

Assignment VII

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Aim of the Problem:

The problem involves the generation of discrete distributions namely geometric, Poisson and Weibull distributions.

Mathematical Analysis/Theory:

Generating from geometric distribution:

It can be shown that with a random number U , then

$$X = \text{Int}(\log(U)/\log(q)) + 1.$$

is indeed geometric with parameter p .

Thus using U we can generate X following geometric distribution.

Generating from Poisson distribution:

For the case of the Poisson, we exploit the recursion property

$$p_{i+1} = (\lambda/(i+1))p_i \text{ for } i \geq 0.$$

The following steps can then be followed to generate from a Poisson with parameter λ :

step 1: generate a random number U .

step 2: set $i = 0$, $p = e^{-\lambda}$ and $F = p$.

step 3: if $U < F$, set $X = i$ and STOP.

step 4: set $p = \lambda p / (i+1)$, $F = F + p$, and $i = i + 1$.

step 5: return to step 3.

The composition of two distributions:

Consider now simulating from a distribution with mass function

$$P(X = j) = \alpha p_j^{(1)} + (1-\alpha)p_j^{(2)}; j \geq 0; 0 < \alpha < 1$$

If X_1 and X_2 are the random variables with respective mass functions, then $p_j^{(2)}$ and $p_j^{(1)}$

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**X = X1 with probability α
X2 with probability $1 - \alpha$**

One approach then to generate from this mixture distribution is :

step 1: generate a random number U1

step 2: generate from X1 and X2 distributions.

step 3: if $U < \alpha$; set $X = X1$.

step 4: else if $U > \alpha$, set $X = X2$;

Part I:

This question wants us to generate 50 random numbers following geometric distribution. For this we take $p=0.4$.

Implementation using R:

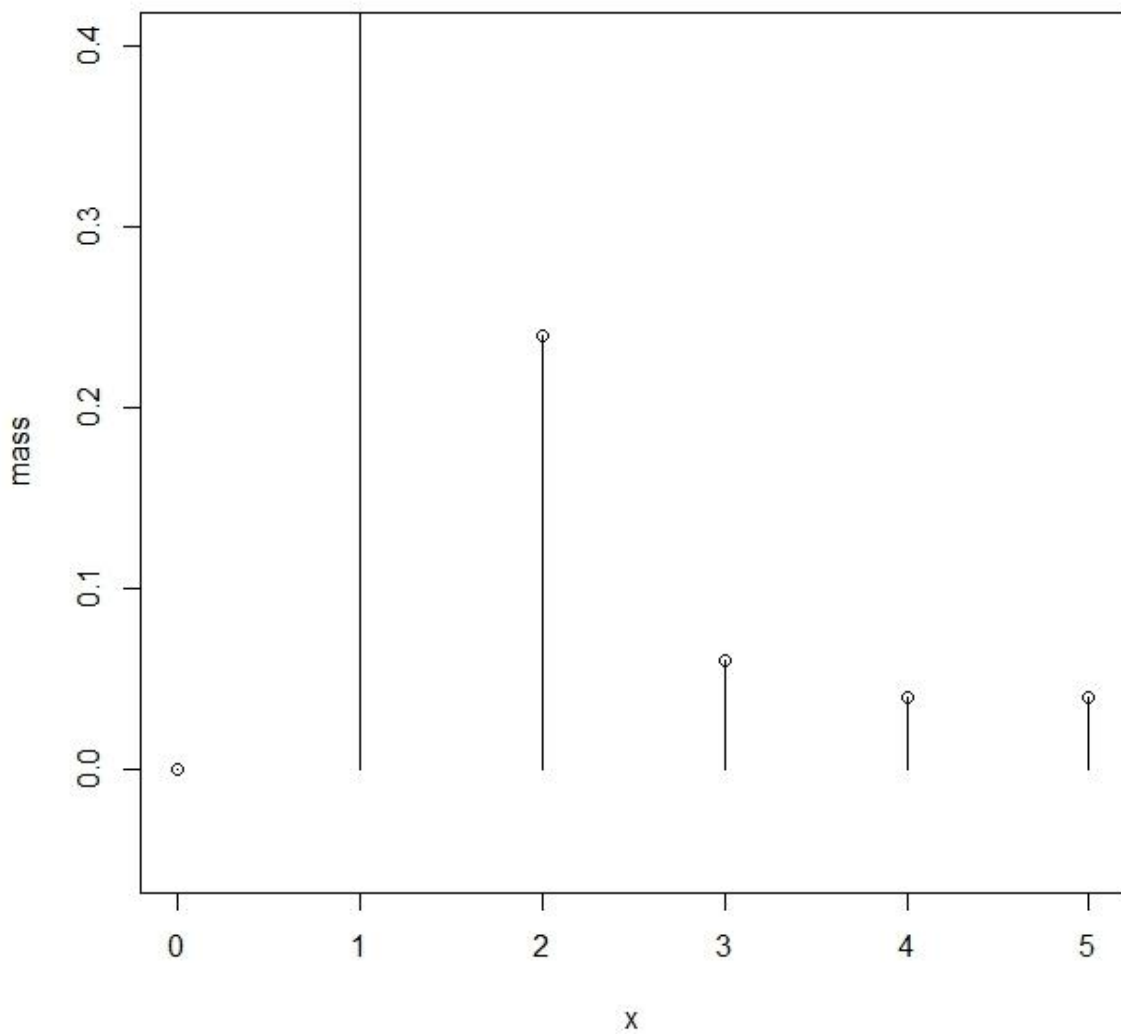
```
geomdist<-function(n)
{
  RN<-NULL;
  for(i in 1:n)
  {
    u1<-runif(1,min=0,max=1);
    p<-0.4;#taking p=0.4
    q<-1-p;
    x<-floor(log(u1)/log(0.4))+1;
    RN<-c(RN,x);
  }
  return(RN);
}
s<-geomdist(50);

mass <- NULL;
for(i in 0:max(s))
```

