

CS & IT ENGINEERING

Types of Graphs Part 3

Lecture No. 5



By- SATISH YADAV SIR

TOPICS TO BE COVERED

01 Isomorphic Graph

02 Hypercube Graph

03 Operation in graph 1

04 Operation in graph 2

05 Operation in graph 3

Types of graph

incident point:

meeting point

$$\underline{G} = (\underline{V}, \underline{E})$$

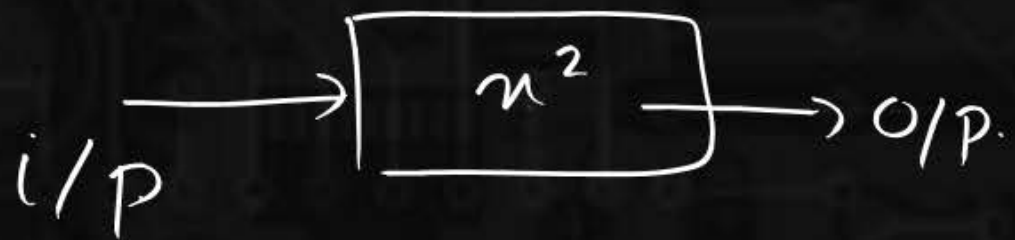
$$G = (V, E, \psi)$$

ψ :

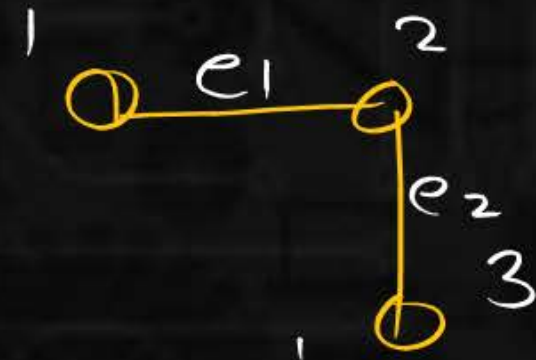
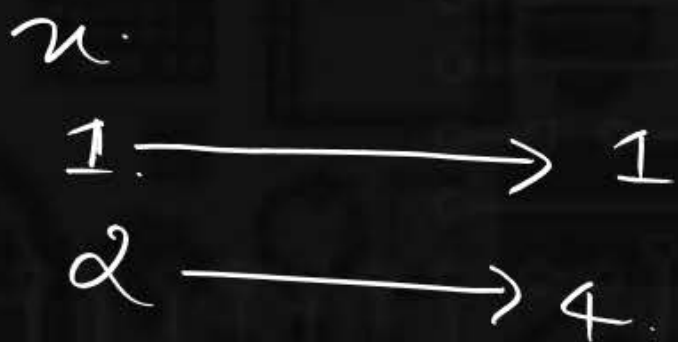
each edge is associated
→ unordered pair

Types of graph

function: $f: \mathbb{Z} \rightarrow \mathbb{Z}$



$$f(n) = n^2$$



$$e_1 \rightarrow (1, 2) \quad G = (V, E, \psi)$$

$$e_2 \rightarrow (2, 3)$$

$$\psi: E \rightarrow V \times V$$

Types of graph

Isomorphic:

same → property

G_1, G_2 are isomorphic to each other if they have same incident property.

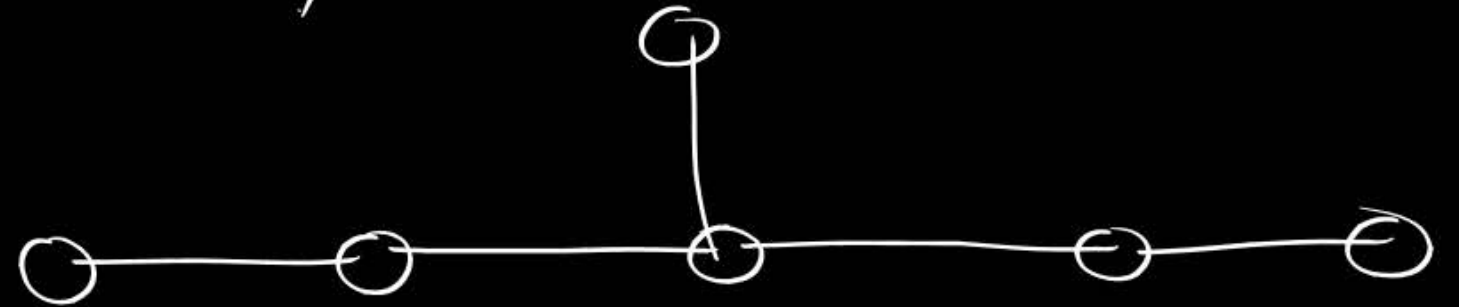
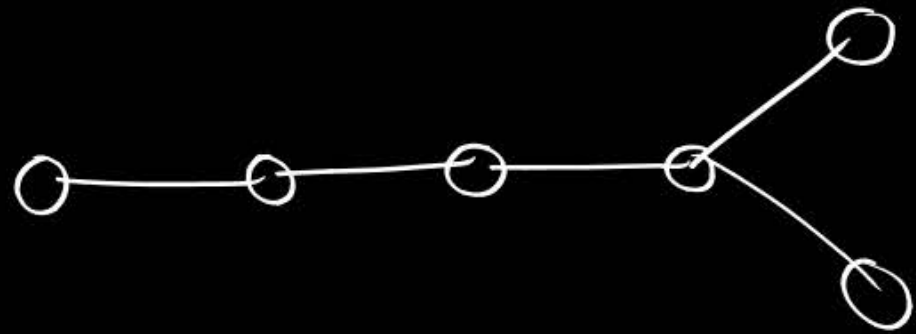
→ same no. of vertices.

→ same no. of edges.

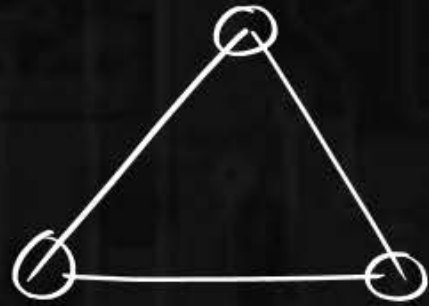
→ Same degree sequence.

isomorphic \rightarrow same no. of v , E , Degree sequence.

Same no. of v , E , degree sequence \nrightarrow isomorphic.



