CS & IT ENGINEERING

GRAPH

Lecture No: 14

Satish Sir

TOPICS TO BE COVERED

- Garphs
- Distance, Radius, Eccentricity

$K_n \longrightarrow$ line Graph $L(K_n)$

<u>note</u>: Degree of each vertex in $L(K_n)$ is $2(n-2)$

Consider a set of vertices $\{1, 2 \ldots\ldots 100\}$

x, y are vertices.

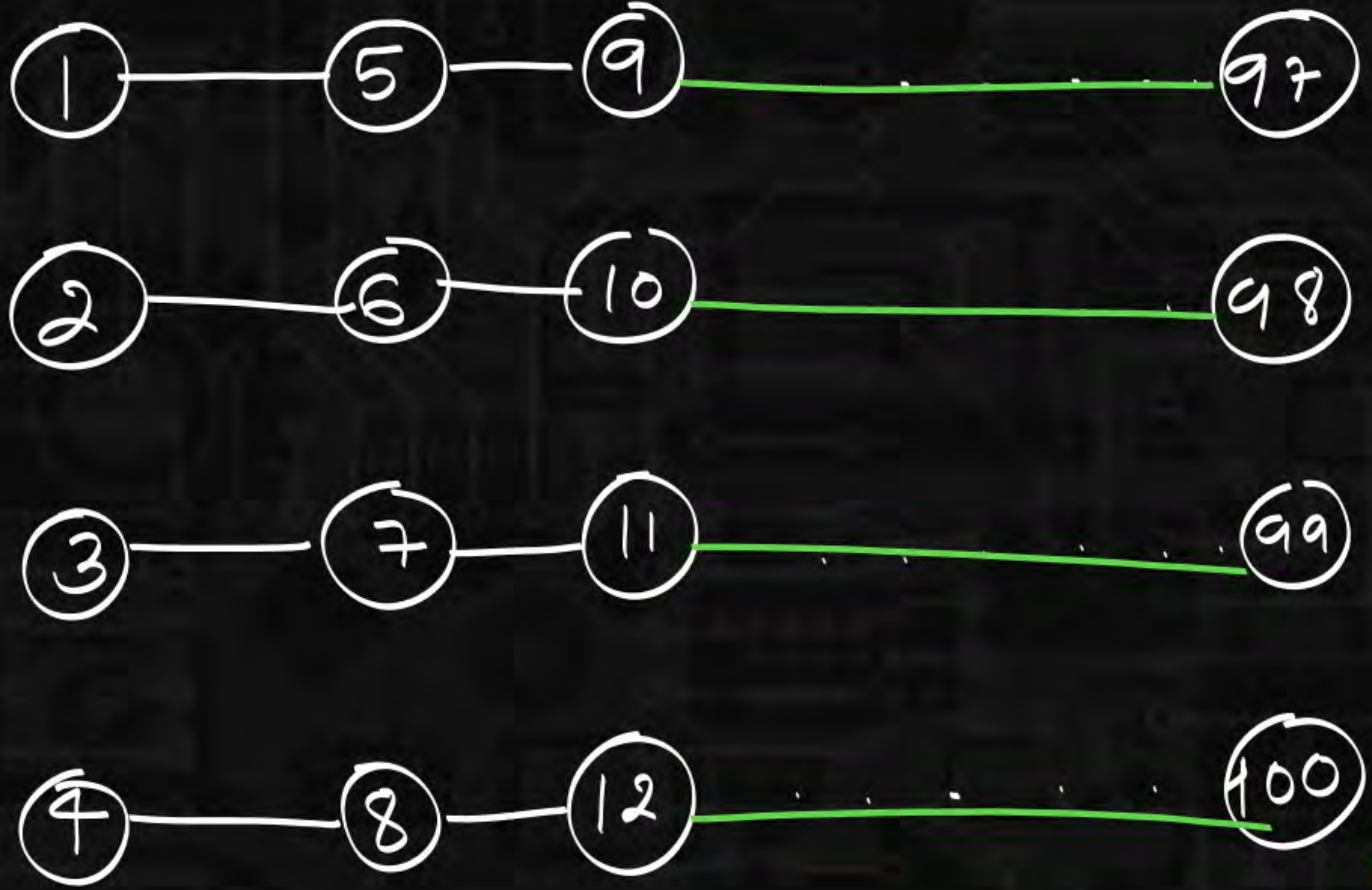
x, y are connected.

$|x - y| = 4$.

what will be total edges in G?

$|1 - 5| = 4$.