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```
{  
  m <- sum(as.integer(i == s));  
  mass <- c(mass,m);  
}  
mass <- mass/50;  
x <- seq(0,max(s));
```

Using the output the following mass function generated:



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Part II:

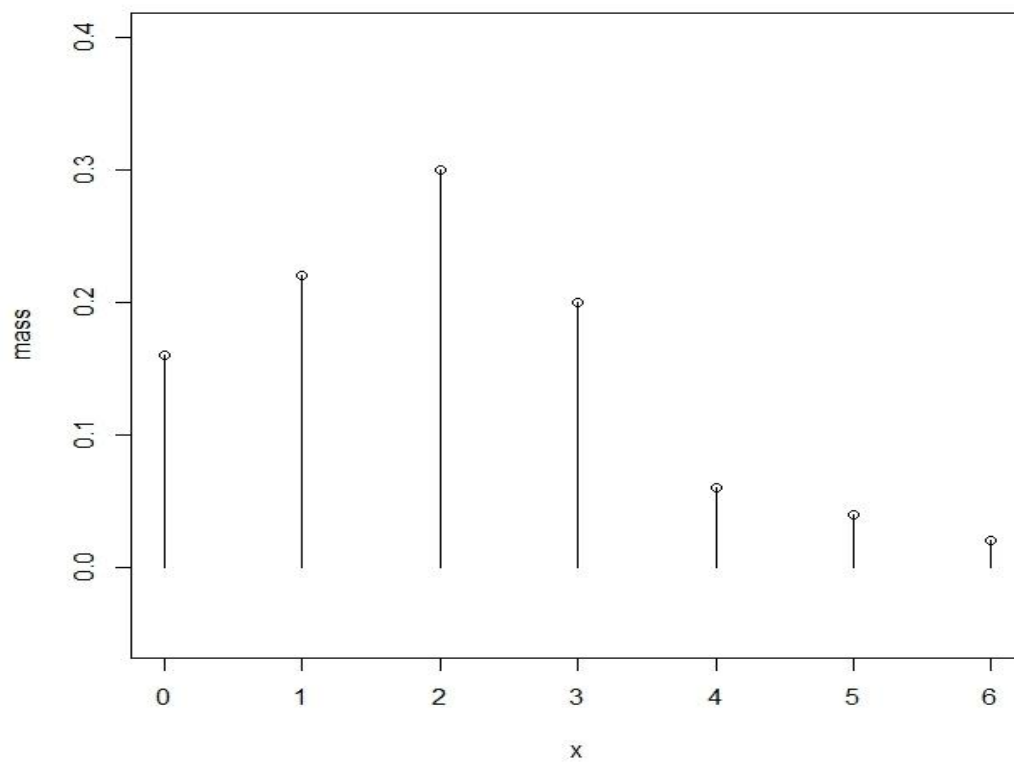
Here we need to generate 50 random numbers following poisson distribution.

Implementation using R:

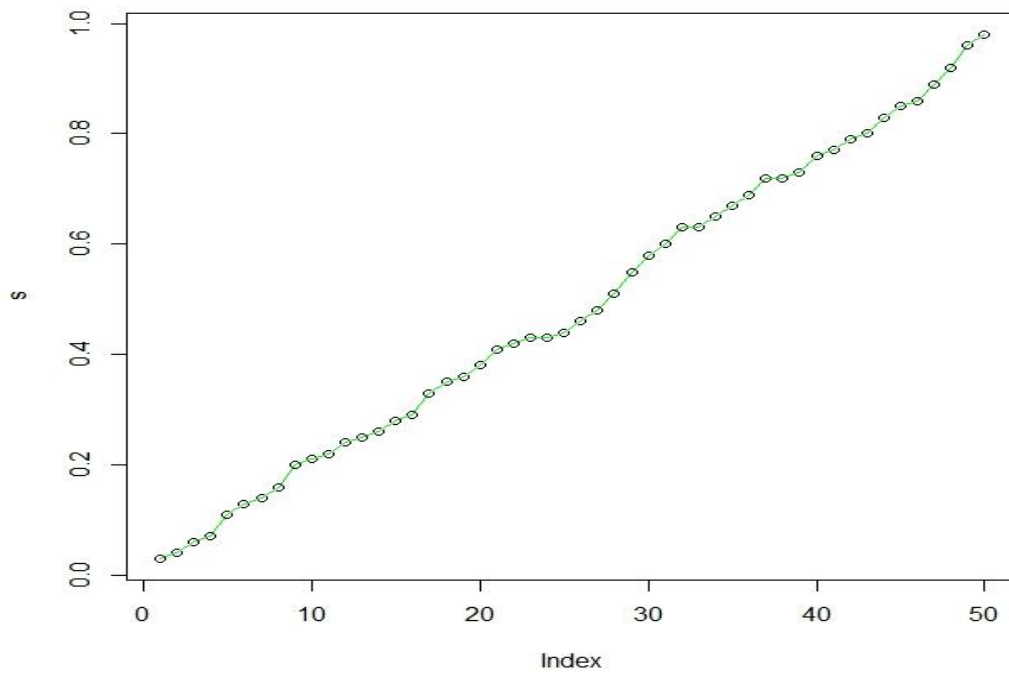
```
poisson<-function(n)
{
  RN<-NULL;
  for(i in 1:n)
  {
    u1<-runif(1,min=0,max=1);
    i<-0;
    p<-exp(-2);#mean=2
    f<-p;
    #x<-i;
    while(u1>f)
    {
      p<-p*2/(1+i);
      f<-f+p;
      i<-i+1;
    }
    x<-i;
    RN<-c(RN,x);
  }
  return (RN);
}
s<-poisson(50);
mass <- NULL;
for(i in 0:max(s))
{
  m <- sum(as.integer(i == s));
  mass <- c(mass,m);
}
mass <- mass/50;
x <- seq(0,max(s));
```

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Using the output the following mass function was generated:



The cumulative distribution of the numbers was like this:



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Part III:

This question asks us to generate joint-Weibull distribution.

The R code:

```
weibull<-function(n){  
  RN<-NULL;  
  for(i in 1:n)  
  {  
    u<-runif(1,min=0,max=1);  
    x1<-sqrt(log(1/u));  
    x2<-(sqrt(log(1/u)))^3;  
    if(u<=0.4)  
    {  
      x<-x1;  
      RN<-c(RN,x);  
    }  
    if(u>0.6)  
    {  
      x<-x2;  
      RN<-c(RN,x);  
    }  
  }  
  return(RN);  
}  
s<-weibull(50);
```

The output was used to generate a histogram:

