Registration No.: | | | | | | | | | | |

Total Number of Pages: 02 **Course: B.Tech/IDD**
Sub_Code: CSBS2001

3rd Semester Regular Examination: 2024-25
SUBJECT: Discrete Mathematics
BRANCH(S): CE, CSIT, CSEAI, CSE, CSE, CSEAIML, CSEDS, CST, IT
Time: 3 Hours
Max Marks: 100
Q.Code: R564
Answer Question No.1 (Part-I) which is compulsory, any eight from Part-II and any two from Part-III.
The figures in the right hand margin indicate marks.

Part-I

Q1 Answer the following questions: **(2 x 10)**

- **a)** Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.
- **b)** Explain bijective function with an example.
- **c)** What rule of inference is used in this argument? If I study hard, then I get A's. I study hard. Therefore I get A's.
- **d)** In how many ways can a party of 5 men and 4 women be seated in a circular table so that no two women sit together?
- **e)** Define cyclic group. Give an example of it.
- **f)** Prove or disprove that every integral domain is a field.
- **g)** Define algebraic structures. Give an example of it.
- **h)** Define Hamiltonian circuits.
- **i)** Explain induced subgraph with a counter example.
- **j)** Define graph isomorphism with an example.

Part-II

Q2 Only Focused-Short Answer Type Questions- (Answer Any Eight out of Twelve) **(6 × 8)**

- **a)** Use mathematical induction to prove that $n^3 - n$ is divisible by 3 whenever n is a positive integer.
- **b)** Find $f \, o \, g$ and $g \, o \, f$, where $f(x) = x^2 + 1$ and $g(x) = x + 2$, both functions are from real numbers to real numbers.
- **c)** When we say a function invertible? Give an example of an invertible function.
- **d)** Find the particular solution of the recurrence relation $a_n + 5a_{n-1} + 6a_{n-2} = 3n^2$.
- **e)** How many poker hands of five cards can be dealt from a standard deck of 52 cards? Also how many ways are there to select 47 cards from a standard deck of 52 cards?
- **f)** Define partial ordering. Show that the inclusion relation $\subseteq$ is a partial ordering on the power set of a set S.
- **g)** If G is a finite group and $a \in G$ then show that $o(a) | o(G)$.

h) If G is a group such that $(a \cdot b)^2 = a^2 \cdot b^2$, for all $a, b \in G$, then show that G is abelian.
i) Show that kernel of homomorphism is a normal subgroup.
j) Show that a graph G is bipartite if it contains no cycle of odd length.
k) Write a short note on Dijkstra's algorithm.
l) Define vertex coloring. Prove that every plannar graph is 6-vertex colorable.

Part-III

Only Long Answer Type Questions (Answer Any Two out of Four)

Q3 **a)** Define propositional equivalence. Show that $(p \to q) \vee (p \to r)$ and $p \to (q \vee r)$ are Logically equivalent. **(8x2)**

b) State Demorgan's laws. Using Venn diagrams prove it.

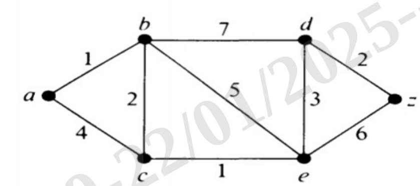
Q4 **a)** Define Pigeon-hole principle. Students are awarded 4 grades $A, B, C,$ and D. How many students must be there in a group so that at least 6 students get the same grade? **(8x2)**

b) Define equivalence relation. Explain it with a suitable example.

Q5 **a)** Show that a homomorphism ϕ of G onto $\bar{G}$ with Kernel K_ϕ is an isomorphism of G into $\bar{G}$ if and only if $K_\phi = (e)$. **(8x2)**

b) Define right coset of a subgroup H in the group G. Show that there is a one-to-one correspondence between any two right cosets of H in G.

Q6 **a)** Define Euler path and Eulerian circuit with example. Find shortest path from a to z in the following graph **(8x2)**



b) Define minimum spanning tree. Find a minimum spanning tree of the following graph