CS & IT ENGINEERING

covering and planarity part 1

Lecture No. 12

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TOPICS TO BE COVERED

01 covering set

02 Covering number

03 Planar Graph

04 Euler's Formula In planarity

05 Sum of Degrees in Region

$$2n \times (2n-2) \times (2n-4) \ldots\ldots$$

$$2 \times n \times 2(n-1) \times 2(n-2) \times 2(n-3) \ldots\ldots$$

$$2^r \quad n \times (n-1) \times (n-2) \ldots 1$$

$$2^n \cdot n!$$

Total no. of P.M in complete Graph $2n$ vertices.

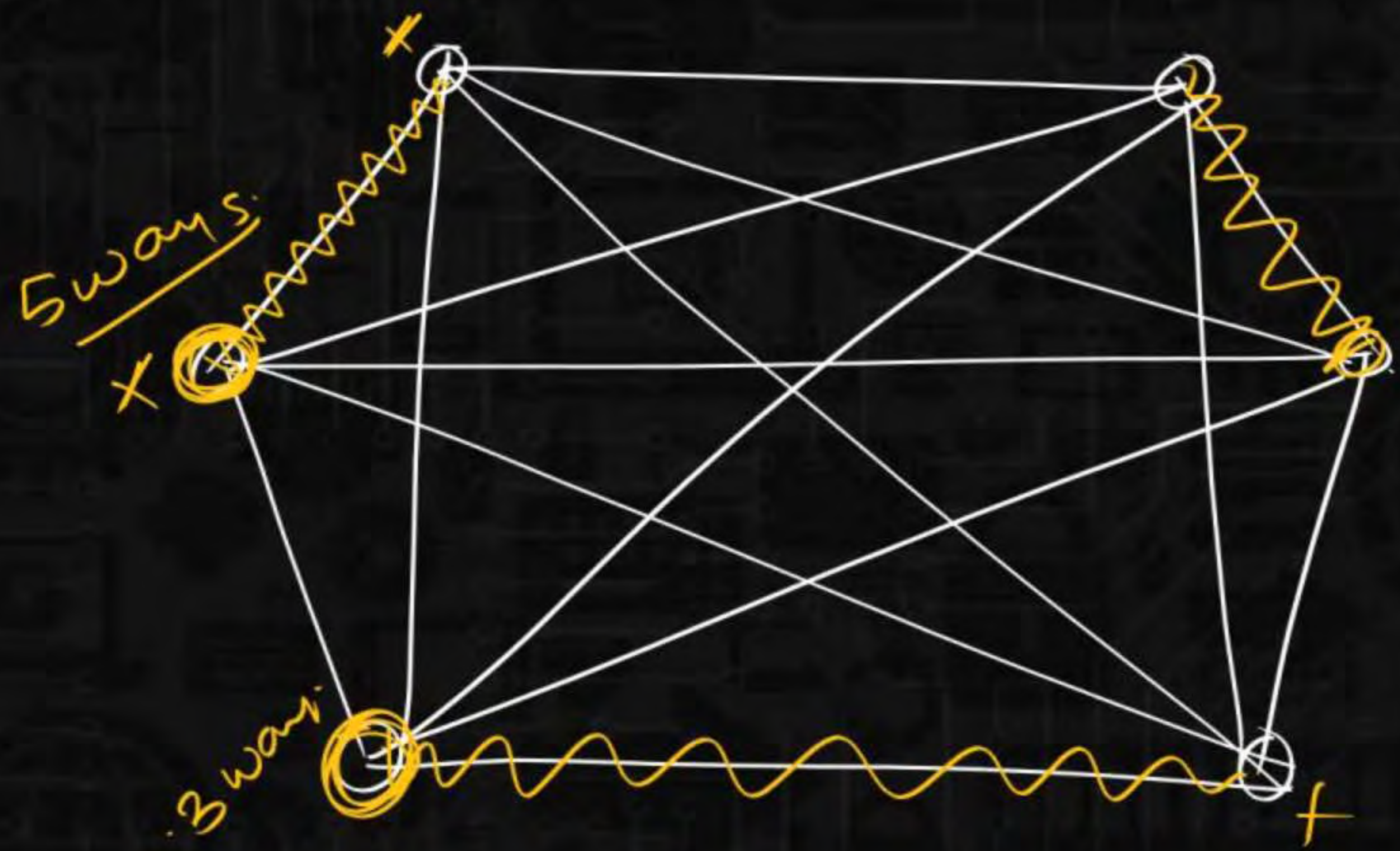
$(2n-1)$ ways. $(2n-3)$ ways

$$(2n-1)\times(2n-3)\times(2n-5)\cdots$$

$$\frac{2n}{2n}\times(2n-1)\times\frac{(2n-2)}{(2n-2)}\times(2n-3)\times\frac{(2n-4)}{(2n-4)}\times(2n-5)$$

$$=\frac{(2n)(2n-1)(2n-2)\times(2n-3)(2n-4)(2n-5)}{2n\times(2n-2)\times(2n-4)}$$

$$=\frac{(2n)!}{2^n\,n(n-1)(n-2)}=\frac{(2n)!}{2^n\cdot n!}$$



5ways × 3ways

GATE

How many p.m exist in complete Graph of 6 vertices?

$2n = 6$

$n = 3$

Ans: 15

$$\frac{(2n)!}{2^n \cdot n!} = \frac{6!}{2^3 \times 3!}$$

$$= \frac{\cancel{6}^{3} \cdot 5 \cdot \cancel{4}^{2} \cdot \cancel{3!}}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{3!}} = \underline{15}$$

Independent set → non adjacent vertices.

Dominating set → me or my friend

matching set → non adjacent edges.

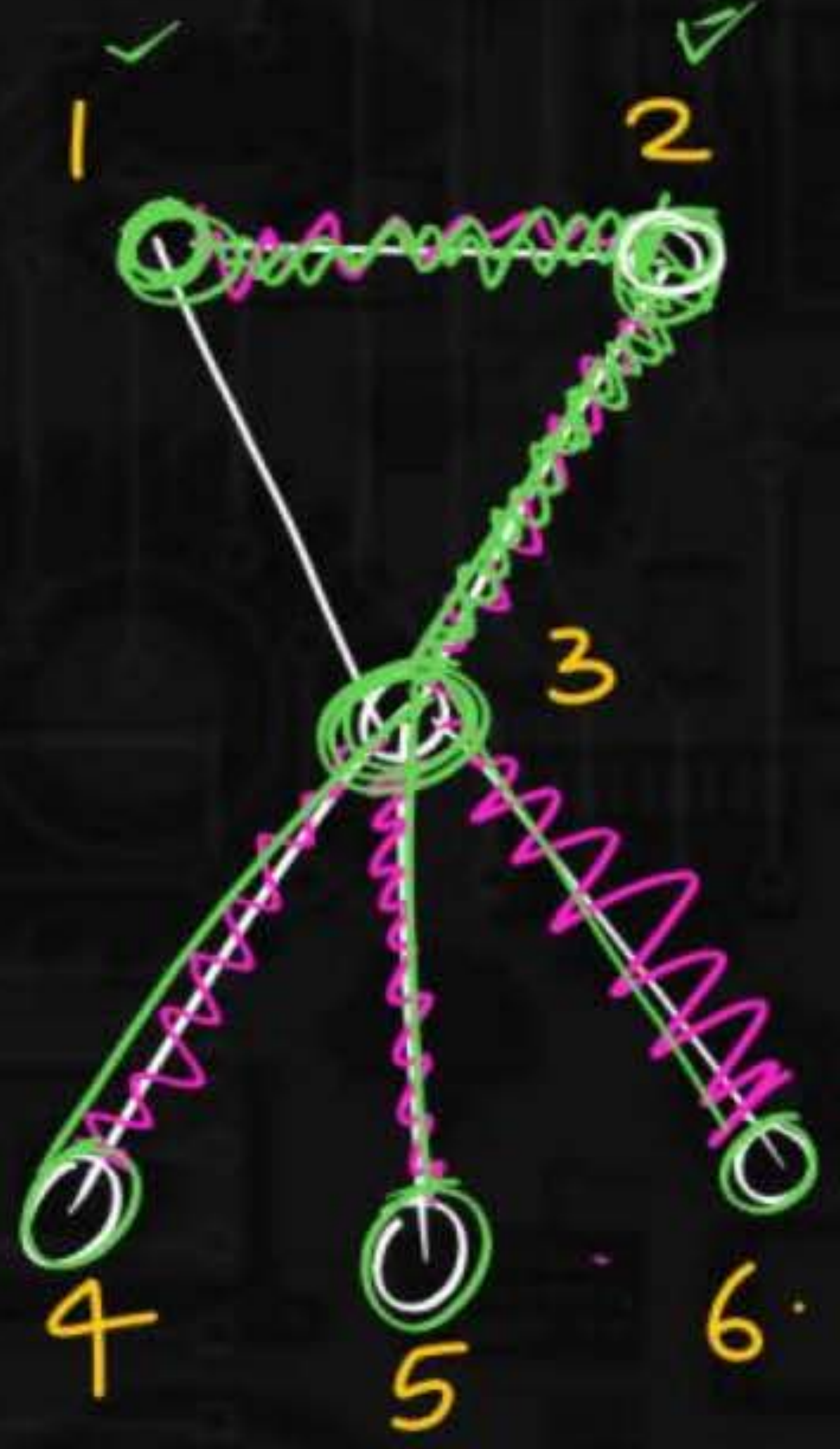
Covering set → at least 1 marriage proposal.

