

CS & IT ENGINEERING

Mathematical Logic

DPP 06
Discussion Notes



[MCQ]

1. Consider a function, $P(x, y, z) = x + y + z = 15$ and domain = \mathbb{Z} , then which of the following is correct?

(a) $\forall x \exists y \exists z P(x, y, z)$

(b) $\exists z \forall x \forall y P(x, y, z)$

(c) $\forall x \exists z \forall y P(x, y, z)$

(d) $\exists z \exists y \forall x P(x, y, z)$

$$\underline{x} + \underline{y} + \underline{z} = 15.$$

$$\forall \underline{x} \exists \underline{y} \exists \underline{z}$$

$$y + z = 15 - x$$

$$y + \underline{z} = 15 - \boxed{\underline{x}}$$

$$y + 0 = 15 - 1$$

$$y = 14$$

[MCQ]

2. Consider an asymmetric function $P(x, y) = \underline{x^2 + y^2 = 10}$ on domain integer, then which of the following is correct?

(a) $\exists x \exists y P(x, y)$ (✓)

$$x^2 + y^2 = 10$$

(b) $\forall x \exists y P(x, y)$

$$x=1 \quad y=3$$

(c) $\forall y \exists x P(x, y)$

(d) None of these

[MSQ]

3. Which of the following is/ are negation of

$$\exists x \forall y \forall z (P(x, y, z) \oplus Q(x, y, z))$$

(A) $\exists x \forall y \exists z (\sim P(x, y, z) \oplus \sim Q(x, y, z))$

(B) $\exists x \forall y \exists z (P(x, y, z) \Rightarrow \sim Q(x, y, z))$

(C) $\exists x \forall y \exists z (P(x, y, z) \Leftrightarrow Q(x, y, z))$

(D) $\exists x \forall y \exists z (\sim P(x, y, z) \Leftrightarrow \sim Q(x, y, z))$

$$\neg (P \oplus Q)$$

$$(P \leftrightarrow Q)$$

$$(\neg P \leftrightarrow \neg Q)$$

[NAT]

4. Consider the following logical expressions

$$\begin{array}{c} \textcircled{1} \\ \boxed{P_1 \vee Q} \\ \downarrow \\ \boxed{P_2 \vee Q} \\ \hline (P_1 \vee P_2 \vee Q) \end{array}$$

$$\begin{array}{c} \boxed{P_1} \\ \downarrow \\ \boxed{P_2} \\ \hline P_1 \wedge P_2 \end{array} \quad \begin{array}{l} \wedge Q. \\ (P_1 \vee P_2) \wedge Q. \end{array}$$

P
W

invalid

(a) $\forall x \forall y P(x, y) \leftrightarrow \exists y \forall x P(x, y)$ (f)

valid
(b) $[\forall x P(x)] \vee Q \leftrightarrow \forall x [P(x) \vee Q]$

valid
(c) $\forall x [P(x) \wedge Q] \leftrightarrow [\forall x P(x)] \wedge Q$

(d) $\exists x [P(x) \vee Q] \leftrightarrow [\exists x P(x)] \wedge Q$

$$\begin{array}{c} \boxed{P_1} \\ \wedge \\ \boxed{P_2} \\ \hline \end{array} \vee Q \quad \leftrightarrow \quad (P_1 \wedge P_2) \vee Q$$

$$\begin{array}{c} \boxed{P_1 \vee Q} \\ \wedge \\ \hline \boxed{P_2 \vee Q} \end{array}$$

$$\begin{array}{c} (P_1 \vee Q) \wedge (P_2 \vee Q) \\ \downarrow \\ (P_1 \wedge P_2) \vee Q \end{array}$$

Total invalid expressions are ____?

$$\begin{array}{c} \boxed{P_1 \wedge Q} \\ \wedge \\ \hline \boxed{P_2 \wedge Q} \end{array}$$

$$\begin{array}{c} (P_1 \wedge Q) \wedge (P_2 \wedge Q) \\ \hline P_1 \wedge P_2 \wedge Q \end{array}$$

$$\begin{array}{c} \boxed{P_1} \\ \wedge \\ \boxed{P_2} \\ \hline \end{array} \wedge Q.$$

$$P_1 \wedge P_2 \wedge Q$$

[NAT]

D: 2

4. Consider the following logical expressions

a) $\exists x((x+3=2) \vee (x-2=1)) \top$

(b) $\exists x ((x \cdot x - 3 =) \vee (x > 3))$

4 > 3

Total invalid expressions are 0?

