CS & IT ENGINEERING

Connectivity in Graphs part 3

Lecture No. 8

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TOPICS TO BE COVERED

01 Euler Graph

02 Theorems in Euler Graph

03 Hamiltonian Graph

04 Theorems in Hamiltonian Graph

05 Subgraph

Connectivity in Graphs

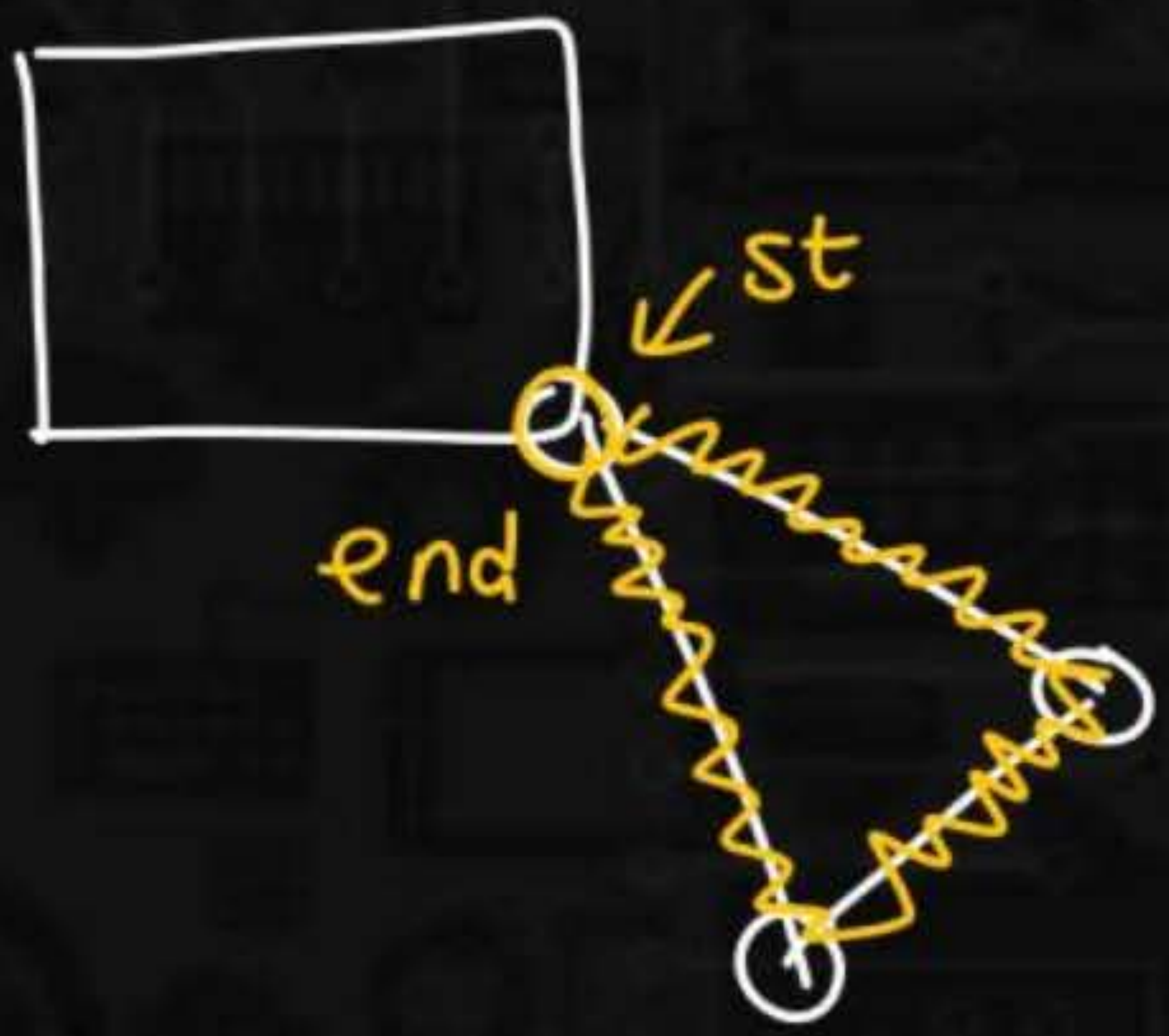
Trail :
alternating sequences of vertices & edges (R.V | R$\times$E)

Closed Trail : Trail + starting = ending vertices.

Euler cycle : Closed Trail + all edges should be covered

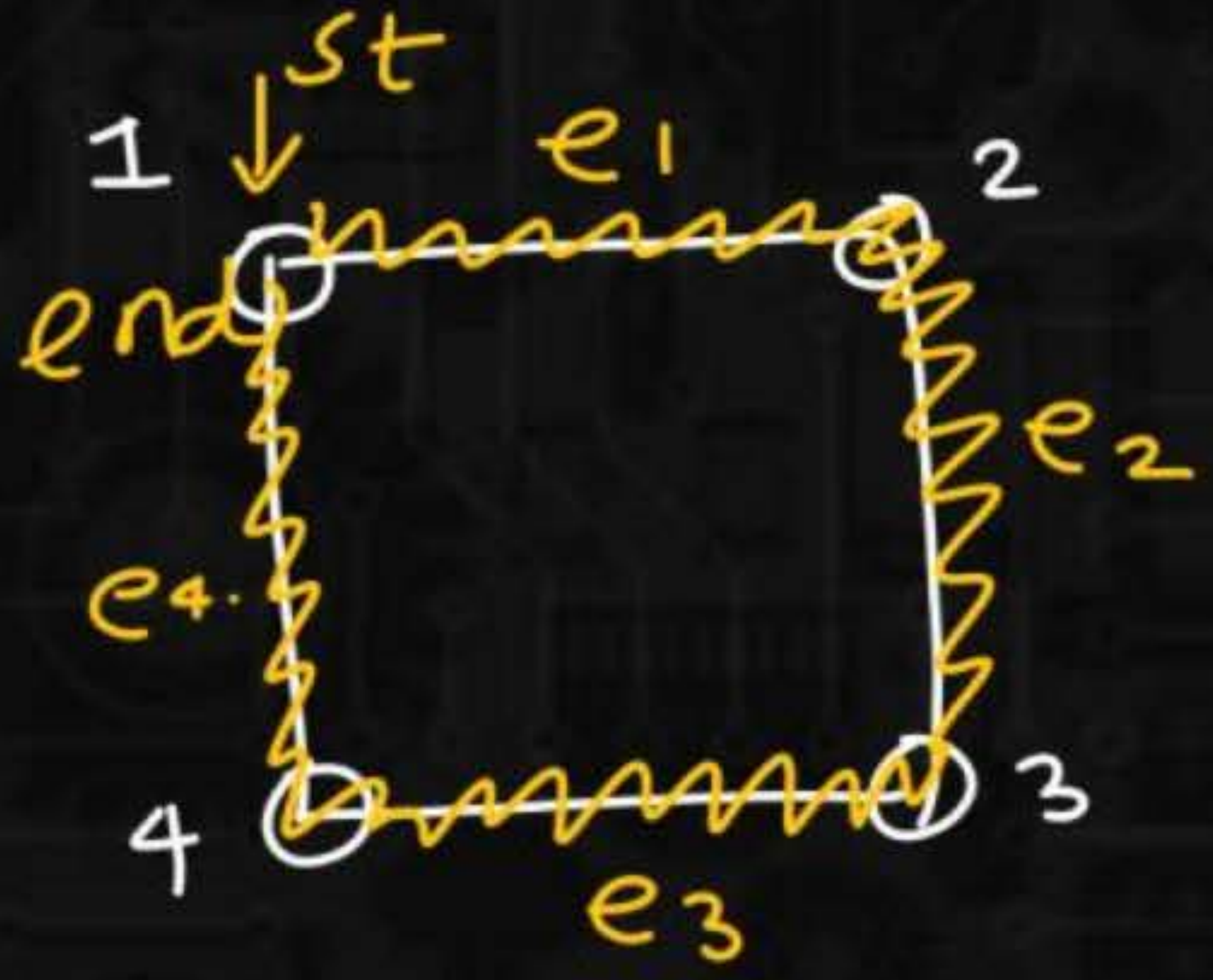
Connectivity in Graphs

closed Trail



Euler cycle: closed Trail + all edges.

Connectivity in Graphs



Euler Graph:

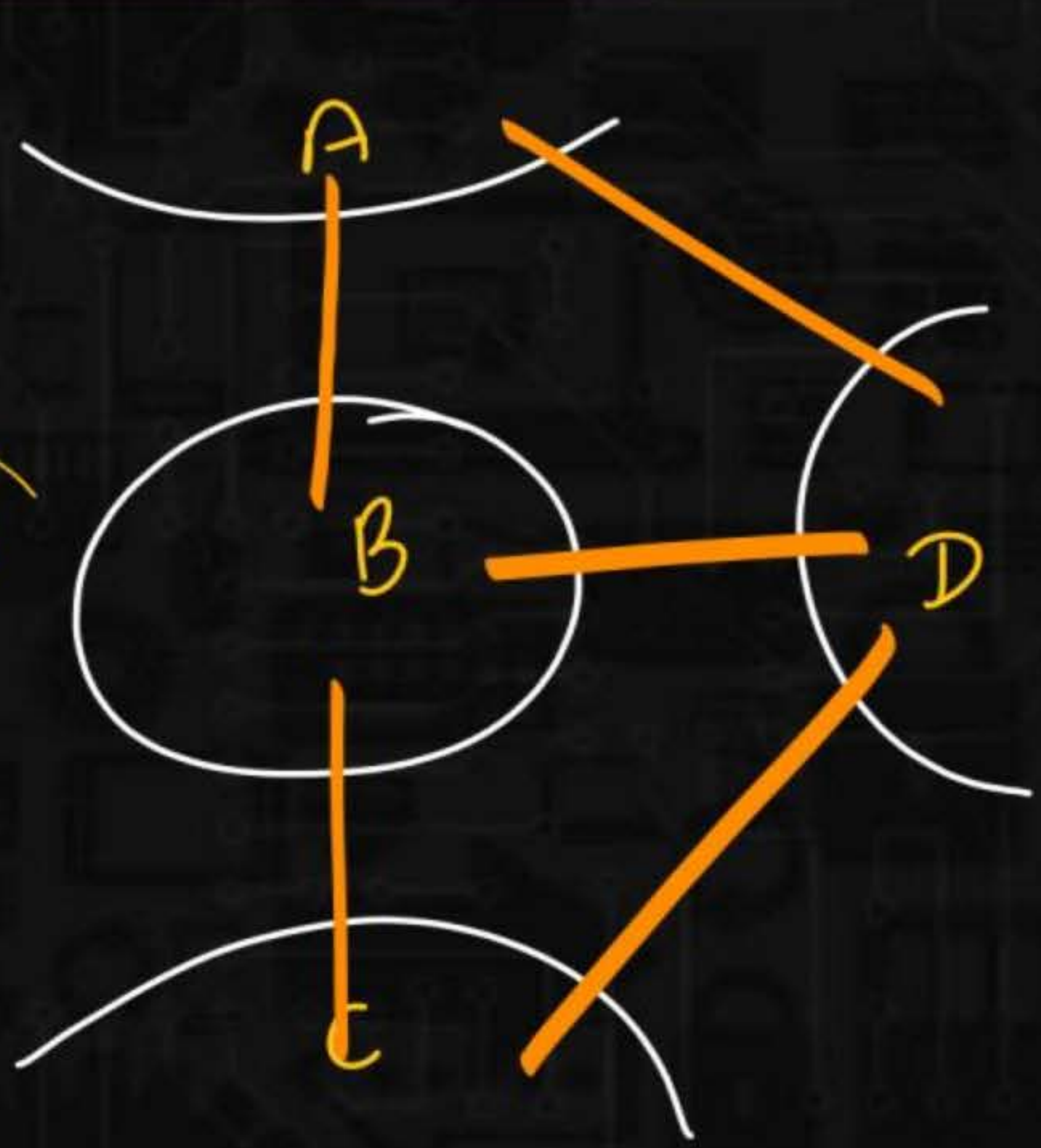
Graph contains euler cycle is called Euler Graph.

→ $1\ e_1\ 2\ e_2\ 3\ e_3\ 4\ e_4\ 1$ (euler cycle)

→ $G = (V, E)$

Connectivity in Graphs

E.C is not possible.



1 → A (closed Trail)

2 → all bridges exactly once (all edges)

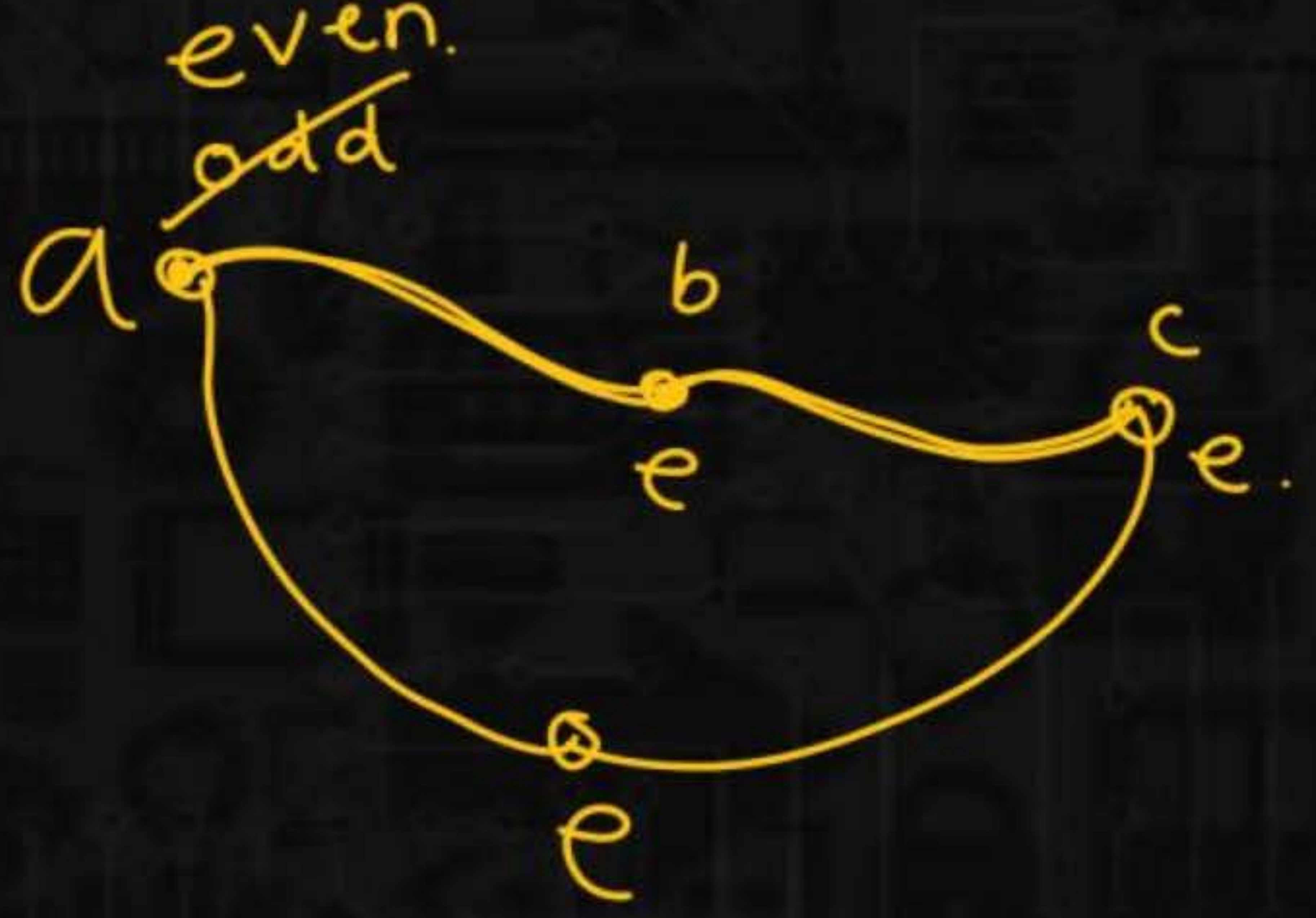
Connectivity in Graphs

Connectivity in Graphs

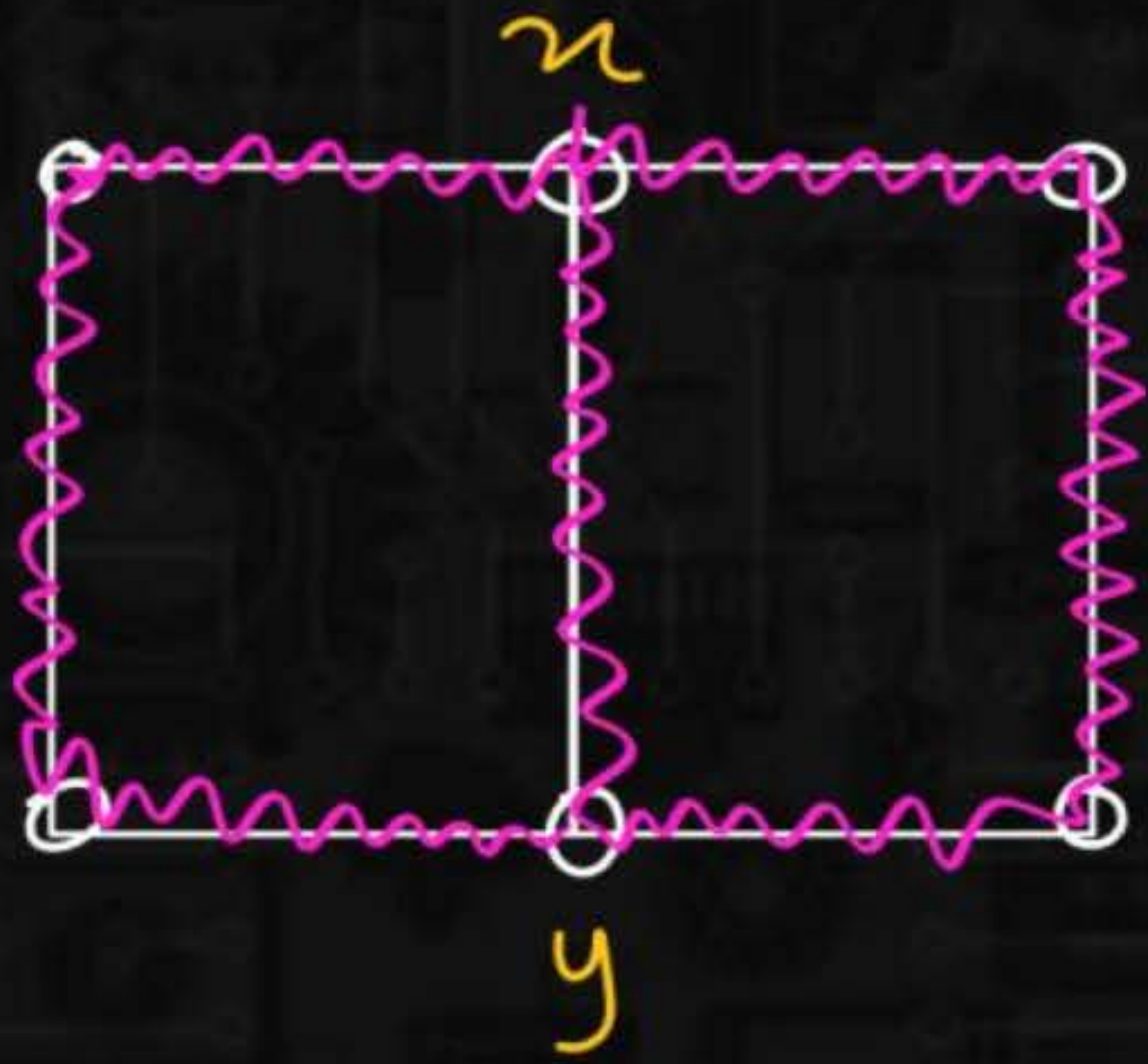
Thm :

Graph contains euler Graph iff degrees of all vertices are even.

