Dominant Strategies Implementation on Prisoner's Dilemma

Trunal Wadibhasme, Sourav Banerjee
Department Of Scientific Computing,Modelling and Simulation
Pune University
August 26,2022

Abstract

We explore how the Dominant Strategy is used in the Prisoners' Dilemma in comparison to other game strategies and evaluate how well it performs. Game theory uses the terms "dominant strategy" and "inferior strategy" to describe the best possible options for each player among all competitive strategy sets, regardless of how that player's opponents may play.[1]
Keywords*:Dominant Strategies,Inferior Strategies*

1. Introduction

The dominant strategy in game theory refers to a situation where one player has superior tactics regardless of how their opponent may play. Holding all factors constant, that player enjoys an upper hand in the game over the opposition. It means, regardless of the strategies employed by the opponent, the dominant player will always dictate the outcome.[1]
Finding the player's preferences might not seem all that difficult at first. It might seem a reasonable first approximation to regard an experimental subject's preferences over the outcomes as tracking the financial payoffs he or she would receive if experimenters ensure that subjects never meet and cannot identify one another, thereby eliminating all kinds of extraneous motives. However, when I read the literature, it becomes clear that this assumption is startlingly inaccurate, even as a first estimate. People's motivations are complex, making it challenging to discern them in many experiments.[1][2]
In game theory, participants use a variety of separate methods to improve their decision-making in effort to defeat the competition. Game theory is frequently used as tactical tool by military, management, customers, players in oligopolies markets, and players in games like The Chase. Although the fundamentals of dominating strategy make Nash analysis largely redundant, a dominant strategy solution may also be in Nash equilibrium. In other words, the incentives for cost and benefit are independent of other actors.[1][2]

1.1. Dominant Strategy Outcomes

1.**Strictly Dominant** Outcome In some situations, one player enjoys a strict advantage over their opponent. It means that, no matter how good the losing party's tactic is, the dominant strategy will always prevail. Here, there is no other possible strategy the opponent can use to alter their odds.[2]
2.**Weakly Dominant** Outcome In a weakly dominant outcome, the dominant player dominates the game but against some strategies, only weakly dominates.[2]
3.**Equivalent Outcome** In an equivalent outcome, none of the actors benefit or lose against each other. They each choose the one optimal result that is fair for both players. In case one of the players selects the alternative, it would mean an outlandish gain or loss.[2]
4.**Intransitive Outcome** In an intransitive outcome, none of the above three outcomes are experienced – no equivalent, strictly, or weak dominant outcome results. The available outcome happens by chance. Either player can win, while the other loses depending on the strategy employed. Therefore, in this outcome, there is no well-defined approach to point to the dominance strategy.[2]

1.2. Practical Example

The prisoner's dilemma is a well-known illustration of the situation two criminals, A and B, find themselves in when they are being persecuted for car theft. The prosecutor believes that the two defendants may have previously committed a burglary offence but were not found guilty.[2][3]
Since there isn't any concrete evidence, the DA uses game theory to coerce the two into confessing. They are given incentive to turn on one another. They will serve a two-year prison sentence for the car theft offence, which is proven beyond a reasonable doubt.If A convicts B of the burglary, the term will be reduced to one year, but B will receive a seven-year sentence for refusing to cooperate. On the other hand, if A admits but B confesses, A will receive a sentence of seven years and B will receive a sentence of one year. Suspect B is given the same deal. However, the sentence will lowered to three years in prison if both admit to of-

		Suspect B			
		Confess		Deny	
Suspect A	Confess	3	3	1	7
	Deny	7	1	2	2

Figure 1. PayOff Matrix[5][8]

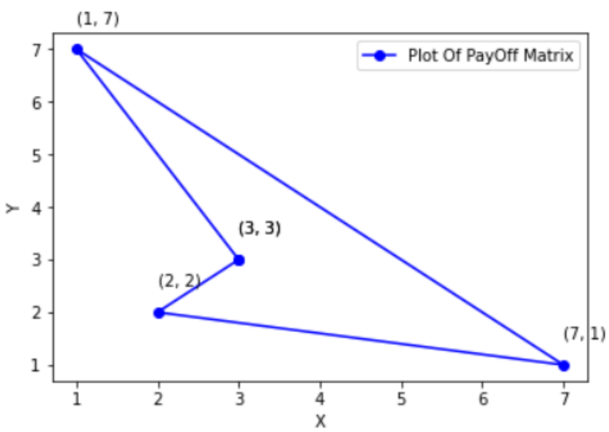


Figure 2. Polygon of PayOff Matrix[5][8]

fence.[2][3]
On the basis of the example, we can create the above-described Payoff matrix table, which reflects a comparable result. This is due to the fact that confessing will be Suspects A and B's primary tactic. The decision between three years and seven years, or one year and two years, will never be easy for either suspect. These can also be shown as a polygon graph, as seen above, and we can use the graphical representation to determine the average score of each function.
Although the fundamentals of a dominating strategy make Nash analysis largely redundant, a dominant strategy solution may also be in Nash equilibrium. Therefore, the cost and benefit incentives are unaffected by the actions of other actors.The optimal strategy for each player is unaffected by the other players' moves in the dominant strategy. This makes the fundamental tenet of the Nash equilibrium—that each actor is aware of the best course of action for the other players—possible but all but useless.[2][3]

The only rational response in this situation would be to use the confessional strategy. No one will want to jeopardise their commitment to the other. The reason for this is that the alternative—seven years in prison rather than one—is harsher. The result is what is known as the Nash equilibrium, a situation where both parties are likely to decide to confess.[2][3]

1.3. Nash Equilibrium Solution

The equilibrium is named after John Forbes Nash, Jr., who authored a one-page article in 1950 (and a longer follow-up in 1951) describing a stable-state equilibrium in a multi-person situation where no participant gains by a change in his strategy as long as the other participants also remain unchanged.[3]In other words, when no other player would take a different action, a Nash equilibrium would occur and each player would remain in their current positions. Each participant decides against moving because doing so would make them worse off.
The prisoner's dilemma is arguably the most well-known instance of a Nash equilibrium. The prisoner's dilemma involves two criminals who are apprehended and questioned separately. Each anticipates the other criminal to confess and come to a plea agreement even though they would both be better off not cooperating with the authorities. Because of this tension between individual and group logic, each criminal is more likely to turn on the other. This illustration has raised some questions concerning Nash equilibrium. The Nash equilibrium can exist whether all group members cooperate or not; the theory is not just applied to circumstances where one party deviates. Numerous games may in fact have different Nash equilibrium.
We have several questions after taking into account both concepts, such as: Why Is an Equilibrium Stable in Dominant Strategies? or Is the Nash Equilibrium the same as the dominant strategy?.Each player is unaware of the other's ideal plan when using a dominant strategy. The best choice available to each participant is chosen. When each player is aware of their opponent's strategy and uses it to inform their own strategy, Nash equilibrium is reached. However, the Nash equilibrium might be the most effective tactic. The best approach used by players is the dominant strategy. When neither party has a reason to change and both parties have dominating tactics, equilibrium is stable.[3]

1.4. Mathematical implicative insinuation of Ascendant Strategy

A strategy X **dominates** a strategy Y if every entry for X is greater than or equal to the corresponding entry for Y. In this case, we say Y is dominated by X. If strategy X dominates strategy Y, we can write $X \geq Y. In\ mathematical form\ let$ a_{ik} be the value in the i^{th} row and k^{th} column. Similarly, a_{jk} is the value in the j^{th} row and k^{th} column. The i^{th} row dominates the j^{th} row if $a_{ik} \geq a_{jk}$ for all k, and $a_{ik} \geq a_{jk}$ for at least one k.
In case of **Nash Equilibrium** a strategy s_i for player i is a best response to the strategy profile si of all other players if for all s'_i

$$u(s_i, s_1) \geq u(s'_i, s_i)$$

In other words, a strategy is the optimum course of action for a certain player if, given what all other players are doing, she is at a loss for what to do. It's important to note that this definition is quite similar to the definition of dominance in that a strategy s_i is the optimal reaction to a certain action s_i of the other players. In the case of dominance, the method has to be preferred above others' actions generally rather than simply in one particular situation.[9]

