CS & IT ENGINEERING

Quantifier 2

Lecture No.06

By- SATISH YADAV SIR

TOPICS TO BE COVERED

01 Negation of Quantifier

02 Nested Quantifier

03 Theorems on Nested Quantifier

04 Problems on Nested Quantifier

05 Nested Quantifeir Relations

Nested quantifier:

$\forall x$

$\exists x$

$\forall x (x^2 + 4 \geq 2)$

D: $\{1, 2, 3\}$

$\forall x (x^2 + y^2 \geq 0)$

$\forall y (1^2 + y^2 \geq 0)$

Type-1.

$P(x): x^2 \geq 0$ $P(x, y): x^2+y^2 \geq 0$ $P(x, y, z): x^2+y^2+z^2 \geq 0$

$\forall x \forall y \, (x \times y \leq 9)$

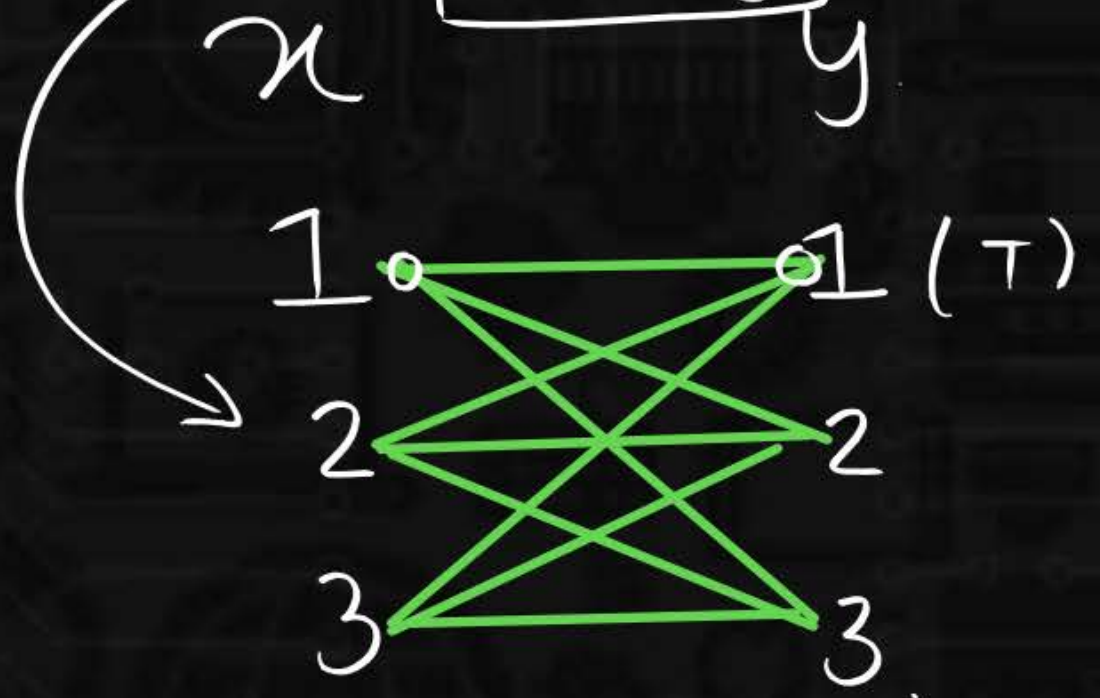
for all of x, all of y such that $P(x, y)$

for each element of x each element of y such that $P(x, y)$

$D: \{1, 2, 3\}$

$P(x, y): x \times y \leq 9$

$\boxed{\forall x \forall y}$



$D: \{1, 2, 3\}$

$\rightarrow x = 1 \quad y = 1$

$1 \times 1 \leq 9$ (T)

$\rightarrow x = 1 \quad y = 2$

$1 \times 2 \leq 9$ (T)

