CS & IT ENGINEERING

Propositional logic

Lecture No.1

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TOPICS TO BE COVERED

01 Propositional statements

02 Different Connectives

03 Satisfiable / Contingency

04 Tautology / contradiction

05 Type 1 Questions in Logic

propositional logic ✓
logical equivalence
Inference Rule.

predicate logic
Quantifier
Quantifier with Inference Rule

1
2
3
4
5
6.

Factual

True. false.

propositional stmt

English.

!

?

Factual

Conclusion.

↓

Thm.

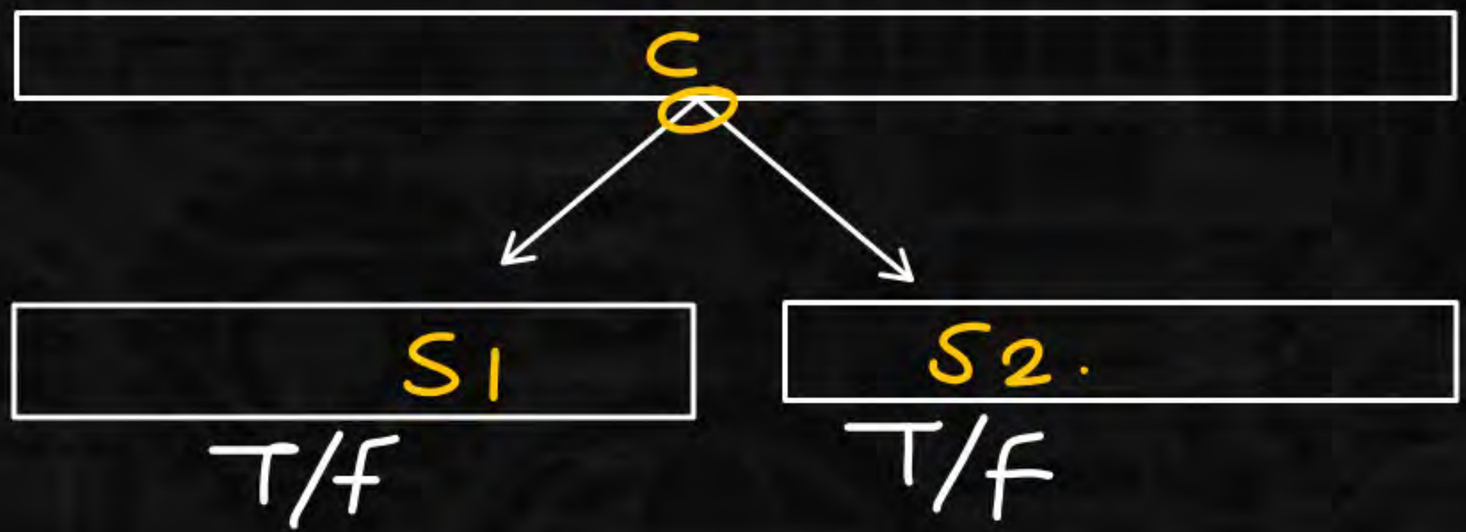
propositional stmt → True / Yes
→ false/no

→ Simple propositional. → we can not break it

→ compound propositional. → breakable.

Simple propositional stmt

eg: $5 > 3$ (T)



eg: $\boxed{5 \geq 3}$

$\boxed{5 > 3}$ OR $\boxed{5 = 3}$

↓

Connectives

conjunction ($\wedge$) (and/but)

and hates false.

and $\equiv$

$\boxed{} \wedge \boxed{}$

$\frac{s_1}{p}$ $\frac{s_2}{q}$

$F \wedge (\_ \wedge \_)$

false

$F \wedge \_$

false

s_1	s_2	c
p	q	$p \wedge q$
T	T	T
T $\wedge$ F		F
F $\wedge$ T		F
F $\wedge$ F		F

OR → Inclusive OR ($\vee$) (one/other/both)

→ Exclusive OR ($\oplus$)

(one/other)

OR LOVES TRUE.