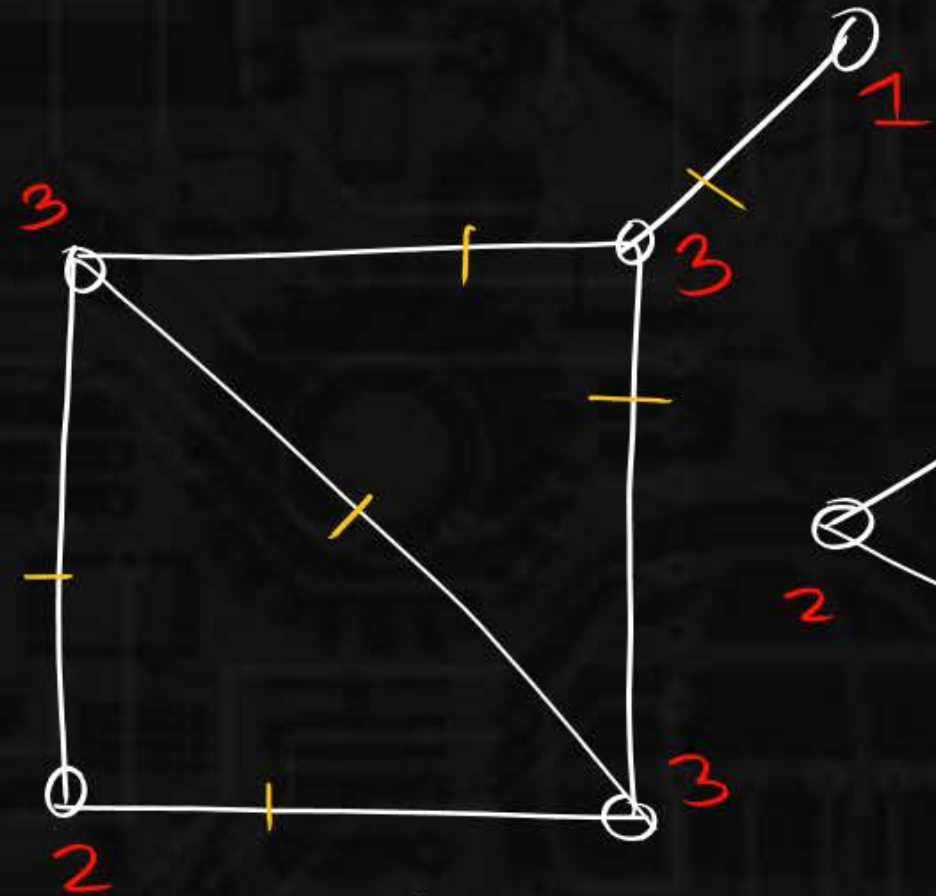
Types of graph



$n = 3$
 $e = 3$
 2 2 2

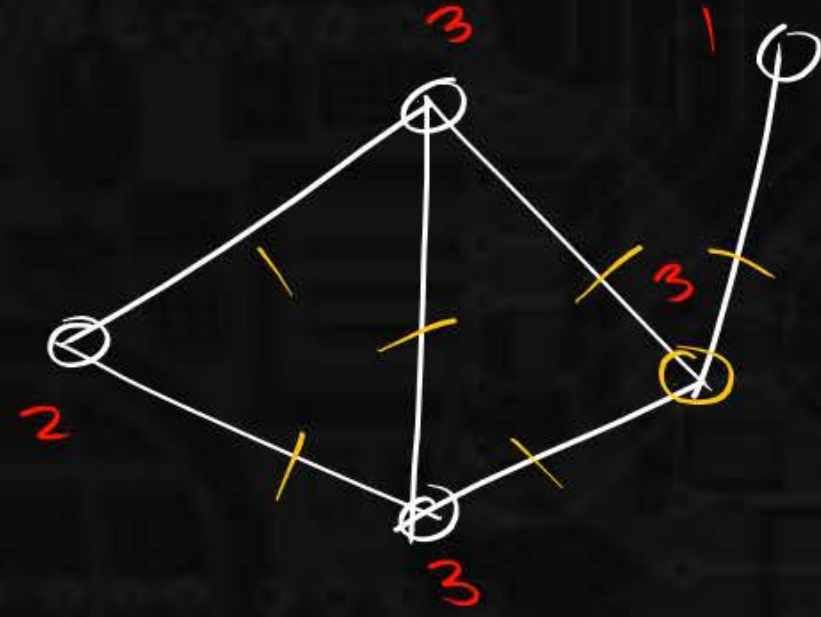


$n = 3$
 $e = 3$
 2 2 2



G_1

$\left\{ \begin{array}{l} n = 5 \\ e = 6 \\ 3 \ 3 \ 3 \ 2 \ 1 \end{array} \right.$



G_2

$\left\{ \begin{array}{l} n = 5 \\ e = 6 \\ 3 \ 3 \ 3 \ 2 \ 1 \end{array} \right.$

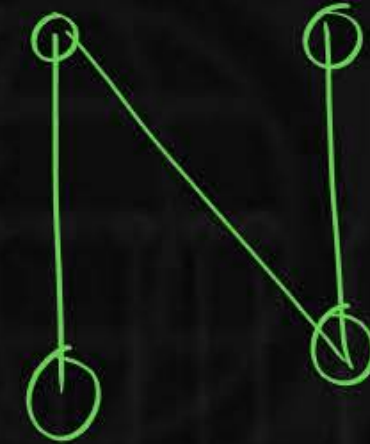
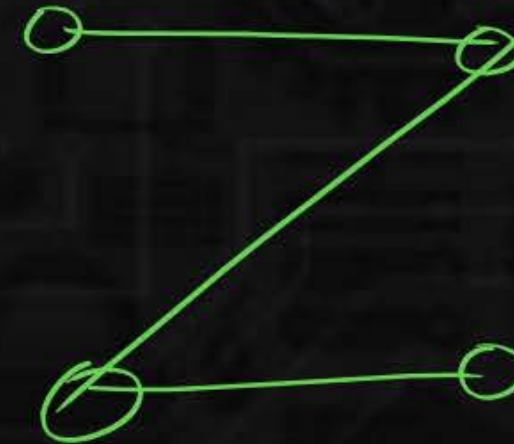
Types of graph

