CS & IT ENGINEERING

Quantifier 1

Lecture No. 05

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01 Theorems on Quantifier

02 English statement to Logic Conversion

03 Problems on Quantifier

04 Type 4

05 Nested Quantifeir

1. $\exists x [P(x) \wedge Q(x)] \rightarrow \exists x P(x) \wedge \exists x Q(x)$ ✓

2. $\exists x [P(x) \vee Q(x)] \leftrightarrow \exists x P(x) \vee \exists x Q(x)$

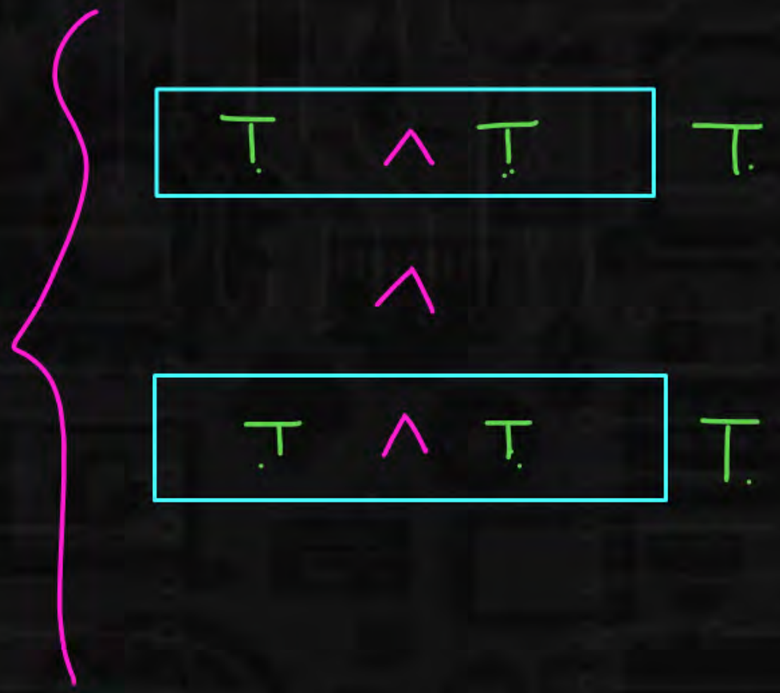
3. $\forall x [P(x) \wedge Q(x)] \leftrightarrow \forall x P(x) \wedge \forall x Q(x)$

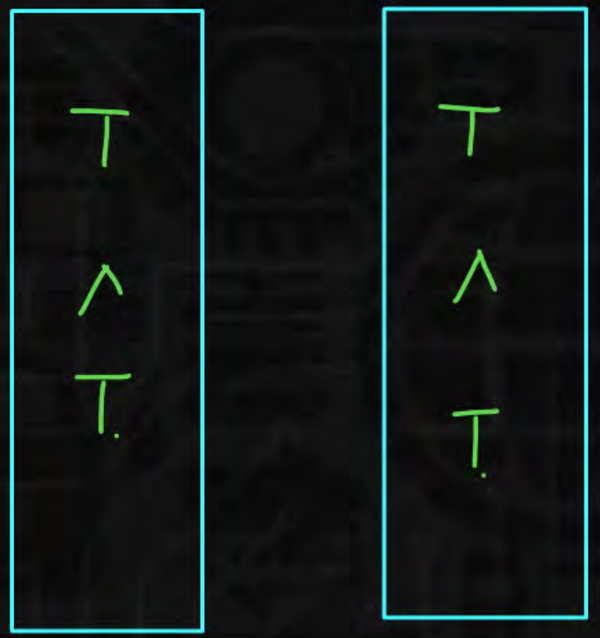
* 4. $\forall x [P(x) \vee Q(x)] \leftarrow \forall x P(x) \vee \forall x Q(x)$