$|\pm x| = x$



24 e

24 e

24 e

24

96 edges

Components = 4

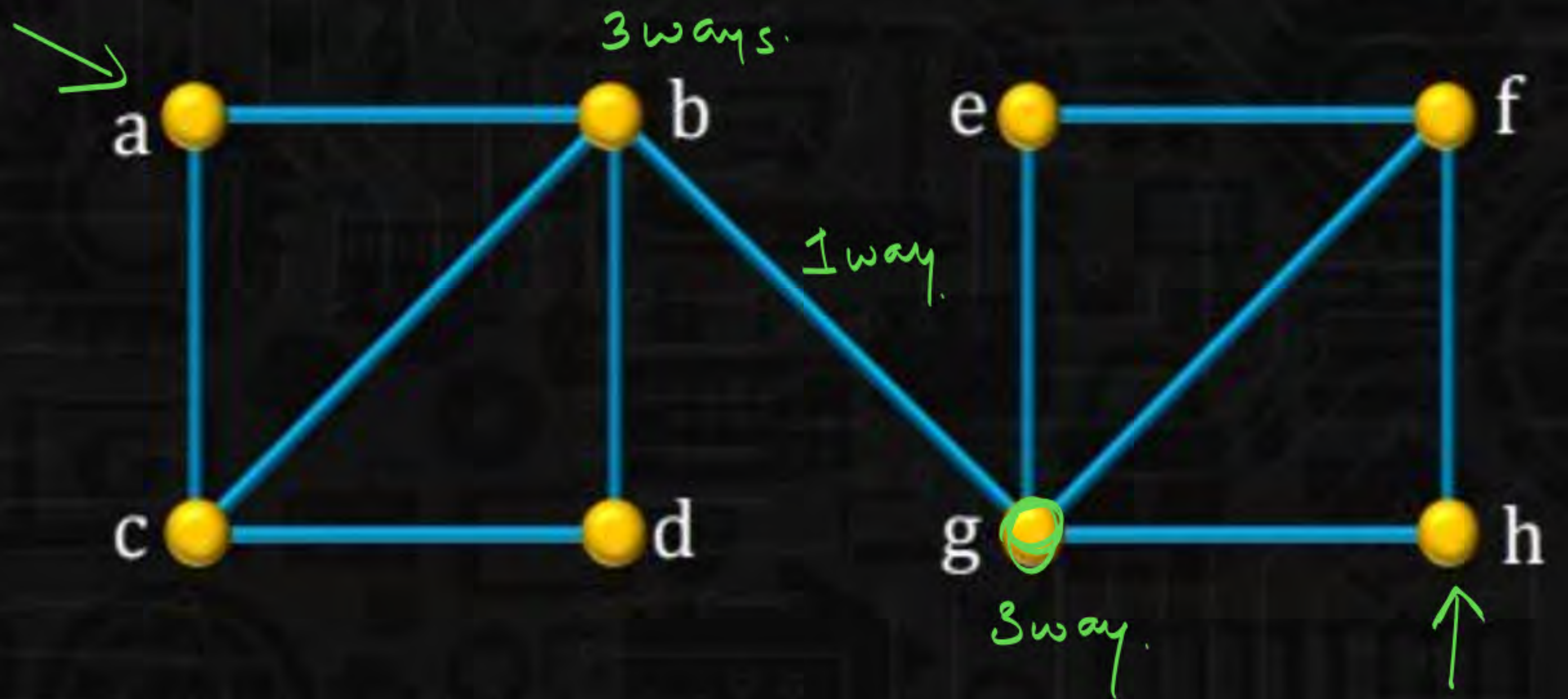
$e(G) = 96$

$K = 4$

$e(\bar{G}) = ?$

$e(G) + e(\bar{G}) = \frac{n(n-1)}{2}$

Q.1 Let G=(V,E) be the undirected graph in figure shown, How many paths are there in G from a to h ? How many of these paths have length 5 ?



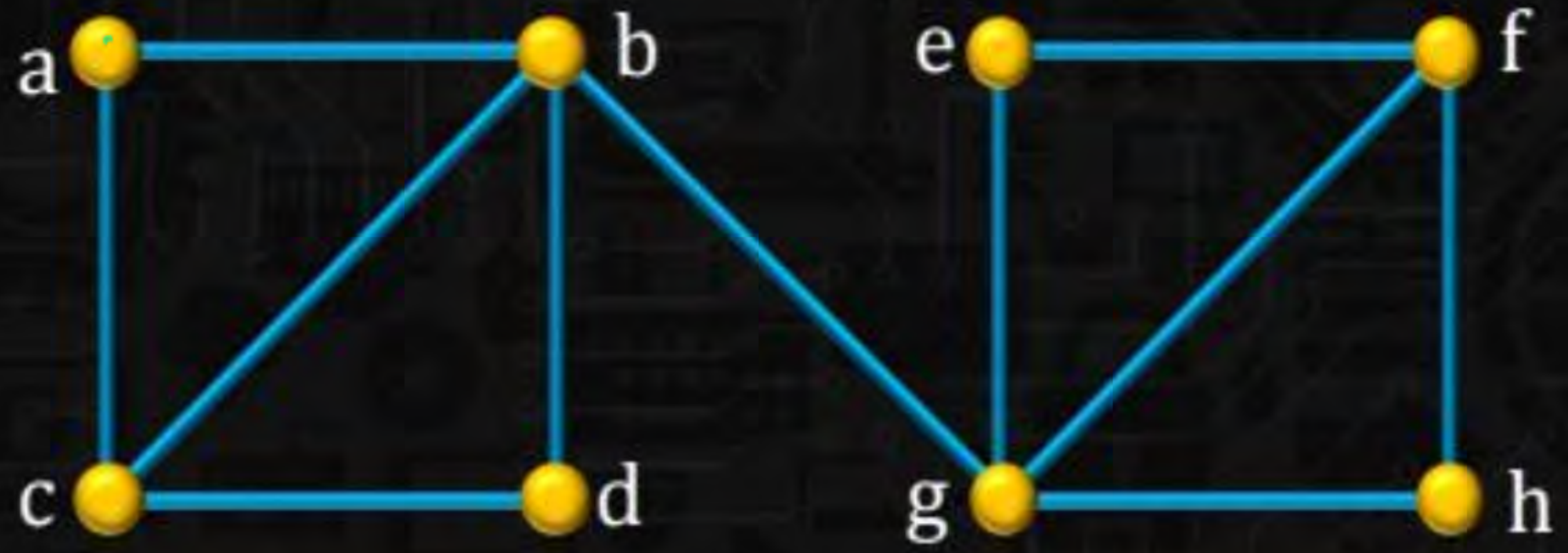
a — b — g — h.

3 ways. × 1 way × 3 ways.

= 9 ways.