1 Graph \rightarrow different drawings.

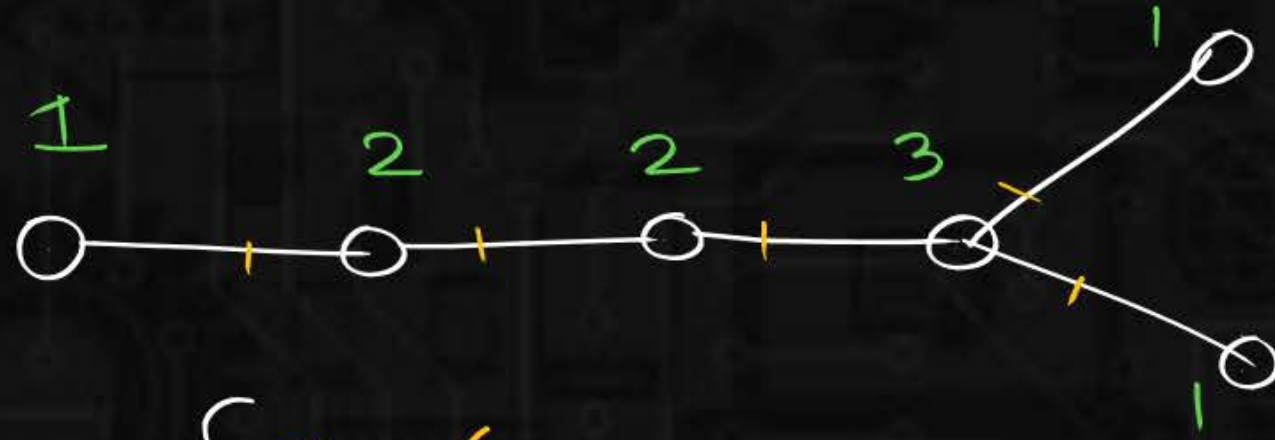
Self-complement

1. $G \equiv \bar{G}$

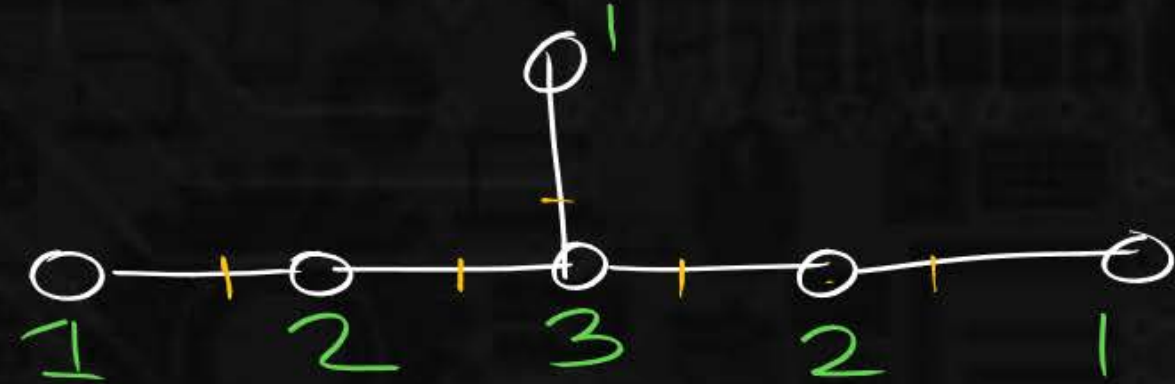
2. if Graph is
isomorphic to
it's own complement.



Types of graph

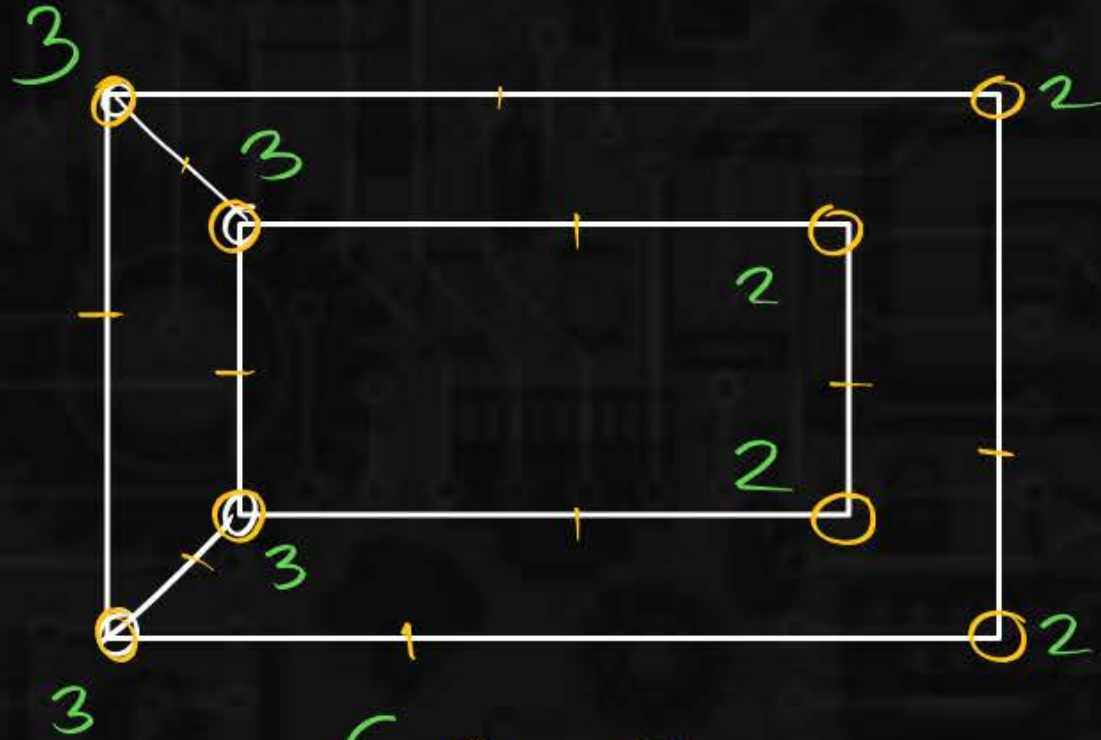


$$\left\{ \begin{array}{l} n = 6 \\ e = 5 \\ 3 \ 2 \ 2 \ 1 \ 1 \end{array} \right.$$

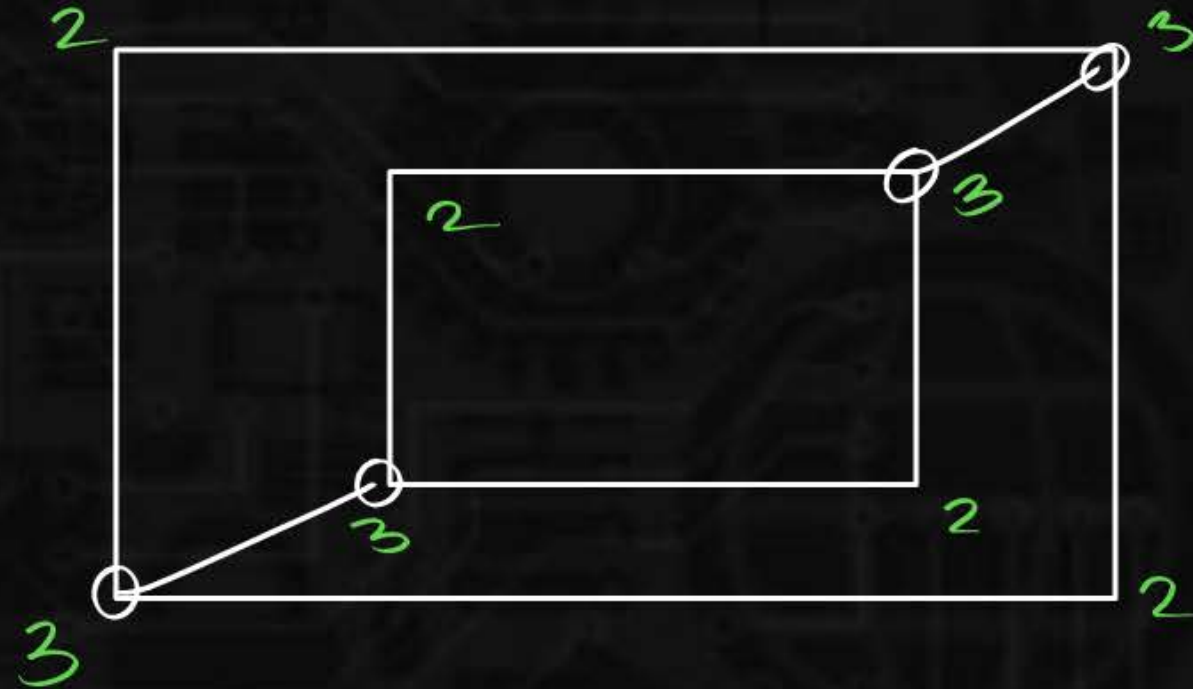


$$\left\{ \begin{array}{l} n = 6 \\ e = 5 \\ 3 \ 2 \ 2 \ 1 \ 1 \end{array} \right.$$

