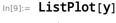
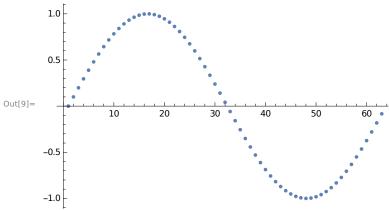
Range[i_{max}] generates the list {1, 2, ..., i_{max} }.

```
Range[i_{min}, i_{max}] generates the list {i_{min}, ..., i_{max}}.
                    Range [i_{min}, i_{max}, di] uses step di.
   In[2]:= Range [10]
  Out[2] = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}
   In[3]:= Range[0, 10, 2]
  Out[3]= \{0, 2, 4, 6, 8, 10\}
   ln[4]:= x = Range[0, 2Pi, 0.1]
  1.5, 1.6, 1.7, 1.8, 1.9, 2., 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 3.,
                     3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, 3.9, 4., 4.1, 4.2, 4.3, 4.4, 4.5, 4.6,
                    4.7, 4.8, 4.9, 5., 5.1, 5.2, 5.3, 5.4, 5.5, 5.6, 5.7, 5.8, 5.9, 6., 6.1, 6.2
 In[12]:= Sin[Pi]
Out[12]= 0
   In[6]:= Sin[x]
  \texttt{Out} \texttt{[6]=} \quad \left\{0.,\, 0.0998334,\, 0.198669,\, 0.29552,\, 0.389418,\, 0.479426,\, 0.564642,\, 0.644218,\, 0.717356,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.479426,\, 0.47942
                     0.783327, 0.841471, 0.891207, 0.932039, 0.963558, 0.98545, 0.997495, 0.999574,
                     0.991665, 0.973848, 0.9463, 0.909297, 0.863209, 0.808496, 0.745705, 0.675463, 0.598472,
                     0.515501, 0.42738, 0.334988, 0.239249, 0.14112, 0.0415807, -0.0583741, -0.157746,
                     -0.255541, -0.350783, -0.44252, -0.529836, -0.611858, -0.687766, -0.756802, -0.818277,
                     -0.871576, -0.916166, -0.951602, -0.97753, -0.993691, -0.999923, -0.996165,
                     -0.982453, -0.958924, -0.925815, -0.883455, -0.832267, -0.772764, -0.70554,
                     -0.631267, -0.550686, -0.464602, -0.373877, -0.279415, -0.182163, -0.0830894
   In[7]:= y = Sin[x];
   In[8]:= ?ListPlot
                     ListPlot[\{y_1, y_2, ...\}] plots points corresponding
```

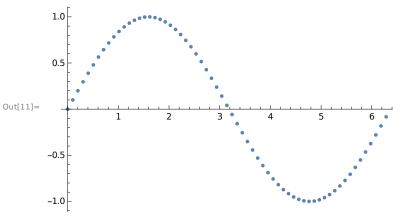
to a list of values, assumed to correspond to x coordinates 1, 2, ListPlot[$\{\{x_1, y_1\}, \{x_2, y_2\}, ...\}$] plots a list of points with specified x and y coordinates.

ListPlot[{ $list_1$, $list_2$, ...}] plots several lists of points.





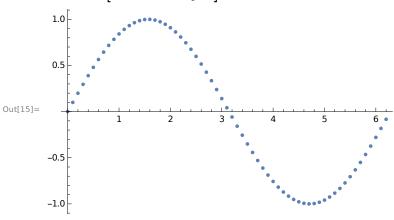
$ln[11]:= ListPlot[y, DataRange \rightarrow \{0, 2Pi\}]$