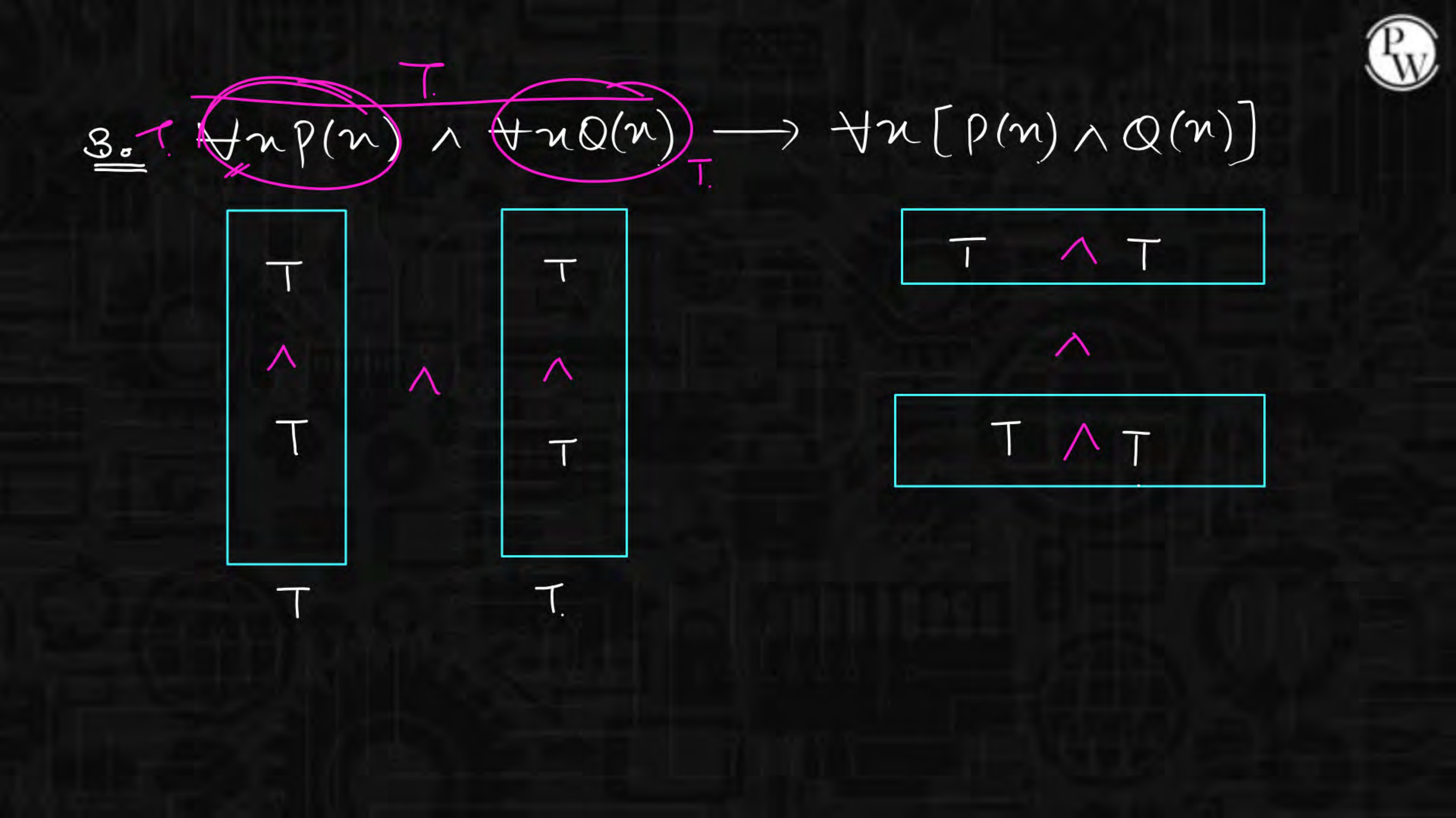
5. $\forall x [P(x) \rightarrow Q(x)] \rightarrow \forall x P(x) \rightarrow \forall x Q(x)$

6. $\forall x [P(x) \leftrightarrow Q(x)] \rightarrow \forall x P(x) \leftrightarrow \forall x Q(x)$

$$\frac{T}{3.\ \forall x\,[P(x) \wedge Q(x)]} \longrightarrow \forall x P(x) \wedge \forall x Q(x)$$