Types of graph

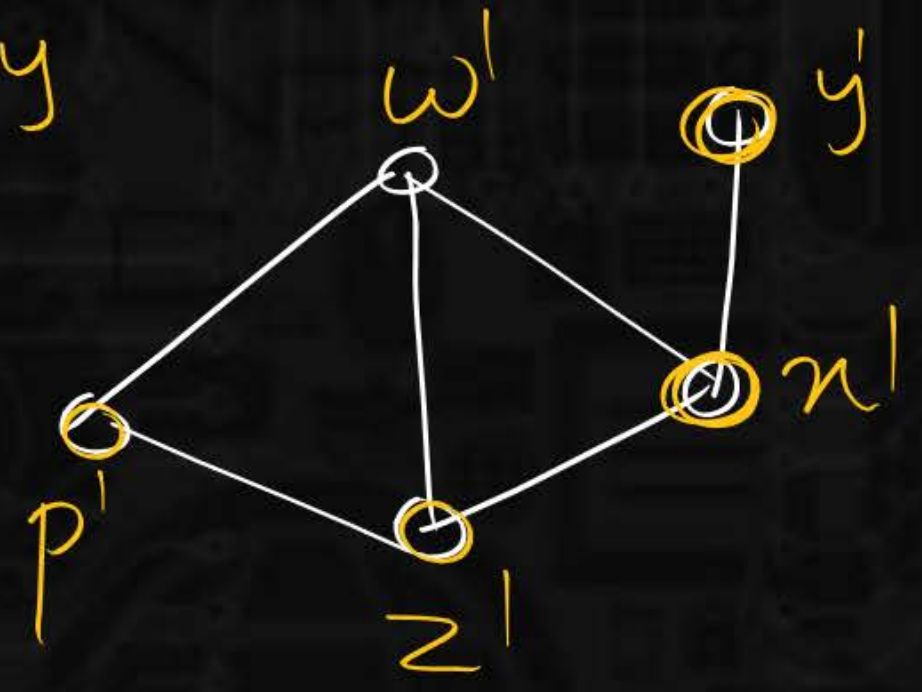
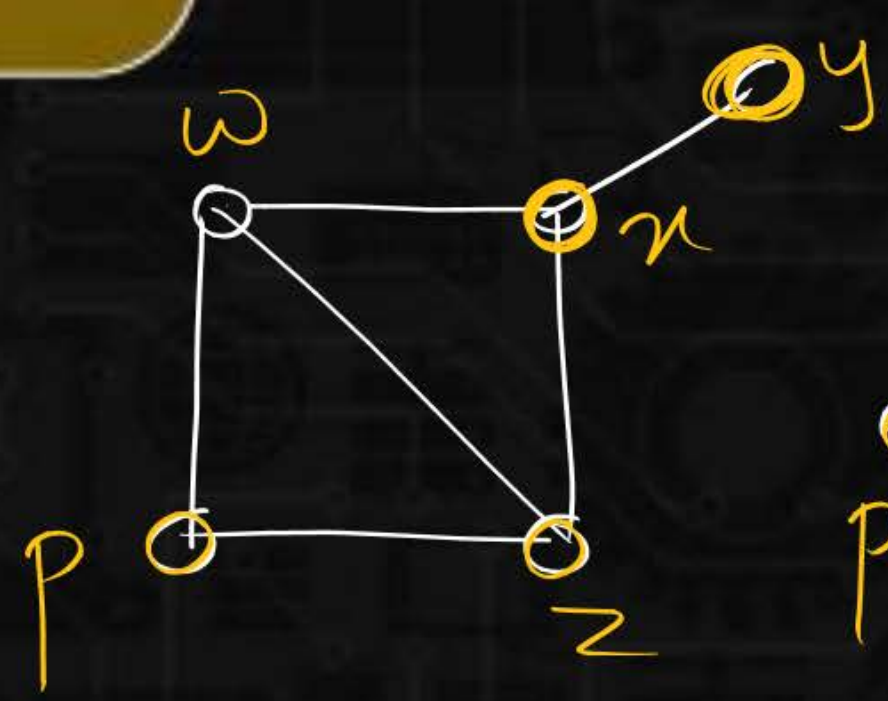
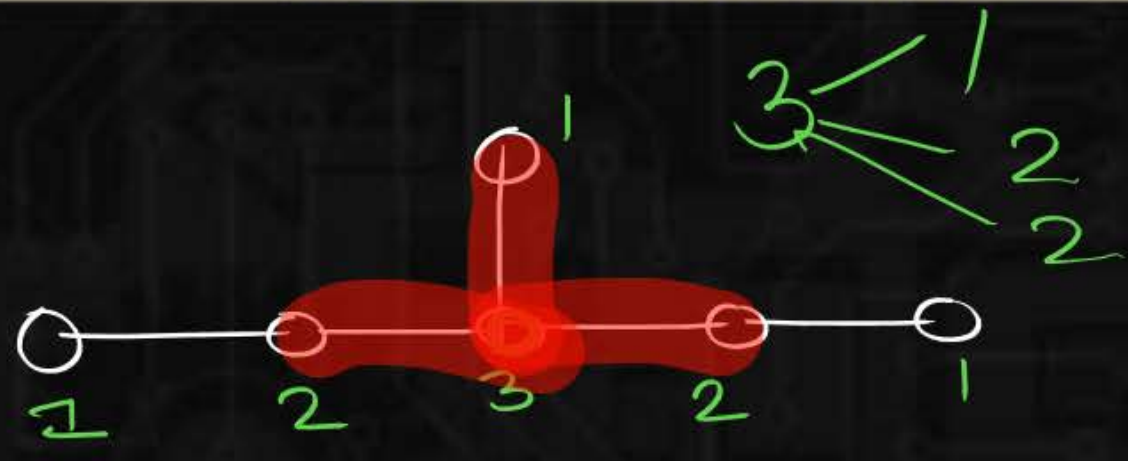


$$\begin{cases} n = 8 \\ e = 10 \\ 3 \ 3 \ 3 \ 3 \ 2 \ 2 \ 2 \ 2 \end{cases}$$



$$\begin{cases} n = 8 \\ e = 10 \\ 3 \ 3 \ 3 \ 3 \ 2 \ 2 \ 2 \ 2 \end{cases}$$

Types of graph



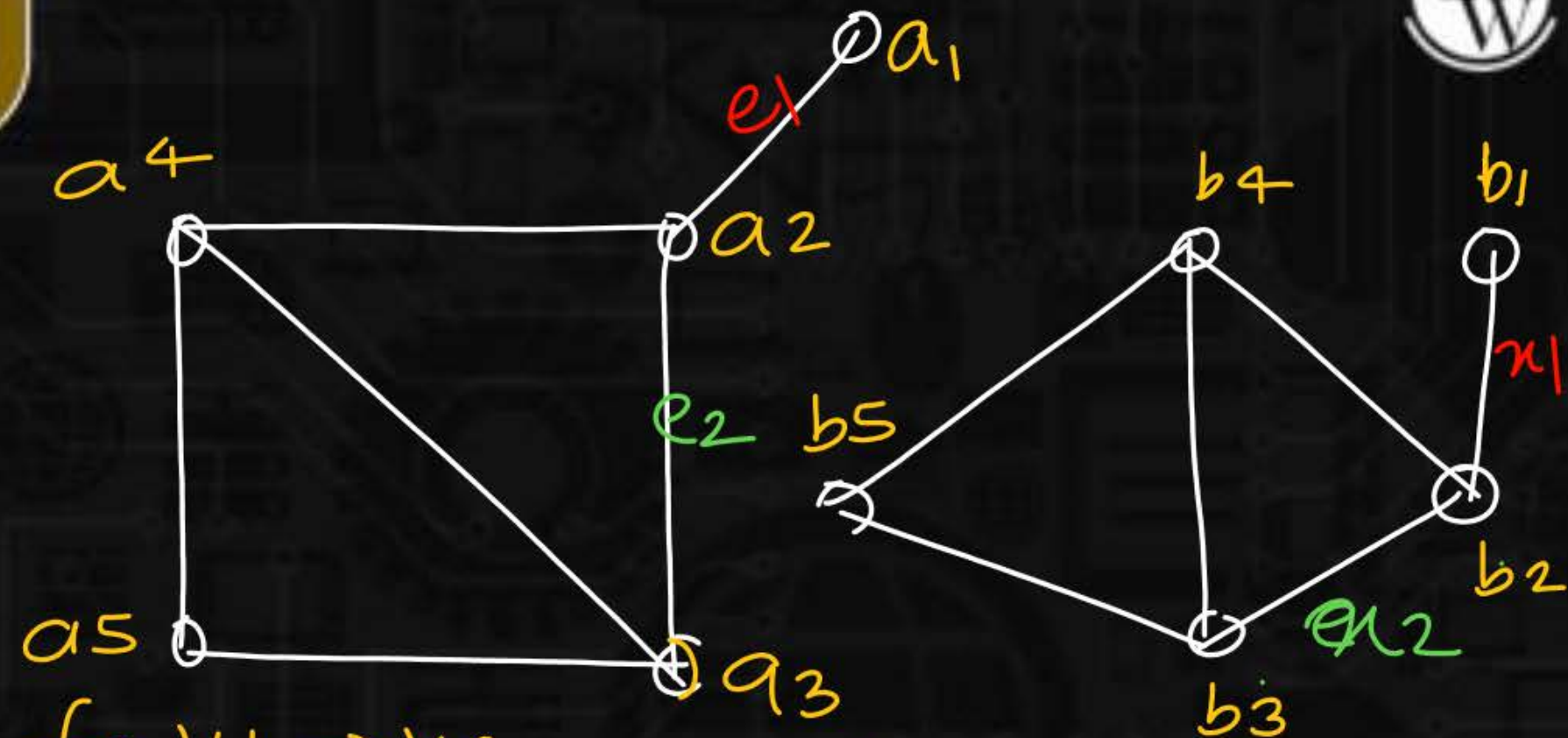
Types of graph

$$G_1(V_1, E_1, \psi_1)$$

$$G_2(V_2, E_2, \psi_2)$$

$$f: G_1 \rightarrow G_2$$

$$\left\{ \begin{array}{l} f: V_1 \rightarrow V_2 \\ f: E_1 \rightarrow E_2 \\ f: \psi_1 \rightarrow \psi_2 \end{array} \right.$$



$$f: V_1 \rightarrow V_2$$

$$a_1 \rightarrow b_1$$

$$a_2 \rightarrow b_2$$

$$f: E_1 \rightarrow E_2$$

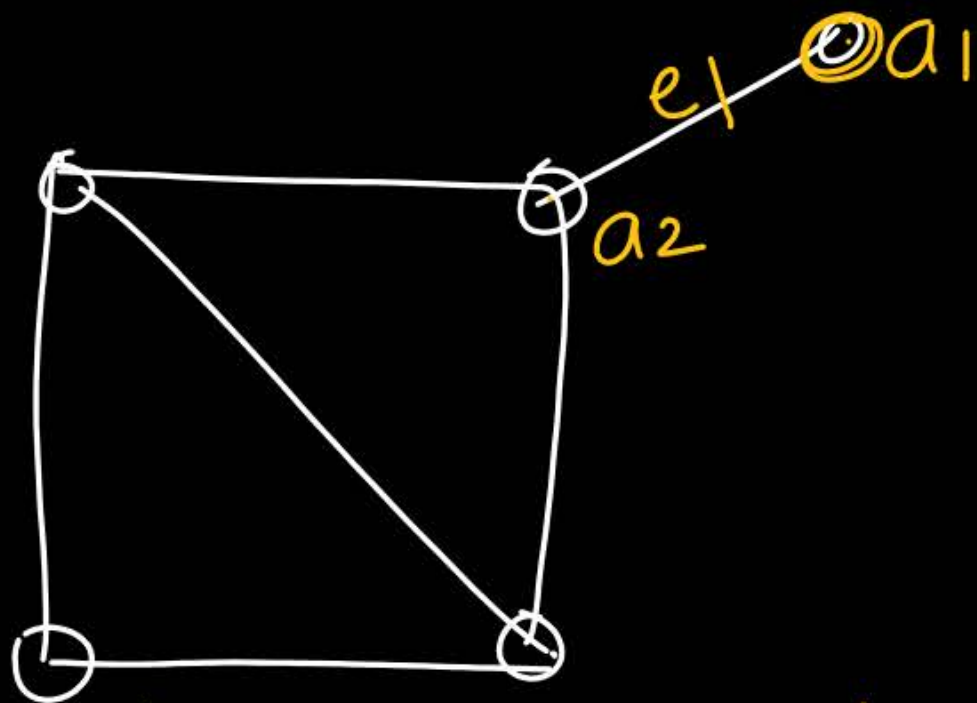
$$e_1 \rightarrow x_1$$

$$e_2 \rightarrow x_2$$

$$e_1 \rightarrow (a_1, a_2)$$

$$b_1$$

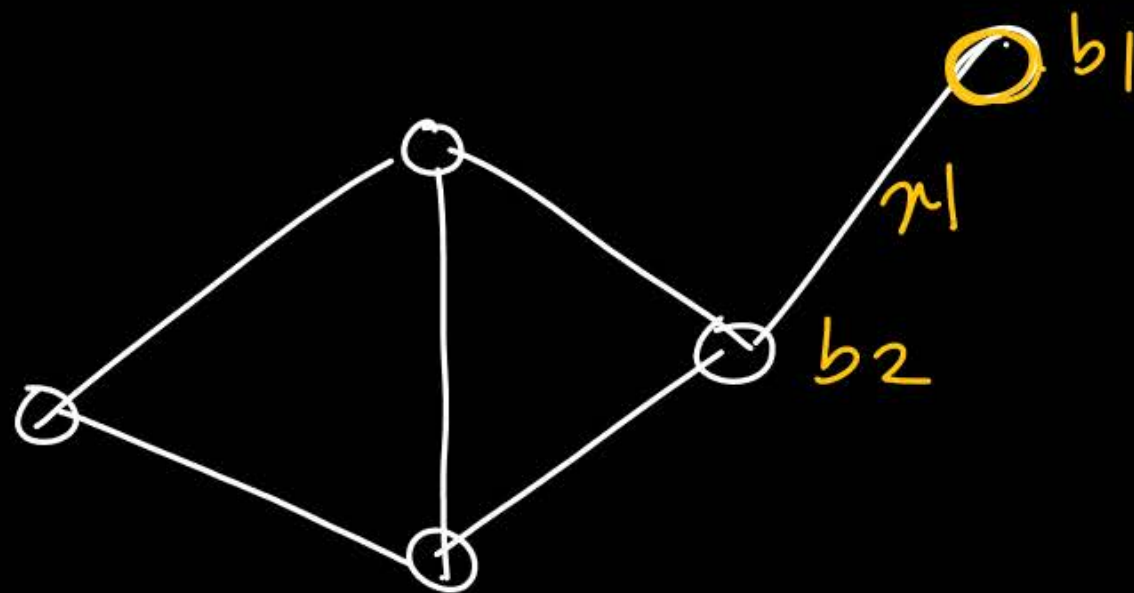
$$x_1 \rightarrow (b_1, b_2)$$



$$f: V_1 \rightarrow V_2$$

$$a_1 \rightarrow b_1$$

$$a_2 \rightarrow b_2$$



$$f: E_1 \rightarrow E_2$$

$$\underline{e_1} \rightarrow \underline{x_1}$$

$$f: \psi_1 \rightarrow \psi_2$$

$$e_1 \rightarrow (a_1, a_2)$$

$$x_1 \rightarrow (b_1, b_2)$$

Types of graph

Consider a graph, vertex which is represented as n -bit signal

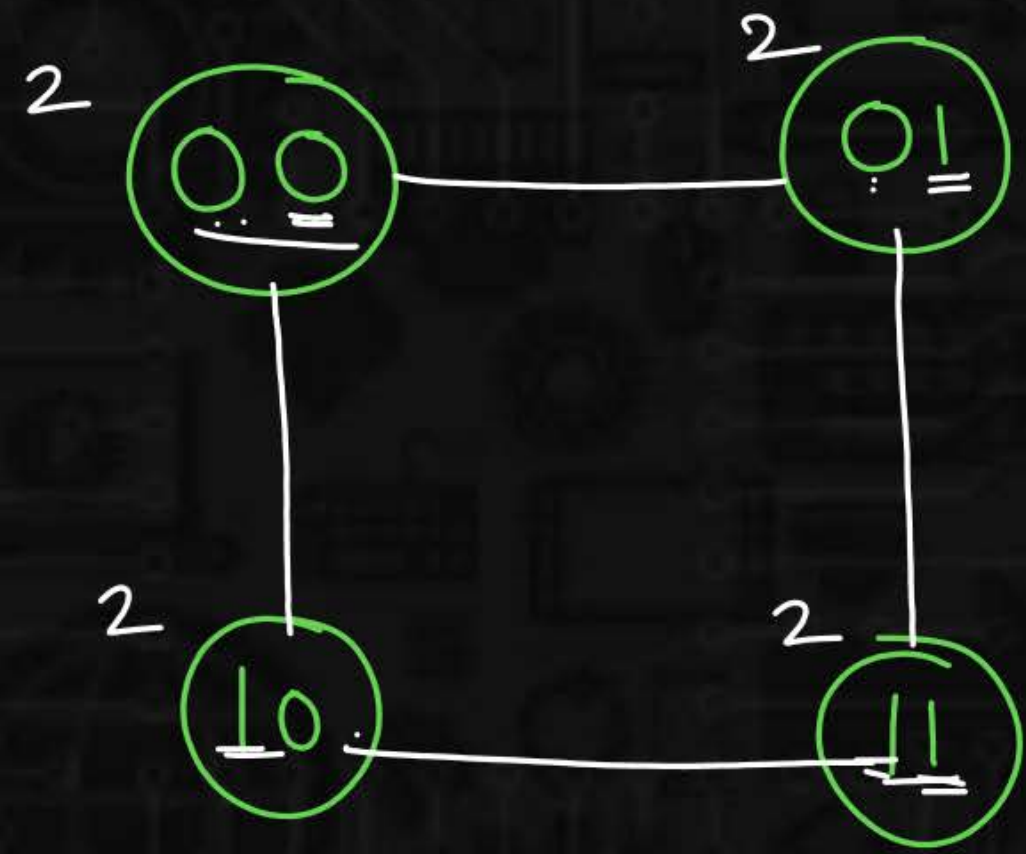
Two vertices are connected, if bit-position changes by 1-bit?

What will be edges in G ?

