Assignment X

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Aim of the Problem:

The problem discusses ways to simulate Brownian Motion in time time intervals [0,5].

We use three techniques:

1) Generate the standard Brownian motion BM(0,1) using recursive technique.
2) Generate the Brownian motion BM(μ,6^2)
3) Generate Brownian motion using time dependent $\mu(t)$ and 6(t).

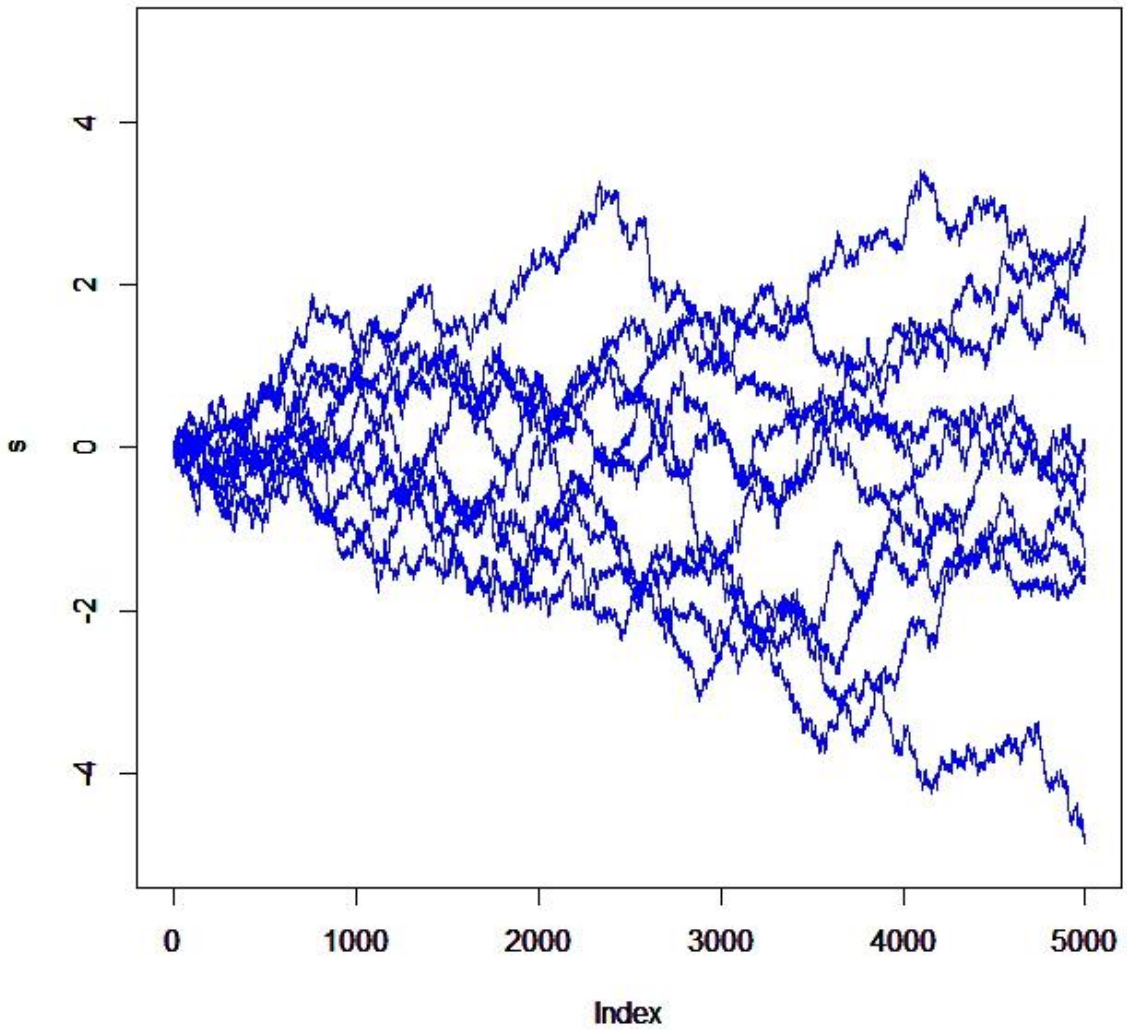
Part I:

This question wants us to generate 5000 BM(0,1) using recursive technique.