1.5. Methodology

Along with all other function we have perform a diminutive code predicated on prisoners dilemma and perform step by step operation which is shown below with basic set of algorithms[8]
1.Define the Strategy function and call it once the game has begun.
2.Define Another Function for storing the scores of the players.
3.Initialize the variable i by assign counter function i=counter_abh().
4.We refer to the function that has assign in it as Temp_list=i().
5.lastEle = temp_list[len(temp_list)-1]
6.store the lastEle value in temp_list
temp_list.append((lastEle)+1)
7.call the counterAb function and store result in variable
cnt = counterAb(temp_list)
8.call the cnt() function amp; store result in some variable
currCn = cnt()
9.curCnt = currCn[len(currCn)-1]
10.call the storeOpAb() function and store the return value
11.Define score function
def score(my_input=None,op_input=None):
12.Conclusively return the Average Score of each Function respectively.
Looking at the average scores of all the functions, we concluded that Trunal, or as we can verbalise our function, is lagging, and in order to compare the results, we will perform EDA to understand the difficulty we are experiencing while playing with other stratergies.

1.6. Results

After repeating the experiments n times, we visually examined some of the characteristics of the various Strategic Functions used in the process.First, we gathered all of the function's average scores, and then we used the scatter plot to examine the function's various characteristics.Scatter plots are habituated to plot data points on a horizontal and a vertical axis in the endeavor to show how much one variable is affected by another. Each row in the data table is represented by a marker whose position depends on its values in the columns set on the X and Y axes.Scatter plots can also be known as scatter diagrams or x-y graphs, and the point of using one of these is to determine if there

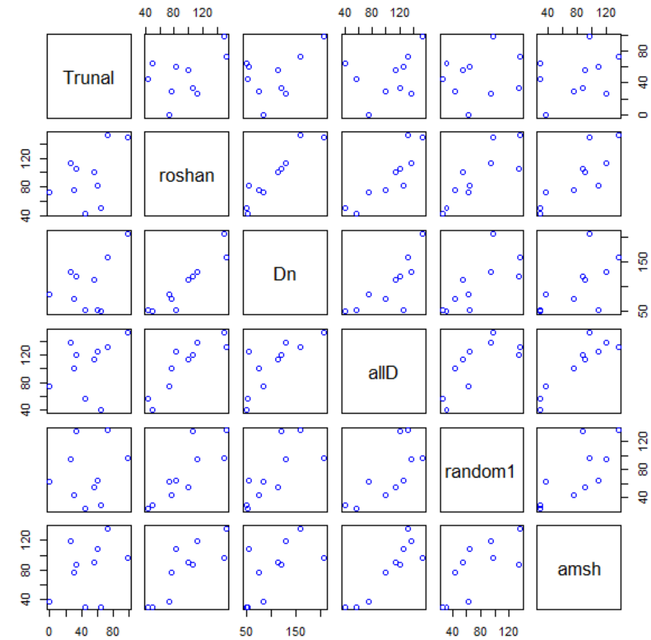


Figure 3. Scatter Plot[8]

are patterns or correlations between two variables.While comparing we have optically canvass that the Function of Roshan,Durgesh,and AllD have exhibiting remotely linear properties as compared to Trunal and Amsh function respectively.[2][3][4][8][9]

