector<int>(N, -1); }
  int get(int x) { return e[x] < 0 ? x : e[x] = get(e[x]); }
  bool sameSet(int a, int b) { return get(a) == get(b); }
  int size(int x) { return -e[get(x)]; }
  bool unite(int x, int y) { // union by size
    x = get(x), y = get(y); if (x == y) return false;
    if (e[x] > e[y]) swap(x, y);
    e[x] += e[y]; e[y] = x; return true;
  }
};
```

DSUrb.h

Description: Disjoint Set Union with Rollback

d21534, 25 lines

```
struct DSUrb {
  vector<int> e;
  vector<array<int,4>> mod;
  DSUrb(int n) { e = vector<int>(n,-1); }
  int get(int x) { return e[x] < 0 ? x : get(e[x]); }
  bool sameSet(int a, int b) { return get(a) == get(b); }
  int size(int x) { return -e[get(x)]; }
  bool unite(int x, int y) { // union-by-rank
    x = get(x), y = get(y);
    if (x == y) {
      mod.push_back({-1,-1,-1,-1});
      return 0;
    }
    if (e[x] > e[y]) swap(x,y);
    mod.push_back({x,y,e[x],e[y]});
    e[x] += e[y]; e[y] = x; return 1;
  }
  void rollback() {
    auto a = mod.back(); mod.pop_back();
    if (a[0] != -1) {
      e[a[0]] = a[2];
      e[a[1]] = a[3];
    }
  }
};
```

7.3.1 SqrtDecompton

HLD generally suffices. If not, here are some common strategies:

- Rebuild the tree after every $\sqrt{N}$ queries.
- Consider vertices with $>$ or $<\sqrt{N}$ degree separately.
- For subtree updates, note that there are $O(\sqrt{N})$ distinct sizes among child subtrees of any node.

Block Tree: Use a DFS to split edges into contiguous groups of size $\sqrt{N}$ to $2\sqrt{N}$.

Mo's Algorithm for Tree Paths: Maintain an array of vertices where each one appears twice, once when a DFS enters the vertex (`st`) and one when the DFS exists (`en`). For a tree path $u \leftrightarrow v$ such that `st[u]<st[v]`,

- If u is an ancestor of v, query `[st[u],st[v]]`.
- Otherwise, query `[en[u],st[v]]` and consider $LCA(u,v)$ separately.

Solutions with worse complexities can be faster if you optimize the operations that are performed most frequently. Use arrays instead of vectors whenever possible. Iterating over an array in order is faster than iterating through the same array in some other order (ex. one given by a random permutation) or DFSing on a tree of the same size. Also, the difference between $\sqrt{N}$ and the optimal block (or buffer) size can be quite large. Try up to 5x smaller or larger (at least).

7.4 DFS Algorithms

DFSBasic.h

Description: Basic DFS implementation for undirected graphs

Time: $\mathcal{O}(N+M)$

14b944, 21 lines

```
struct DFS {
  int N;
  vector<bool> visited;
  vector<vector<int>> adj;
  DFS(int _N) {
    N = _N;
    adj.resize(N);
    visited.resize(N);
  }
  void ae(int u, int v) {
    adj[u].push_back(v);
    adj[v].push_back(u);
  }
  void dfs(int u) {
    visited[u] = true;
    for (auto& v: adj[u]) {
      if (visited[v]) continue;
      dfs(v);
    }
  }
};
```

SCCK.h

Description: Kosaraju's Algorithm, DFS twice to generate strongly connected components in topological order. a, b in same component if both $a \to b$ and $b \to a$ exist.

Time: $\mathcal{O}(N+M)$

4e9f2b, 18 lines

```
struct SCC {
  int N; V<vi> adj, radj;
  vi todo, comp, comps; V<bool> vis;
  void init(int _N) { N = _N;
    adj.rsz(N), radj.rsz(N), comp = vi(N,-1), vis.rsz(N); }
  void ae(int x, int y) { adj[x].pb(y), radj[y].pb(x); }
  void dfs(int x) {
    vis[x] = 1; each(y,adj[x]) if (!vis[y]) dfs(y);
    todo.pb(x); }
  void dfs2(int x, int v) {
    comp[x] = v;
    each(y,radj[x]) if (comp[y] == -1) dfs2(y,v); }
  void gen() { // fills allComp
    F0R(i,N) if (!vis[i]) dfs(i);
    reverse(all(todo));
    each(x,todo) if (comp[x] == -1) dfs2(x,x), comps.pb(x);
  }
};
```

SCCT.h

Description: Tarjan's, DFS once to generate strongly connected components in topological order. a, b in same component if both $a \to b$ and $b \to a$ exist. Uses less memory than Kosaraju b/c doesn't store reverse edges.

Time: $\mathcal{O}(N+M)$

a36e0c, 22 lines

