CS & IT ENGINEERING

LOGIC

Lecture No: 09

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TOPICS TO BE COVERED

- Inference Rule
- Quantifier
- Negation of Quantifier
- Nested Quantifier

Q.1

$p \rightarrow q$

$\neg q$

$\neg r$

$\therefore \neg(p \vee r)$

1. $p \rightarrow q \equiv \neg q \rightarrow \neg p$

2. $\neg q \longrightarrow \neg q$ (m.p)

$\neg r$

$$\frac{\neg q \rightarrow \neg p \qquad \neg q}{\neg p}$$

$\neg r \longrightarrow$ 3 premises

(Conjunction) $\neg p \wedge \neg r \equiv \neg(p \vee r)$

$\neg p \vee q$

$\neg q$

$\neg p$

$\neg r$

$\neg p \wedge \neg r$

$\equiv \neg(p \vee r)$

Q.2

$$p \to q$$
$$r \to \neg q$$
$$r$$
$$\overline{\therefore \ \neg p}$$

$$\left.\begin{array}{l} \neg p \vee q \\ \neg r \vee \neg q \\ r \end{array}\right\} \to \left.\begin{array}{l} \neg p \vee q \\ \neg q \end{array}\right\} \to \neg p$$

2: $$\begin{array}{ll} p \to q & p \to q \\ R \to \neg q & \equiv q \to \neg R \\ R & \end{array} \Bigg\} \to \begin{array}{l} p \to \neg R \\ R \end{array} \equiv \begin{array}{l} R \to \neg p \\ R \\ \hline \neg p \end{array}$$

Q.3

$p \rightarrow (q \rightarrow r)$

$\neg q \rightarrow \neg p$

$p \wedge p$

$\therefore r$

$p \wedge p \wedge p \equiv p.$

$\underline{p \wedge p} \equiv \underline{\underline{p}}.$

$$\frac{\begin{array}{ll} p \rightarrow (q \rightarrow r) & \text{(Given 1)} \\ p. & (3) \end{array}}{q \rightarrow r \;\; (2A)}$$

→ Type-1

→ Type-3 (IR)

$\neg q \rightarrow \neg p \equiv p \rightarrow q \; (2)$

$$\frac{q \rightarrow r \; (2A)}{p \rightarrow r}$$

$$\frac{p}{r.}$$

Q.3

$$p \to (q \to r)$$

$$\neg q \to \neg p$$

$$p$$

$$\therefore\ r$$

$$\overset{T}{\overline{\left[\overset{T}{\overline{p \to (q \to R)}} \wedge \overset{T}{\overline{(\neg q \to \neg p)}} \wedge \overset{T}{\overline{P}}\right]}} \to \overset{F}{\overline{R}}$$

$(T \to F)$

$(T \longrightarrow F)$

F

$F \quad F$

$P = T$

$R = F$

$q = T$

$F \longrightarrow$

Q.4

$p \wedge q$

$p \rightarrow (r \wedge q)$

$r \rightarrow (s \vee t)$

$\neg s$

$\therefore t$

7) $s \vee t$

8) $\neg s$ (Given)

---

t (D.S)

1) $p \wedge q$

2) p (1, simpl)

3) $p \rightarrow (R \wedge q)$ (Given)

modus ponens.

---

4) $R \wedge q$.

$\downarrow$

5) R (4, simpl)

6) $R \rightarrow (s \vee t)$ (Given)

modus ponens.

---

7) $s \vee t$

Q.5

$p \rightarrow (q \rightarrow r)$

$p \vee s$

$t \rightarrow q$

$\neg S$

$\therefore \neg r \rightarrow \neg t$

1) $p \vee s$ (Given)
2) $\neg s$ (Given)

---

p

$p \rightarrow (q \rightarrow R)$ (Given) (modus ponens)

$t \rightarrow q$ Given

$q \rightarrow R$) $\rightarrow$ $t \rightarrow R \equiv \neg R \rightarrow \neg t$

Q.6

$$
\begin{array}{l}
p \lor q \\
\neg p \lor r \\
\neg r \\
\hline
\therefore\ q
\end{array}
$$

Q.7 $(\neg p \vee \neg q) \rightarrow (r \wedge s)$

$r \rightarrow t$

$\neg t$

$\therefore\ p$

$(\neg p \vee \neg q) \rightarrow (R \wedge S)$

(Contrapositive)

$\neg(R \wedge S) \rightarrow \neg(\neg p \vee \neg q)$

$(\neg R \vee \neg S) \longrightarrow p \wedge q$

$\neg R \vee \neg S$

$$\frac{(\neg R \vee \neg S) \rightarrow p \wedge q \qquad \neg R \vee \neg S}{p \wedge q}$$

modus ponens.