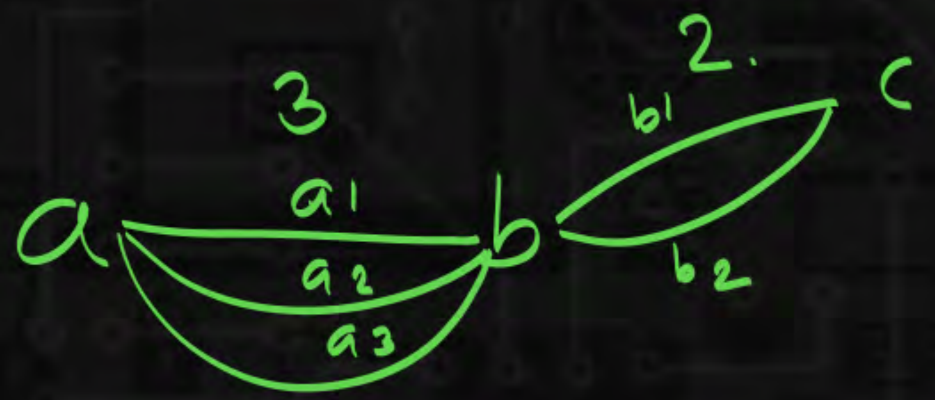
9 path.

Q.1 Let G=(V,E) be the undirected graph in figure shown, How many paths are there in G from a to h ? How many of these paths have length 5 ? (Ans : 3)



1

counting:



$a_1 < b_1, b_2$

$a_2 < b_1, b_2$

$a_3 < b_1, b_2$

$a \to c : 3 \times 2 = 6$ ways

Q.2 If G_1, G_2 are (loop-free) undirected graphs, prove that G_1, G_2 are isomorphic if and only if $\overline{G_1}, \overline{G_2}$ are isomorphic.

$$\boxed{G_1 \equiv G_2 \iff \overline{G_1} \equiv \overline{G_2}}$$

T T

F F.

$G_1 \equiv G_2$

$deg(x) = 2$ $deg(a) = 2$ $deg(a') = 2.$

x a x' a'

$x \leftrightarrow a$ $deg(x') = 3.$ $x' \leftrightarrow a'$

Q.3 Let G be a cycle on n vertices. Prove that G is self complementary if and only if $n=5$.

$$e = \frac{n(n-1)}{4}$$

$n = 4 \quad e = \frac{\cancel{4} \cdot 3}{\cancel{4}} = 3$

$\boxed{n = 5} \quad e = \frac{5 \cdot \cancel{4}}{\cancel{4}} = 5$

$G \quad + \quad \bar{G} \quad = \quad K5$

$n = 5 \qquad n = 5$

$e = 5 \qquad e = 5$

$2\,2\,2\,2\,2 \qquad 2\,2\,2\,2\,2 \qquad 4\,4\,4\,4\,4$

$(c_5) \qquad (c_5)$

$n = 5$

$2\,2\,2\,2\,2 \qquad 2\,2\,2\,2\,2$

$G + \bar{G} = K5$

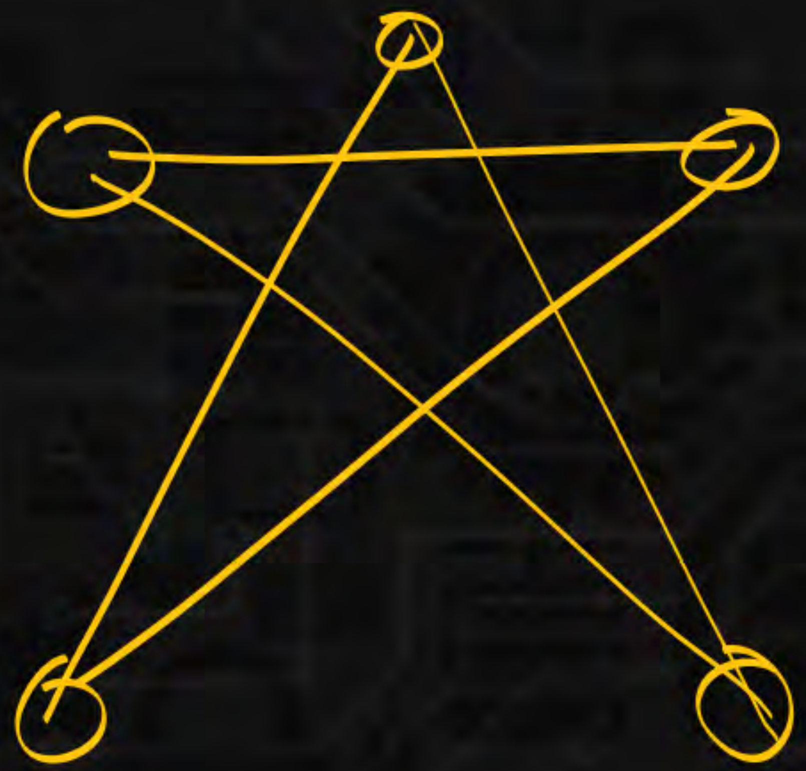
$\boxed{4\,4\,4\,4\,4}$

$G, \bar{G}$ are same Graph.

C_5

$n = 5$
$e = 5$

$+$



C_5

$n = 5$
$e = 5$

$=$

$e = \frac{13 \cdot \cancel{12}^{3}}{\cancel{4}}$

$= \underline{13 \times 3}$.

C_3, C_4, C_5 $n=5$ $e=5$. → Self complement

C_6, C_7, C_8 C_{13} $n=13$ $e=13$.

$n=6$ $e=6$.

C_{1000}

$n = 5$

$e = \frac{5 \times 4}{4} = 5$.

Self complement

$e = \frac{n(n-1)}{4} = \frac{6 \cdot 5}{4} =$

Q.4 a) How many subgraphs H=(V,E) of K_6 satisfy |V|= 3? (If two subgraphs are isomorphic but have different vertex sets, consider them distinct.)

$^6C_3 \times 2^{\frac{3\times 2}{2}}$

b) How many subgraphs H=(V,E) of K_6 satisfy |V|=4 ?