Covering set :

set of edges such that all vertices should incident on at least 1 edge

set of edges → all vertices → at least 1 edge.

{ }

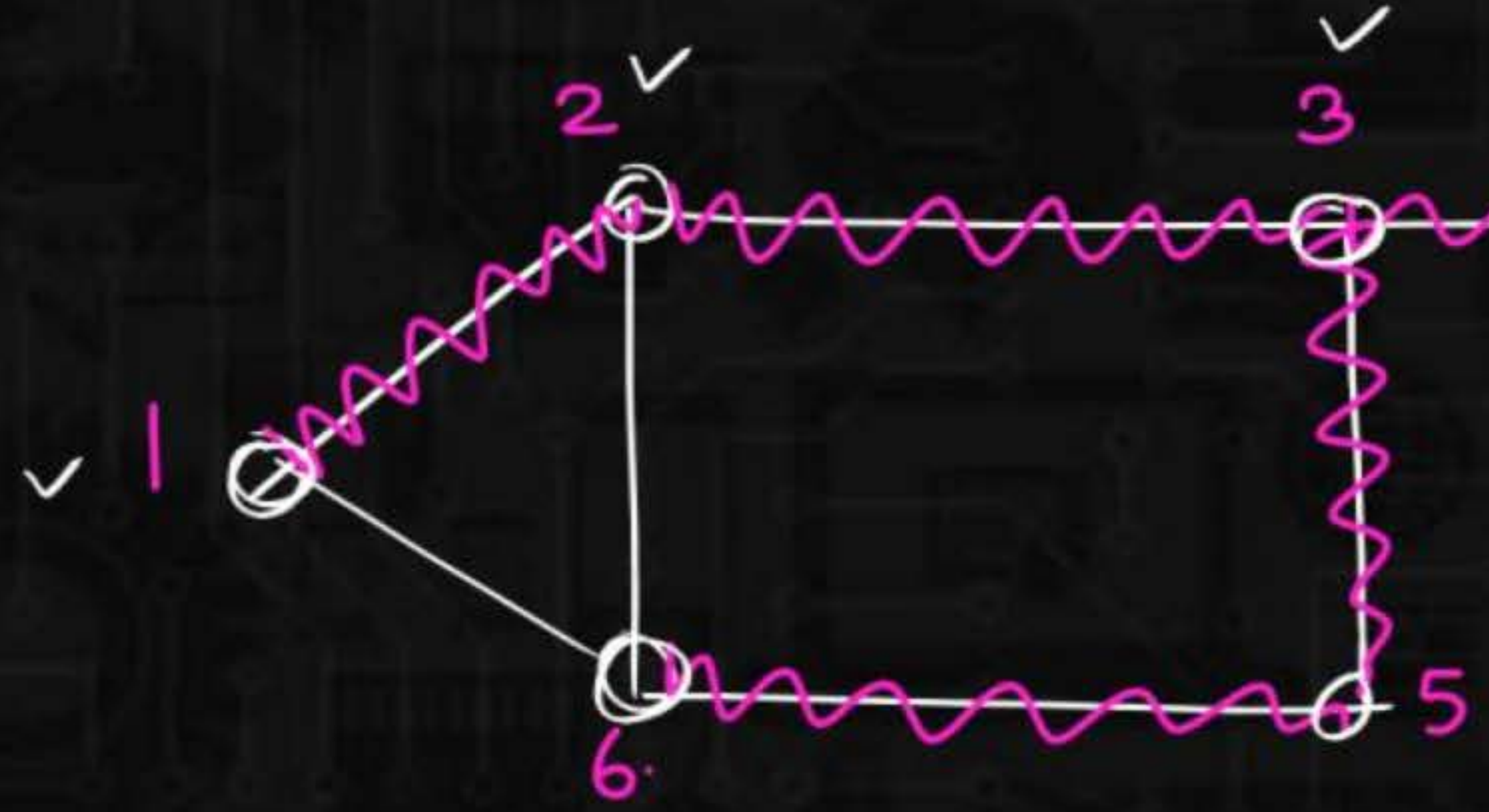


{12, 23, 34, 35, 36}

all edges → covering set.