3. $\forall x P(x) \wedge \forall x Q(x) \longrightarrow \forall x [P(x) \wedge Q(x)]$
T
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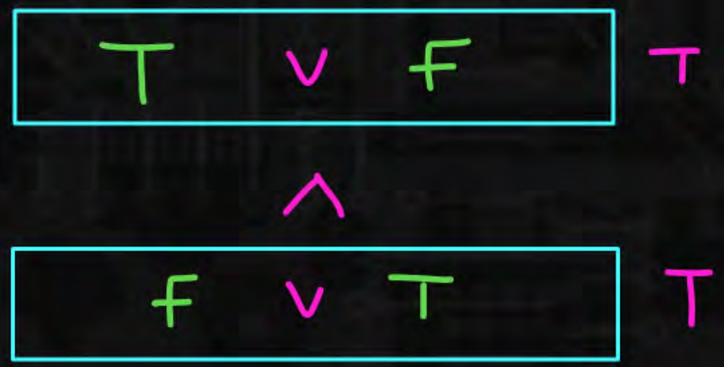
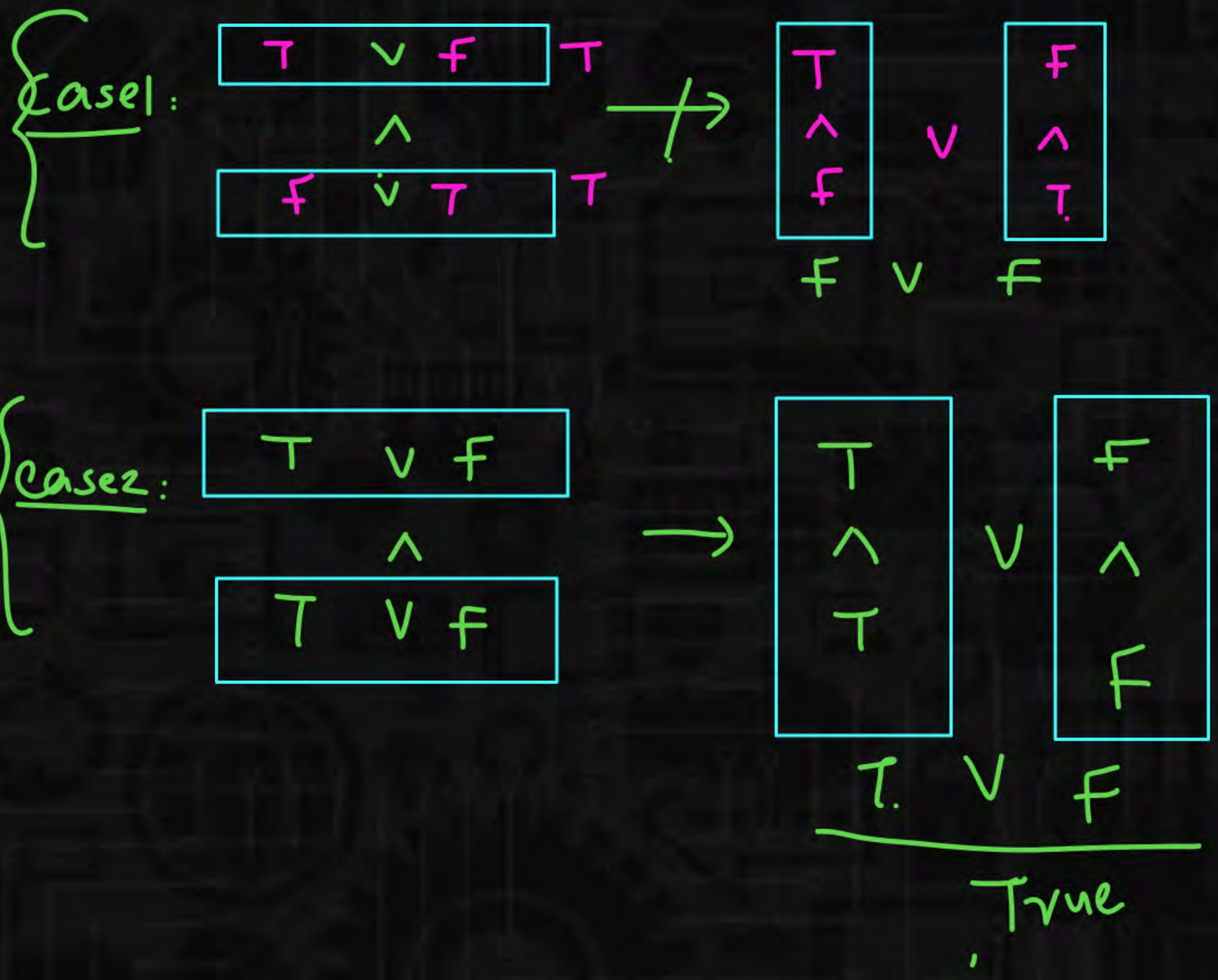
T.