$\rightarrow x = 1 \quad y = 3$

$1 \times 3 \leq 9$

$x = 2 \quad y = 1$

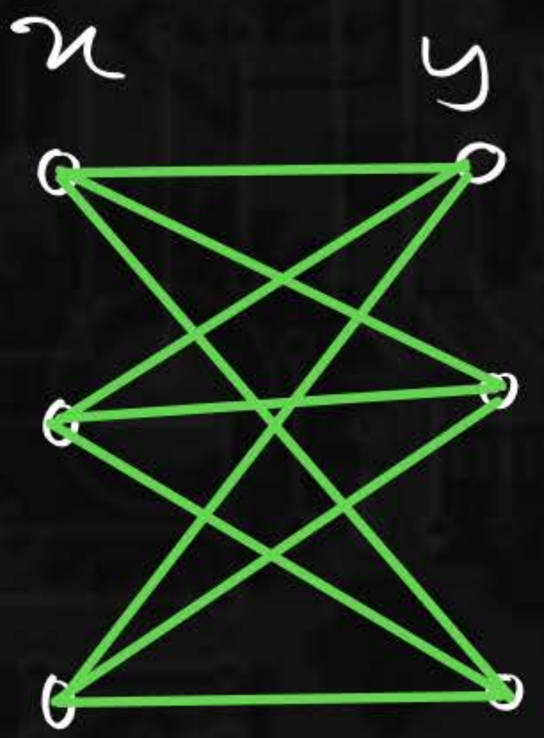
$2 \times 1 \leq 9$ (T)

$D : \{1, 2, 3\}$

$P(x, y) : x \times y \leq 9$

$\forall x \forall y \, (x \times y \leq 9) \rightarrow$ True

$\forall x \forall y \rightarrow$ false when at least 1 edge is false.