Types of graph

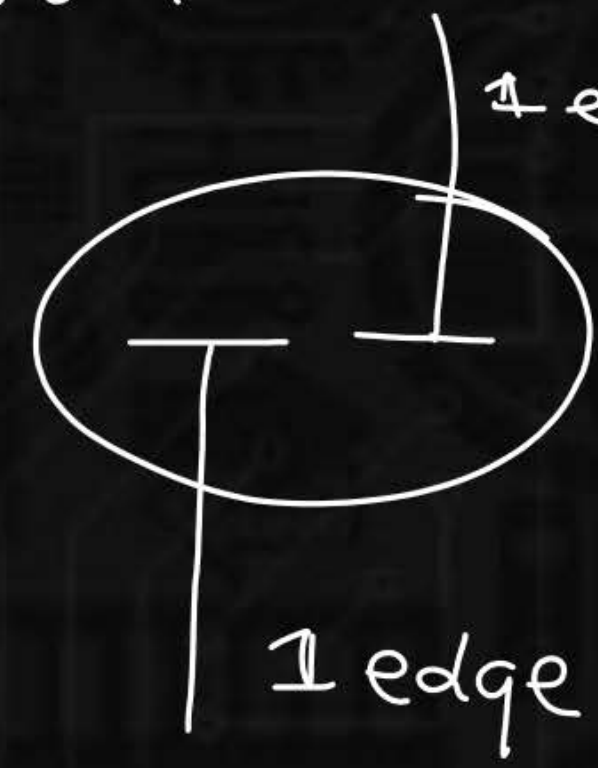
$$n = 2(\text{bit})$$

Total vertices = 4



$$n = 2$$

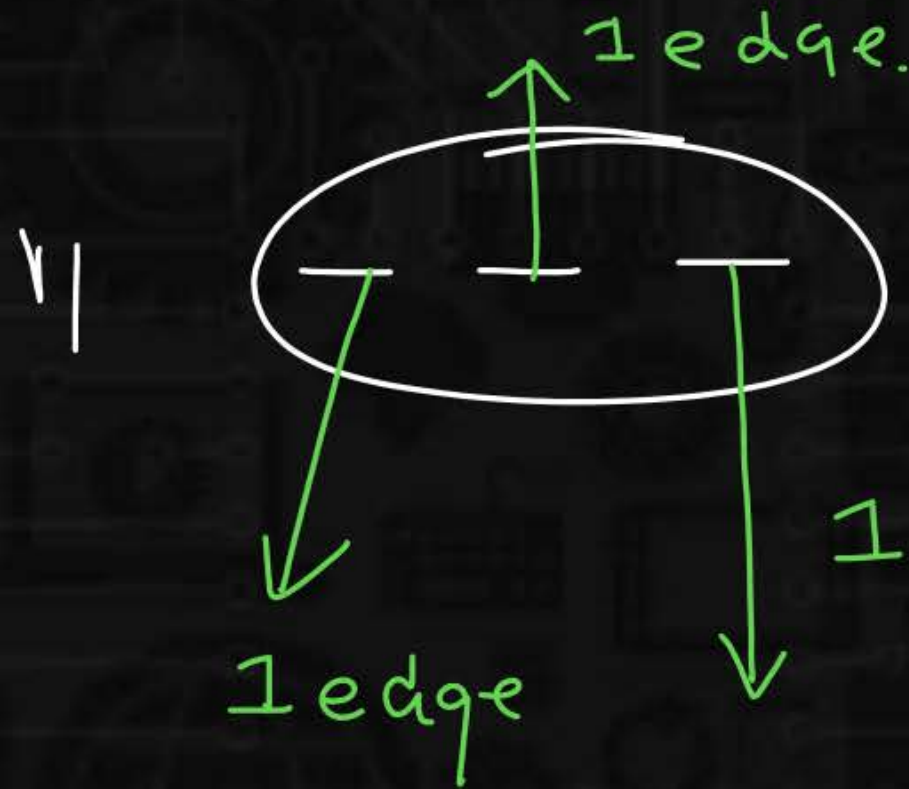
Total vertices = $2^2 = 4$ $e = ?$



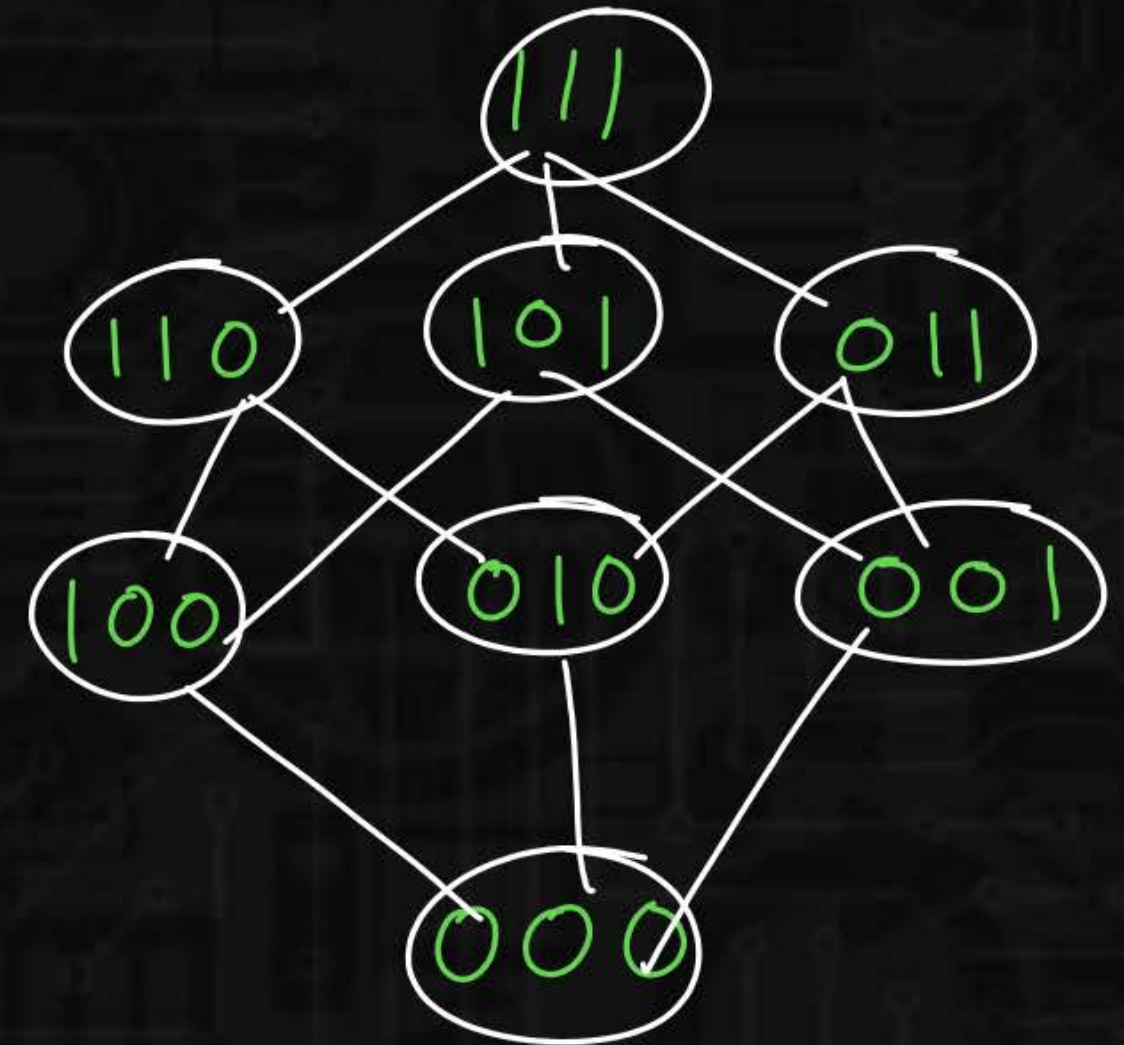
Types of graph

$n = 3$ (bit signal)

Total vertices = $2^3 = 8$



Degree of each vertex is 3.



Types of graph

n -bit signal.