Ex. $\forall x[P(x) \vee Q(x)] \not\rightarrow \forall x P(x) \vee \forall x Q(x)$



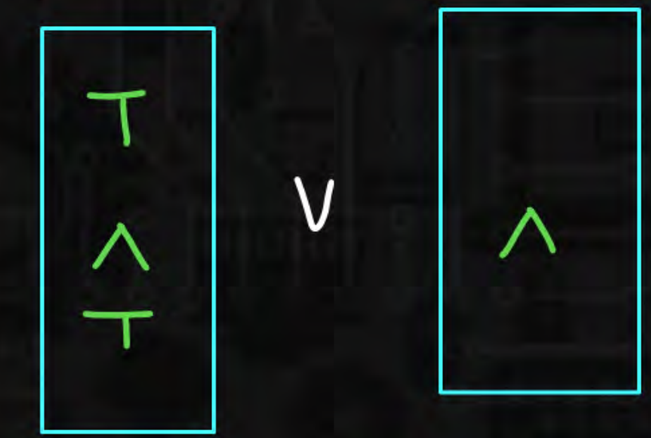
T ∨ f

f ∨ F.

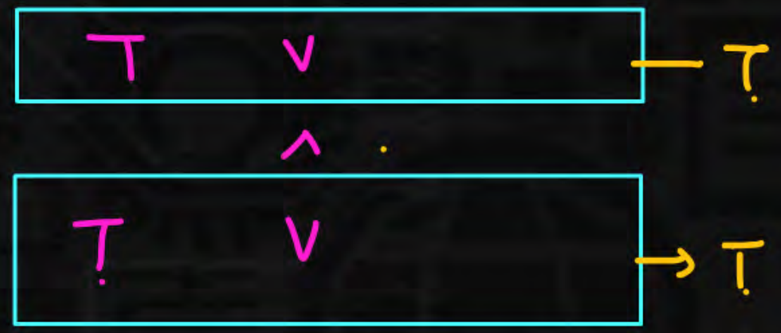

Case1:
T ∨ f
T
∧
f ∨ T
T
T ∧ f
∨
f ∧ T
f ∨ f
Case2:
T ∨ f
∧
T ∨ f
T ∧ T
∨
f ∧ F
T ∨ F
True

T.

$\forall x P(x) \vee \forall x Q(x) \longrightarrow \forall x [P(x) \vee Q(x)]$

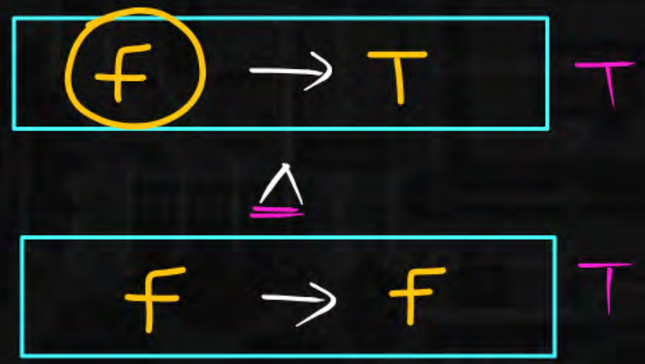


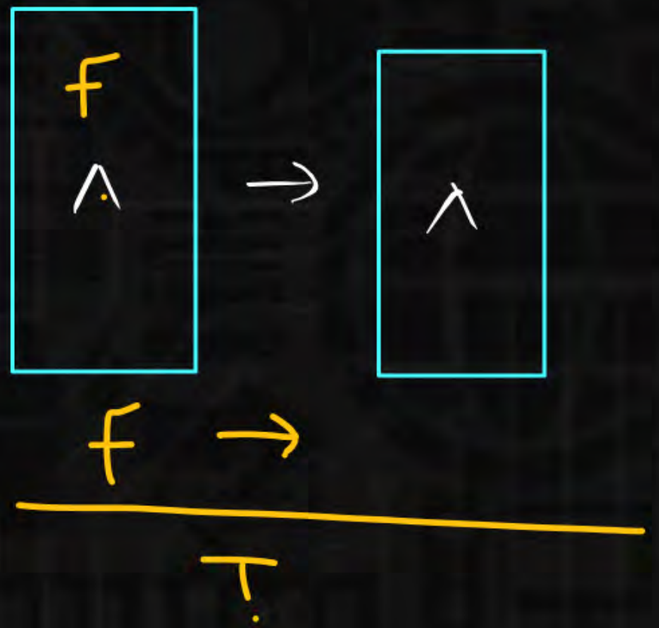
T.