$\forall x \forall y \rightarrow$ True when all edges will be True

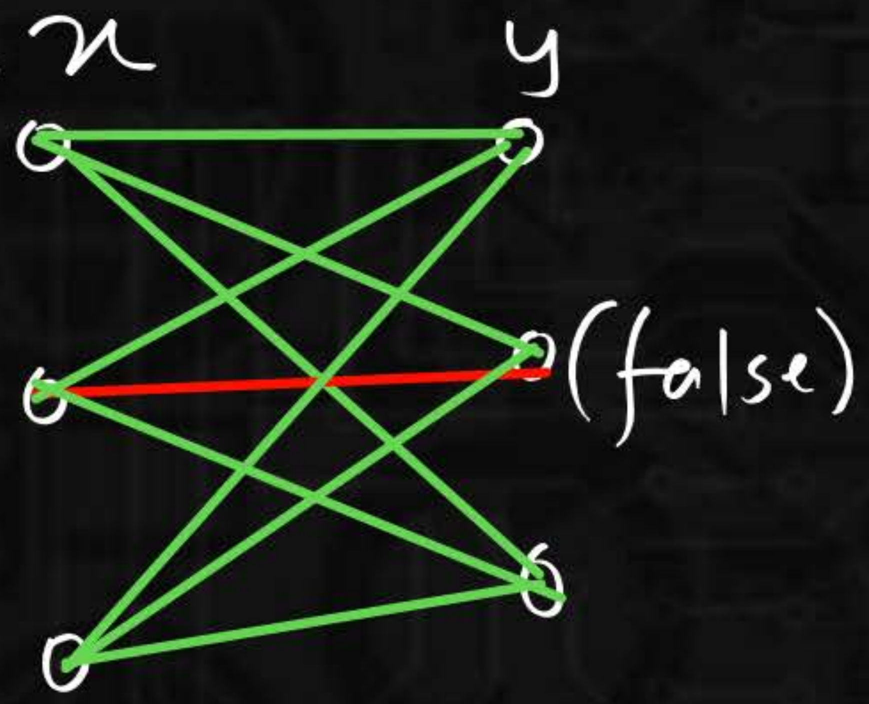
1 —— 1

$\wedge$

1 —— 2

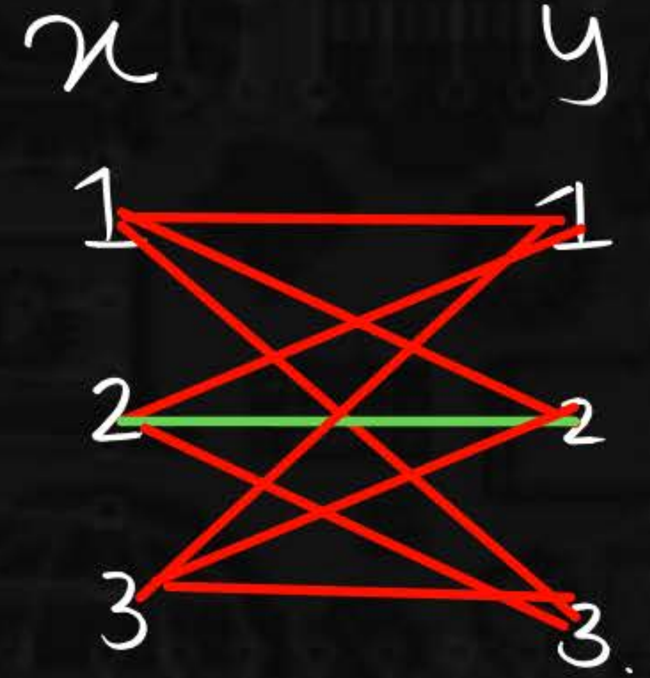
$\wedge$

1 —— 3



$D : \{1, 2, 3\}$

$P(x, y) : x \times y = 4.$



eg : $D : z.$

$\forall x \forall y \, (x \cdot y = 1)$

$\hookrightarrow$ false

$D : \{1, 2, 3\}$

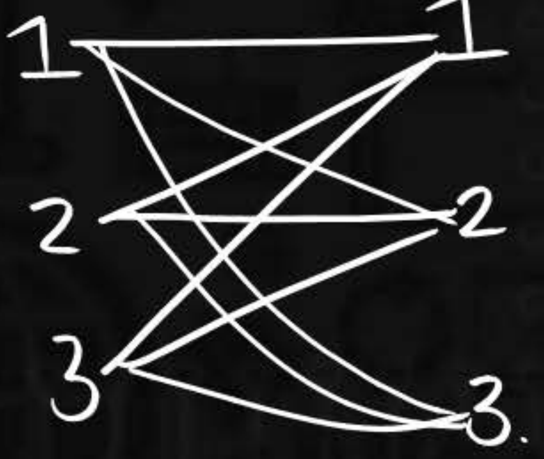
$P(x, y) : x \times y \leq 9.$

$\forall y \forall x$

$y = 1 \quad x = 1.$

$1 \times 1 \leq 9.$

$y \qquad x$



$\forall y \forall x \rightarrow$ True

When all edges will be True

$\forall y \forall x \rightarrow$ false

When at least 1 edge is false

$D : \{1, 2, 3\}$

$P(x, y) : x \times y \leq 9.$

$$\underset{F}{\overset{T}{\forall x \forall y}} \longrightarrow \underset{F}{\overset{T}{\overline{\forall y \forall x}}}$$

$$9 \left\{ \begin{array}{cc} x & y \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{array} \right.$$

