Homework 1

Problem 1

Consider a satellite of mass m orbiting the Earth with mass $M_\oplus$ in a circular orbit with orbital distance r. Consider that the satellite–Earth system is isolated and no other perturbatios exist from other Solar system bodies.

1. Derive the potential, kinetic, and total energies of the satellite. (10)
2. Consider that due to air drag the satellite's orbit loses energy. Show that the kinetic energy of the satellite will increase. Assume that the changes are adiabatic and the satellite always moves from one circular Keplerian orbit to another circular Keplerian orbit. (10)

Problem 2

Consider the derivation of interaction cross-section in Lecture 2. Find the rate of interactions with pericenter distance r_p for a single star of mass $M_\star$ in a star cluster with number density of n. (10)

1. Assume that all stars in a star cluster of density $n = 10^5\,\mathrm{pc}^{-3}$, velocity dispersion $v_\sigma = 10\,\mathrm{kms}^{-1}$, and total $N = 10^5$ have the following properties: $M_\star = 0.5\,M_\odot$, $\frac{R_\star}{R_\odot} = \left(\frac{M_\star}{M_\odot}\right)^{0.7}$.
   (a) Find out the expected rate of star-star physical collisions for any one of these stars. (10)
   (b) What is the rate of physical collisions by all stars in this star cluster? (10)
   (c) Recalculate the above numbers without taking into account gravitational focusing. Which term in the cross section dominates? (10)
2. If 10% of the cluster stars are binaries with semimajor axis $a = 1\,\mathrm{AU}$, calculate the rates of total binary-single and binary-binary encounters with $r_p \leq a$. (10)

Problem 3

Show that heat capacity is not negative for a central force law of the form r^{-1}. (30)

Hints:

1. What was the key assumption that resulted in a negative heat capacity in a r^{-2} force law?

2. What would be the equivalent virial theorem for r^{-1} force law?

3. Does negative heat capacity follow?