True

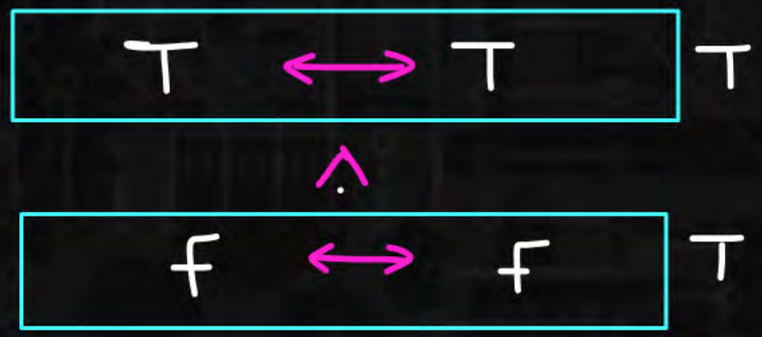
$$\overset{T.}{\overline{\forall x\,[P(x) \to Q(x)]}} \longrightarrow \left(\forall x P(x) \to \forall x Q(x)\right)$$



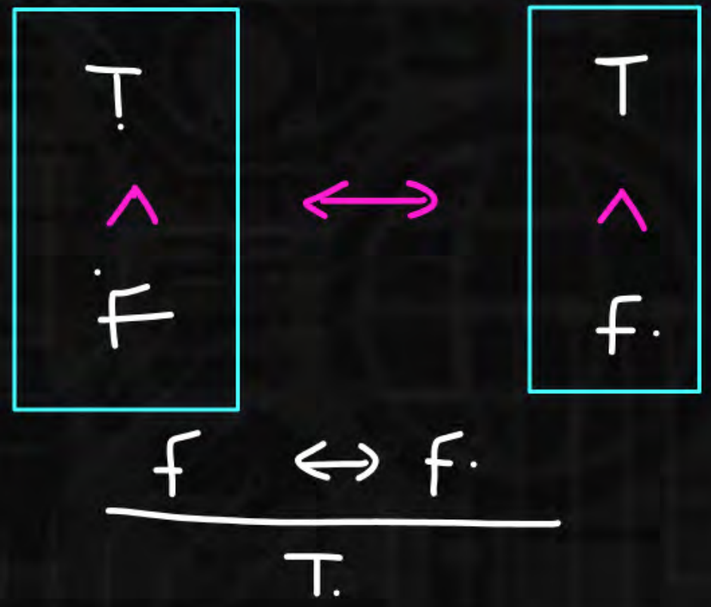


T.

$\forall x [P(x) \leftrightarrow Q(x)]$



$\forall x P(x) \leftrightarrow \forall x Q(x)$



D: 2.

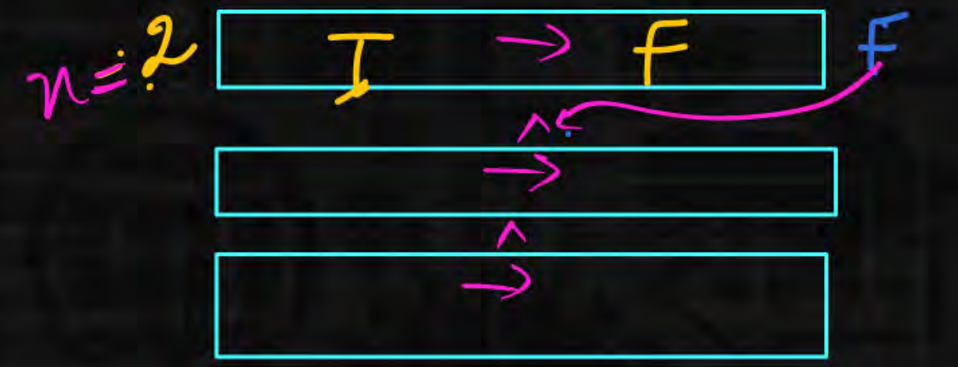
$P(x)$: $x^2 - 5x + 6 = 0 \rightarrow x = 2, 3$.