($\underline{T}$ $\vee$ ___) True

($\underline{T}$ $\vee$ ___ $\vee$ ___) True

p		q		$p \vee q$
T	$\vee$	F	$\equiv$	T
F	$\vee$	T	$\equiv$	T
T	$\vee$	T		T
F		F		F

Conditional stmts ($\rightarrow$)

$\boxed{\text{if } p \text{ then } q.}$

$\boxed{p} \rightarrow \boxed{q}$

if you will win the match (p)
then i will give pizza. (q)

win $\rightarrow$ pizza

$p \rightarrow q$

if p then q

if p, q

$\boxed{q \text{ if } p}$

q when p

q whenever p

p implies q

$\boxed{\begin{array}{l} p \text{ only if } q \\ q \text{ unless } \neg p. \end{array}}$

if you will win the match then i will give pizza party.

p	q	$p \to q$
win (T)	pizza (T)	T.
win (T)	~~pizza~~ (F)	F.
~~win~~ (F)	pizza (T)	T
~~win~~ (F)	~~pizza~~ (F)	T

$T \to T \equiv T.$

$\boxed{T \to F \equiv F.}$

$F \to T \equiv T$

if she do the dishes then i will cook.

wash $\longrightarrow$ cook.

$$T \longrightarrow T \equiv T$$

$$T \longrightarrow F \equiv F$$

$$F \rightarrow \qquad T.$$

if p.m exist in Graph then no of vertices will be even.

$$\boxed{\underline{P}.M \rightarrow \underline{\underline{Even}}}$$

$$\overset{T}{p.m} \xrightarrow{} \overset{T}{Even} \equiv T$$

$$\overset{T}{p.m} \rightarrow \cancel{E}ven^{(F)} \equiv F$$

$$p.\overset{(F)}{\cancel{m}} \rightarrow \overset{T}{Even} \equiv T$$

$$p.\underset{(F)}{\cancel{m}} \rightarrow \underset{(F)}{E\cancel{v}en} \equiv T$$

$$T \rightarrow T \equiv T$$

$$T \rightarrow F \equiv F$$

$$\left\{\begin{array}{l} \underline{F} \rightarrow F \equiv \underline{\underline{T}} \\ F \rightarrow T \equiv T \end{array}\right.$$

$P \to Q$

$$\begin{cases} T \to T \equiv T. \\ T \to F \equiv F. \\ F \to T \equiv T. \\ F \to F \equiv T. \end{cases}$$

$$\begin{cases} T \to T \equiv T. \\ T \to F \equiv F. \\ F \to \quad \equiv T. \end{cases}$$

Implication

$\overset{1\rightarrow 2}{\underline{p \rightarrow q}} \equiv \underline{P.m} \rightarrow \underline{\text{Even}}\ (T)$

Converse

$\overset{2\rightarrow 1}{q \rightarrow p}$

$\underline{\text{Even}} \rightarrow \underline{P.m}$

$p \rightarrow q \neq q \rightarrow p$

Inverse

$\overset{\neg 1\rightarrow \neg 2}{\neg p \rightarrow \neg q}$

$P.\cancel{m} \rightarrow \text{Ev}\cancel{e}\text{n}$

$p \rightarrow q \neq \neg p \rightarrow \neg q$

Contrapositive

$\neg q \rightarrow \neg p$

$\underline{\text{Ev}\cancel{e}\text{n}} \rightarrow P.\cancel{m}$

$\underline{\text{odd}} \rightarrow P.\cancel{m}$

$\underline{p \rightarrow q \equiv \neg q \rightarrow \neg p}$

win $\rightarrow$ pizza.

$p \rightarrow q$

<u>Converse</u>: $q \rightarrow p$ pizza $\rightarrow$ win $p \rightarrow q \neq q \rightarrow p$.

<u>Inverse</u>: $\neg p \rightarrow \neg q$ ~~win~~ $\rightarrow$ ~~pizza~~ $p \rightarrow q \neq \neg p \rightarrow \neg q$.

<u>Contrapositive</u> $\neg q \rightarrow \neg p$. ~~pizza~~ $\longrightarrow$ ~~win~~ $\underline{p \rightarrow q = \neg q \rightarrow \neg p}$.