***

$$\boxed{\forall x \forall y \equiv \forall y \forall x}$$

$\forall x \forall y \rightarrow \forall y \forall x$

$\forall y \forall x \rightarrow \forall x \forall y$

$T \longrightarrow F$

$D : \{1, 2, 3\}$

$P(x, y) : x \times y = 4$

$\exists x \; \exists y$

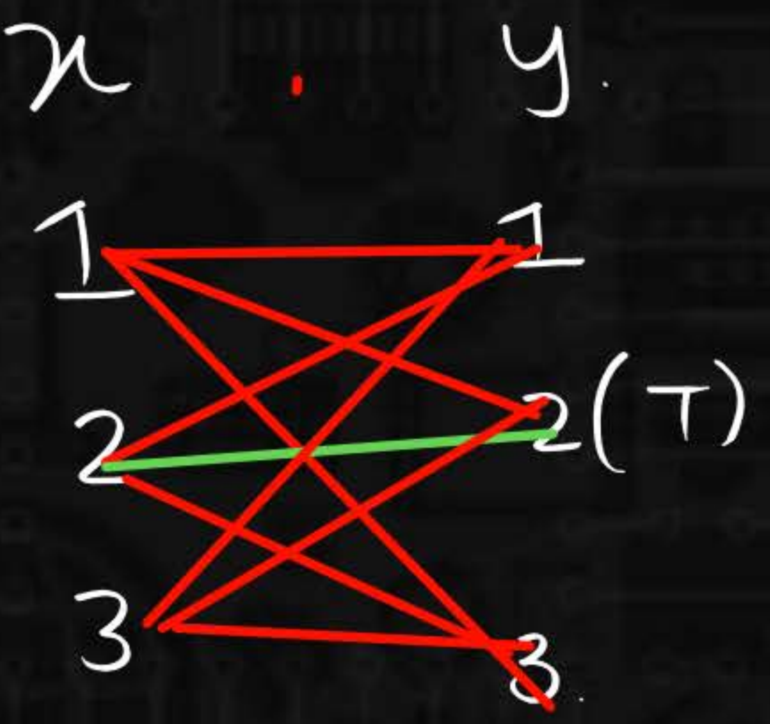
→ there exist x there exist y $P(x, y)$

→ atleast 1 value of x, at least 1 value of y $P(x, y)$

→ some value of x, some value of y $P(x, y)$

$D : \{1, 2, 3\}$

$p(x, y) : \underline{x \times y} = 4$



$x = 1 \quad y = 1$

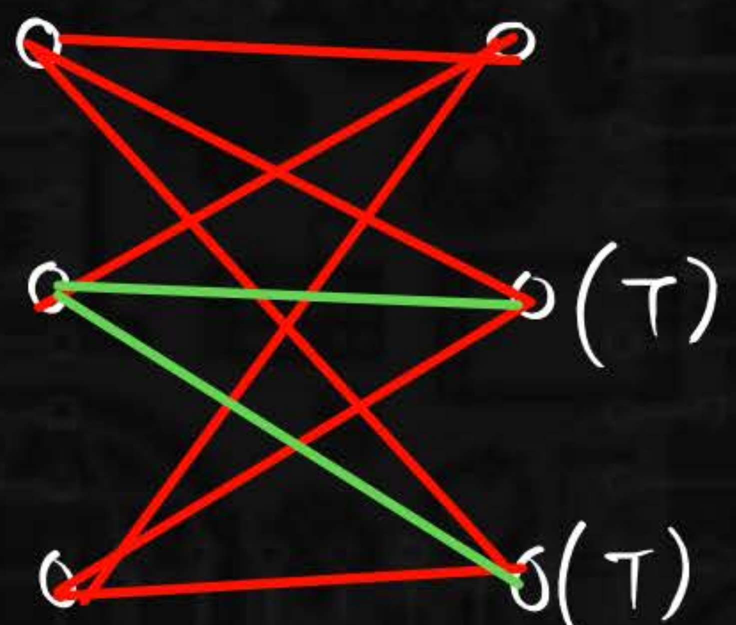
$1 \times 1 = 4 \text{ (F)}$

$x = 1 \quad y = 2$

$1 \times 2 = 4 \text{ (F)}$

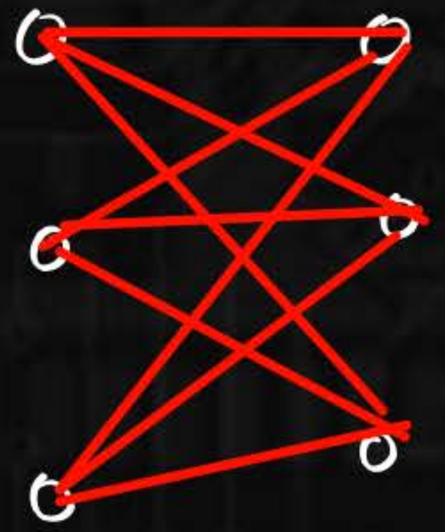
$\exists x \exists y \Rightarrow$ True

When at least 1 edge is True



$\exists x \exists y \rightarrow$ false

When all edges will be false.