$Q(x)$: $x < 0$ $\rightarrow$ -ve values.

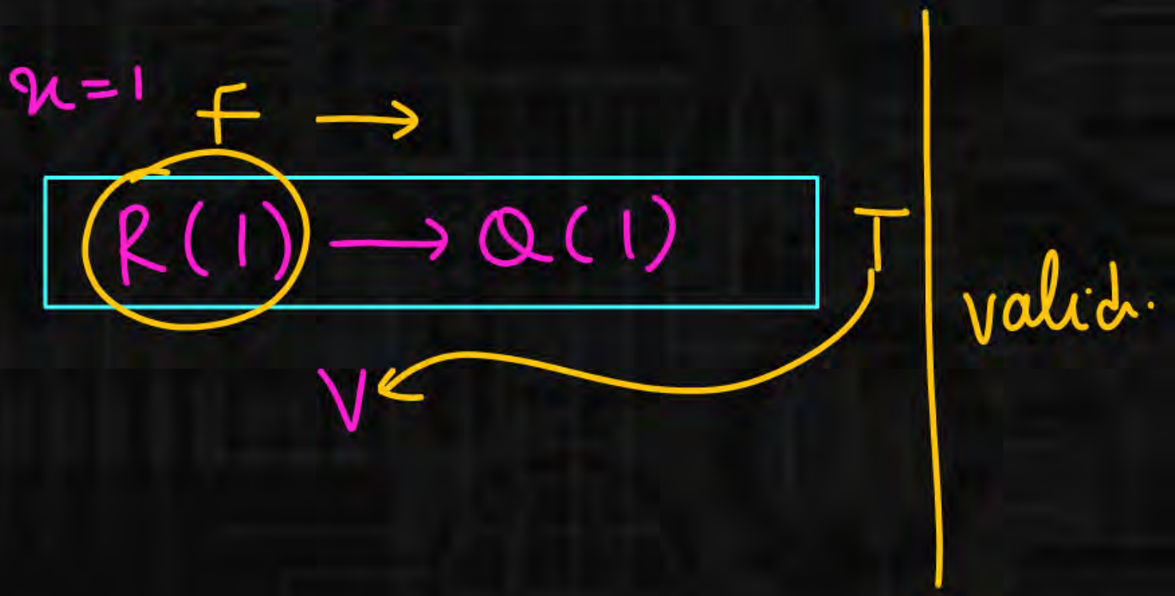
$R(x)$: x is even no.

D: 2.