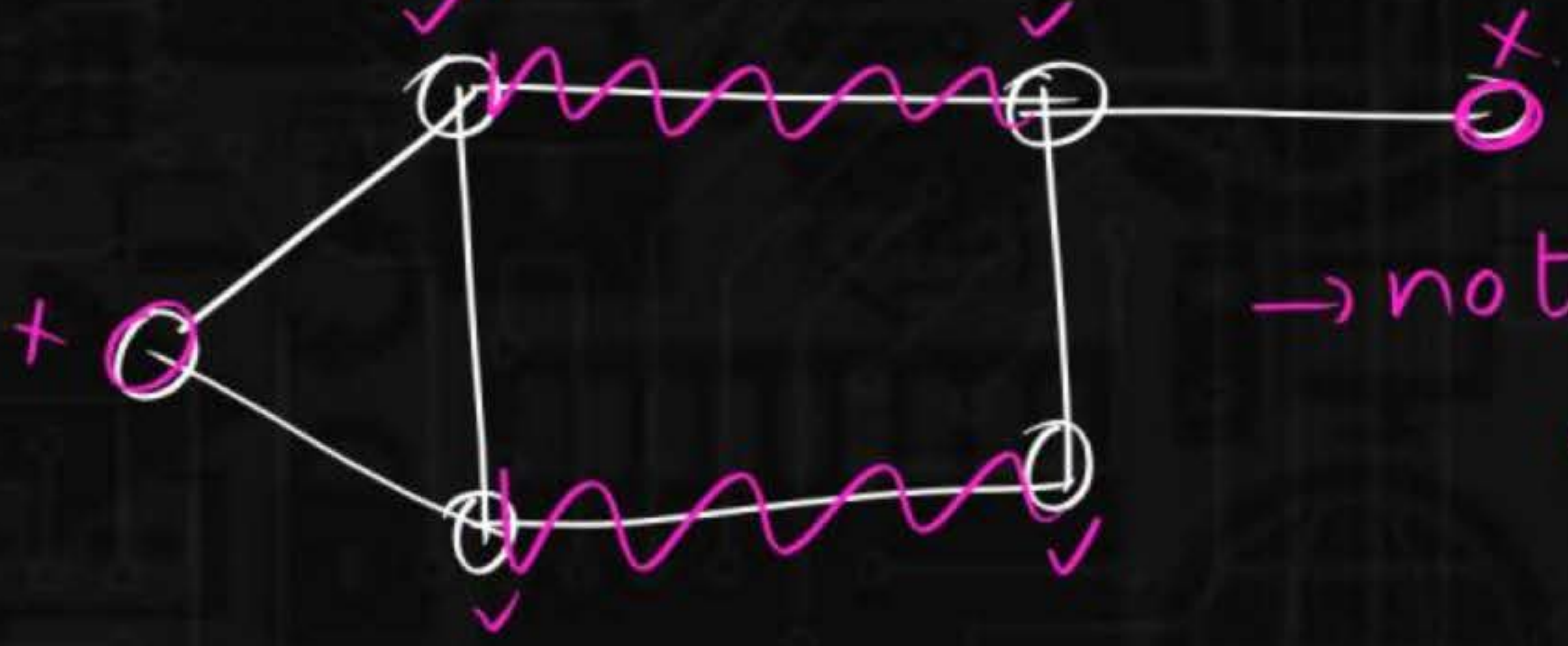
→ not minimal covering set

minimal covering set

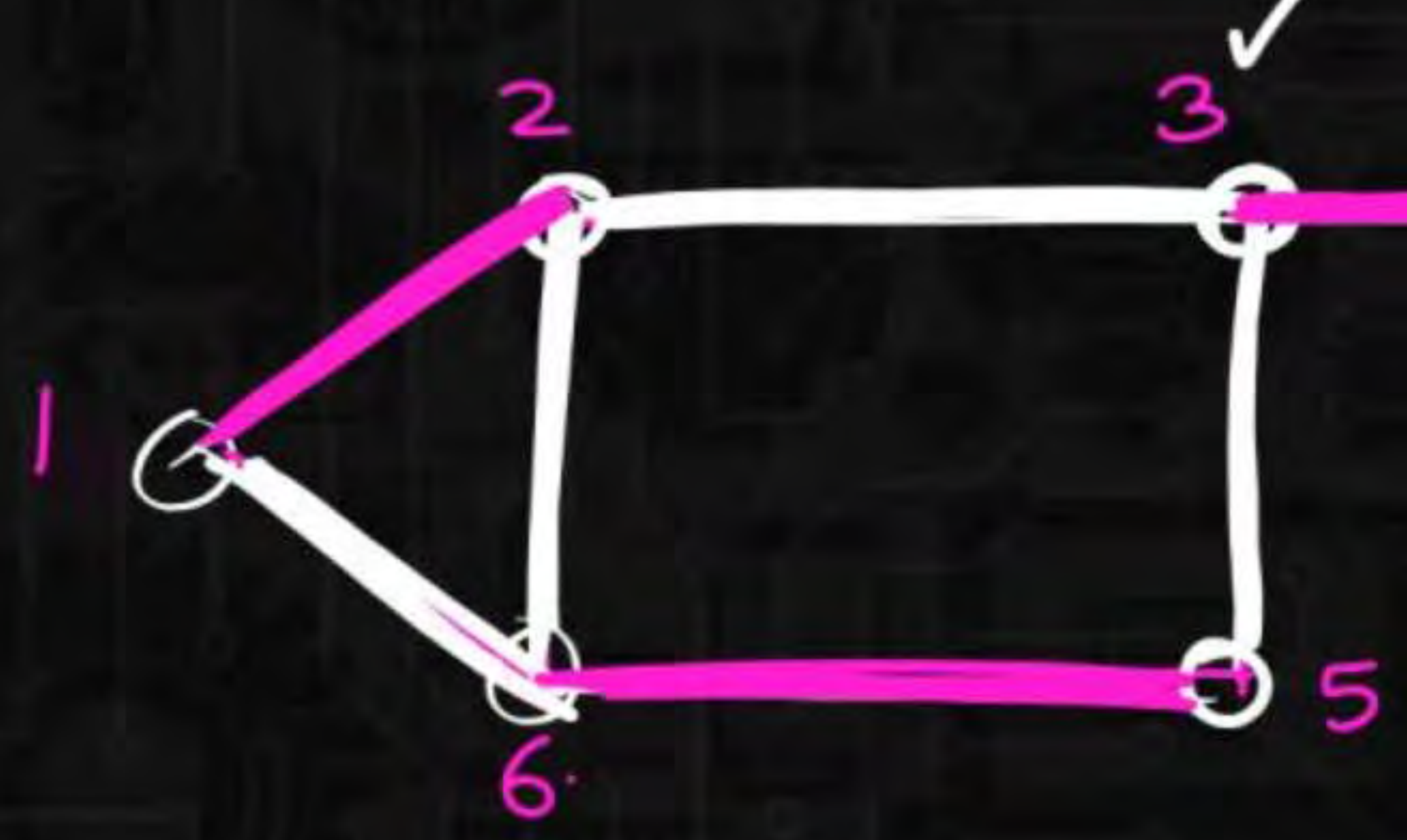


→ covering set

all vertices → at least 1 edge



→ not covering set



all edges → covering set

cs {12, 23, 34, ~~35~~, 26, 16, 65}

cs {12, 23, 34, ~~26~~, 16, 65}

cs {12, 23, 34, ~~16~~, 65}

cs {12, ~~23~~, 34, 65}

cs {12, 34, 65}

{minimal covering set}

covering set :

minimal covering set : $\{12, 23, 35\}$

covering set such that we can not remove new edge from this.

covering no : smallest minimal covering set
$C(G)$

can not add

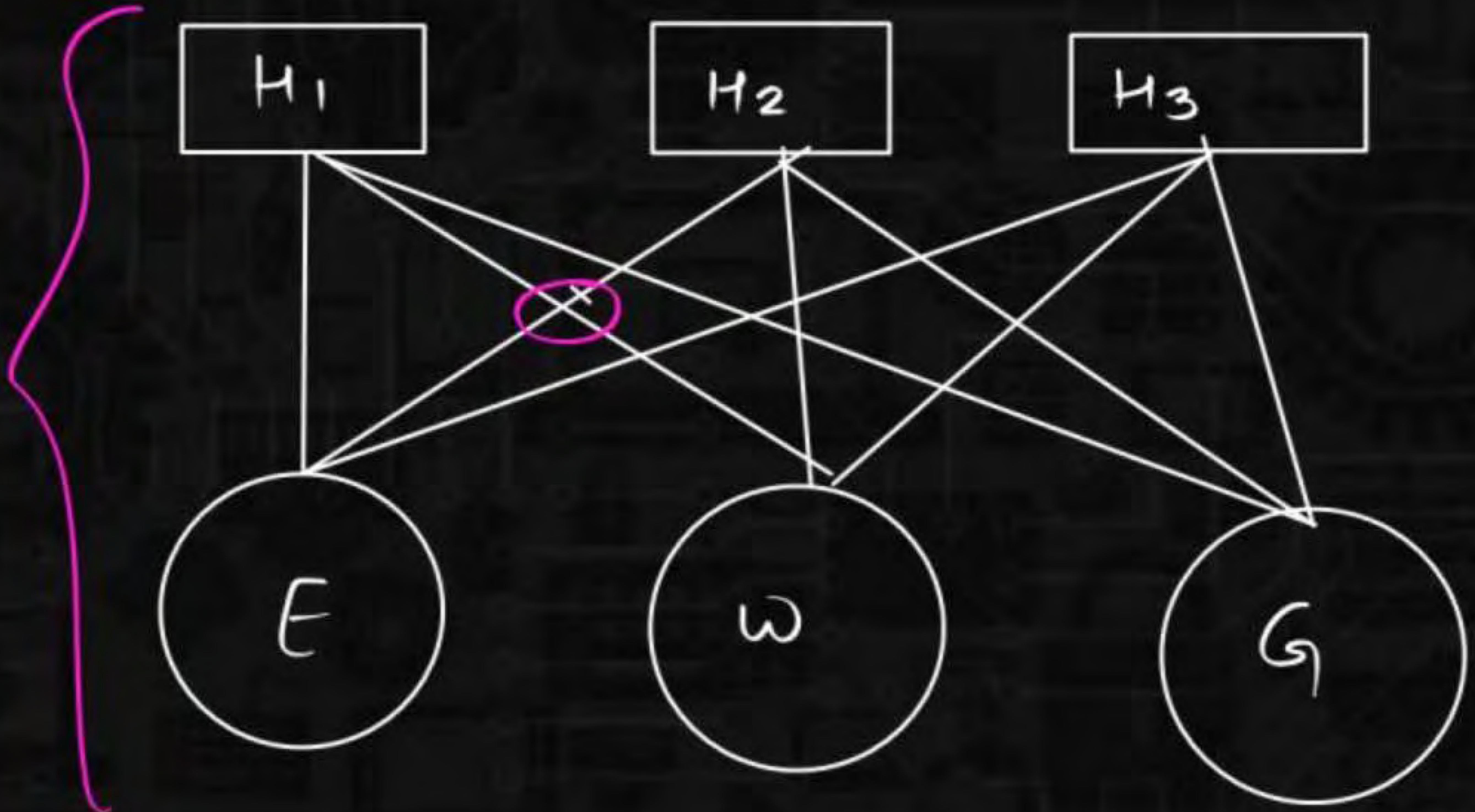
Independent set → maximal Independent set → Independence no.
(largest MIS)