$p \wedge q \longrightarrow p$

(Simplification)

$R \rightarrow t \equiv \neg R \vee t$

$\neg t$

$$\frac{\neg R \vee t \qquad \neg t}{\neg R}$$

$\neg R$

$\Downarrow$ (Addition)

$\neg R \vee \neg S$

$p = T$

$$\frac{T}{p} \rightarrow \frac{T}{p \vee q}$$

$$\frac{T \vee -}{T}$$

Q.8

$$p$$
$$\neg p \vee q$$
$$q \to (r \to s)$$
$$t \to r$$

$$\therefore \neg s \to \neg t$$

$\longrightarrow q$

$q \to (r \to s)$

$r \to s \longrightarrow t \to s$

$\neg s \to \neg t$

Q.9

$$u \to r$$
$$(r \wedge s) \to (p \vee t)$$
$$q \to (u \wedge s)$$
$$\neg t$$
$$\underline{\quad q \quad}$$
$$\therefore\ p$$

1) $q \to (u \wedge s)$ (Given)
2) q (Given)

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3) $u \wedge s$
4) u (3, simpl)
5) $u \to R$ (Given)

---

6) R
7) s (3, simpl)
8) $R \wedge s$ (6, 7, conjunction)

9) $R \wedge s \to p \vee t$
10) $R \wedge s$ (step 8)

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11) $p \vee t$ (modus p)
12) $\neg t$ (Given)

---

p

Q.10 Negate and simplify each of the following.

1) $\exists x[p(x) \vee q(x)]$

negate

$\neg \exists x[p(x) \vee q(x)]$

$\forall x[\neg p(x) \wedge \neg q(x)]$

A. $\exists x[p(x) \vee q(x)]$

B. $\forall x[p(x) \wedge \neg q(x)]$

C. $\forall x[p(x) \rightarrow q(x)]$

D. $\exists x[(p(x) \vee q(x)) \rightarrow r(x)]$

Q.11 Let p(x), q(x), and r(x) be the following open statements.

p(x): $x^2 - 7x + 10 = 0$ ($x = 2, 5$)

q(x): $x^2 - 2x - 3 = 0$ ($x = -1, 3$)

r(x): $x < 0$ (negative)

D : Z

$\neg R(x)$: +ve

T → F

Determine the truth or falsity of the following statements, where the universe is all integers.

A. $\forall x[p(x) \rightarrow \neg r(x)]$

B. $\forall x[q(x) \rightarrow r(x)]$

C. $\exists x[q(x) \rightarrow r(x)]$

D. $\exists x[p(x) \rightarrow r(x)]$

T → F

a) $\forall x\,[p(x) \rightarrow \neg R(x)]$

$x = 2$: $\boxed{\overset{T}{P(2)} \rightarrow \overset{T}{\neg R(2)}}$

$\wedge$

$x = 5$: $\boxed{T \rightarrow T}$

$x = 1$: F →

Q.11 Let p(x), q(x), and r(x) be the following open statements.

p(x): $x^2 - 7x + 10 = 0$ ($x = 2, 5$)

q(x): $x^2 - 2x - 3 = 0$ ($x = -1, 3$)

r(x): $x < 0$ (negative)

Determine the truth or falsity of the following statements, where the universe is all integers.

$D : \mathbb{Z}$

$\neg R(x)$: +ve

$T \to F$

$R(x): x < 0$

b) $\forall x \, (q(x) \to R(x))$

$x = 3$: $\boxed{\underset{T}{q(3)} \to \underset{F}{R(3)}}$ F

$R(3): 3 < 0$ (false)

false

A. $\forall x[p(x) \to \neg r(x)]$

B. $\forall x[q(x) \to r(x)]$ (false)

C. $\exists x[q(x) \to r(x)]$

D. $\exists x[p(x) \to r(x)]$

Q.11 Let p(x), q(x), and r(x) be the following open statements.

p(x): $x^2 - 7x + 10 = 0$ ($x = 2, 5$) D : Z

q(x): $x^2 - 2x - 3 = 0$ ($x = -1, 3$) $\neg R(x)$: +ve T → F

r(x): $x < 0$ (negative)

Determine the truth or falsity of the following statements, where the universe is all integers.

C) $\exists x\,(q(x) \to R(x))$

A. $\forall x[p(x) \to \neg r(x)]$ $x = 999$

B. $\forall x[q(x) \to r(x)]$

C. $\exists x[q(x) \to r(x)]$ (True)

D. $\exists x[p(x) \to r(x)]$ (True)

$\boxed{q(999) \to R(999)}$ F → T

∨

T

$\boxed{F \to}$ T

∨

$\boxed{\to}$

∨

$\boxed{-1 \to}$

∨

$\boxed{3 \to}$

Q.11 Let p(x),q(x), and r(x) be the following open statements.

p(x): $x^2 - 7x + 10 = 0$ ($x = 2, 5$) $D: Z$

q(x): $x^2 - 2x - 3 = 0$ ($x = -1, 3$) $\neg R(x)$: +ve $T \to F$

r(x): $x < 0$ (negative)

Determine the truth or falsity of the following statements, where the universe is all integers.

d) $\exists x\,(p(x) \to R(x))$ T

$x=1$ $\boxed{P(1) \to R(1)}$ T

F →

V.