$^6C_4 \times 2^{\frac{4\times 3}{2}}$

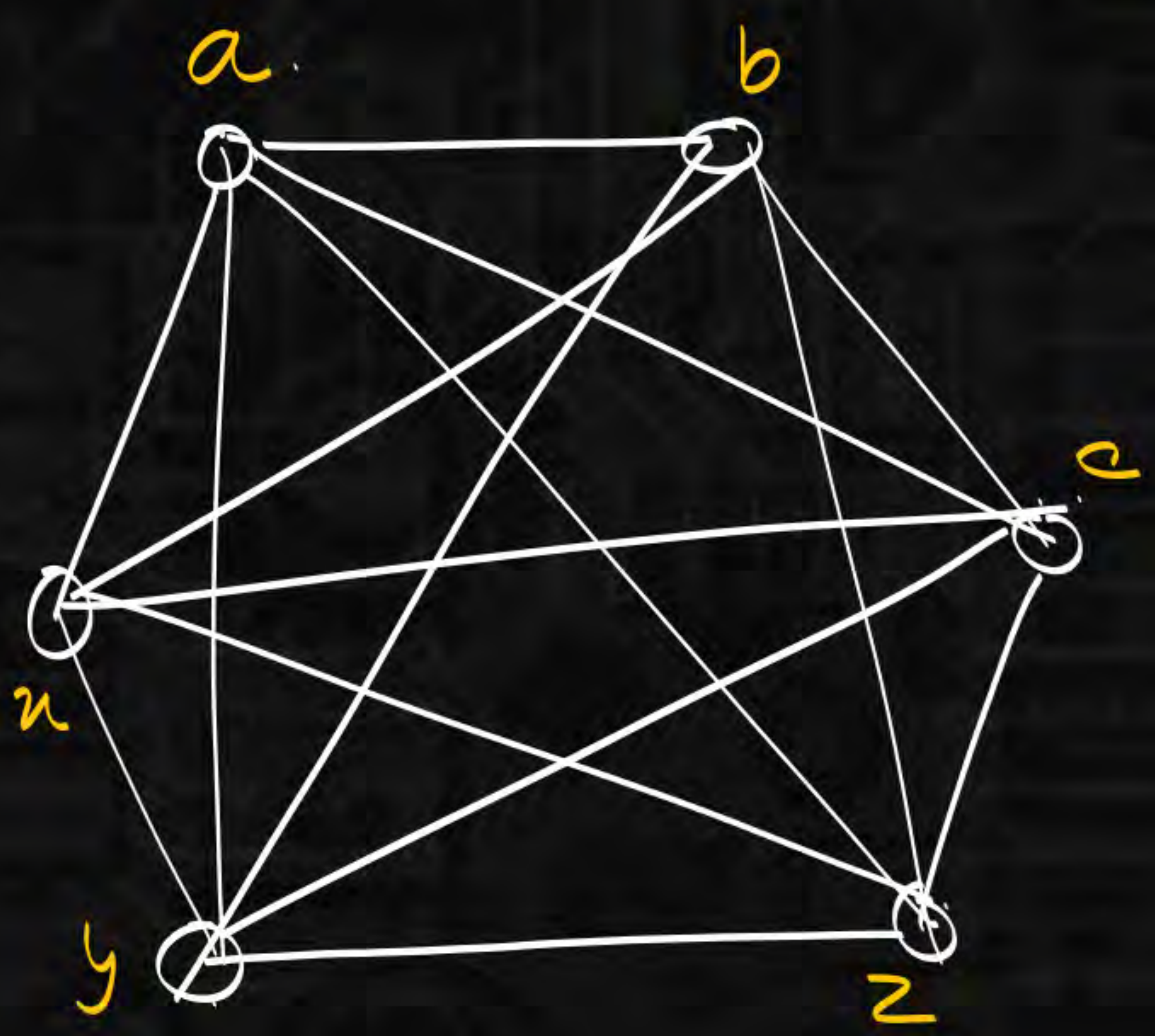
c) How many subgraphs does K_6 have?

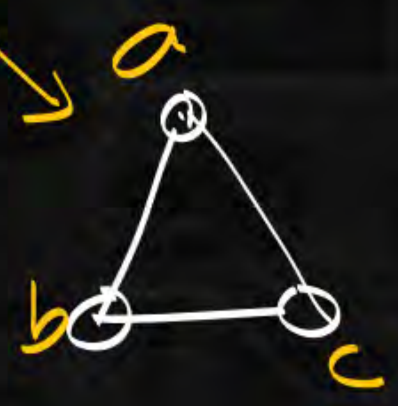
d) For n≥3, how many subgraphs does K_n have?

c) $^6C_1 \times 2^{\frac{1\times 0}{2}} + {}^6C_2 \times 2^{\frac{2\times 1}{2}} + {}^6C_3 \times 2^{\frac{3\times 2}{2}} \cdots {}^6C_6 \times 2^{\frac{6\times 5}{2}}$