Implementation using R:

```
#this calculates the standard brownian motion BM(0,1)

for(j in 1:10)

{

	z<-rnorm(5000,mean=0,sd=1);

	w<-NULL;wm1<-NULL;wm2<-NULL;

	t<-seq(0,5,length.out=5000);

	w[1]=0;

	for(i in 2:5000)

	{

		w[i]=w[i-1]+(sqrt(t[i]-t[i-1]))*(z[i]);

	}

	wm1<-c(wm1,w[2]);

	wm2<-c(wm2,w[5]);

	s<-w;

	plot(s,type="l",col="blue",ylim=c(-5,5));

	par(new=TRUE);

}

k1<-mean(wm1);

k2<-mean(wm2);
```

The following plot was obtained for 10 paths:



Part II:

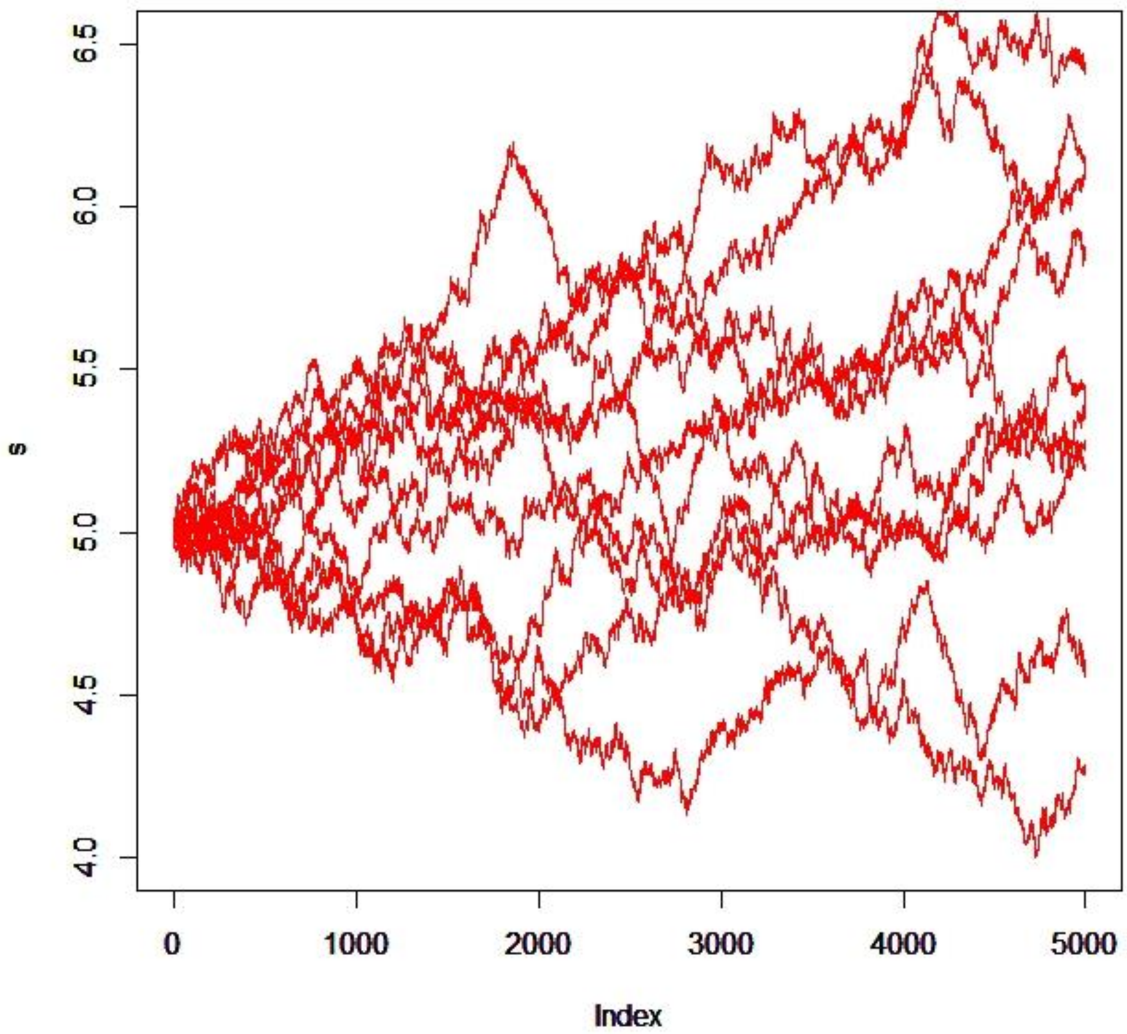
Here we have to repeat the same exercise using μ=0.06 and 6=0.3.