```
struct SCC {
  int N, ti = 0; V<vi> adj;
  vi disc, comp, stk, comps;
  void init(int _N){ N = _N, adj.rsz(N);
    disc.rsz(N), comp.rsz(N,-1);}
  void ae(int x, int y) { adj[x].pb(y); }
  int dfs(int x) {
    int low = disc[x] = ++ti; stk.pb(x);
    each(y,adj[x]) if (comp[y] == -1) // comp[y] == -1,
      ckmin(low,disc[y]?:dfs(y));  // disc[y] != 0 -> in stack
    if (low == disc[x]) { // make new SCC
      // pop off stack until you find x
      comps.pb(x); for (int y = -1; y != x;)
        comp[y = stk.bk] = x, stk.pop_back();
    }
    return low;
  }
  void gen() {
    F0R(i,N) if (!disc[i]) dfs(i);
    reverse(all(comps));
  }
};
```

Geometry (8)

8.1 Geometric primitives

Point.h

Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

47ec0a, 28 lines

```
template <class T> int sgn(T x) { return (x > 0) - (x < 0); }
template<class T>
struct Point {
  typedef Point P;
  T x, y;
  explicit Point(T x=0, T y=0) : x(x), y(y) {}
  bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
```

```
  bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
  P operator+(P p) const { return P(x+p.x, y+p.y); }
  P operator-(P p) const { return P(x-p.x, y-p.y); }
  P operator*(T d) const { return P(x*d, y*d); }
  P operator/(T d) const { return P(x/d, y/d); }
  T dot(P p) const { return x*p.x + y*p.y; }
  T cross(P p) const { return x*p.y - y*p.x; }
  T cross(P a, P b) const { return (a-*this).cross(b-*this); }
  T dist2() const { return x*x + y*y; }
  double dist() const { return sqrt((double)dist2()); }
  // angle to x-axis in interval [-pi, pi]
  double angle() const { return atan2(y, x); }
  P unit() const { return *this/dist(); } // makes dist()=1
  P perp() const { return P(-y, x); } // rotates +90 degrees
  P normal() const { return perp().unit(); }
  // returns point rotated 'a' radians ccw around the origin
  P rotate(double a) const {
    return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
  friend ostream& operator<<(ostream& os, P p) {
    return os << "(" << p.x << "," << p.y << ")"; }
};
```

8.2 Polygons

ConvexHull.h

Description:
Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull.

Time: $\mathcal{O}(n \log n)$

"Point.h" 310954, 13 lines

```
typedef Point<ll> P;
vector<P> convexHull(vector<P> pts) {
  if (sz(pts) <= 1) return pts;
  sort(all(pts));
  vector<P> h(sz(pts)+1);
  int s = 0, t = 0;
  for (int it = 2; it--; s = --t, reverse(all(pts)))
    for (P p : pts) {
      while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t--;
      h[t++] = p;
    }
  return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1])};
}
```

Strings (9)

9.1 Light

KMP.h

Description: f[i] is length of the longest proper suffix of the i-th prefix of s that is a prefix of s

Time: $\mathcal{O}(N)$ 70d0fa, 14 lines

```
vector<int> kmp(str s) {
  int N = sz(s); vi f(N+1); f[0] = -1;
  FOR(i,1,N+1) {
    for (f[i]=f[i-1];f[i]!=-1&&s[f[i]]!=s[i-1];)f[i]=f[f[i]];
    f[i]++;
  }
  return f;
}
vi getOc(str a, str b) { // find occurrences of a in b
  vi f = kmp(a+"@"+b), ret;
  FOR(i,sz(a),sz(b)+1) if (f[i+sz(a)+1] == sz(a))
    ret.pb(i-sz(a));
  return ret;
}
```

Z.h

Description: f[i] is the max len such that s.substr(0,len) == s.substr(i,len)

Time: $\mathcal{O}(N)$ 566170, 15 lines

```
vi z(str s) {
  int N = sz(s), L = 1, R = 0; s += '#';
  vi ans(N); ans[0] = N;
  FOR(i,1,N) {
    if (i <= R) ans[i] = min(R-i+1,ans[i-L]);
    while (s[i+ans[i]] == s[ans[i]]) ++ans[i];
    if (i+ans[i]-1 > R) L = i, R = i+ans[i]-1;
  }
  return ans;
}
vi getPrefix(str a, str b) { // find prefixes of a in b
  vi t = z(a+b); t = vi(sz(a)+all(t));
  each(u,t) ckmin(u,sz(a));
  return t;
}
```

Various (10)

10.1 Binary search

fstTrue.h

Description: Finds the first value x in [lo, hi] such that predicate f(x) is true.

Usage: int ans = fstTrue()

Time: $\mathcal{O}(\log(hi - lo))$ 2654fa, 9 lines

