Research on Storage Space Allocation Model of Containers in Container Yard Based on Mixture Storage

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Abstract - **With the development of containerization, the reqnirement for operations optimization in container terminals has become prevalent and necessary. How to improve the efficiency of container yard and yard handling system is really an important snbject for container terminals to tackle with. In this paper, a systematic method of container yard is proposed in order to solve both storage space allocation problem and yard crane schednling problem. Based on mixtnre storage statns of container yard, a mix-integer programming model, which considers varions constraints related to operations of yard cranes and storage space allocation rules of yard blocks, is developed. Finally, nnmerical experiments are condnced to valid this model.**

Keywords - **Container yard, storage space allocation, yard crane schednling**

I. INTRODUCTION

With the development of containerization, the competition among container terminals has become critical. In a container terminal, a yard stock is a special place which is used to store containers. Usually the yard stock is separated into different blocks. Each block consists of 20-30 yard-bays. Each yard-bay contains several rows. The configuration of a block could be specified by three parameters, they are, the number of bays, the number of rows, and the number of tiers. Thus, the position of a slot in the block may be specified by bay, row and tier, as shown in Fig.1.

For container yard operations optimization, it should be decided which block and which slot have to be selected for a container (import/export) to be stored. Because container ships seek a short turn-around time at container terminals, how to allocate a reasonable storage space for each container and also to schedule yard cranes (YCs) in a high efficiency become quite important.

What's more, in consideration of high space usage of yard block, containers usually pile up. As a result, rehandles may occur when containers are placed on top of the target one. Therefore, stacking logistics is becoming more and more complicated and sophisticated due to uncertain factors of container terminals [1].

A great deal of research has been done in the field of storage space allocation for containers in container yard. According to different attributes of containers, many optimization models and strategies have been proposed. Zhang [2] studied the storage space allocation problem in the storage yards of terminals using a rolling-horizon approach. Lim and Xu [3] proposed a critical-shaking neighborhood search for the yard allocation problem. Preston and Kozan [4] discussed the optimal storage strategy for various container-handling schedules. Considering the weight of export containers, Kim [5] formulated a dynamic programming model to determine the storage location to minimize the number of relocation movements expected for the loading operation of export containers. Kim and Kim [6] studied how to allocate storage space for import containers based on the Lagrangian relaxation technique. Lee and Chao [7] proposed a model to develop a movement plan to improve the layout of containers in a bay, aiming to improve operational efficiency by pre-marshaling the containers in such an order that it fits the loading sequence. Zhang [8] proposed an optimization model of gantry crane loading, with an objective function designed to minimize the number of unnecessary movements, a heuristic algorithm based on least spanning tree is used to solve the model. Based on the graphic search technology and the pattern recognition theory, Hao [9] presented a stacking bay optimization model of hybrid sequence in order to optimize the stacking efficiency. Based on both a branch and bound (B & B) method and a heuristic search algorithm, Kim and Park [10] discussed the problem of scheduling quay cranes (QCs).

However, these researches have not solved the following problems. First, mixed operation of loading/unloading of containers at the same time, which is the real status of maritime port, is not considered. Actually, import/export containers carried by one same ship need to be operated simultaneously in order to minimize the stay time of container ships in container terminal. Therefore, the combined operation of import/export containers is a major problem of yard planning. Second, storage space allocation problem and yard crane scheduling problem of container yard are usually discussed separately. As a result, integration of container yard as a whole is not considered and a high efficiency performance of container terminal could not be achieved. Finally, mixture storage problem of containers, especially in many space limited container terminals, such as in Korea, Hong Kong and some other container terminals in Asia, is rarely paid great attention to.

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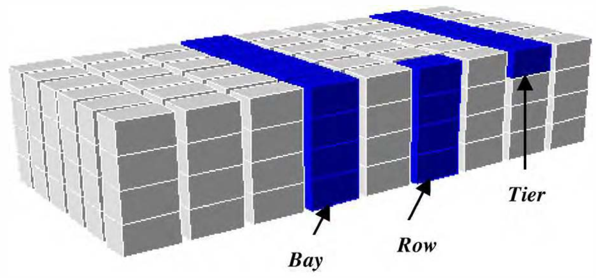

Fig. 1. Configuration of a container yard block.

In this paper, in order to solve the problem of storage space allocation and yard crane scheduling simultaneously, a mix-integer programming model, considering various constraints related to operations of container yard, is developed based on mixture storage status. Then, numerical experiments are provided in order to valid this model.

The optimization model of storage space allocation problem is described in section 2. Section 3 demonstrates the numerical experiments. The final section is the conclusion.

II. OPTIMIZATION MODEL

The goal of studying the storage space allocation problem is to determine the exact slot for import/export containers. In order to improve the space utilization of yard block, and the efficiency of operation equipments of container yard, an appropriate method is to combine the storage space allocation problem with the scheduling problem of YCs, so that a systematic optimal method of yard block is developed in this paper.

In fact, containers could be categorized by their different attributes, such as, type, length, discharge port and weight. A common strategy for determining the storage space for containers is to pile up a container on top of another one with the same categories. Here, containers with the same attributes are regarded as the same group. After loading/unloading by QCs at allocated berths, containers are transported to yard block for storage.

TABLE I
LOAD PLAN OF QCS

Sub-tour number	Container type	Number of containers	Type of task	Starting time	Ending time
1	A	20	D	09:00	09:47
2	C	18	L	09:47	10:29
3	B	22	D	10:31	11:03
4	A	24	L	11:03	11:11
5	C	30	L	11:11	11:19
6	B	26	L	11:19	11:35

It should be paid great attention to that there are operation precedence relationships among different groups due to both ship operation rules and yard operation rules. These kinds of operation rules lead to the sub-tour sequence in loading/unloading operation.

Table I illustrates a QCs schedule plan which is the known information for YCs scheduling.

This paper defines a "task" as a loading/unloading operation of a container in yard block. It is assumed that once a YC starts to load/unload a container to/from a space of yard block, it will not stop until all the containers belongs to the load plan of QCs are operated [10]. Although there are many similarities between the m-parallel machine scheduling problem and the YC scheduling problem, the YC scheduling problem has several unique characteristics which are related to the operation rules of container terminals. For example, there are precedence relationships which should be obeyed among different groups of container. What's more, the parallel YCs of a block should keep a suitable distance according to the bay, in order to avoid interference.

The following notations are used for a mathematical formulation.

Indices:

i : Sub-tour index, $i \in I = \{1,2,...,n\}$

j :Container (import/export)index, $j \in J = \{1,2,...,N\}$

There are n_i containers of sub-tour i . For all containers of n sub-tours, it is regulated that the container u of sub-tour i is ordered as $\sum_{i=1}^{i} n_{i-1} + u$, $n_0 = 0$, $N = \sum_{i \in I} n_i$.

k : YC index, $k \in K = \{0,1\}$

YCs are ordered in an increasing order of their relative locations in the direction of increasing bay numbers of block.

m, m' : Storage space index, $m, m' \in M = \{b, r, t\}$

b : Bay index of yard block, bays are ordered in an increasing order from one side of block. $b \in B = \{0,1,2,...,n_b - 1\}$

n_b : Total number of bays of yard block

r : Row index of yard block, $r \in R = \{0,1,2,...,5\}$

t : Tier index of yard block, $t \in T = \{0,1,2,3\}$

Problem data:

n_i : Total number of containers of sub-tour i

s_i :The starting operation time of containers of sub-tour i

f_i : The ending operation time of containers of sub-tour i

sz_i : Size of containers of sub-tour i

sct_i : Customer of containers of sub-tour i

ie_j : =1, if j is an import container;

=0, if j is an export container;

O_j : Operation of task j. A task is defined as a loading/unloading operation of container j. O_0 , O_{N+1} : Operation of the virtual task. Respectively represent the first and the last operation of YCs. $O_j \in J^+ = \{0,1,2,...,N+1\}$

$J^s : (j, j') \in J^s$. The pre-defined precedence between task j of sub-tour i and task j' of sub-tour ($i+1$). That is, operation $O_{j'}$ is performed immediately after operation O_j is performed. The starting operation time of all tasks should be satisfied by J^s.

J' :Containers already placed in block. $J' = \{1,2,...,N'\}$

ps : The standard taking up and putting down operation time of a container (known)

pr : The standard rehandles time resulted from loading/unloading container j (known)

$tr_{j,j',k}$: The traveling time of YC k from space of container j to space of container j', if YC k performs $O_{j'}$ immediately after performing O_j. $tr_{0,j',k} = tr_{j,N+1,k} = 0$.

b_j : Bay of container of operation O_j

$b_j \in B = \{0,1,2,...,n_b - 1\}$

r_j : Row of container of operation O_j

$r_j \in R = \{0,1,2\}$

t_j : Tier of container of operation O_j

$t_j \in T = \{0,1,2,3\}$

ht_{b_j,r_j} : Height of stowage of bay b_j and row r_j

tp_b : Type of container of bay b

ct_b : Customer of bay b

H : A sufficient large constant

Decision Variables

$X_{j,j',k} = 1$, if YC k performs $O_{j'}$ immediately after performing O_j; 0, otherwise

$Y_{j,m} = 1$, if container j is an import container and it is to be allocated to storage space m ; 0, otherwise

$Z_{j,m,m'} = 1$, if loading/unloading operation of container j to/from storage space m leads to rehandles of the original container of space m moving to another space m' ; 0, otherwise

C_j The ending operation time of O_j. $C_0 = 0.0$

p_j The operation time of O_j, which could be divided into two parts: the standard taking up and putting down operation time and possible rehandles time

$V_{j,j'} = 1$, if $C_{j'} \geq C_j - p_j$ & $C_j \geq C_{j'} - p_{j'}$; 0, otherwise. $V_{j,j'}$ is a variable which is used to decide whether containers j and j' are performed simultaneously.

In this paper, the storage space allocation problem could be solved with the YC scheduling problem at the same time. The objective is to minimize the total operation time of YCs, which can be formulated as follows:

$$Minimize\left(\sum_{k \in K} \sum_{j' \in J^+, j' \neq j} \sum_{j \in J^+} t_{j,j',k} X_{j,j',k} + \sum_{j \in J^+} p_r \cdot Z_{j,m,m'}\right) \quad (1)$$

Subject to

$$C_j - p_j \geq 0, \forall j \in J^+ \quad (2)$$

$$C_{j'} - p_{j'} \geq C_j - p_j, \forall (j, j') \in J^s \quad (3)$$

$$(C_{j'} - p_{j'}) - (C_j + t_{j,j',k}) + (1 - X_{j,j',k})H \geq 0, \forall j, j' \in J^+, \forall k \in K \quad (4)$$

$$C_j - C_{j'} - (1 - V_{j,j'})H \leq 0, \forall j, j' \in J \quad (5)$$

$$C_j - (C_{j'} - p_{j'}) + (1 - V_{j,j'})H \geq 0, \forall j, j' \in J \quad (6)$$

$$p_j = ps + pr \times Z_{j,m,m'}, \forall j \in J, \forall m, m' \in M \quad (7)$$

$$Y_{j,m} \leq ie_j, \forall j \in J, \forall m \in M \quad (8)$$

$$\sum_{j \in J} Y_{j,m} \leq 1, \forall m \in M \quad (9)$$

$$\sum_{m \in M} Y_{j,m} = 1, \forall j \in J \quad (10)$$

$$\sum_{k \in K} \sum_{j' \in J^+} X_{j,j',k} = 1, \forall j \in J \quad (11)$$

$$\sum_{k \in K} \sum_{j \in J^+} X_{j,j',k} = 1, \forall j' \in J \quad (12)$$

$$\sum_{j' \in J^+} X_{j,j',k} = 1, \forall j \in J, \forall k \in K \quad (13)$$

$$\sum_{j' \in J^+} X_{0,j',k} = 1, \forall k \in K \quad (14)$$

$$\sum_{j' \in J^+} X_{N+1,j',k} = 0, \forall k \in K \quad (15)$$

$$\sum_{j' \in J^+} X_{j',j,k} = 1, \forall j \in J, \forall k \in K \quad (16)$$

$$\sum_{j' \in J^+} X_{j',0,k} = 0, \forall k \in K \quad (17)$$

$$\sum_{j' \in J^+} X_{j',N+1,k} = 1, \forall k \in K \quad (18)$$

$$\sum_{j' \in J} X_{j,j',k} - \sum_{j' \in J} X_{j',j,k} = 0, \forall j \in J^+, \forall k \in K \quad (19)$$

$$b_j - b_{j'} + 1 \le H(1 - V_{j,j'}), \forall b_j \le b_{j'}, \forall j, j' \in J \quad (20)$$

$$(1 - V_{j,j'})H \ge \sum_{k \in K} \sum_{j' \in J} k \cdot X_{j,j',k} - \sum_{k \in K} \sum_{j' \in J} k \cdot X_{j',j,k},$$

$$\forall j, j' \in J^+, j' \ne j, \forall b_j + 1 \le b_{j'}, \forall b_j, b_{j'} \in B \quad (21)$$

$$ht_{b_j, r_j} + 1 - t_j \le H(1 - Y_{j,m}), \forall j \in J, \forall m \in M \quad (22)$$

$$tp_{b_j} - sz_i \le H(1 - Y_{j,m}),$$

$$\forall i \in I, \forall j \in (\sum_{i=1}^{i} n_{i-1} + 1, \sum_{i=1}^{i} n_i), \forall m \in M \quad (23)$$

$$ct_{b_j} - sct_i \le H(1 - Y_{j,m}),$$

$$\forall i \in I, \forall j \in (\sum_{i=1}^{i} n_{i-1} + 1, \sum_{i=1}^{i} n_i), \forall m \in M \quad (24)$$

$$s_i \le C_j - p_j, \forall j \in (\sum_{i=1}^{i} n_{i-1} + 1, \sum_{i=1}^{i} n_i), \forall i \in I \quad (25)$$

$$C_j \le f_i, \forall j \in (\sum_{i=1}^{i} n_{i-1} + 1, \sum_{i=1}^{i} n_i), \forall i \in I \quad (26)$$

$$0 \le b_j - b_{j'} \le H(1 - Z_{j,m,m'}),$$

$$\forall j \in J, \forall j' \in J', \forall m, m' \in M, \forall b_j, b_{j'} \in B \quad (27)$$

In the objective function (1), the total operation time of YCs of container yard could be divided into two parts. The main part is the traveling time of YCs between the successively operated containers, and another part which could not be ignored is the possible rehandles time, which is actually very important in determining an appropriate space for every container.

Constraints (2) - (8) define decision variables. Constraints (2) - (4) define C_j, constraints (5) and (6) define $V_{j,j'}$, and constraints (7) and (8) respectively define p_j and $Y_{j,m}$.

Both constraints (9) and (10) are quantity constraints. Constraint (9) ensures that every storage space should be placed at most only one container, and constraint (10) ensures that every container must be placed at one storage space.

Constraints (11) - (21) define YCs operations. Constraints (11) and (12) ensure that every container must be operated by exactly one YC. Constraints (13) - (18) ensure that there are at most only one preceding operation and one subsequent operation to every container among the loading/unloading operation sequence. Constraint (19) is a flow balance constraint, guaranteeing that loading/unloading operations are performed in well-defined sequences. Constraints (20) and (21) are unique constraints of YC scheduling problem of yard block, differing from the typical m-parallel machine scheduling problem. By constraint (20), interference among YCs could be avoided. It is regulated that there are at least one bay distance between two simultaneously performed YCs. In similar, it is regulated by constraint (21) that two simultaneously performed YCs should not cross each other.

Constraints (22) - (24) are also unique constraints related to the storage space operation of container yards. Constraint (22) guaranteed that every container could only be placed on top of another container, or just be placed on the ground. Constraint (23) defines that containers with the same size should be placed at the same bay. Constraint (24) defines that containers of the same customer should be placed at the same bay.

Both constraints (25) and (26) are operation time constraints. Constraint (25) restricts the starting time of each operation, and constraint (26) restricts the ending time of each operation similarly.

Constraint (27) is a very important constraint, which defines rehandles. It is regulated that if a rehandle is happened during loading/unloading operation of a container, then the rehandle could only be accepted within the same bay.

III. CASE STUDY

In order to testify the above optimization model for both allocating storage space and scheduling YCs of container yard, we design different cases to run by Cplex10.1 software. All calculations run on a Dell D630 notebook with a double 2G CPU and a 2G memory.

Base on two different operation types, separated operation and mixed operation, the outcome and its optimized solution of allocating storage space of containers and scheduling YCs of yard block are illustrated in Table 2 according to different operated quantity of import/export containers. Fig.2 and Fig.3 demonstrate separately the computation time of CPUs and the objective value of operations for different number of containers.

We could find that mixed operation has a much better performance compared with separated operation of loading/unloading of containers. Mixed operation has an obvious advantage that it could make full use of attributes of different containers. It could be estimated that the greater the number of containers to be operated, the better performance the mixed operation.

TABLE II
OUTCOME AND ITS OPTIMIZED SOLUTION OF STORAGE SPACE ALLOCATION

Number of Containers (Import/Export)	Separated Operation		Mixed Operation	
	CPU(s)	Objective Value	CPU(s)	Objective Value
20(12/8)	101.17	112	133.47	94
25(15/10)	131.79	114	132.12	108
30(18/12)	149.27	108	148.24	104
35(21/14)	563.99	132	201.97	120
40(24/16)	6235.12	140	5768.09	136

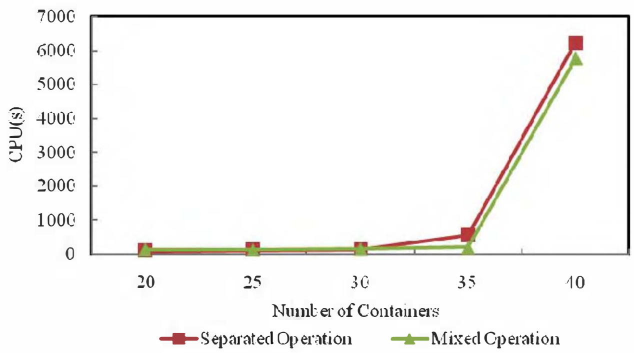

Fig. 2. Computation time (CPU(s)) of operations for different number of containers.

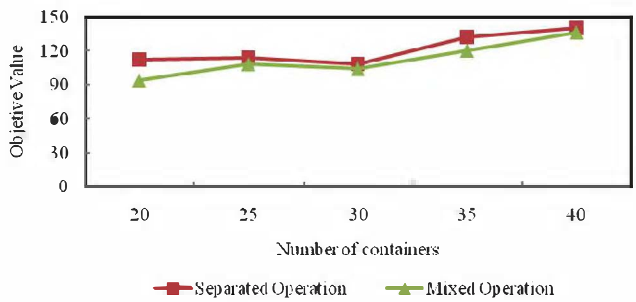

Fig. 3. Objective value of operations for different number of containers.

IV. CONCLUSION

Container terminals are main inter-modal interfaces in the global transportation network. A reasonable planning of container yard is really an important subject of container terminals.

In our study, based on mixture storage status, an optimization model of storage space allocation problem as well as scheduling YCs of container yard is put forward. The formulation of constrains (1)-(27) is a mixed-integer linear program. The optimization model is testified to be effective by the computational experiments in this paper. However, it is shown that the computational time is excessive for practical use for container terminals. Therefore, practical heuristic algorithm will be developed as an extension of this study to solve the systematic optimal problem for allocating both terminal space and equipment resources reasonably.

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