Implementation using R:

```
#this calculates BM with mean=0.06 and s.d.=0.3,starting from 5
for(j in 1:10)
{
        z<-rnorm(5000,mean=0,sd=1);
        w<-NULL;wm1<-NULL;wm2<-NULL;
        t<-seq(0,5,length.out=5000);
        w[1]=5;
        sig<-0.3;
        mu<-0.06;
        for(i in 2:5000)
```

```
        {
                w[i]=w[i-1]+mu*(t[i]-t[i-1])+sig*(sqrt(t[i]-t[i-1]))*(z[i]);
        }
        wm1<-c(wm1,w[2]);
                wm2<-c(wm2,w[5]);
        s<-w;
        plot(s,type="l",col="red",ylim=c(4,6.5));
        par(new=TRUE);
}
k1<-mean(wm1);
k2<-mean(wm2);
```

The following plot was obtained for 10 paths:

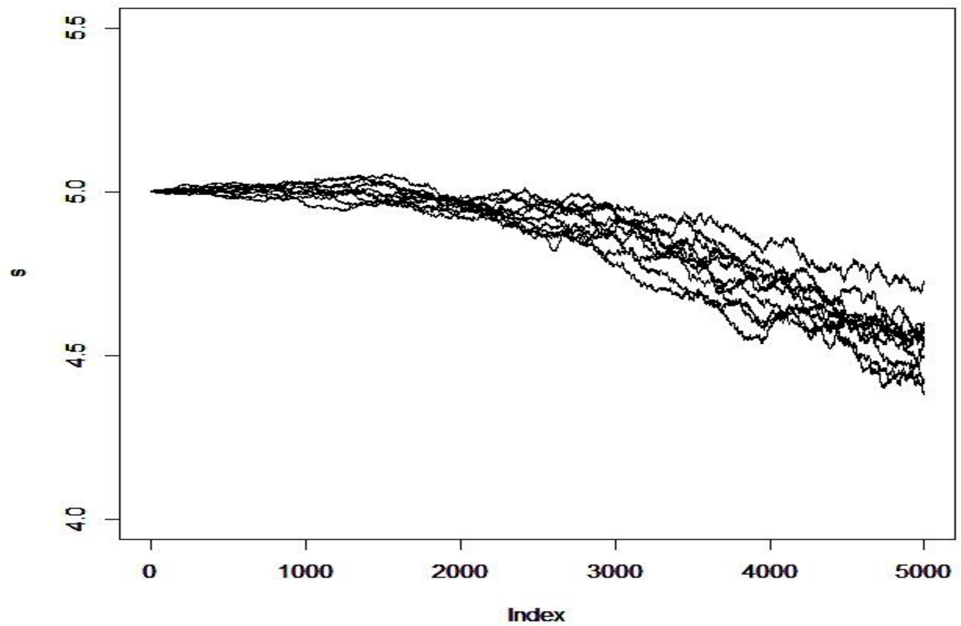


Part III:
Here we have to use Euler approximation to do the same.

Implementation using R:

```
#this uses Euler approximation to generate BM
for(j in 1:10)
{
        z<-rnorm(5000,mean=0,sd=1);
        w<-NULL;wm1<-NULL;wm2<-NULL;
        t<-seq(0,5,length.out=5000);
        w[1]=5;
        #sig<-0.3;
        #mu<-0.06;
        for(i in 2:5000)
        {
                mu<-0.0325-(0.05*t[i]);
                sig<-0.012+(0.0138*t[i])+(0.00125*t[i]*t[i]);
                w[i]=w[i-1]+mu*(t[i]-t[i-1])+sig*(sqrt(t[i]-t[i-1]))*(z[i]);
        }
        wm1<-c(wm1,w[2]);
                wm2<-c(wm2,w[5]);
        s<-w;
        plot(s,type="l",col="black",ylim=c(4,5.5));
        par(new=TRUE);
}
k1<-mean(wm1);
k2<-mean(wm2);
```

this plot was obtained from ten different paths:



The following table gives the E[W(2)] and E[W(5)] values:

Technique	E[W(2)]	E[W(5)]
BM(0,1)	0.04425319	0.01414178
BM(μ,6²)	5.01423	4.993528
Euler	4.99983	5.00049