T

A. $\forall x[p(x) \to \neg r(x)]$

B. $\forall x[q(x) \to r(x)]$

C. $\exists x[q(x) \to r(x)]$ (True)

D. $\exists x[p(x) \to r(x)]$ (True)

Q.12 Consider the open statement

$p(x,y): y - x = y + x^2$

$p(0,0)$ $\quad y - x = y + x^2$

$x = 0 \quad y = 0$ $\quad 0 - 0 = 0 + 0$

where the universe for each of the variables x,y comprises all integers. Determine the truth value for each of the following statements. $p(x,y) \quad x = 1$

b) $p(1,1) \quad y = 1$

a) $p(0,0)$ (T)

b) $p(1,1)$ (F)

c) $p(0,1)$

d) $\forall y p(0,y)$

e) $\exists y p(1,y)$

f) $\forall x \exists y p(x,y)$

g) $\exists y \forall x p(x,y)$

h) $\forall y \exists x p(x,y)$

$p(x,y): y - x = y + x^2$

$1 - 1 = 1 + 1^2$

$0 = 2$

Q.12 Consider the open statement

$x=0$

$p(x,y): y - x = y + x^2$

d) $\forall y(0,y)$ $y-x=y+x^2$

$\forall y \exists x(x,y) \rightarrow y-0=y+0^2$

where the universe for each of the variables x,y comprises all integers. Determine the truth value for each of the following statements.

c) $P(0,1)$ $x=0$ $y=1$

$y-x=y+x^2$

$1-0=1+(0)^2$

a) $p(0,0)$ (T)

b) $p(1,1)$ (F)

c) $p(0,1)$ (T)

d) $\forall y p(0,y) \rightarrow$

e) $\exists y p(1,y)$

f) $\forall x \exists y p(x,y)$

g) $\exists y \forall x p(x,y)$

h) $\forall y \exists x p(x,y)$

Q.12 Consider the open statement

$p(x,y): y - x = y + x^2$

e) $\exists y(1, y)$ $\quad y - x = y + x^2$

$x = 1$ $\quad y - 1 = y + 1.$ (false)

where the universe for each of the variables x,y comprises all integers. Determine the truth value for each of the following statements.

$y - 2 = y + 4.$

a) $p(0,0)$ (T)

b) $p(1,1)$ (F)

c) $p(0,1)$ (T)

d) $\forall y p(0,y) \rightarrow$

e) $\exists y p(1,y)$ (false)

f) $\forall x \exists y p(x,y)$ (false)

g) $\exists y \forall x p(x,y)$

h) $\forall y \exists x p(x,y)$

$\forall x \exists y (y - x = y + x^2)$

x $\quad$ y

$y - 1 = y + 1.$

1 $\rightarrow$

2 $\rightarrow$ 0

3 $\rightarrow$ 0

Q.12 Consider the open statement

$p(x,y): y - x = y + x^2$

where the universe for each of the variables x,y comprises all integers. Determine the truth value for each of the following statements.

g) $\exists y \forall x \left(\underline{\underline{y}} - x = \underline{\underline{y}} + x^2 \right)$

a) $p(0,0)$ (T

b) $p(1,1)$ (F)

c) $p(0,1)$ (T)

d) $\forall y p(0,y)$ →

e) $\exists y p(1,y)$

f) $\forall x \exists y p(x,y)$

g) $\exists y \forall x p(x,y)$ (false)

h) $\forall y \exists x p(x,y)$

y $\quad$ x

0 $\quad$ -1

Q.12 Consider the open statement

$p(x,y): y - x = y + x^2$

where the universe for each of the variables x,y comprises all integers. Determine the truth value for each of the following statements.

a) $p(0,0)$ (T)

b) $p(1,1)$ (F)

c) $p(0,1)$ (T)

d) $\forall y\, p(0,y)$ →

e) $\exists y\, p(1,y)$

f) $\forall x \exists y\, p(x,y)$

g) $\exists y \forall x\, p(x,y)$

h) $\forall y \exists x\, p(x,y)$ (True)

$x = 0$

h) $\forall y \exists x\, (y - x = y + x^2)$

$y = y$

Q.13 Determine whether each of the following statements is true or false.

The universe comprises all integers.

a) $\forall x \exists y \exists z (x = 7y + 5z)$ (T)

b) $\forall x \exists y \exists z (x = 4y + 6z)$ (F)

1. $r \to s$

$\sim s$

$\therefore \sim r$

2. $r \to s$

$p \to q$

$r \vee p$

$\therefore s \vee q$

3. q

Q

$(p \wedge q)$

4. $p \to (r \to s)$

$\sim r \to \sim p$

p

$\therefore s$

5. $(p \wedge q) \to \sim t$

$w \vee r$

$w \to p$

$r \to q$

$\therefore (w \vee r) \to \sim t$

6. $\sim t \to \sim r$

$\sim s$

$t \to w$

$r \vee s$

$\therefore w$

7. $(p \wedge q) \to \sim t$

$w \vee r$

$w \to p$

$r \to q$

$\therefore (w \vee r) \to \sim t$

8. p

$p \to q$

$q \to r$

$\therefore r$

9. $\sim r$

$p \to q$

$q \to r$

$\therefore \sim p$

10. $\sim p$

$p \to q$

$q \to r$

$\therefore \sim r$

11. r

$p \to q$

$q \to r$

$\therefore p$


THANK YOU GW
SOLDIERS !