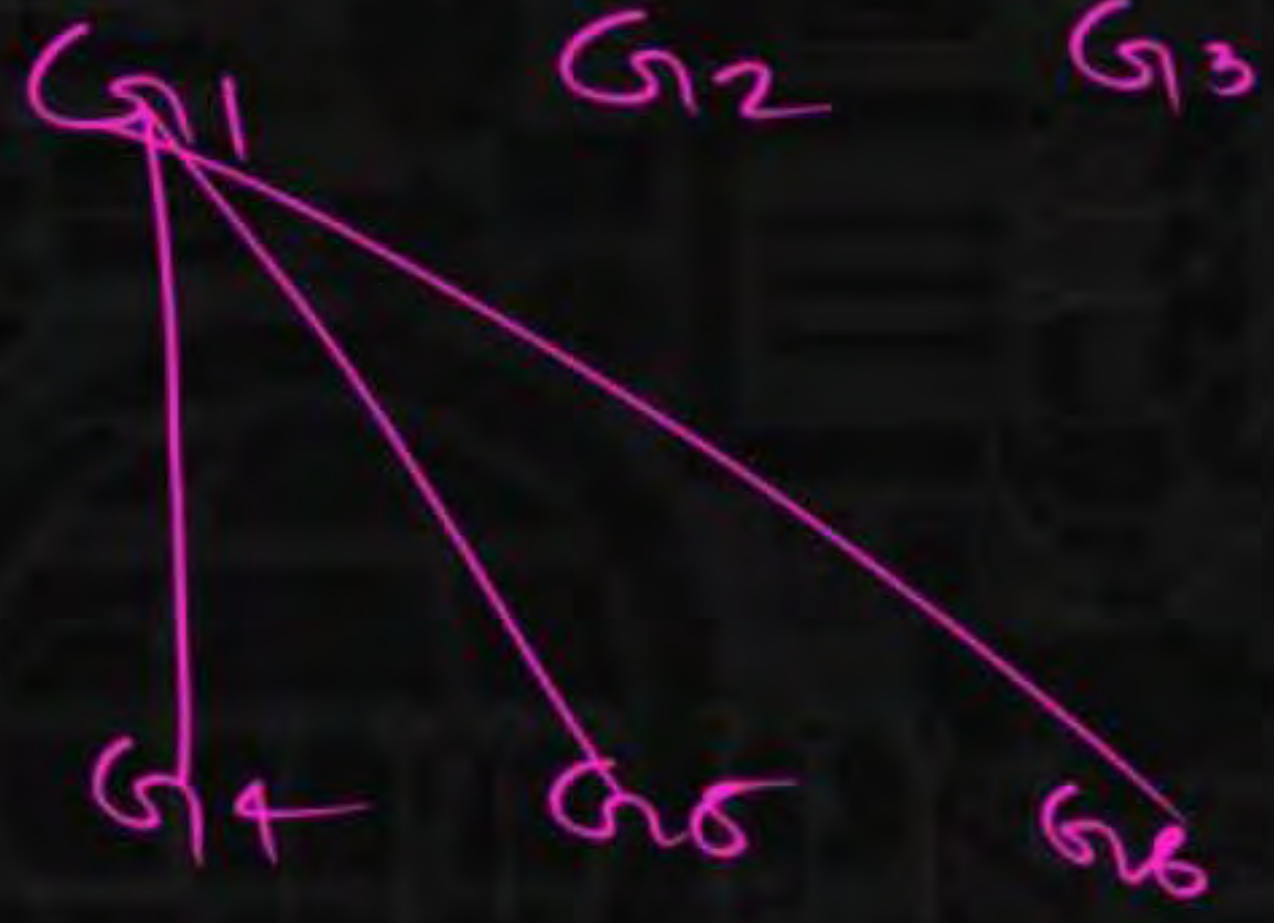
can not remove.

Dominating set → minimal dominating set → Dominance no.
(smallest MDS)

matching set → MMS → matching no.
(largest MMS)

Covering set → minimal covering set → covering no.
(smallest MCS)


H1
H2
H3
E
ω
G


G1
G2
G3
G4
G5
G6

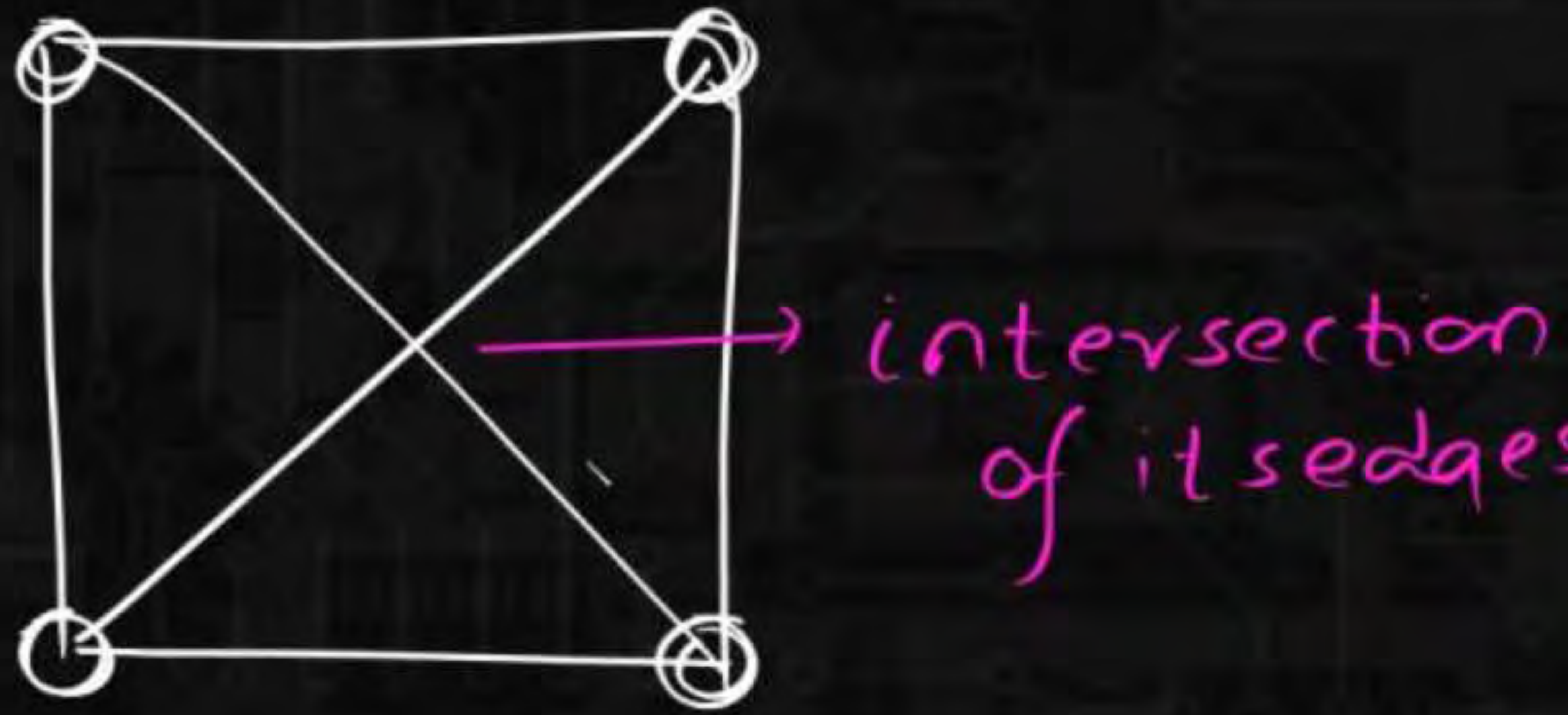
Graph

Planar Graph.

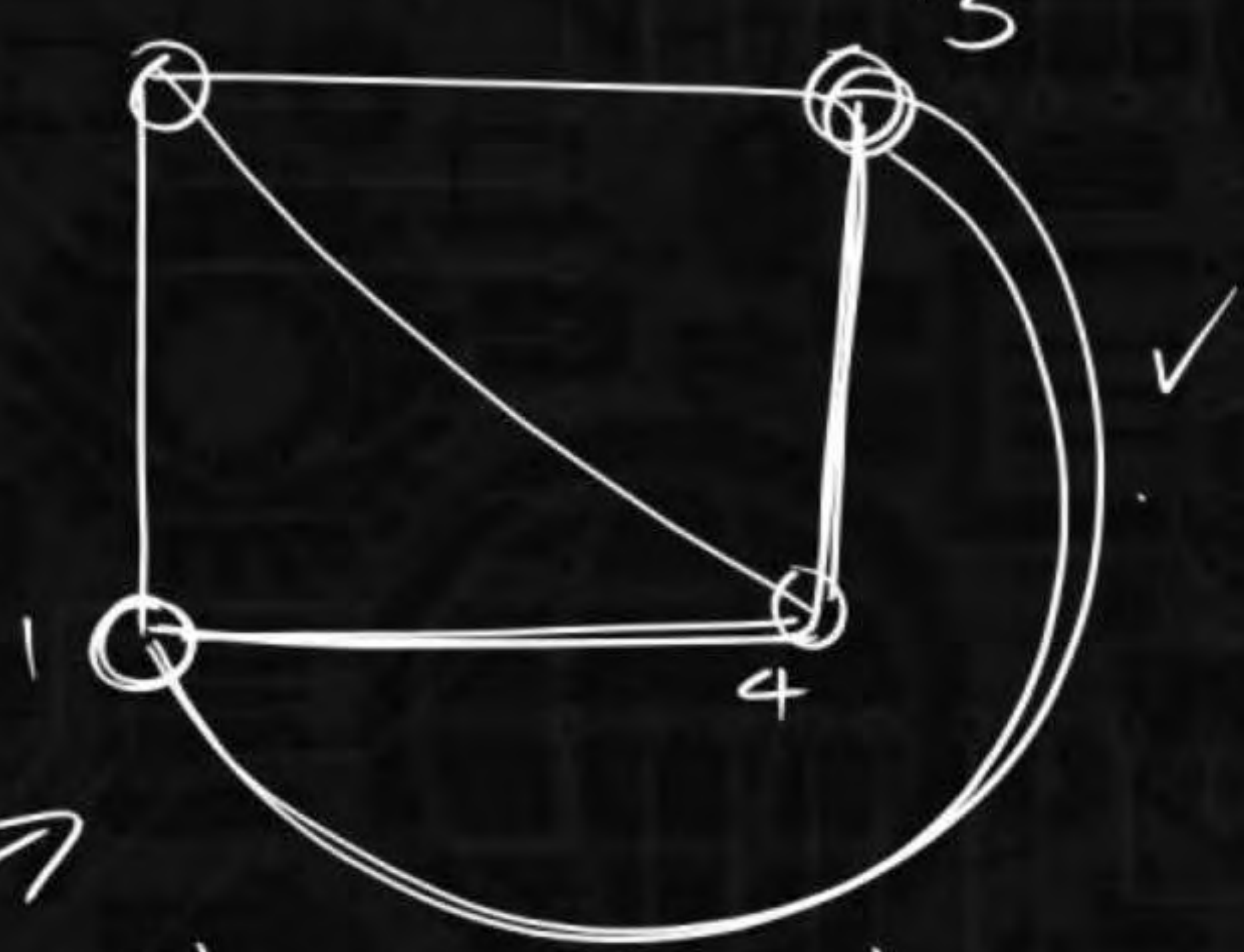
if we can draw graph without intersection of its edges