Using Scatter Plot visualization is not properly clear so **we have to do other EDA(Exploratory Data Analysis)** in order to observe any patter of randomness within the function and can able to predict where the function is lagging.
The Fig4 plot below depicts the relationship between the number of matches played and the average score for each Strategic function. As we can optically discern from the graph, our curve is red when compared to other strategic functions, and the green curve, which represents the Dn Function set, achieves the highest average Score.
While comparing,our graph doesn't perform well but keep on incriminating as the number of matches played in the games and form a pattern like Peaks and troughs(zigzag) perform Peak and Tough analysis to determine the lag.We should be aware of consolidation in the study of peaks and troughs to recognize this sideways pattern, avoiding the mistake of thinking the prevailing trend is about to reverse.In this plot, In this plot, I can see that the functions that choose more Deny have a higher chance of winning than those that choose Confess and red curve perform in periodic manner as we have visually examine that from match 2 to 4 it decrements then from 4 to 6 increases and then de-

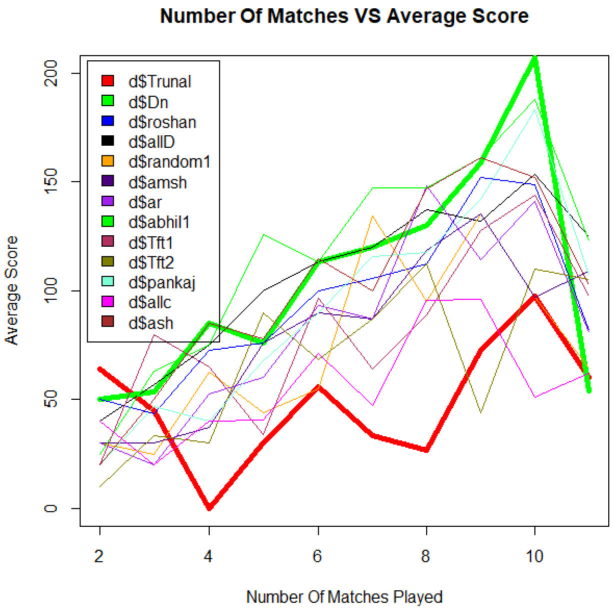


Figure 4. Curve Plot[8]

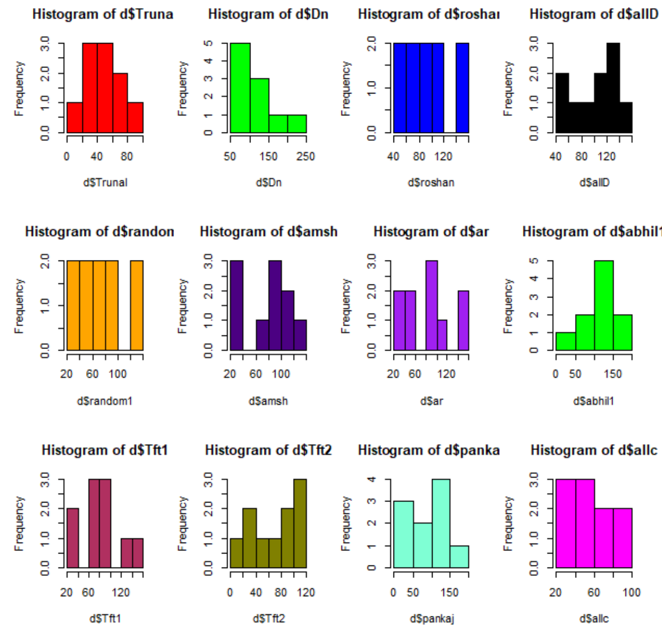
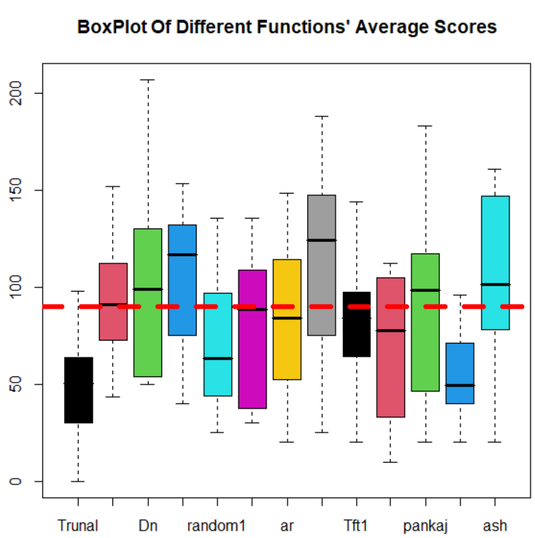