In[13]:= Thread[{x, y}]

```
Out[13] = \{\{0., 0.\}, \{0.1, 0.0998334\}, \{0.2, 0.198669\}, \{0.3, 0.29552\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.389418\}, \{0.4, 0.489418\}, \{0.4, 0.489418\}, \{0.4, 0.489418\}, \{0.4, 0.489418\}, \{0.4, 0.489418\}, \{0.4, 0.489418\}, \{0.4, 0.489418\}, \{0.4, 0.489418\}, 
                          [0.5, 0.479426], {0.6, 0.564642}, {0.7, 0.644218}, {0.8, 0.717356}, {0.9, 0.783327},
                         {1., 0.841471}, {1.1, 0.891207}, {1.2, 0.932039}, {1.3, 0.963558}, {1.4, 0.98545},
                         {1.5, 0.997495}, {1.6, 0.999574}, {1.7, 0.991665}, {1.8, 0.973848}, {1.9, 0.9463},
                          [2., 0.909297], {2.1, 0.863209}, {2.2, 0.808496}, {2.3, 0.745705}, {2.4, 0.675463},
                          [2.5, 0.598472], {2.6, 0.515501}, {2.7, 0.42738}, {2.8, 0.334988}, {2.9, 0.239249},
                          [3., 0.14112], {3.1, 0.0415807}, {3.2, -0.0583741}, {3.3, -0.157746}, {3.4, -0.255541},
                          [3.5, -0.350783], {3.6, -0.44252}, {3.7, -0.529836}, {3.8, -0.611858},
                          [3.9, -0.687766], {4., -0.756802}, {4.1, -0.818277}, {4.2, -0.871576},
                         [4.3, -0.916166], [4.4, -0.951602], [4.5, -0.97753], [4.6, -0.993691],
                         {4.7, -0.999923}, {4.8, -0.996165}, {4.9, -0.982453}, {5., -0.958924},
                          [5.1, -0.925815], {5.2, -0.883455}, {5.3, -0.832267}, {5.4, -0.772764},
                         {5.5, -0.70554}, {5.6, -0.631267}, {5.7, -0.550686}, {5.8, -0.464602},
                         {5.9, -0.373877}, {6., -0.279415}, {6.1, -0.182163}, {6.2, -0.0830894}}
```

In[15]:= ListPlot[Thread[{x, y}]]



In[18]:= ? Thread

Thread[f[args]] "threads" f over any lists that appear in args.

Thread[f[args], h] threads f over any objects with head h that appear in args.

Thread [f[args], h, n] threads f over objects with head h that appear in the first n args.

In[19]:= **? Table**

Table [expr, n] generates a list of n copies of expr.

Table [expr, $\{i, i_{max}\}$] generates a list of the values of expr when i runs from 1 to i_{max} .

Table [expr, $\{i, i_{min}, i_{max}\}$] starts with $i = i_{min}$.

Table $[expr, \{i, i_{min}, i_{max}, di\}]$ uses steps di.

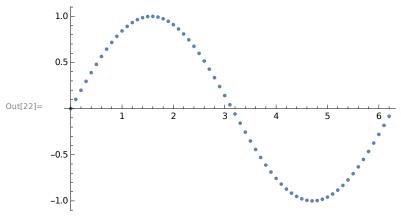
Table [expr, $\{i, \{i_1, i_2, ...\}\}$] uses the successive values $i_1, i_2,$

Table [expr, $\{i, i_{min}, i_{max}\}$, $\{j, j_{min}, j_{max}\}$, ...] gives a nested list. The list associated with i is outermost.

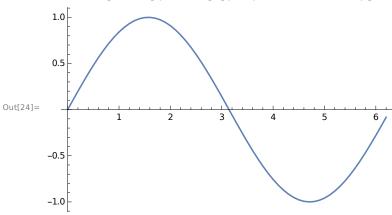
$$In[23]:= Table[{i, j}, {i, 5}, {j, 5}]$$

$$\begin{array}{l} \text{Out[23]=} & \left\{ \left\{ \left\{ 1\,,\,1 \right\},\, \left\{ 1\,,\,2 \right\},\, \left\{ 1\,,\,3 \right\},\, \left\{ 1\,,\,4 \right\},\, \left\{ 1\,,\,5 \right\} \right\},\\ & \left\{ \left\{ 2\,,\,1 \right\},\, \left\{ 2\,,\,2 \right\},\, \left\{ 2\,,\,3 \right\},\, \left\{ 2\,,\,4 \right\},\, \left\{ 2\,,\,5 \right\} \right\},\, \left\{ \left\{ 3\,,\,1 \right\},\, \left\{ 3\,,\,2 \right\},\, \left\{ 3\,,\,3 \right\},\, \left\{ 3\,,\,4 \right\},\, \left\{ 3\,,\,5 \right\} \right\},\\ & \left\{ \left\{ 4\,,\,1 \right\},\, \left\{ 4\,,\,2 \right\},\, \left\{ 4\,,\,3 \right\},\, \left\{ 4\,,\,4 \right\},\, \left\{ 4\,,\,5 \right\} \right\},\, \left\{ \left\{ 5\,,\,1 \right\},\, \left\{ 5\,,\,2 \right\},\, \left\{ 5\,,\,3 \right\},\, \left\{ 5\,,\,4 \right\},\, \left\{ 5\,,\,5 \right\} \right\} \right\} \end{array}$$