Total vertices = $2^n = v$

Degree of each vertex = n .

$$\sum d(v_i) = 2e$$

$$2^n \times n = 2e$$

$$e = \frac{n \times 2^n}{2}$$

$$\sum d(v_i) = 2e.$$

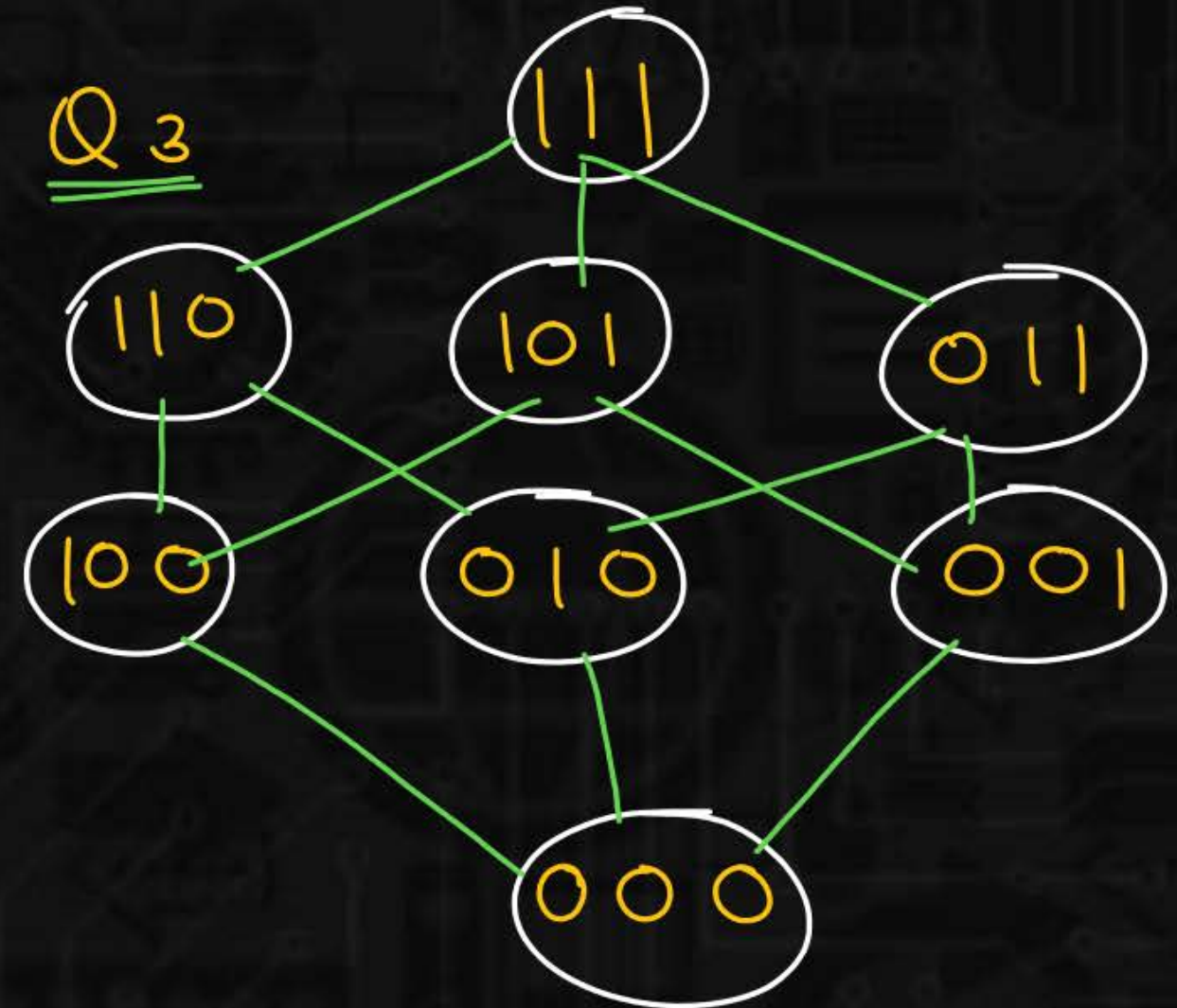
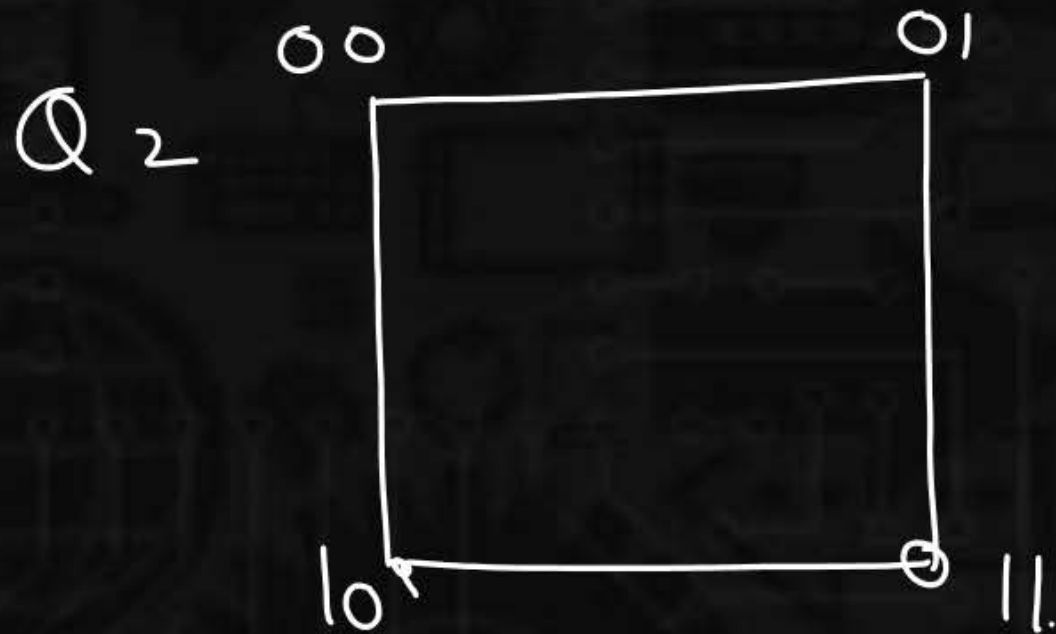
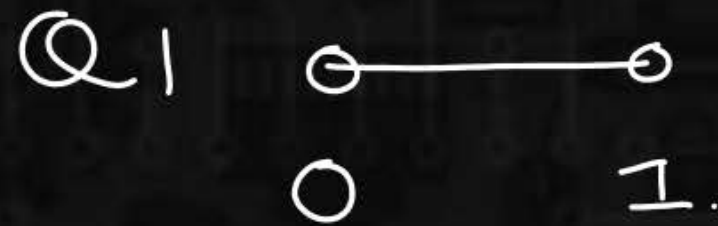
$$2^n \times n = 2e$$

$$e = n \times \frac{2^n}{2}$$

$$e = n \times 2^{n-1}$$

Types of graph

Hypercube (Q_n)
 \rightarrow n bit signal.



n -bit signal.

$$\text{Total vertices} = 2^n = v$$

$$\text{Degrees of all vertices} = n.$$

$$K_{2^n, 2^n-1, 2^n-1, 2^n-1, \dots}$$

$$\text{Total vertices} = 2^n.$$

$$G \rightarrow \underbrace{n, n, n, \dots, n}_n.$$

$$(G) 2^n-1-n, 2^n-1-n, 2^n-1-n, \dots$$

$$e(G) = n \cdot 2^{n-1}.$$

$$e(G) + e(\overline{G}) = \frac{v(v-1)}{2} = \frac{2^n(2^n-1)}{2}$$

↓

$$n \cdot 2^{n-1} + e(\overline{G}) = \frac{2^n(2^n-1)}{2}.$$

$$e(\overline{G}) = \frac{2^n(2^n-1)}{2} - n \cdot 2^{n-1}.$$