non planar Graph.

$n = 4.$

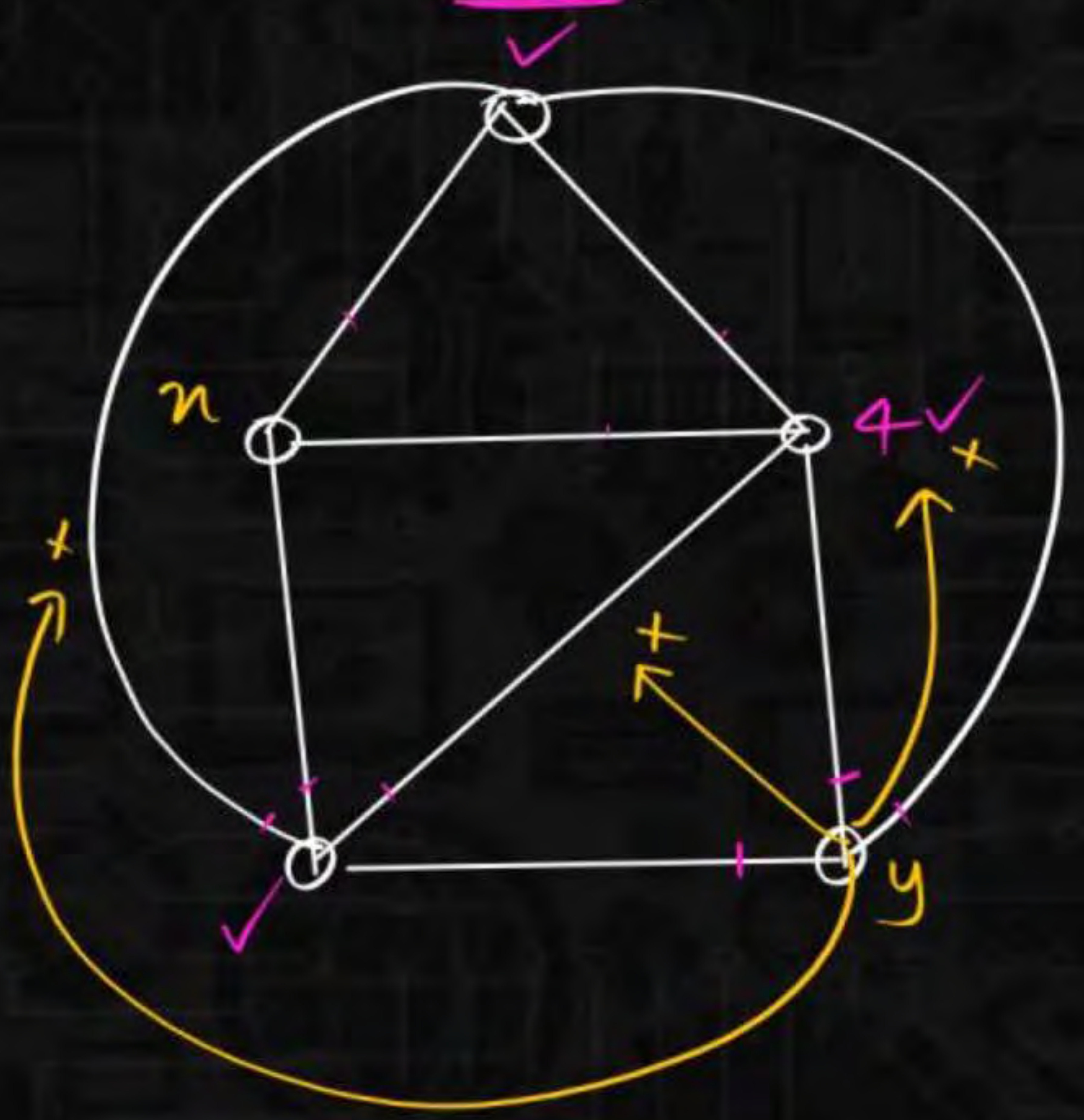


Planar Graph.



Planar Graph

$$\left.\begin{array}{l} n = 5 \\ e = 10 \end{array}\right) K_5$$



<u>K_5 is non planar Graph.</u>

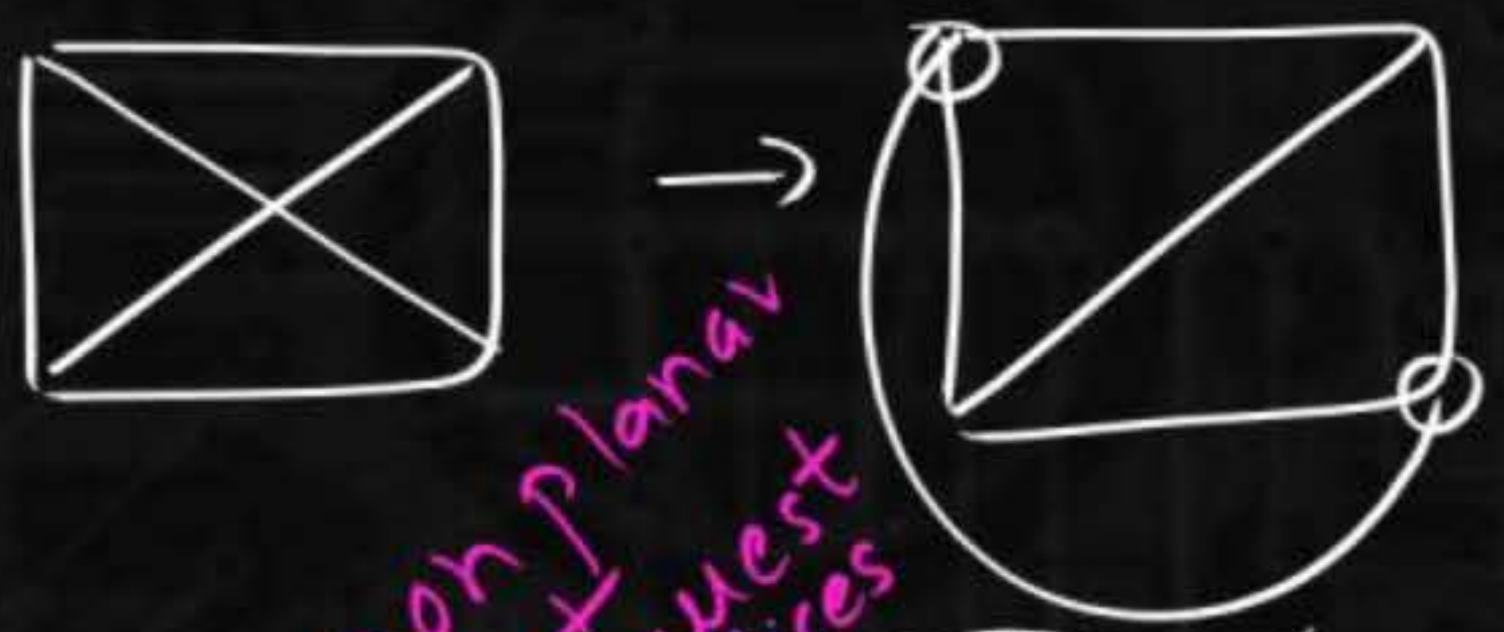
Kurowtski's first graph is non planar.