(connected)



Connectivity in Graphs



1. closed. (×)
2. all edges covered. ✓

Trail

Open Trail:

Euler line $\left\{\begin{array}{l}\text{Open Trail} \\ + \\ \text{all edges in the graph.}\end{array}\right.$

Connectivity in Graphs

covering all edges

Closed Trail

↓

Euler cycle *

Euler Graph.

Degrees of all vertices even.

Open Trail.

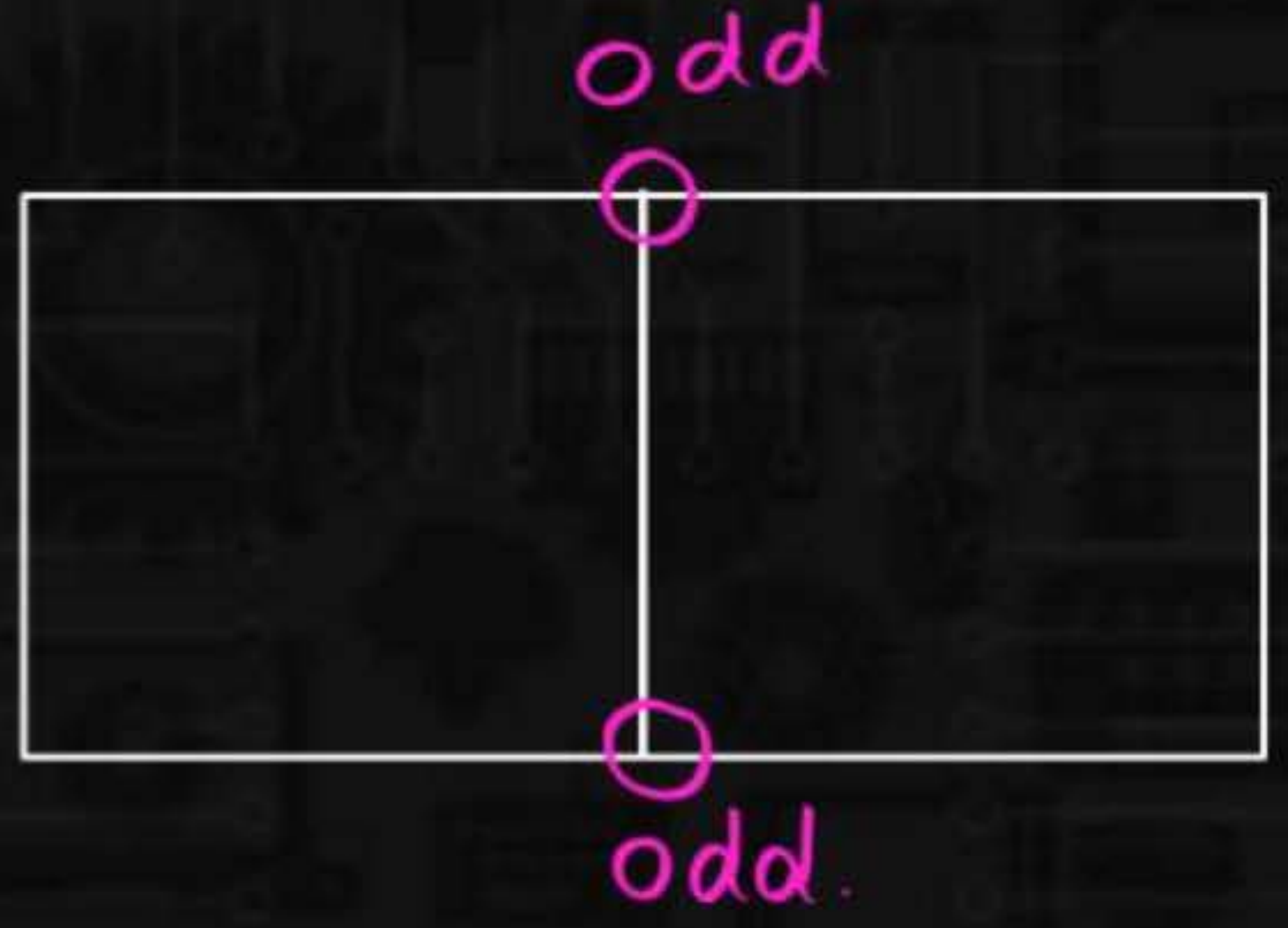
↓

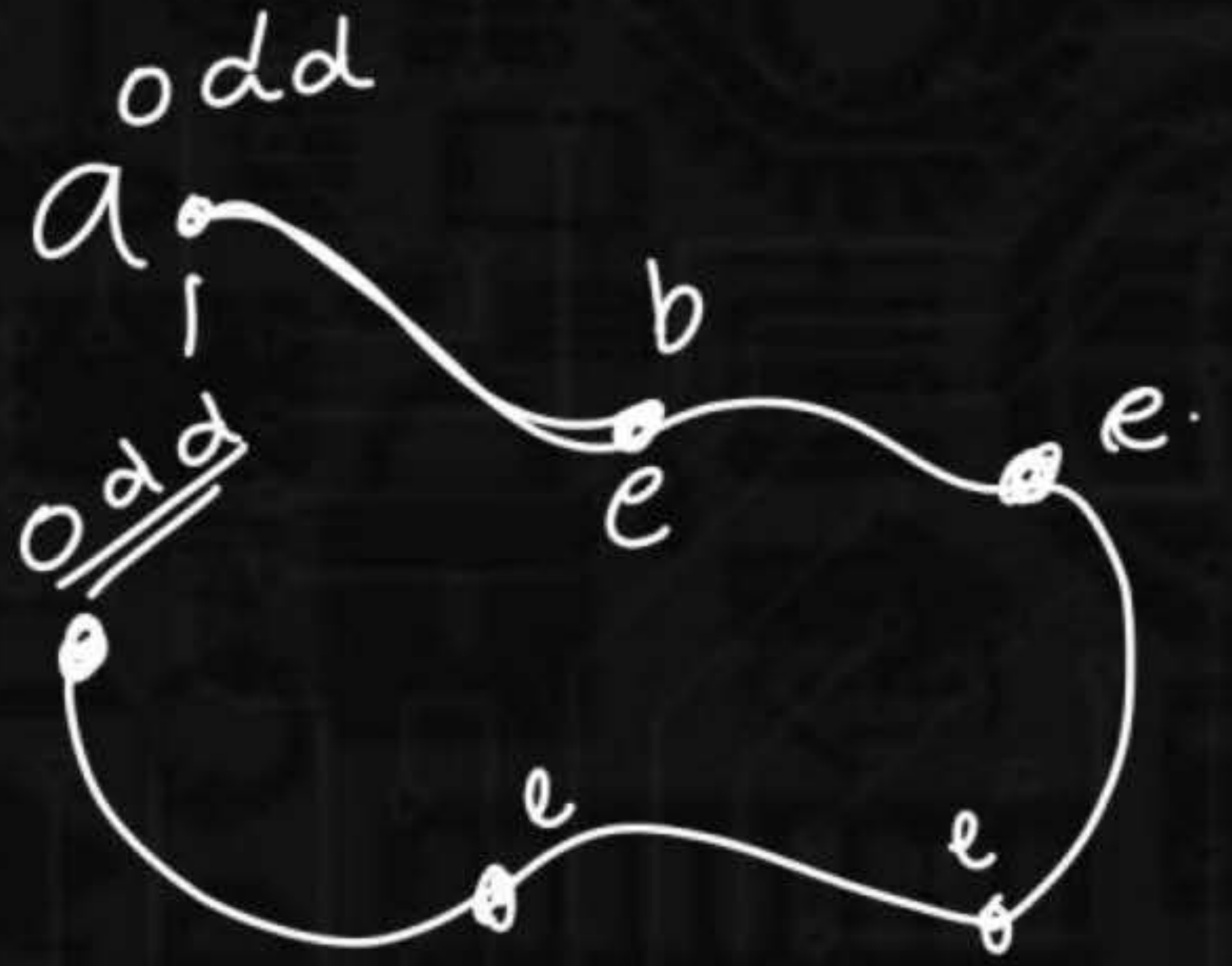
euler line/Euler path.

exactly 2 odd degree vertices.