$\forall x [R(x) \rightarrow Q(x)]$

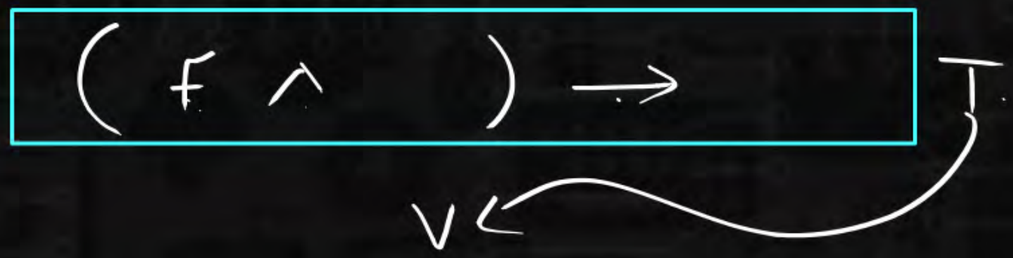


$\exists x [R(x) \rightarrow Q(x)]$



$$\exists x\,[(P(x) \wedge Q(x)) \rightarrow R(x)]$$

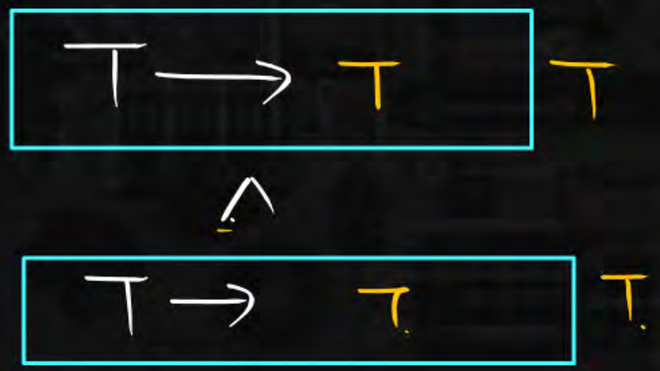
$x = 1$



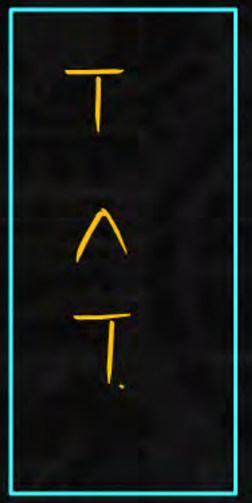
T.

$$\forall x\,(A \rightarrow P(x)) \equiv A \rightarrow \forall x\, P(x)$$

A = True

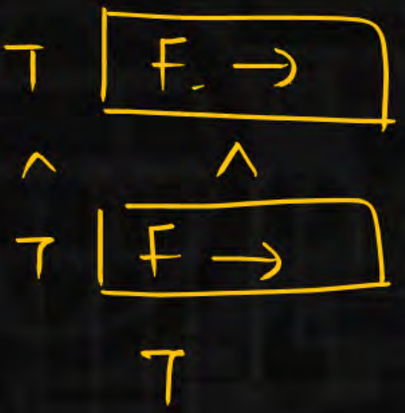


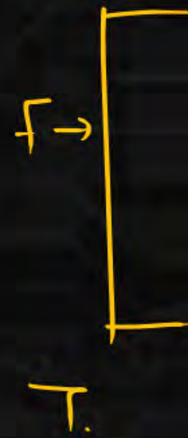
T →



T → T.

A = False.





All mothers are female.

↓

All of x | x is mother | x is female.