$p \to q \equiv \neg q \to \neg p$.

$p \to q \neq q \to p$.

p	q	$\neg p$	$\neg q$	$p \to q$	$q \to p$	$\neg p \to \neg q$	$\neg q \to \neg p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

A B

if G is $wn \rightarrow e = 2(n-1)$

<u>Converse</u>: $q \rightarrow p$ $\quad e = 2(n-1) \rightarrow wn$ (false) $\quad p \rightarrow q \neq q \rightarrow p$.

<u>Inverse</u>: $\neg p \rightarrow \neg q$ $\quad \cancel{wn} \rightarrow e \neq 2(n-1)$ false

<u>Contraposition</u> $\quad e \neq 2(n-1) \rightarrow \cancel{wn}$ True

A	B
T	T
F	F
T	T
F	F

A, B are having same value
OR
Same behaviour

$\boxed{A \equiv B}$.

$$p \rightarrow q \equiv \neg q \rightarrow \neg p.$$

Biconditional ($\Leftrightarrow$)

if and only if

iff

p	q	$p \Leftrightarrow q$
T	T	T
F	F	T
T	F	F
F	T	F

A	B	$A \leftrightarrow B$
T	T	T
F	F	T
T	T	T
F	F	T

$A \equiv B$

same behaviour.

T
T
T
T

→ Tautology

Valid

T → atleast 1 True

→ Satisfiable

eg2:

Satisfiable.

p	q	$p \vee q$
T	F	T
F	T	T
F	F	F
T	T	T

eg1:

p	q	$p \wedge q$ → Satisfiable
T	F	F
F	T	F
F	F	F
T	T	T

all validis are satisfiable.

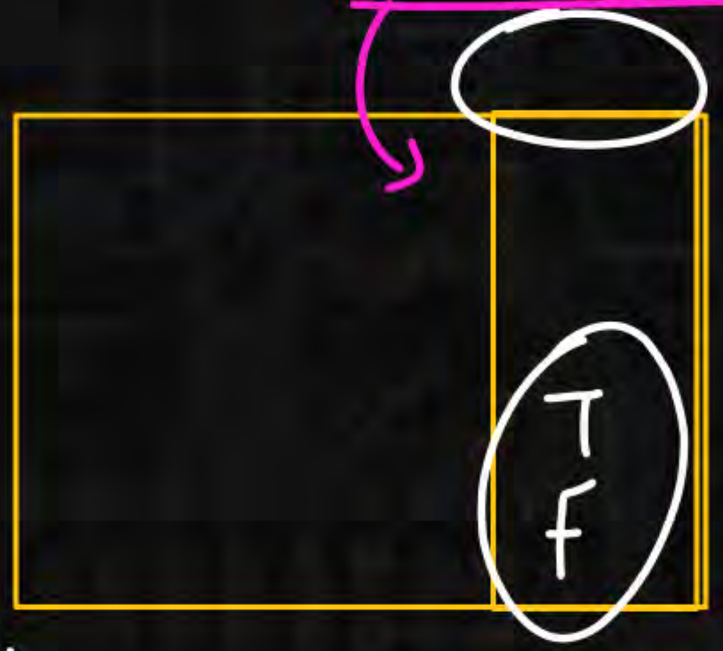
all satisfiables are not valid.

→ all contingency are satisfiable.

→ all satisfiable's are contingency.
(false)

Contradiction.

Contingency.



neither Tauto- nor contradiction.

T
T
T
F

→ satisfiable.
/ contingency.

T
T
T
T

valid.
satisfiable.
not contingency.

Satisfiable → atleast 1. True.

if all are True's no problem

Contingency → $\boxed{\begin{array}{c} T \\ F. \end{array}}$ → atleast 1 combi

Satisfiable.

T
T
T
T

$\text{Valid/Tautology} \Longrightarrow \text{Satisfiable}$

$\text{Satisfiable} \not\Longrightarrow \text{Valid/Tautology}$

Valid/Tautology

→ all are T

T → atleast 1 True

not tautology.

not contradiction.

Contingency → satisfiable.


THANK YOU GW
SOLDIERS !