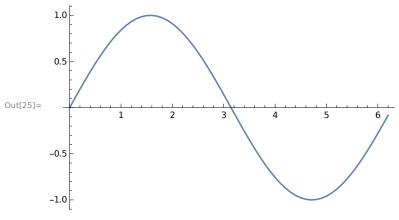
 $\label{eq:local_local_local_local} \mbox{ln[22]:= ListPlot[Table[{i, Sin[i]}, {i, 0, 2Pi, 0.1}]]}$



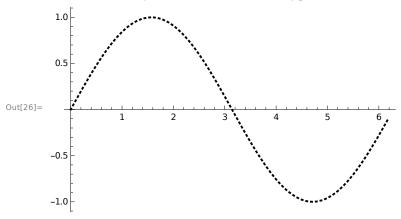
 $\label{eq:loss_loss} \mbox{ln[24]:= ListPlot[Table[{i, Sin[i]}, {i, 0, 2Pi, 0.1}], Joined \rightarrow True]}$



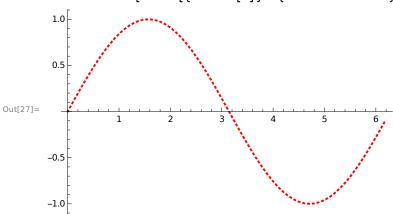
 $\label{eq:local_local_local_local} \mbox{ln[25]:= ListLinePlot[Table[\{i, Sin[i]\}, \{i, 0, 2Pi, 0.1\}]]}$



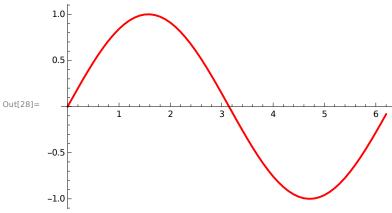
 $\label{eq:local_local_local_local} \mbox{ln[26]:= ListLinePlot[Table[\{i, Sin[i]\}, \{i, 0, 2Pi, 0.1\}],}$ PlotStyle → {Thick, Dotted, Black}]



 $\label{eq:local_$



 $\label{eq:local_$

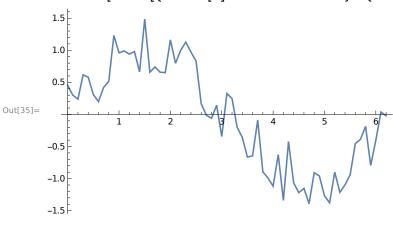


In[29]:= **? Random**

Random[] gives a uniformly distributed pseudorandom Real in the range 0 to 1.

Random[type, range] gives a pseudorandom number of the specified type, lying in the specified range. Possible types are: Integer, Real and Complex. The default range is 0 to 1. You can give the range {min, max} explicitly; a range specification of max is equivalent to {0, max}.

$$\label{eq:loss_limit} $$ \ln[35] := ListPlot\big[Table\big[\big\{i, \, Sin\big[i\big] \, + \, Random\big[\big] \, - \, 0.5\big\}, \, \big\{i, \, 0, \, 2\,Pi, \, 0.1\big\}\big], \, Joined \rightarrow True\big] $$$$



In[33]:= ? Show

Show[graphics, options] shows graphics with the specified options added. Show[$g_1, g_2, ...$] shows several graphics combined.

$$\label{eq:linear_loss} \begin{split} & \inf[37] := Show[ListPlot[Table[\{i, Sin[i] + Random[] - 0.5\}, \ \{i, \ 0, \ 2\,Pi, \ 0.1\}], \\ & \quad Joined \rightarrow True, \ PlotStyle \rightarrow \left\{Blue\right\}] \\ & \quad , Plot[Sin[x], \ \left\{x, \ 0, \ 2\,Pi\right\}, \ PlotStyle \rightarrow \left\{Red, \ Thick\right\}]] \end{split}$$