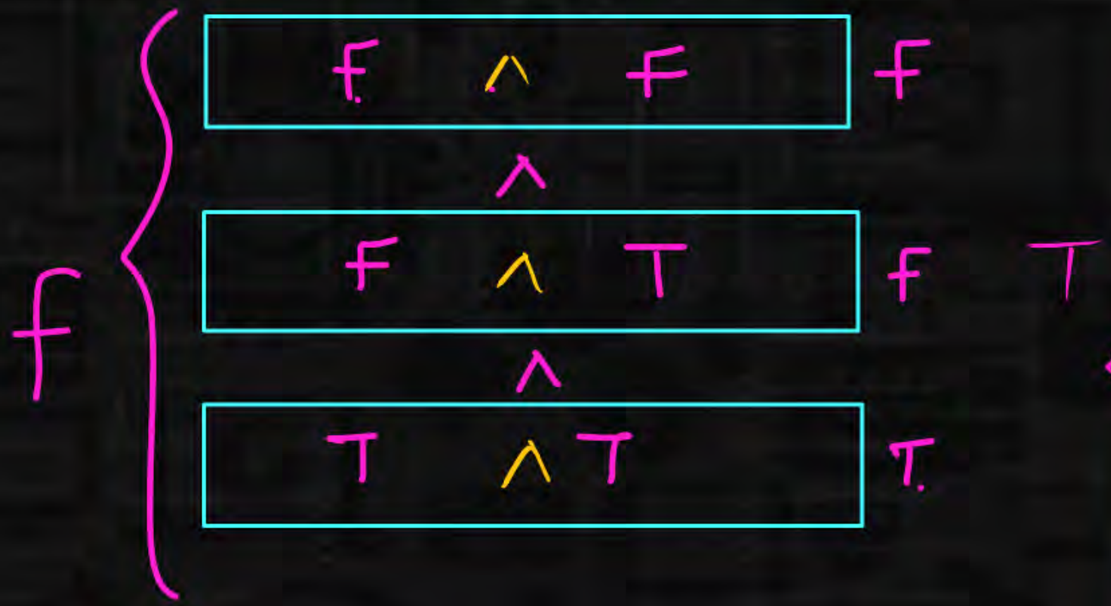
↓ | ↓ | ↓

$\forall x \; (MT(x) \quad F(x))$

$\{ \overrightarrow{\wedge}$ -

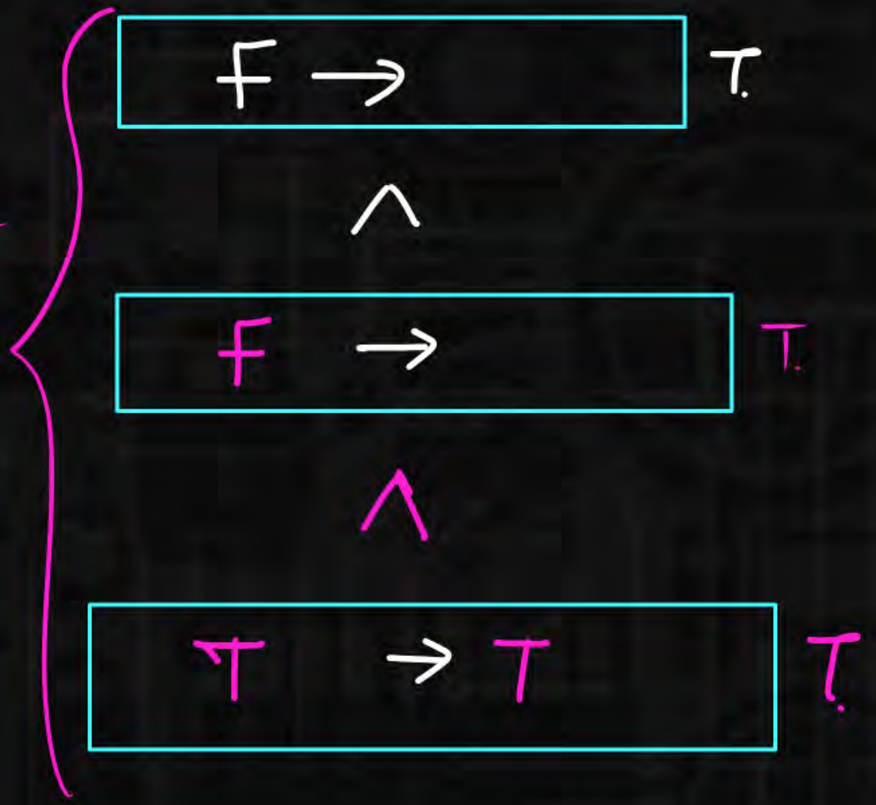
Case1:

$\forall x [mT(x) \wedge F(x)]$



Case2

$\forall x [mT(x) \rightarrow F(x)]$



- BABA
- Single female
- mT/F

ALL →

Some cats are black.

Some of x x is cat x is black.

$\exists x$ $\{CT(x)$ $BL(x)\}$

Some → ∧ ∨ ↔

T T T T T

F T F T T

Some cats are black

True

$\exists x\,[CT(x) \rightarrow BL(x)]$

T → F F

∨

T → T T

$\exists x\,[CT(x) \vee BL(x)]$

T ∨ T

∨

∨

$\exists x\,(CT(x) \wedge BL(x))$

T ∧ F F

∨

T ∧ T T

$\exists x\,(CT(x) \leftrightarrow bl(x))$

T ↔ F F

∨

T ↔ T T

Some cats are black.

$\exists x [CT(x) \rightarrow BL(x)]$

$\exists x [CT(x) \wedge bl(x)]$

→ kat/w
→ maqqie/w
→ cow

false.

T. { T → F : F ; ∨ ; T → F : F ; ∨ ; F → : T. }

f { T ∧ F : F ; ∨ ; T ∧ F : f ; f ∧ : f }

Some ∧

ALL →

all graphs are connected.

$\forall x\,[G(x) \rightarrow C(x)]$

not(all graphs are connected)

$\neg\,\forall x\,[G(x) \rightarrow C(x)]$

- All graphs are disconnected.
  $\forall x\,[G(x) \rightarrow \neg C(x)]$
- no graphs are connected.

all of my friends are perfect

$\forall x [F(x) \rightarrow \neg P(x)]$

not(all of my friends are perfect) $\neq$ none of my friends are perfect

$\neg \forall x [F(x) \rightarrow P(x)]$

=

All of my friends are not perfect

no/none $\forall x[\ \rightarrow \neg\ ]$

not all $\neg \forall x[\ \rightarrow\ ]$

Type-4 → box method


THANK YOU GW
SOLDIERS !