$x \times y = 4$

$x = 1 \quad y = 1$

$P(1,1) \vee P(1,2) \vee$

$P(1,3) \vee P(2,1)$

$\vee$

$\vee$ ✓

$D : \mathbb{Z}$

$P(m, n) : m \times n = 1$

$\exists m \exists n (m \cdot n = 1) \rightarrow$ True

$m = 1 \quad n = 1$

$D : \mathbb{Z}$

$$P(x, y) : \left(\frac{x}{y} \in \mathbb{Z}\right)$$

$$\exists x \exists y \left(\frac{x}{y} \in \mathbb{Z}\right) \rightarrow \text{True}$$

$x = 4 \quad y = 2$

$x = 6 \quad y = 2$

D: 2.

$\exists x \exists y (2x + y = 6 \wedge 3x + 2y = 10)$ $\begin{cases} x = 2 \\ y = 2. \end{cases}$

True

$(2x + y = 6) \times 2.$

$3x + \cancel{2y} = 10$

$4x + \cancel{2y} = 12.$

$-x = -2.$

$x = 2$

$2(2) + y = 6.$

$4 + y = 6.$

$y = 2$

$\exists x \exists y \equiv \exists y \exists x.$

$\forall x \forall y \equiv \forall y \forall x.$

D: $P(x, y)$

fixed:

- Domain
- open stmt
- predicate variable

$\forall x \forall y \rightarrow \boxed{\exists x \exists y}$

$\equiv$

$\forall y \forall x \rightarrow \boxed{\exists x \exists y}$

$\forall x \forall y \rightarrow \boxed{\exists y \exists x}$

$\equiv$

$\forall y \forall x \rightarrow \boxed{\exists y \exists x}$

Type 1:

$\forall x \forall y$

Type 2

$\exists x \exists y$

$$P \to Q \equiv \neg P \vee Q$$

negation of quantifier :

$\underline{Q}$: $\forall x\, [P(x) \to Q(x)]$

what will be negation of this ?

$$\neg \forall x\, [P(x) \to Q(x)]$$

$$\exists x\, \boxed{\neg}\, [P(x) \to Q(x)]$$

$$\exists x\, \neg [\neg P(x) \vee Q(x)]$$

$$\exists x\, [P(x) \wedge \neg Q(x)]$$

negation:

$\exists x [P(x) \vee Q(x)]$

$\neg \exists x [P(x) \vee Q(x)]$

$\forall x \neg [P(x) \vee Q(x)]$

$\forall x [\neg P(x) \wedge \neg Q(x)]$

$\forall x [(P(x) \vee Q(x)) \longrightarrow R(x)]$

$\forall x \forall y [(x = y) \longrightarrow Q(x, y)]$

$\neg \forall x \forall y (\neg (x = y) \vee Q(x, y))$

$\exists x \exists y ((x = y) \wedge \neg Q(x, y))$

1. $\neg \exists x [P(x) \vee Q(x)]$

$\forall x [\neg P(x) \wedge \neg Q(x)]$

$\exists x \left[(P(x) \vee Q(x)) \wedge \neg R(x) \right)$

2.

$\neg \forall x [\overbrace{(P(x) \vee Q(x))}^{P} \xrightarrow{} \overset{Q}{R(x)}]$

$\exists x \neg [\neg (P(x) \vee Q(x)) \vee R(x))$

$$\forall x\,[\,(P(x) \vee Q(x)) \rightarrow R(x)\,]$$

$$\neg\,\forall x\,[\,\neg\,\boxed{(P(x) \vee Q(x))} \vee R(x)\,]$$

$$\exists x\,[\,\boxed{P(x) \vee Q(x)} \wedge \neg R(x)\,]$$


THANK YOU GW
SOLDIERS !