$$\sum_{k=1}^{6} {}^6C_k \times 2^{\frac{k(k-1)}{2}}$$

$\{abcxyz\}$

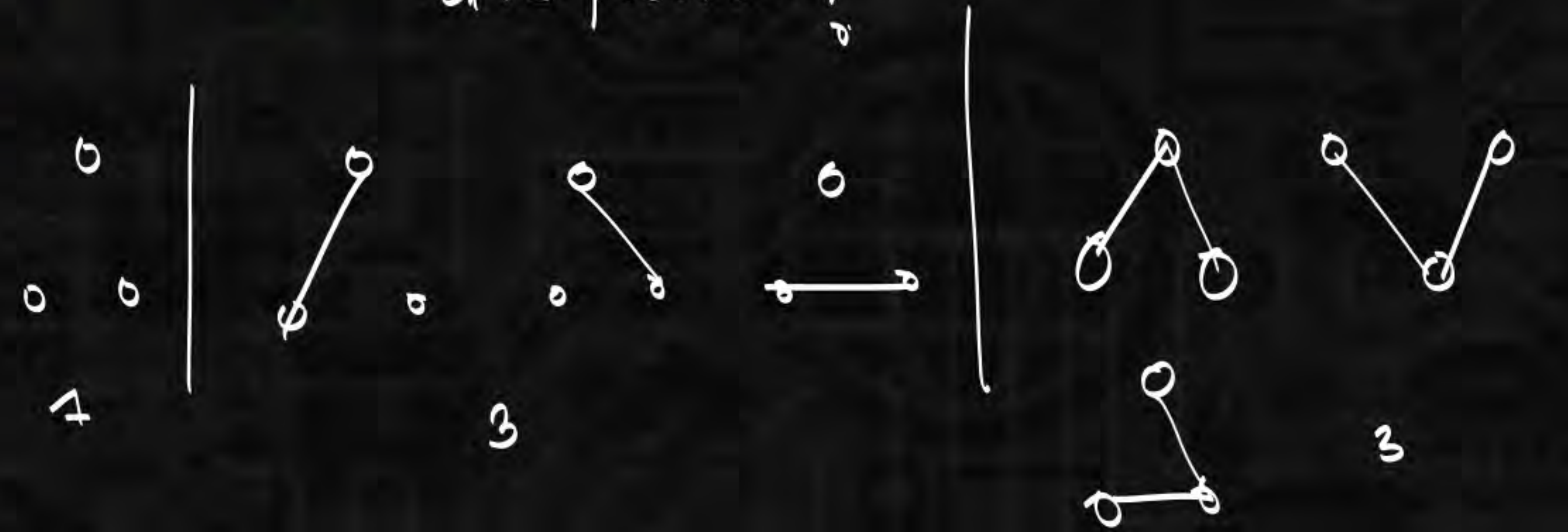


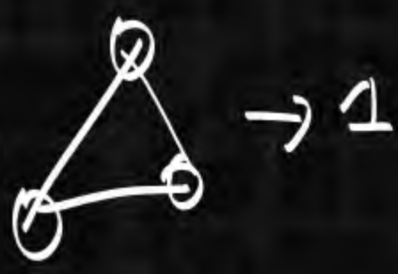


$^6C_3 \times 2^3$

how many graphs are possible?

$2^{\frac{3\times 2}{2}} = 2^3 = 8$





$1+3+3+1 = 8$

Q.5 A certain graph G has order 14 and size 27. The degree of each vertex of G is 3,4 or 5. There are six vertices of degree 4. How many vertices of G have degree 3 and how many have degree 5 ?

5 vertices.

3 vertices.

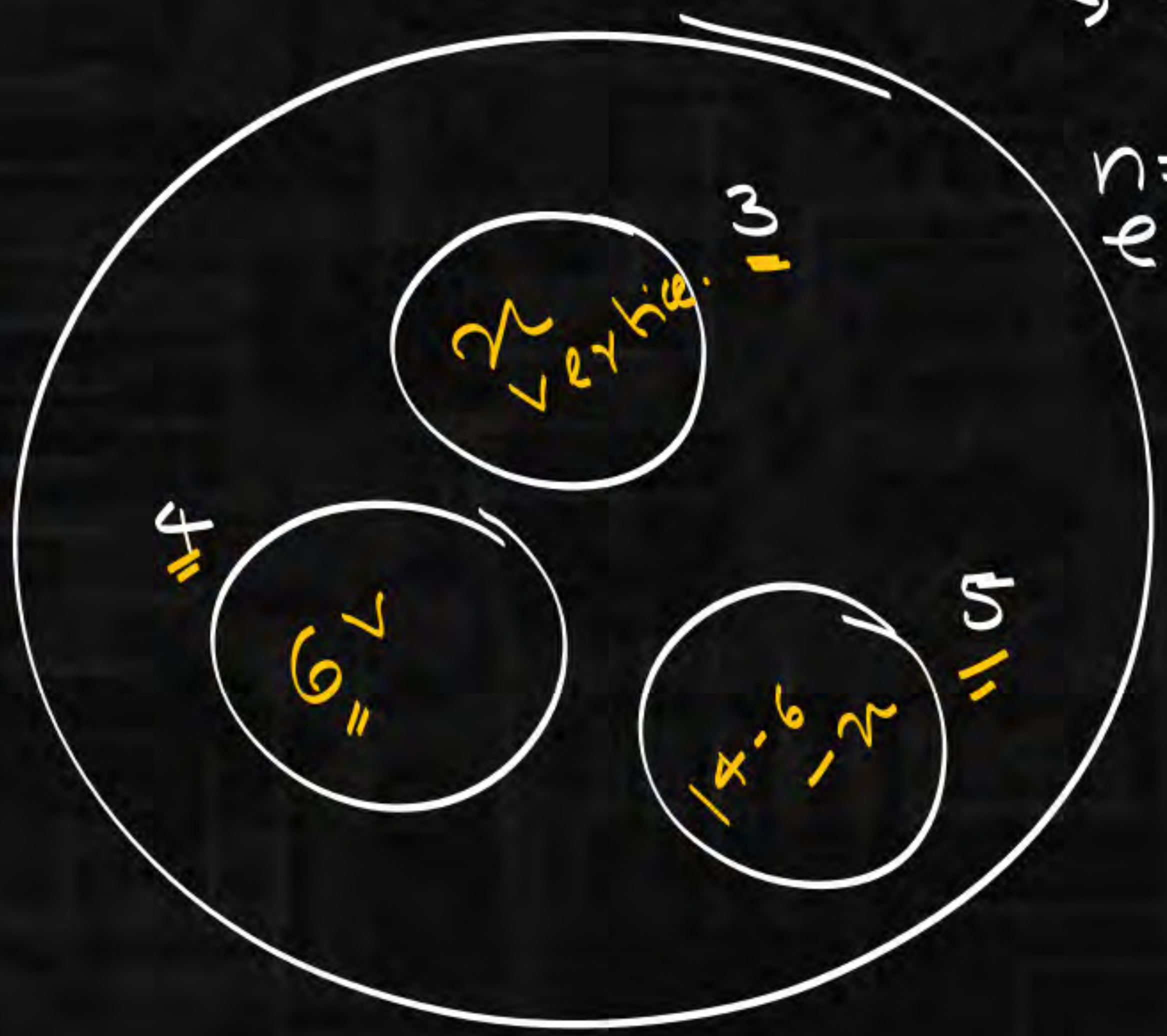
Order = vertices = 14

$n = 14$
$e = 27.$

size = edges = 27

$$6\times 4 + 3\times x + 5(14-6-x) = 2\times e = 2\times 27$$

$x =$



Q.6 The degree of each vertex of a certain graph of order 12 and size 31 is either 4 or 6. How many vertices of degree 4 are there?