```
template<typename T, typename U> T fstTrue(T lo, T hi, U f) {
  ++hi;
  assert(lo <= hi);
  while (lo < hi) {
    T mid = lo + (hi - lo) / 2;
    f(mid) ? hi = mid : lo = mid + 1;
  }
  return lo;
};
```

lstTrue.h

Description: Finds the last value x in [lo, hi] such that predicate f(x) is true.

Usage: int ans = fstTrue()

Time: $\mathcal{O}(\log(hi - lo))$ 8aff85, 9 lines

```
template<typename T, typename U> T lstTrue(T lo, T hi, U f) {
  --lo;
  assert(lo <= hi);
  while (lo < hi) {
    T mid = lo + (hi - lo + 1) / 2;
    f(mid) ? lo = mid : hi = mid - 1;
  }
  return lo;
};
```

realTrue.h

Description: Binary search on continuous (floating-point) domain.

Usage: int ans = realTrue()

Time: $\mathcal{O}(\log(precision))$ — write 100 iterations and have a relax 691e1d, 7 lines

```
template<typename T, typename U>T realTrue(T lo, T hi, U f) {
  for (int i = 0; i < 100; ++i) {
    T mid = 0.5 * (hi + lo);
    f(mid) ? lo = mid : hi = mid;
  }
  return lo;
};
```

10.2 Dynamic programming

When doing DP on intervals:
$a[i][j] = \min_{i<k<j}(a[i][k] + a[k][j]) + f(i,j)$, where the (minimal) optimal k increases with both i and j,

- one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $a[i][j]$ only between $p[i][j-1]$ and $p[i+1][j]$.
- This is known as Knuth DP. Sufficient criteria for this are if $f(b,c) \le f(a,d)$ and $f(a,c) + f(b,d) \le f(a,d) + f(b,c)$ for all $a \le b \le c \le d$.
- Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.

10.3 Debugging tricks

- `signal(SIGSEGV, [](int) { _Exit(0); });` converts segfaults into Wrong Answers. Similarly one can catch SIGABRT (assertion failures) and SIGFPE (zero divisions). `_GLIBCXX_DEBUG` violations generate SIGABRT (or SIGSEGV on gcc 5.4.0 apparently).
- `feenableexcept(29);` kills the program on NaNs (1), 0-divs (4), infinities (8) and denormals (16).

10.4 Optimization tricks

10.4.1 Bit hacks

- `x & -x` is the least bit in x.
- `for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after x with the same number of bits set.
- `F0R(b,k) F0R(i,1<<K) if (i&1<<b) D[i] += D[i^(1<<b)];` computes all sums of subsets.

10.4.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize for loops and optimizes floating points better (assumes associativity and turns off denormals).
- `#pragma GCC target ("avx,avx2")` can double performance of vectorized code, but causes crashes on old machines. Also consider older `#pragma GCC target ("sse4")`.
- `#pragma GCC optimize ("trapv")` kills the program on integer overflows (but is really slow).

FastInput.h

Description: Read an integer from stdin. Usage requires your program to pipe in input from file.

Usage: `./a.out < input.txt`

Time: About 5x as fast as cin/scanf. 7b3c70, 17 lines

```
inline char gc() { // like getchar()
  static char buf[1 << 16];
  static size_t bc, be;
  if (bc >= be) {
    buf[0] = 0, bc = 0;
    be = fread(buf, 1, sizeof(buf), stdin);
  }
  return buf[bc++]; // returns 0 on EOF
}

int readInt() {
  int a, c;
  while ((a = gc()) < 40);
  if (a == '-') return -readInt();
  while ((c = gc()) >= 48) a = a * 10 + c - 480;
  return a - 48;
}
```

10.5 Other languages

Python3.py

Description: Python review. 40 lines

```
from math import *
import sys, random
def nextInt():
  return int(input())
def nextStrs():
  return input().split()
def nextInts():
  return list(map(int,nextStrs()))

n = nextInt()
v = [n]
def process(x):
  global v
  x = abs(x)
  V = []
  for t in v:
    g = gcd(t,x)
    if g != 1:
      V.append(g)
    if g != t:
      V.append(t//g)
  v = V
for i in range(50):
  x = random.randint(0,n-1)
  if gcd(x,n) != 1:
    process(x)
  else:
    sx = x*x%n # assert(gcd(sx,n) == 1)
    print(f"sqrt {sx}")
    sys.stdout.flush()
    X = nextInt()
    process(x+X)
    process(x-X)
print(f'! {len(v)}',end='')
for i in v:
  print(f' {i}',end='')
print()
sys.stdout.flush() # sys.exit(0) -> exit
# sys.setrecursionlimit(int(1e9)) -> stack size
# print(f'{ans:=.6f}') -> print ans to 6 decimal places
```