Figure 5. Histogram[8]

creases and goes on continuously and if we perform for sizably voluminous number of matches maybe red curve can victoriously triumph.[6]
To visually examine any pattern, whether normal, uniform, or exponential, we must first perform a histogram to determine the distribution roughly as shown in fig 5 and can optically canvass that red and green histogram show some short of uniform distribution while in case of green histogram it establish Exponential,While the orange and blue histogram shows a uniform distribution.[2][3][4][8][9]



. Figure 6.BoxPlot[8]

From all the three EDA plots we have conclude that the function which is having more occurrence of Gainsay have the better chance of acquiring victory in comparison with the Confess one,Even thought we are performing Dominant strategies still we require more number of games in order to reach at top level.
The Box plot shows the variation of different average score function Strategies and the average score lies between 90 showen by dotted lines.It consist of 3 Quantiles and one median along with some outliers.The plot such as AbhiI1,AR and Amsh have astronomically immense variation while compared to others and Allc,Trunal and Tft1 have less variation.SO we can conclude that our function has less variation.For an accurate prediction of the model, algorithms need a low variance and low bias. But this is not possible because bias and variance are related to each other: If we decrease the variance, it will increase the bias. If we decrease the bias, it will increase the variance.

1.7. Shortcomings

Both methods have in common one major shortcoming: they do not always narrow down what may happen in a game to a tractably small number of possibilities. For example, a game has an equilibrium in dominant strategies only if all players have a dominant strategy. If this is not the case, this solution concept is not very useful.
Similarly, some games may not have any strategies that can be deleted via iterated deletion. Even among games that

do have some dominated strategies, the remaining set of rationalize strategies may be very large and so the predictive power may not be precise enough to be useful.
A drawback of the dominant strategy solution concept is, however, that it will often fail to exist. Hence if we wish to develop a predictive theory of behavior in games then we must consider alternative approaches that will apply to a wide variety of games.[4]

1.8. Conclusion

1.A player has a dominant strategy if that strategy gives them a higher payoff than anything else they could do, no matter what the other players are doing. If a player has a dominant strategy, expect them to use it!
2.A player has a strictly dominated strategy if that strategy gives them a lower payoff than any other strategy they could use, no matter what the other players are doing. If you have a strictly dominated strategy, expect other players to anticipate you'll never play it and choose their actions accordingly.
3.After conducting the experiments, we can only conclude that more matches should be played in order to win in the case of dominant strategy.
4.The function with the most Deny or any strategy that gives the player more Deny has a better chance of winning.
5.According to game theory, the dominant strategy is the optimal move for an individual regardless of how other players act.
6.A Nash equilibrium describes the optimal state of the game where both players make optimal moves but now consider the moves of their opponent.
7.A well-known example of where the Nash equilibrium plays out in game theory is the prisoner's dilemma.
8.Although independent concepts, the dominant strategy could also be the Nash equilibrium.
9.Nash equilibrium can occur when a group fully cooperates or when no members of a group cooperate.[2][3][6][8]

1.9. References

[1]Ruisheng Li,Publication: Science Direct,Dominant Strategy

[2]CFI Team,CFI, Dominant Strategy

[3]CHRISTINA MAJASKIinvestopedia,Comparing a Dominant Strategy Solution vs. Nash Equilibrium Solution

[4]Michael Kingston,Towards Data science Dominance, Domination and Iterative Deletion

[5]Jacob Koehler,youtube,Nash Equilibrium with Python

[6]Mary Hall ,investopedia,Peak-and-Trough Analysis

[7]Stcakoverflow,StackOverflow

[8]Github,souravBanerjCodes

[9]Mathematically Formula