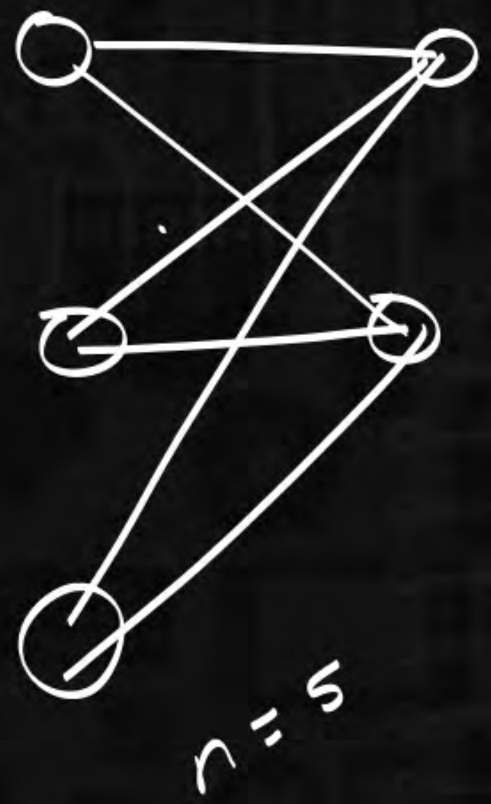
Q.7 Prove that any subgraph of a bipartite graph is bipartite.

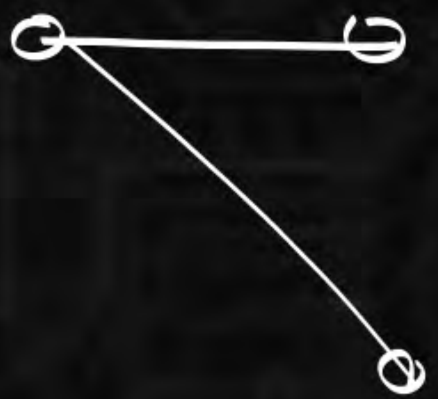
B.P

B.P does not contains odd length cycle

→ Tree

→ even length cycle.





B.P.

Q.8 Let G=(V,E) be a loop-free connected graph with $|V| = v$. If $|E| > \left(\frac{v}{2}\right)^2$, prove that G cannot be bipartite.

Q.9 If $G = (V, E)$ is a loop-free undirected graph with $|V| = n \geq 3$, and if $|E| \geq \binom{n-1}{2} + 2$, then G has a Hamilton cycle.

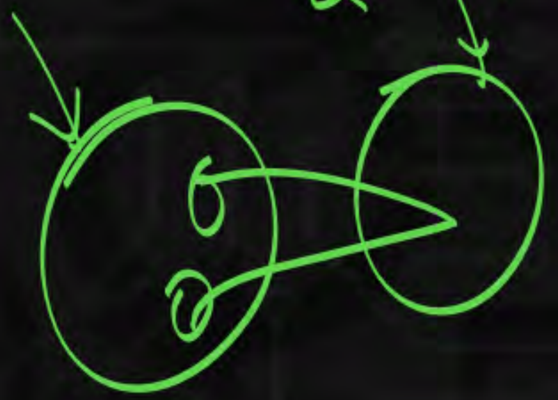
$$e = \frac{(n-1)(n-2)}{2} + 1$$

$$e = \frac{(n-k)(n-k+1)}{2}$$

$k = 2$

$$e = \frac{(n-1)(n-2)}{2}$$

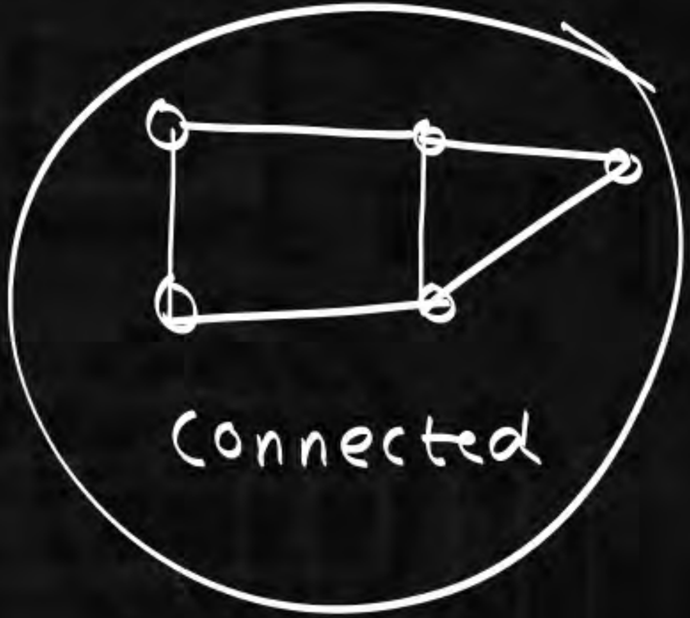
$$e = \frac{(n-1)(n-2)}{2} + 2$$

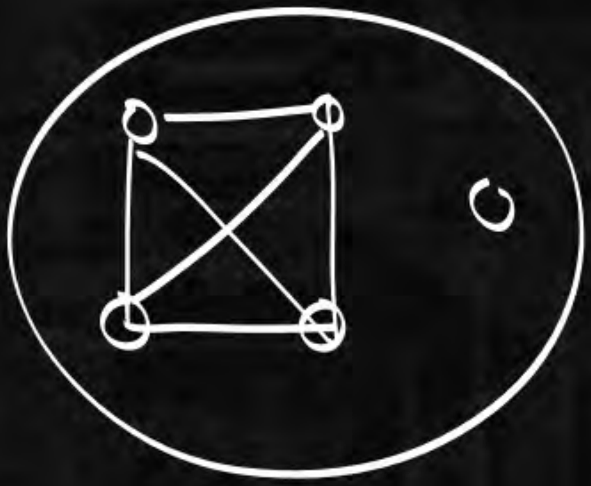


exactly

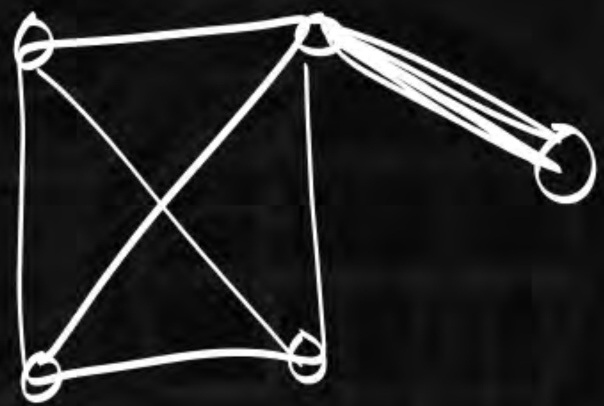
$$e = \frac{(n-1)(n-2)}{2} \to \begin{cases} \text{connected} \\ \text{disconnected} \end{cases}$$

$n = 5$, $e = \frac{4 \cdot 3}{2} = 6$





$$\left(e = \frac{(n-1)(n-2)}{2} + 1\right)_{\text{connected}}$$



$$e = \frac{(n-1)(n-2)}{2} + 2 \quad \left(\begin{matrix} H.P \\ H.C \end{matrix}\right)$$

Q.10 a) How many edge-disjoint Hamilton cycles are there in K_{21}?

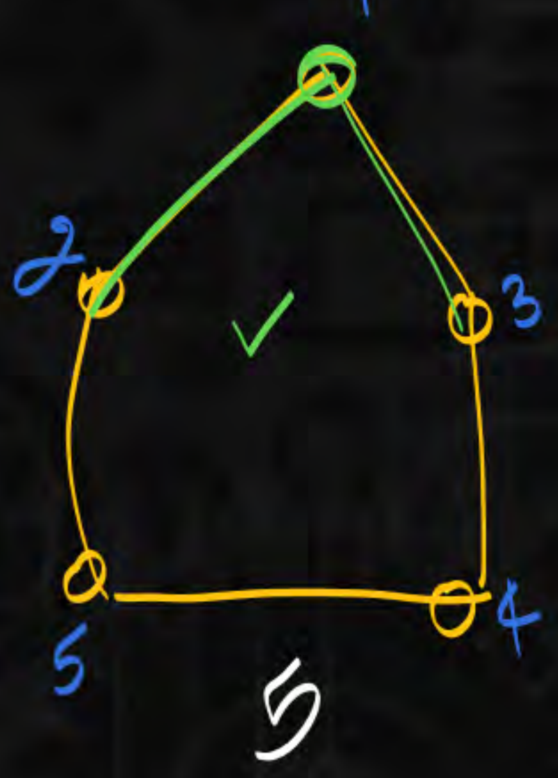
$\frac{(21-1)}{2} = \frac{20}{2} = 10$

b) Nineteen students in a nursery school play a game each day where they hold hands to form a circle. For how many days can they do this with no student holding hands with the same playmat twice?

$\frac{19-1}{2} = \frac{18}{2}$

$\rightarrow$ 9 days.

$10/5 =$



edge-disjoint Hamiltonian cycle

Complete graph.

$$e = \frac{n(n-1)}{2}$$

H. C

$\downarrow$

n edges.

$$\frac{n(n-1)}{2} \Big/ n \longrightarrow \left\{ \frac{(n-1)}{2} \right.$$

edge disjoint
cycle

Q.11 Suppose that $G=(V,E)$ is a loop-free undirected graph. If G is 5 -regular and $|V|=10$, prove that G is nonplanar.

$5 \times 10 = 2e$

$e = 25$

$25 > 3(10) - 6 \longrightarrow$ nonplanar

$\left\{ \underline{e > 3n-6} \rightarrow \left\{ \text{non planar} \right. \right.$

$p \rightarrow q$

$\equiv \neg q \rightarrow \neg p \rightarrow$ if planar $\longrightarrow e \leq 3n-6$