$n = 1$ ✓

$n = 2$ ✓

$n = 3$ ✓

$n = 4$ ✓



nonplanar + smallest edges

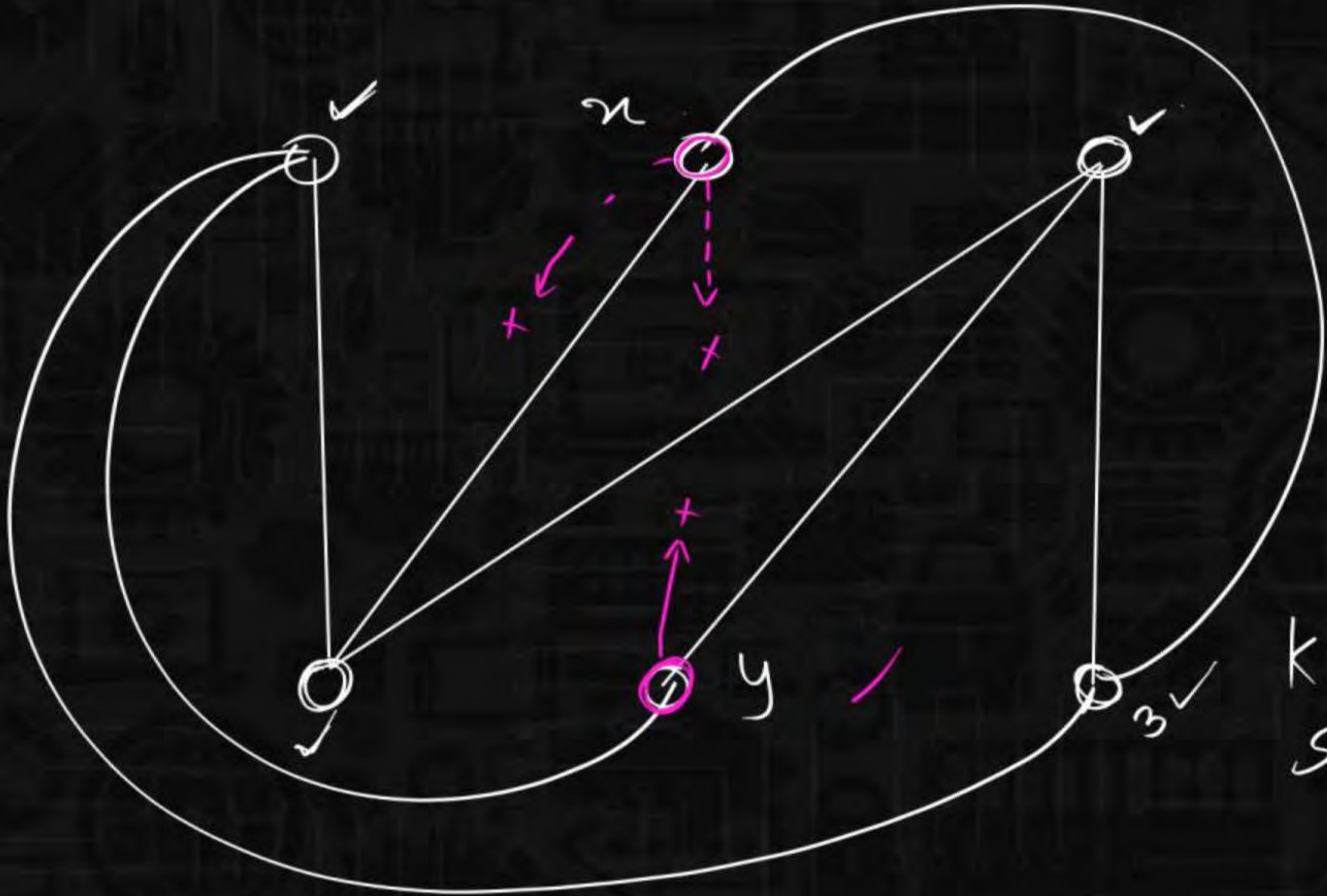
nonplanar + smallest vertices

$n = 5$ .... K_5 ✓ $\rightarrow$ nonplanar

$e = 10$

$n = 6$ .... $K_{3,3}$ ✓

$e = 9$



$K_{3,3}$

$K_{3,3}$ is non planar

Kurowtski's second graph is non planar

$\underline{\underline{K_5}}$ $\underline{K_{3,3}}$

Observations:

1. Both graphs are regular Graphs.

*** 2. 1st Graph (K_5) → nonplanar + smallest vertices.
   2nd Graph $(K_{3,3})$ → nonplanar + smallest edges.

3. if we remove single edge from both the graphs then both graphs will become planar.

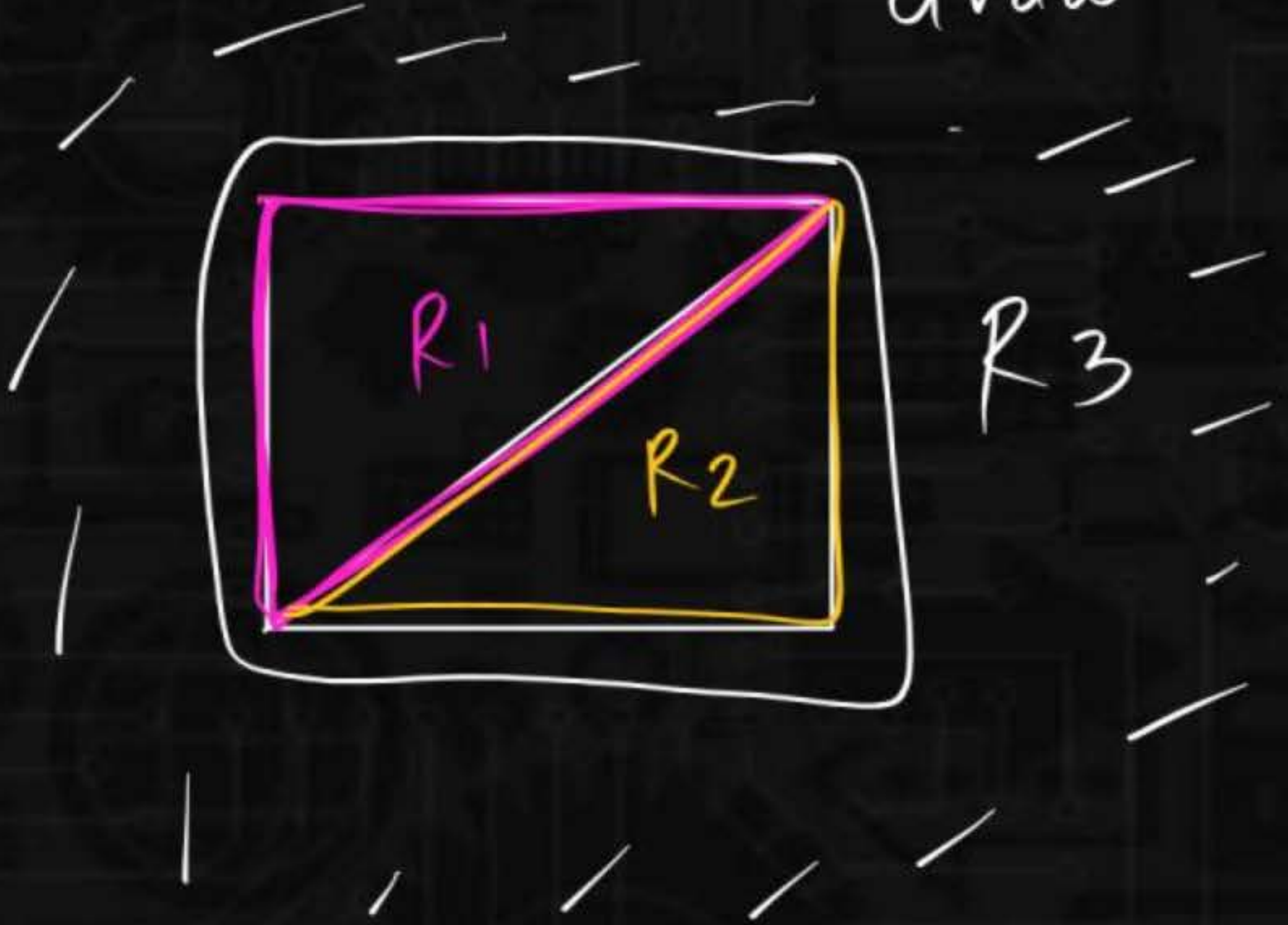
embedding

Planar Graph $\xrightarrow[\text{draw}]{\quad}$ plane $\rightarrow$ creates Regions (R)