$\downarrow$

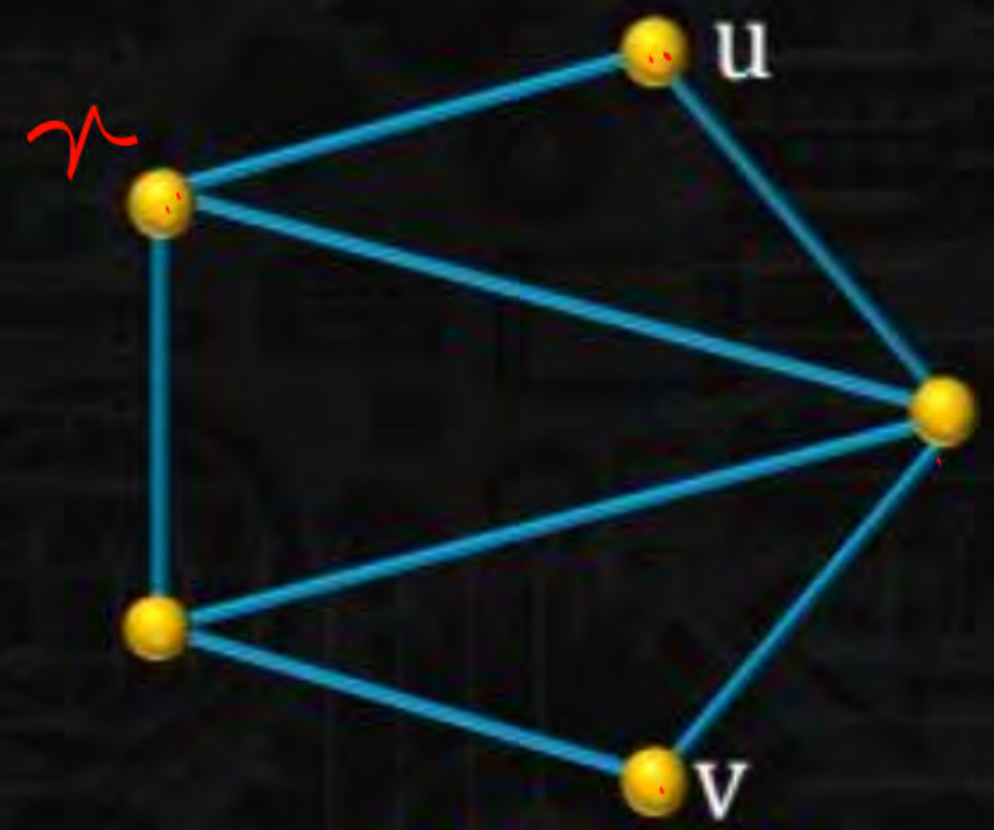
contrapositive

$e \nleq 3n-6 \longrightarrow$ nonplanar

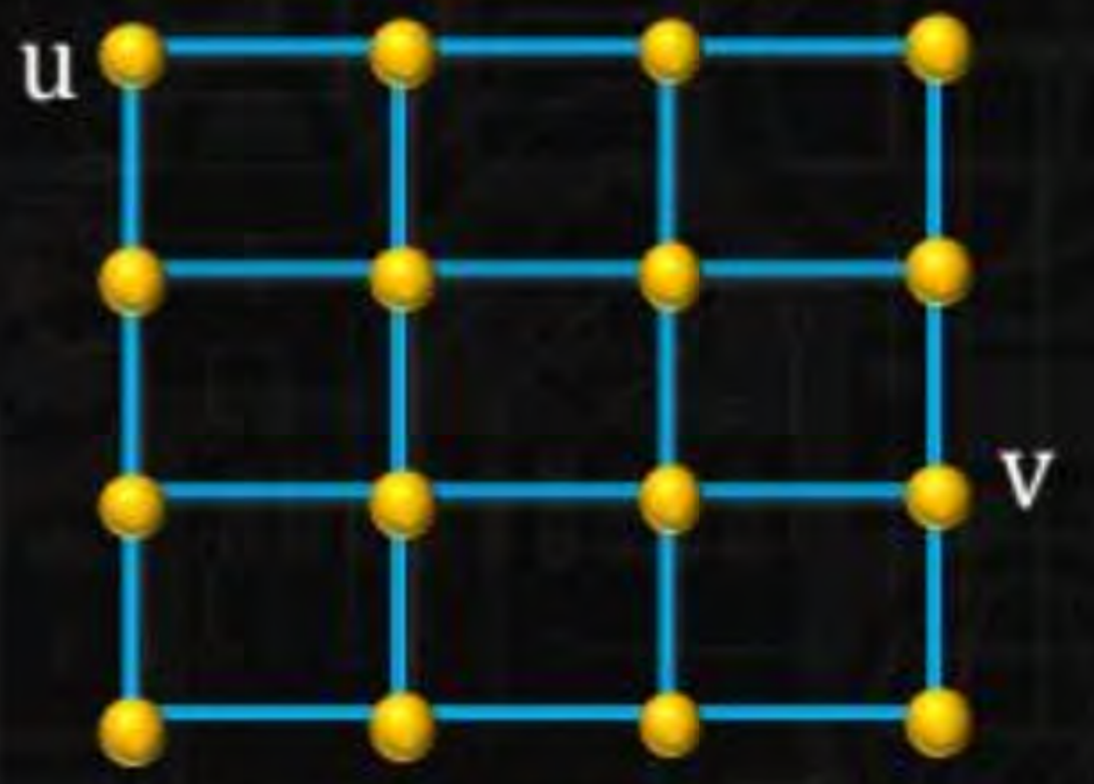
Q.12 Prove that for every integer x with $0 \leq x \leq 5$, the sequence x, 1, 2, 3, 5, 5 is not graphical.

Q.13 If the sequence $x,7,7,5,5,4,3,2$ is graphical, then what are the possible values of $x (0 \leq x \leq 7)$?

- **Length** number of edges contained in the graph
- **The distance** between two vertices in a graph is the
  Shortens
  number of edges in a shortest path
- This is also known as the geodesic distance or shortest-path distance.



d(u, v) = 2



d(u, v) = 5

length :

Distance

shortest distance

Eccentricity of graph

max (distance)

Eccentricity of a node u is the maximum of the distances of any other node in the graph from u.
The radius of a graph is the minimum of the eccentricity values among all the nodes of the graph.

Radius of this graph is 2

max (
a to b → 1
a to c → 1
a to d → 2
a to e → 3
a to f → 3
)

Therefore eccentricity of node is 3

- The **diameter** d of a graph is the maximum eccentricity of any vertex in the graph.
- The **girth** of an undirected graph is the length of a shortest cycle contained in the graph

- If the graph does not contain any cycles (that is, it is a forest), its girth is defined to be infinity. For example, a 4-cycle (square) has girth 4. A grid has girth 4 as well, and a triangular mesh has girth 3. A graph with girth four or more is triangle-free.


THANK YOU GW
SOLDIERS !