OR

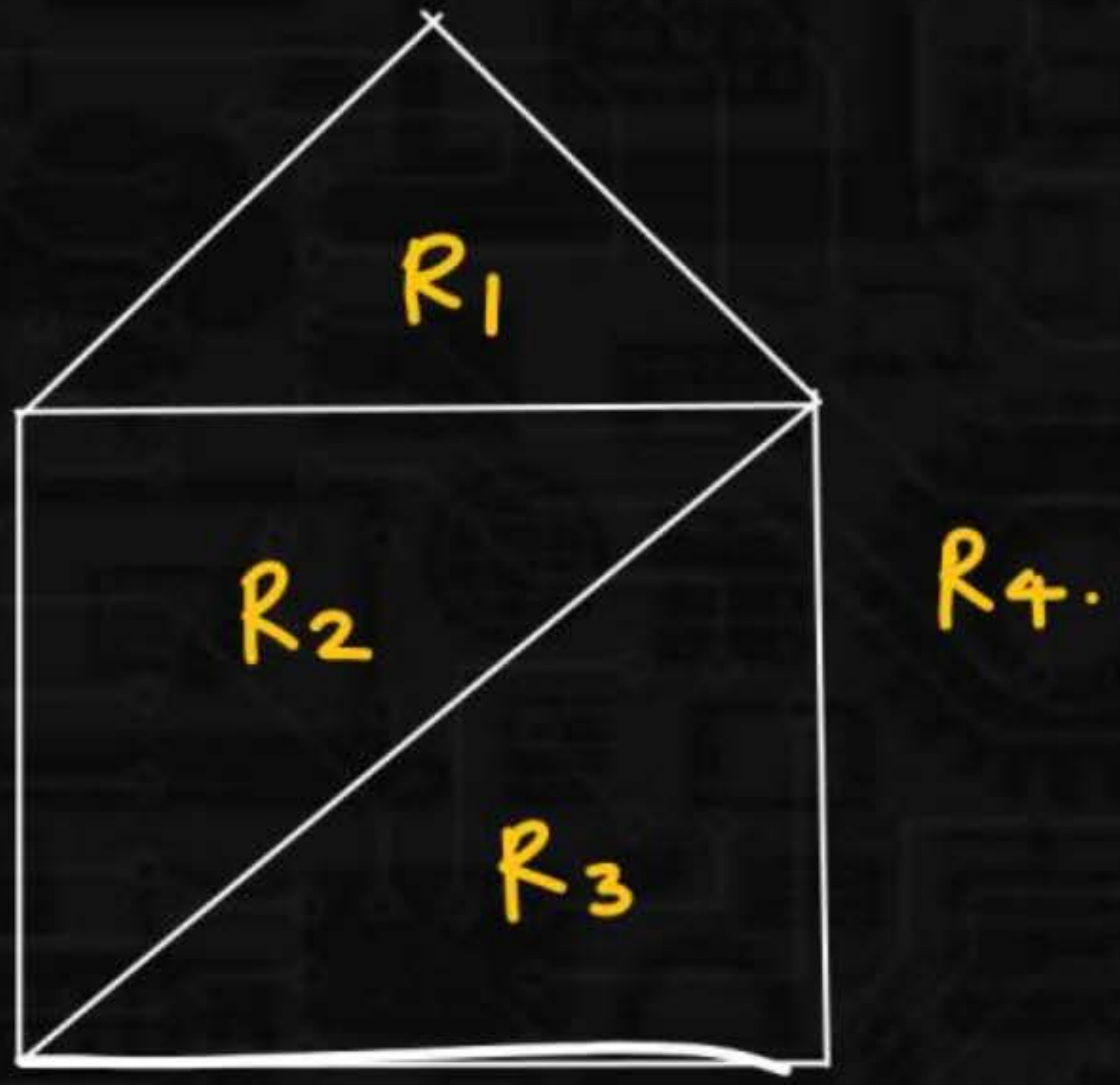
faces (f)

- finite (bounded): R_1, R_2
- Infinite (unbounded)



B.R : R_1

U.R : R_2



B.R = R_1, R_2, R_3

U.R = R_4.

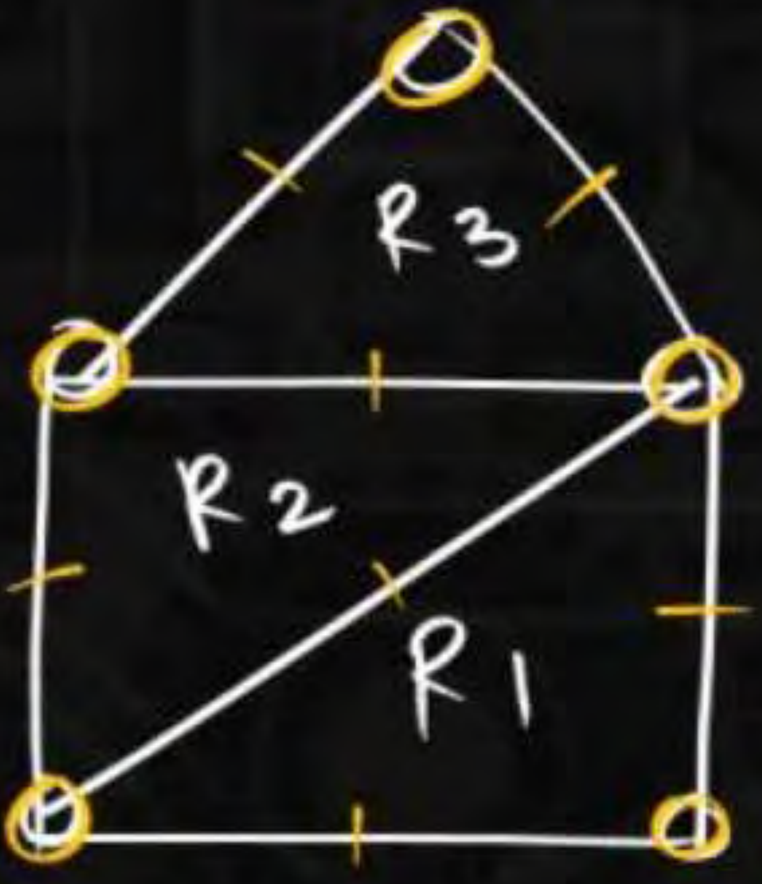
Euler's formula :

$$n - e + f = 2$$

n = Total vertices

e = Total edges

f = Total faces/Region



$$\begin{cases} n = 5 \\ e = 7 \end{cases}$$

$$n - e + f = 2$$

$$5 - 7 + f = 2$$

$$f = 4$$

Total no. of bounded Region

$= 4 - 1 = 3$

$n = 2$

$e = 1$

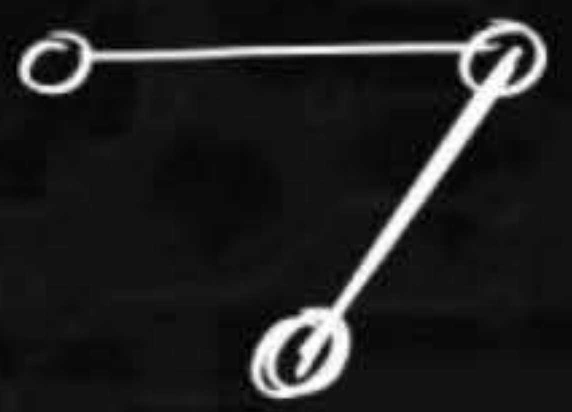
$n - e + f = 2$

$2 - 1 + f = 2$

$1 + f = 2$

$f = 1$

Case1:



$n\uparrow$

$e\uparrow$

✓

$n = 3$

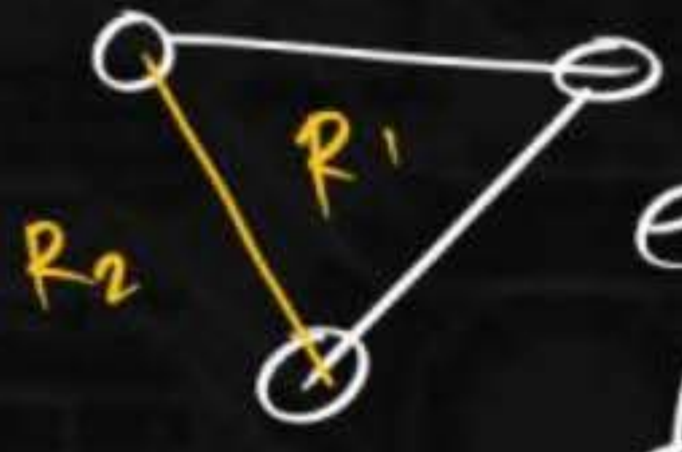
$e = 2$

$n - e + f = 2$

$3 - 2 + f = 2$

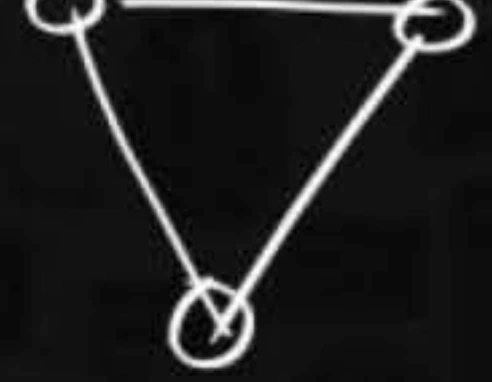
$f = 1$

Case2:



$e\uparrow$

$f\uparrow$



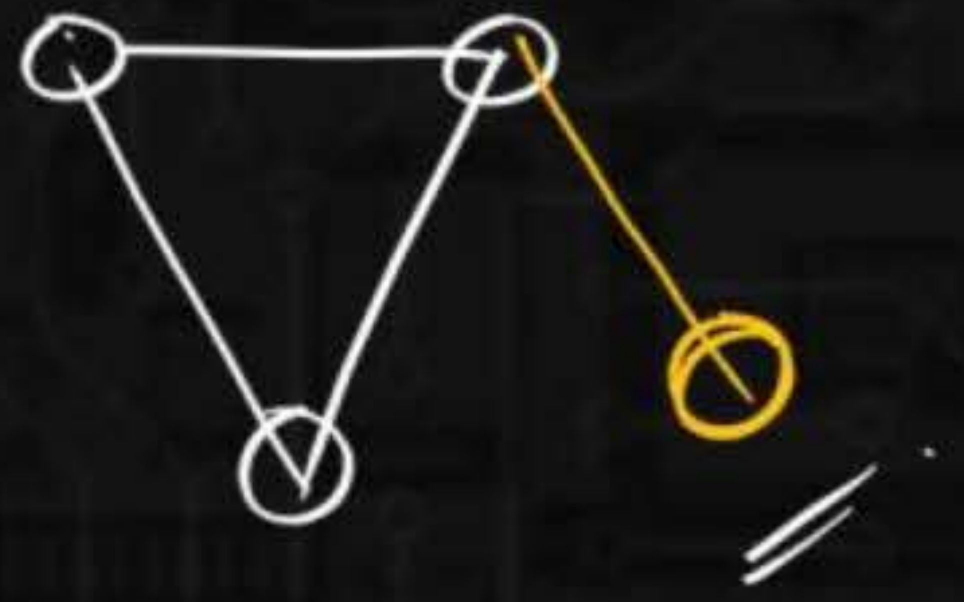
$n = 3$

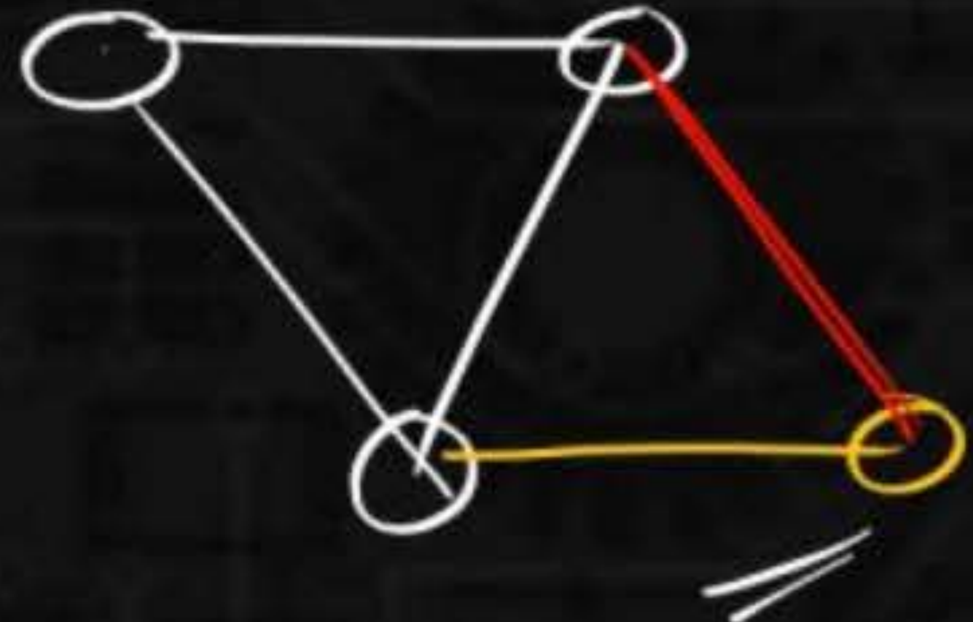
$e = 3$

$n - e + f = 2$

$3 - 3 + f = 2$

$f = 2$





$n - e + f = 2$

$n\uparrow \; e\uparrow$

$e\uparrow \; f\uparrow$

$n\uparrow$

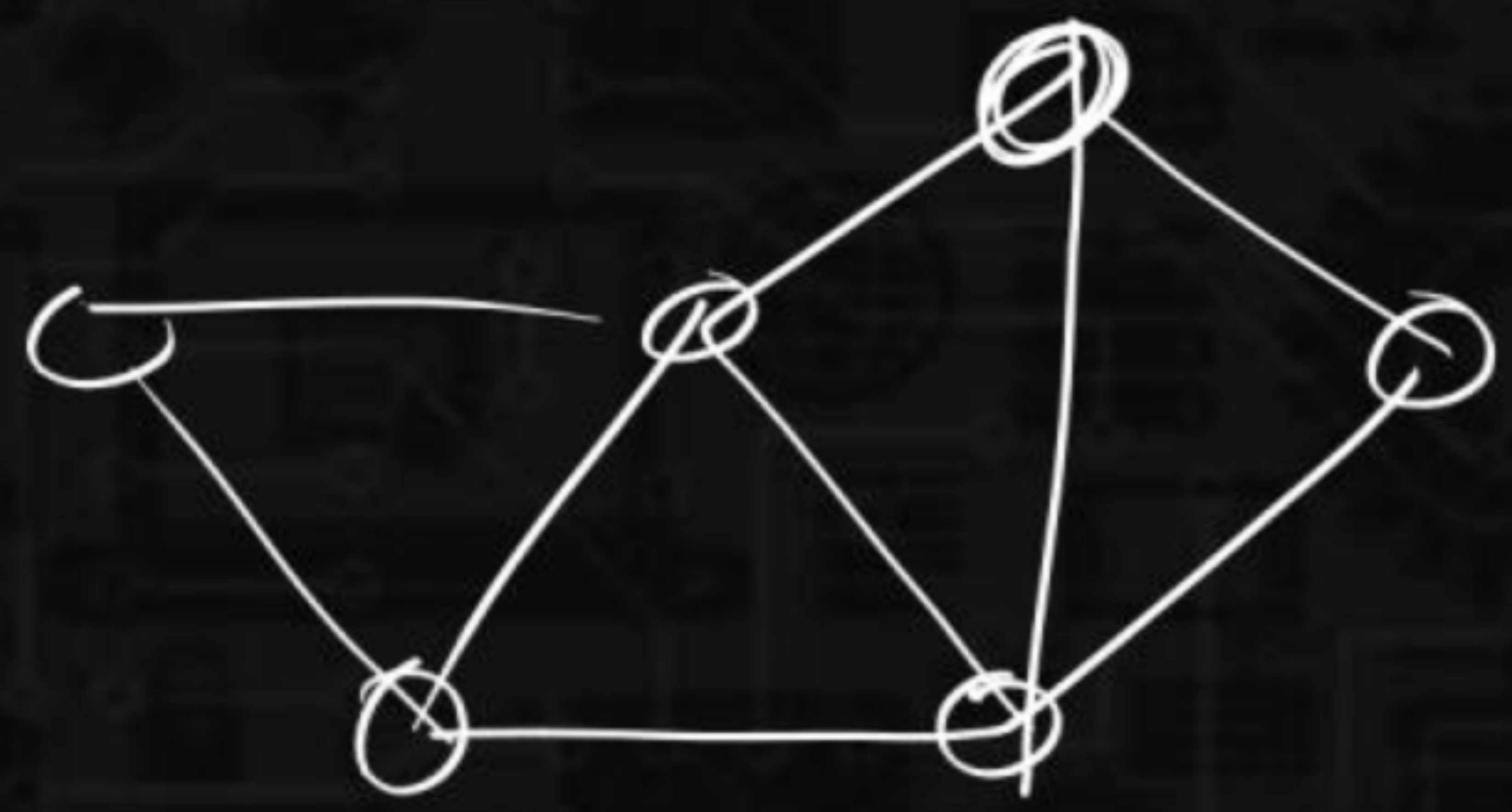
$e\uparrow$

$n - e + f = 2$ ✓

$n - e + f = 2$.

$e\uparrow$

$f\uparrow$




THANK YOU GW
SOLDIERS !