Connectivity in Graphs

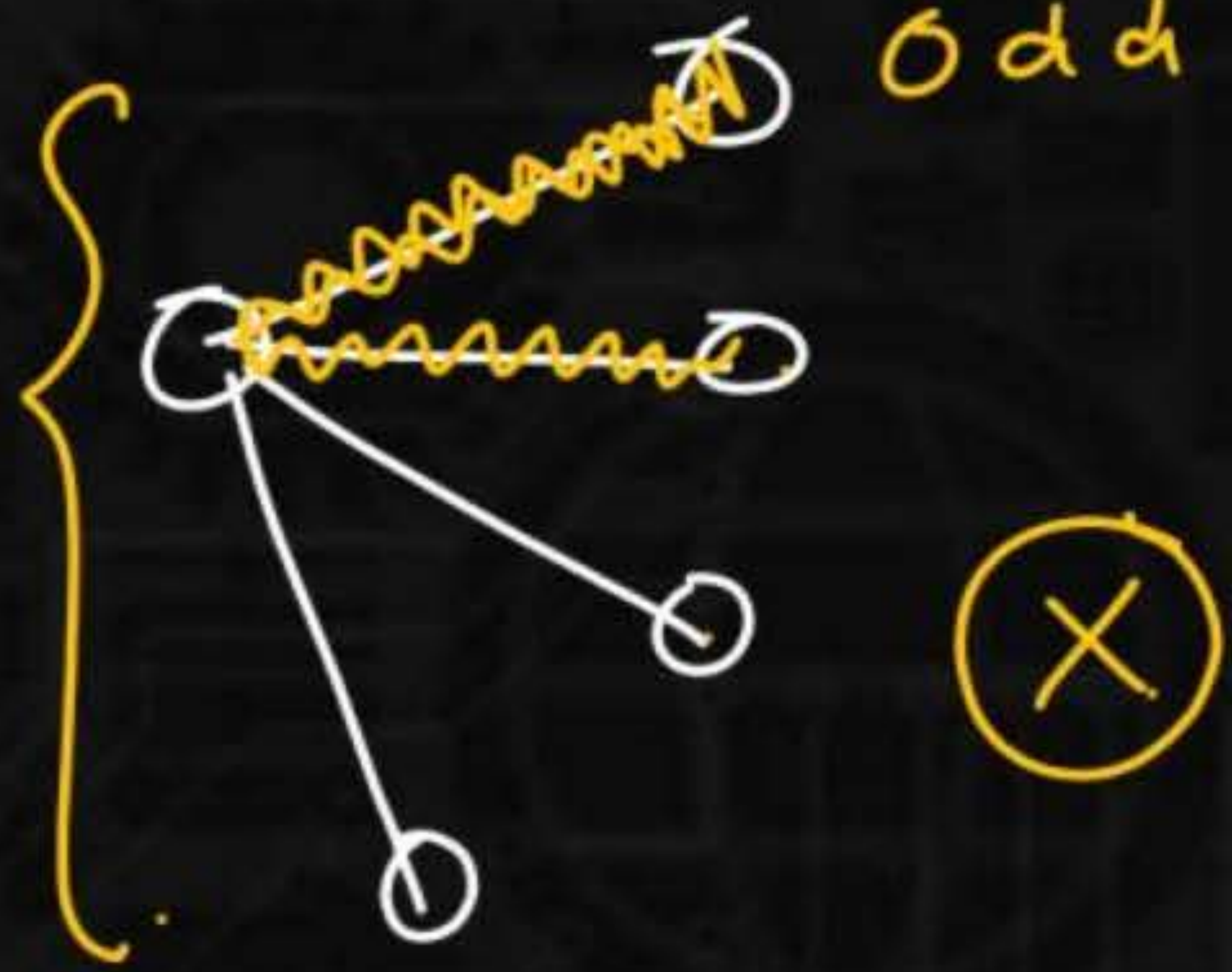
<u>Thm</u>: Graph contains euler path iff it contains exactly 2 odd degree vertices.





Connectivity in Graphs

all edges + open → euler path.



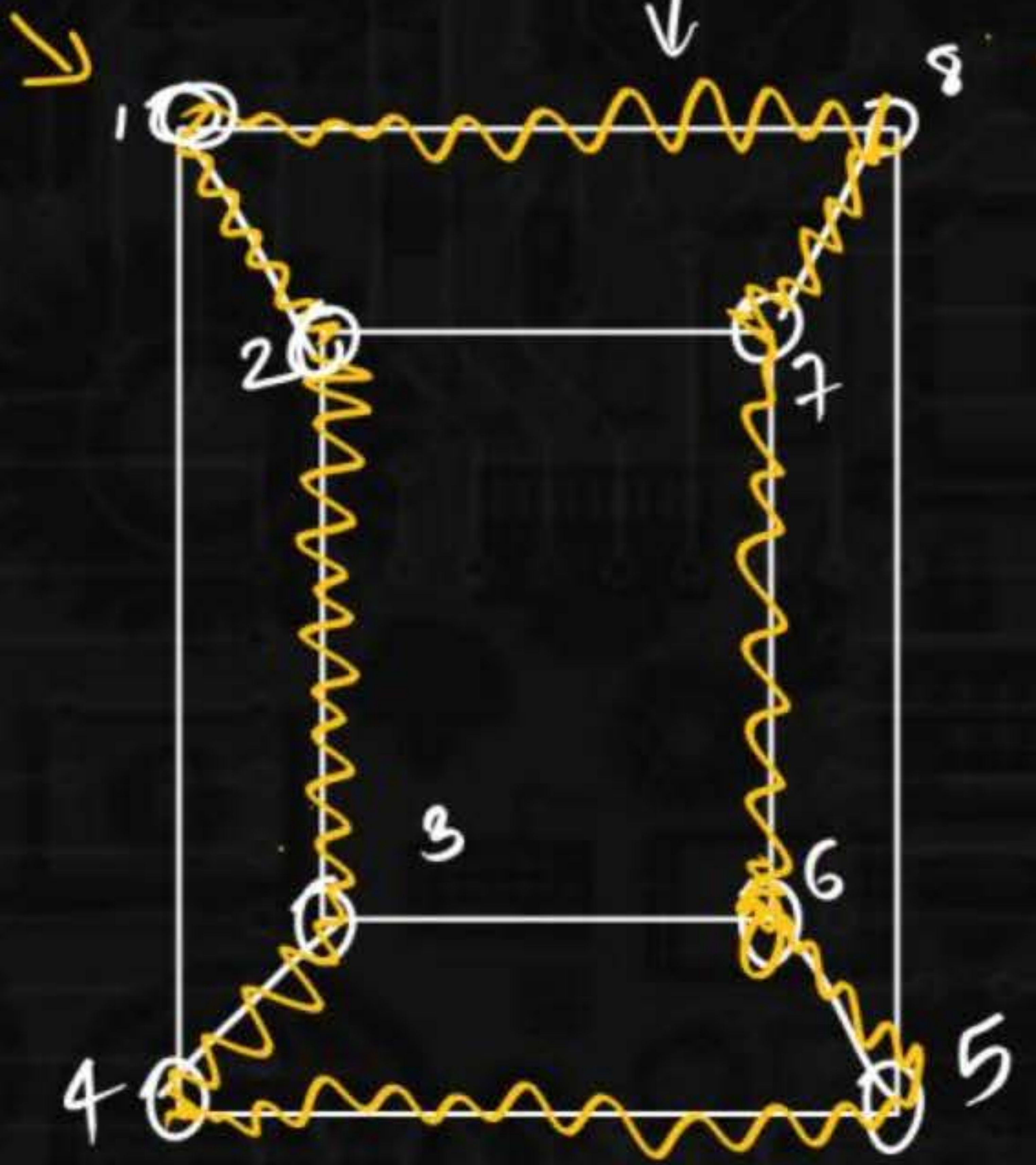
Connectivity in Graphs

Path :

Closed Path : Path + st = ending vertex.

Hamiltonian cycle : Closed Path + all vertices should be covered

Connectivity in Graphs



closed ✓
all vertices are covered ✓

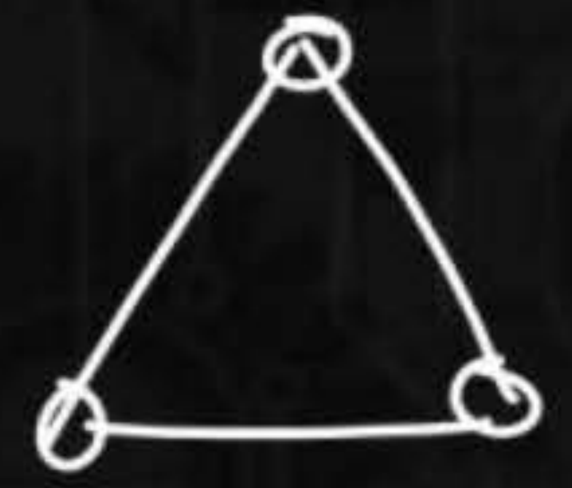
Hamiltonian cycle

Graph contains H.C then Hamiltonian Graph.

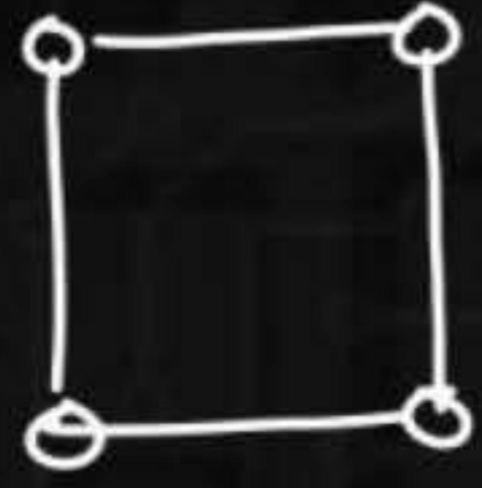
1—2—3—4—5—6—7—8—1.

Connectivity in Graphs

C_n :



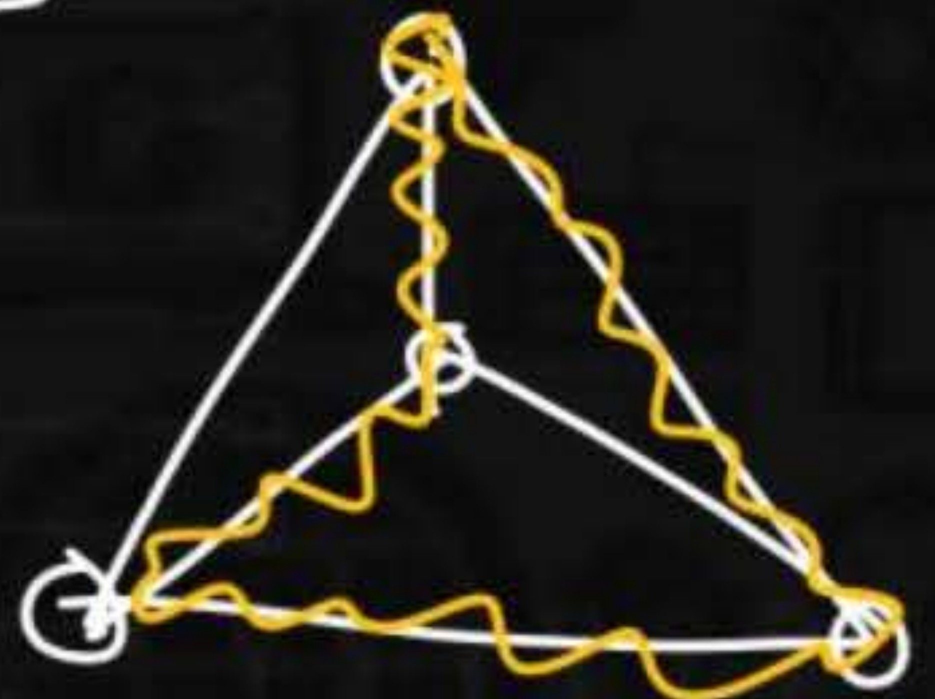
C_3

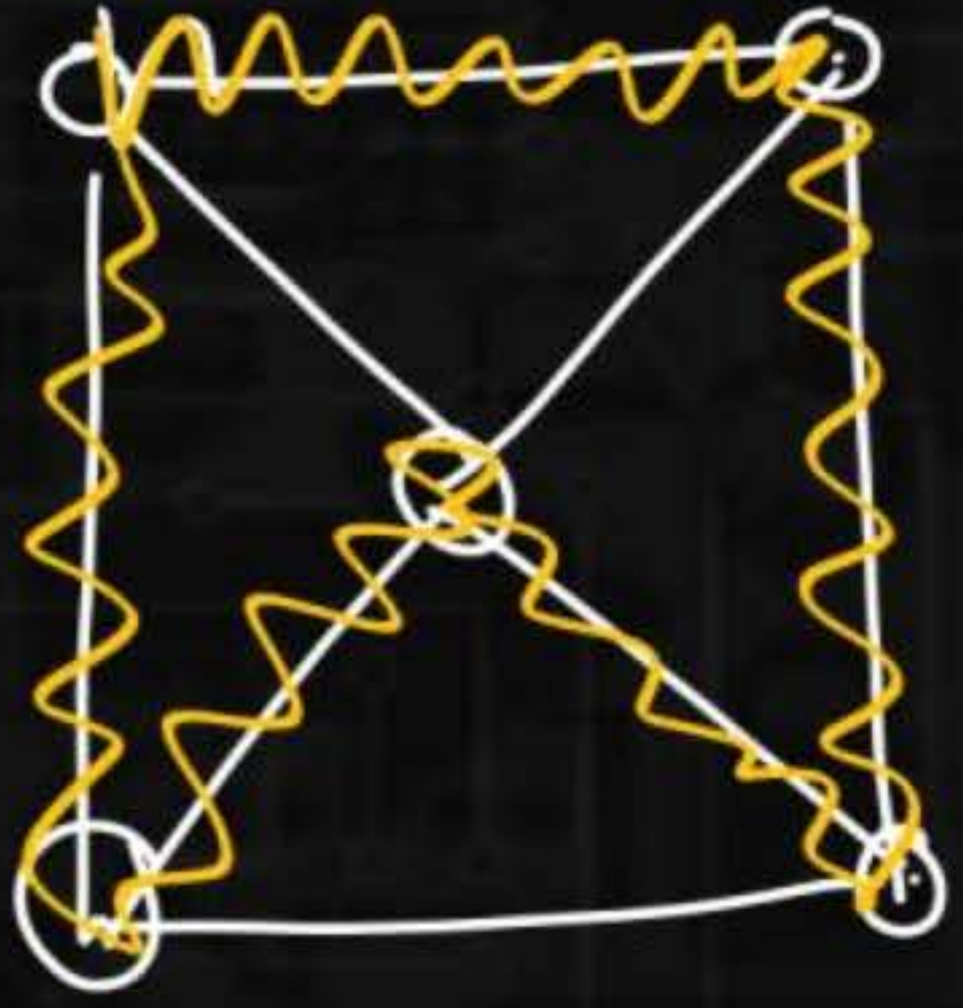


C_4.

* Every C_n is always Hamiltonian as well as Euler Graph.

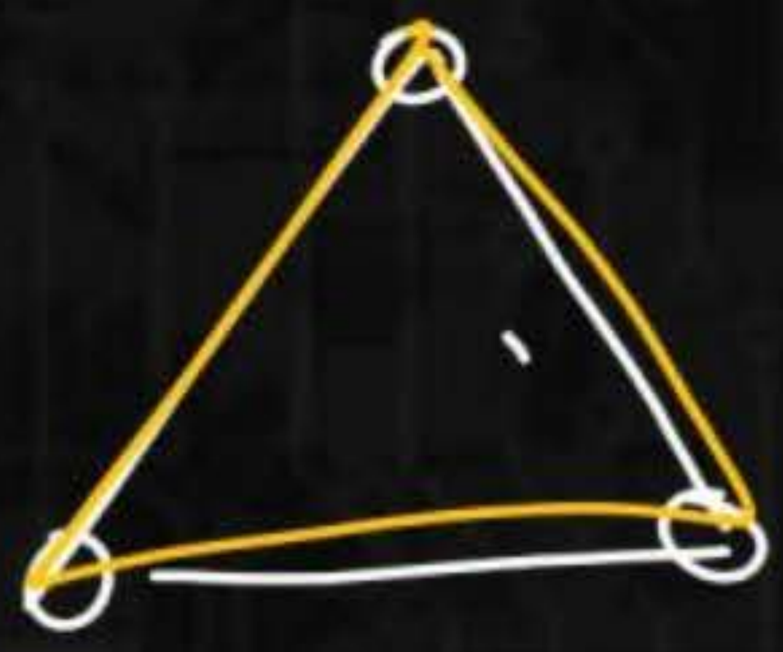
W_n. 3



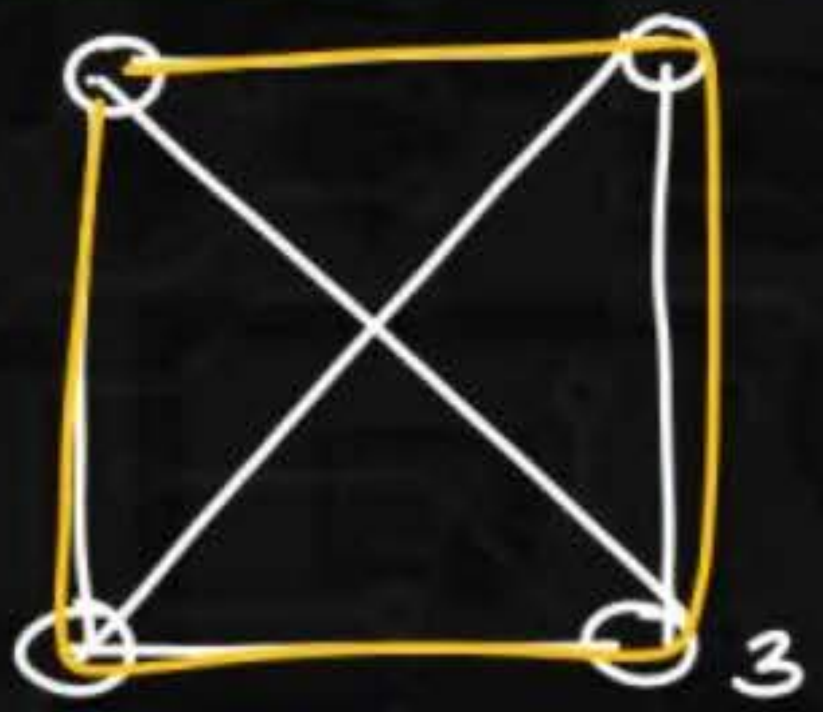


Every wheel Graph is Hamiltonian but not euler.

Connectivity in Graphs



K_3



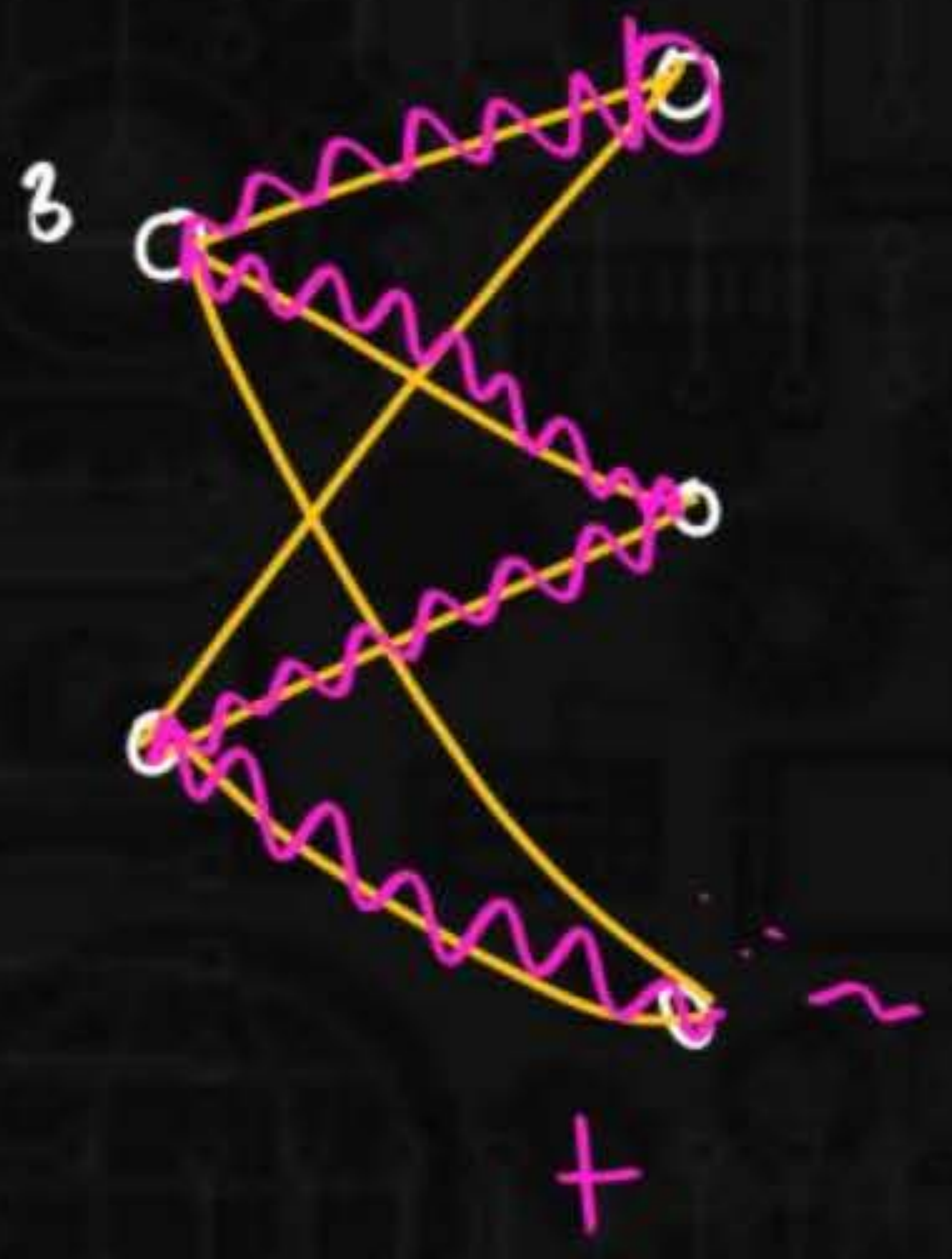
K_4

* every K_n is Hamiltonian
($n \geq 3$)

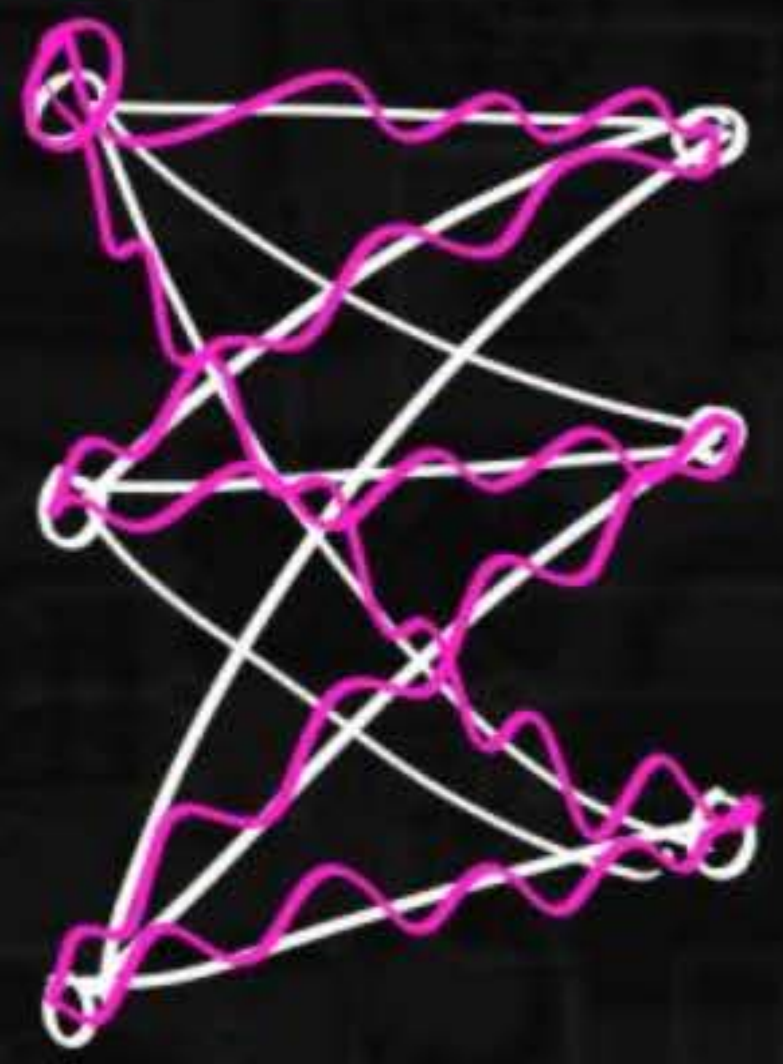
* every K_n is euler Graph.
$n \to$ odd ($n \geq 3$)

Connectivity in Graphs

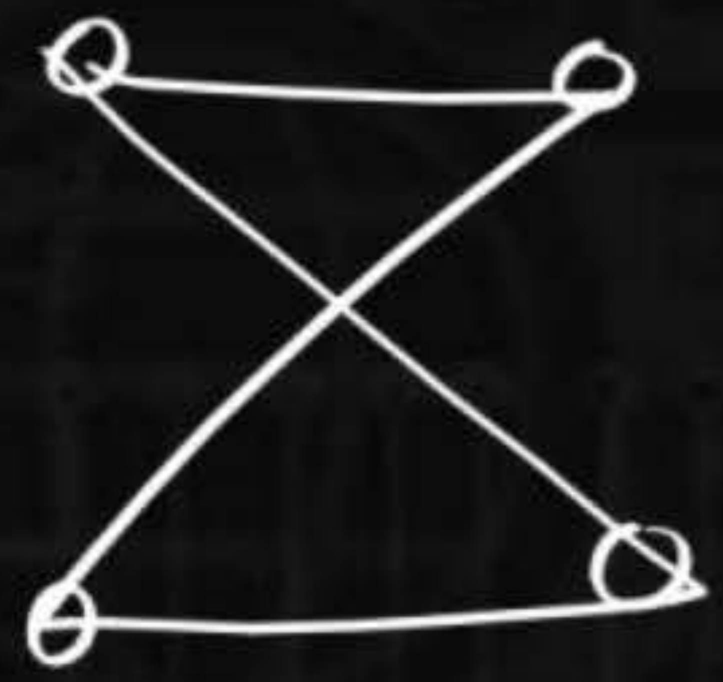
$K_{2,3}$ H.C × E.C ×



$K_{3,3}$ E.C × H.C ✓

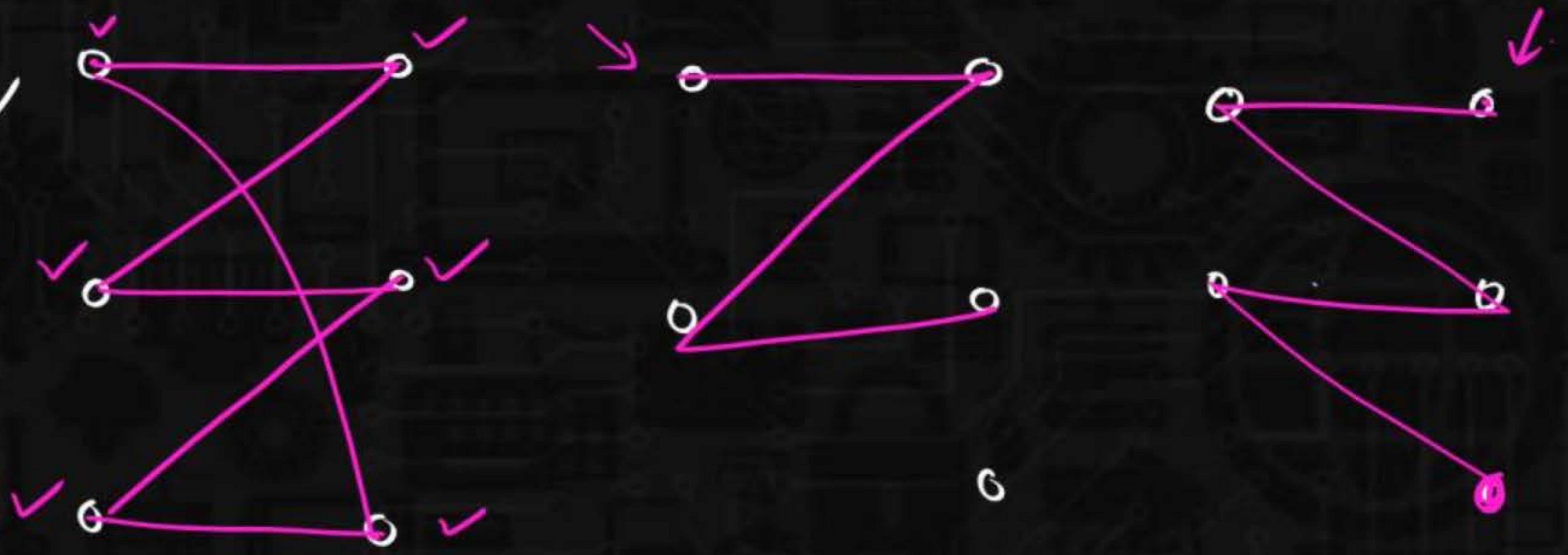


$K_{2,2}$ E.C ✓ H.C ✓



Connectivity in Graphs

H.C ✓



Connectivity in Graphs

$K_{n,n}$ H.C ✓ $(n \geq 2)$

✓ E.C (n is even)

$(n \geq 2)$

C_n E.C ✓

H.C ✓

W_n E.C ✗

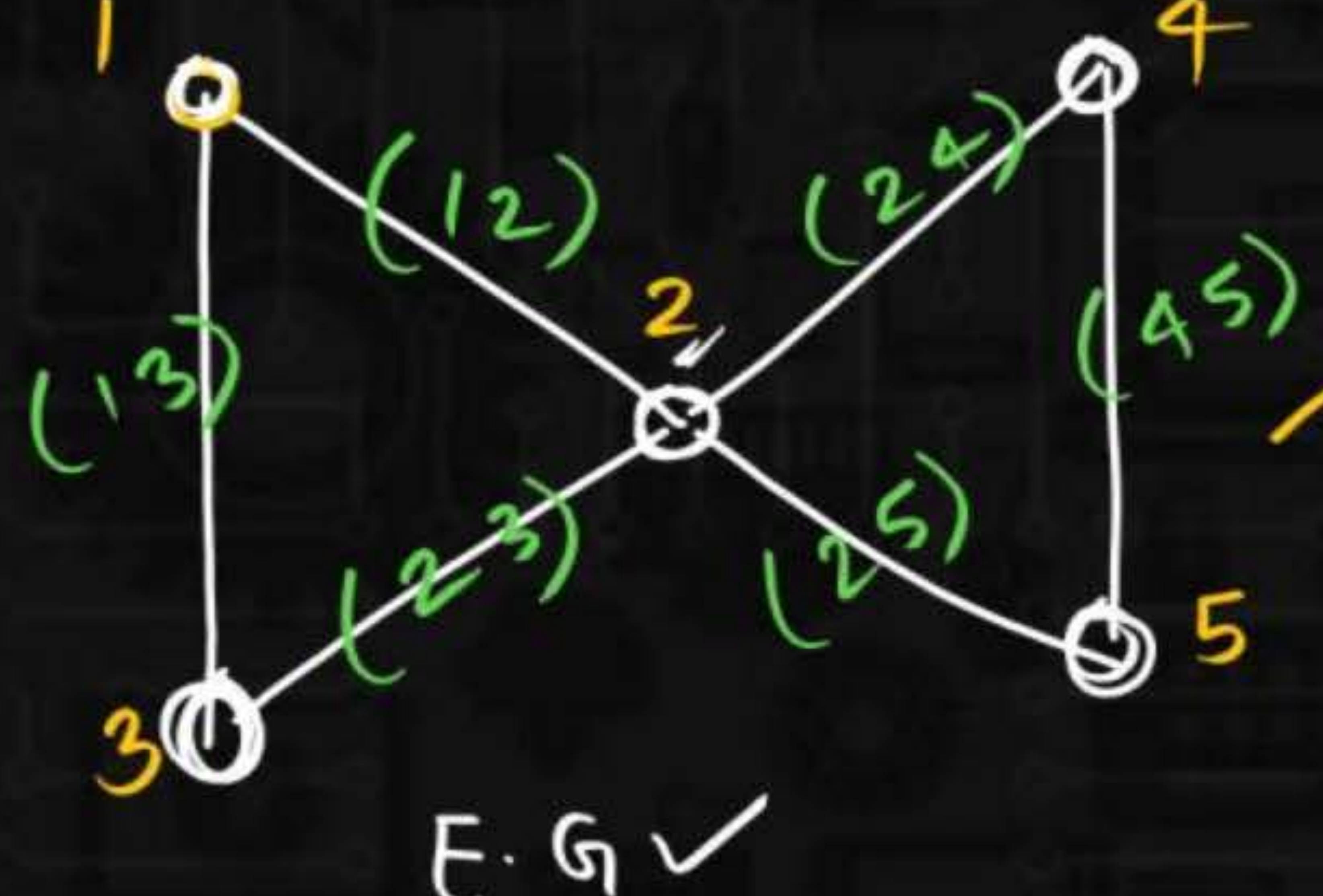
H.C ✓

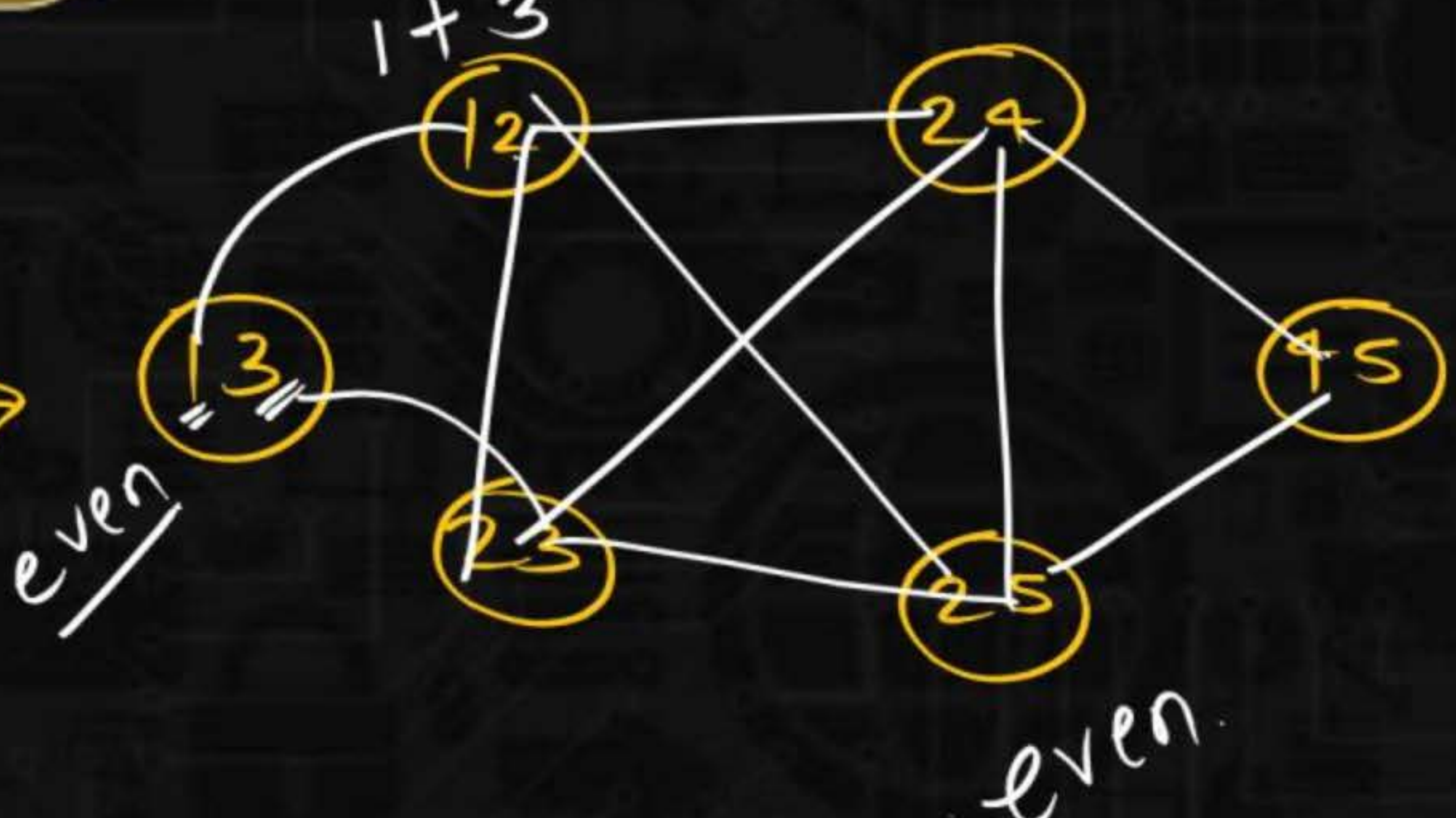
K_n H.C ✓ $(n \geq 3)$

E.C ✓ (n is odd)

$(n \geq 3)$

Connectivity in Graphs





Connectivity in Graphs

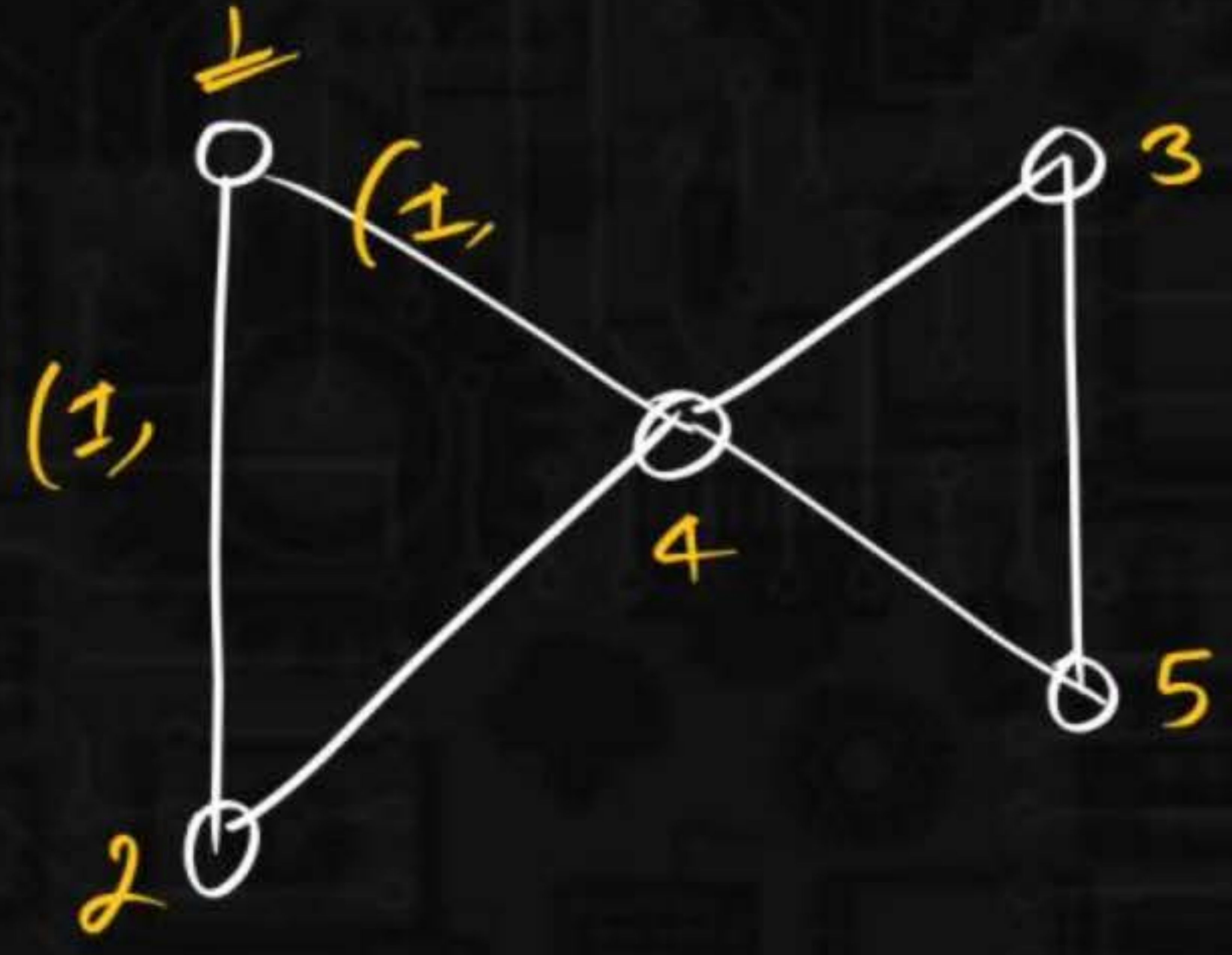
E.G ✓

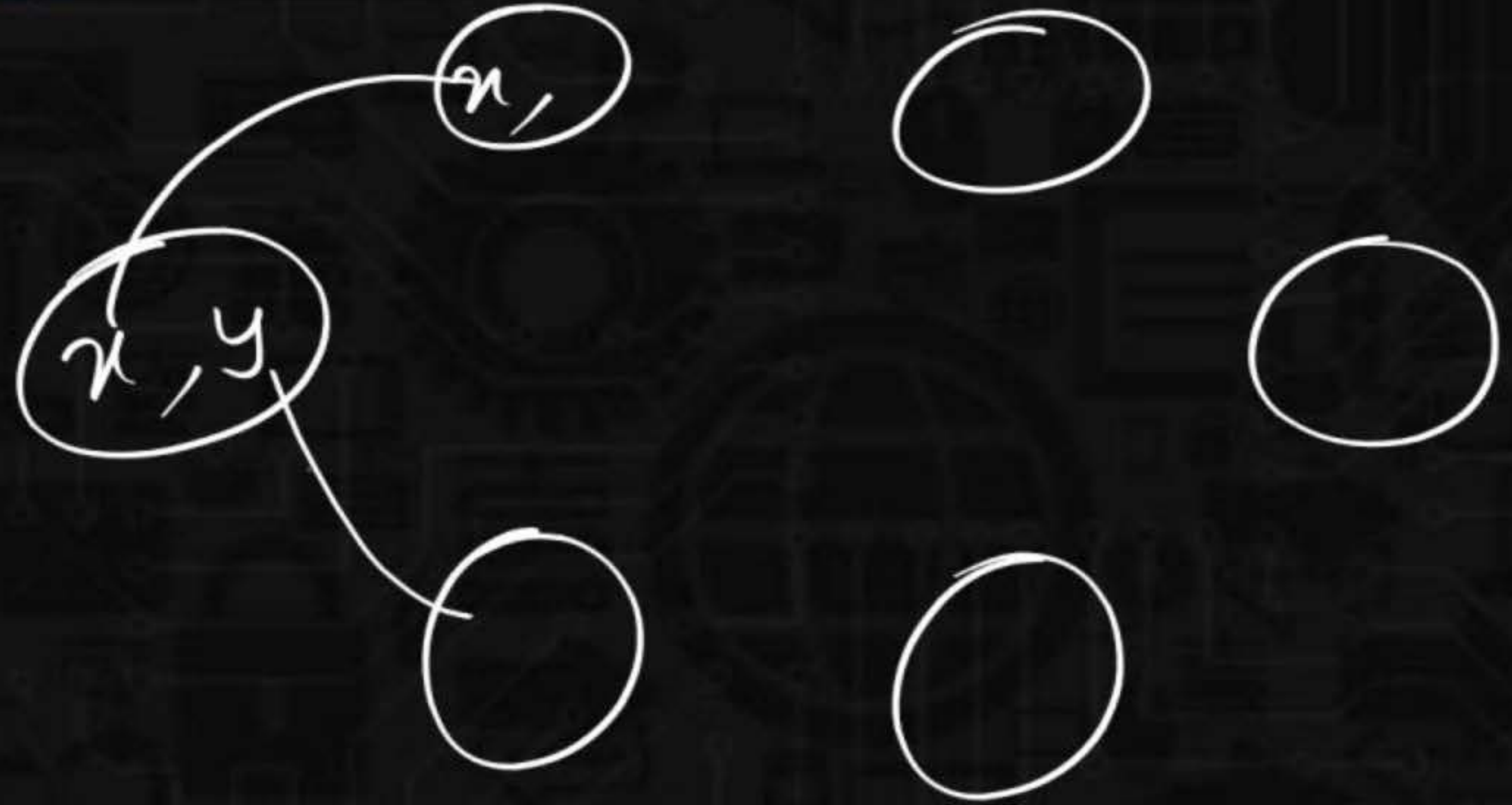
↓

all edges.

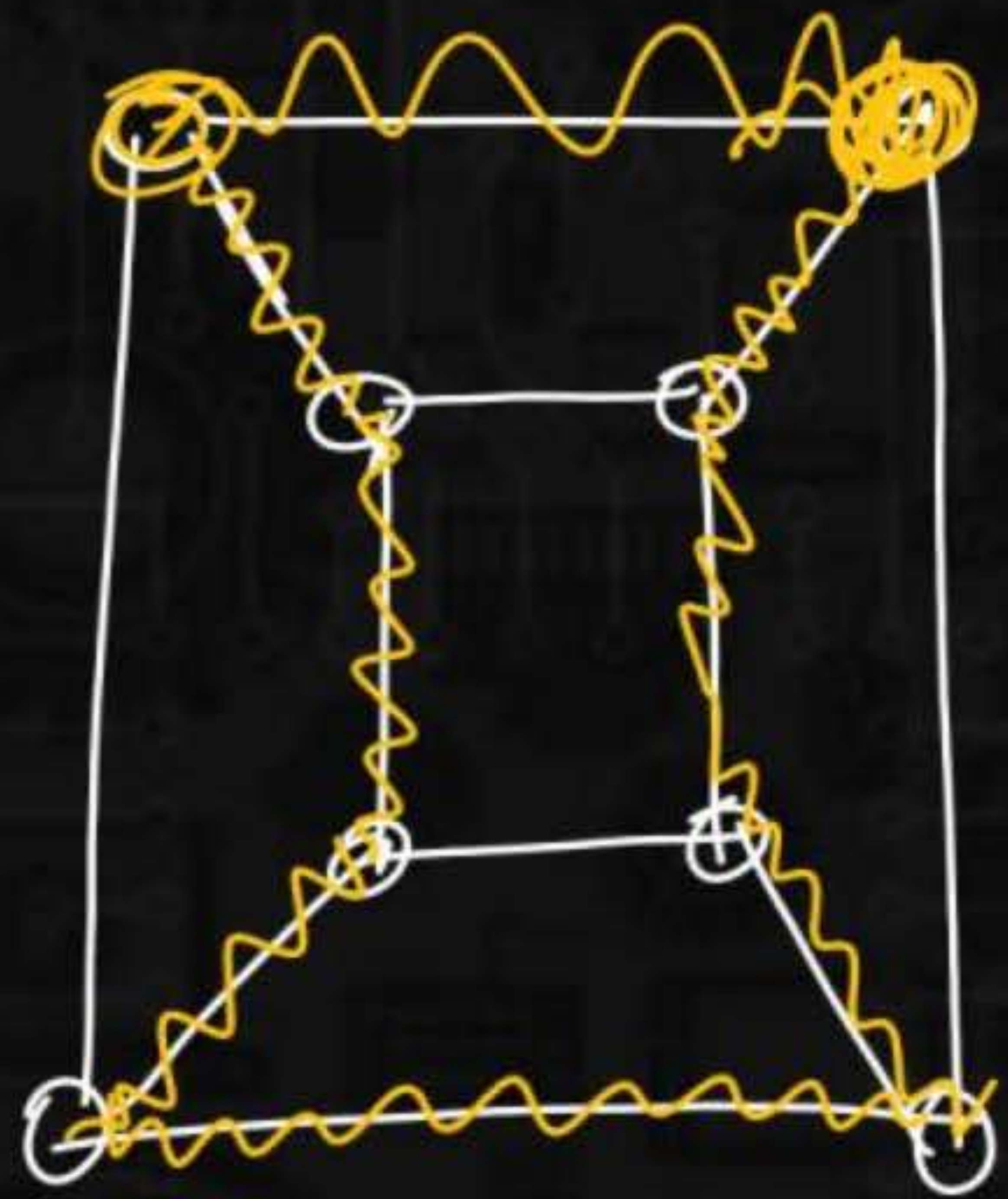
** Line Graph of Euler in Euler as well as Hamiltonian Graph.

Connectivity in Graphs





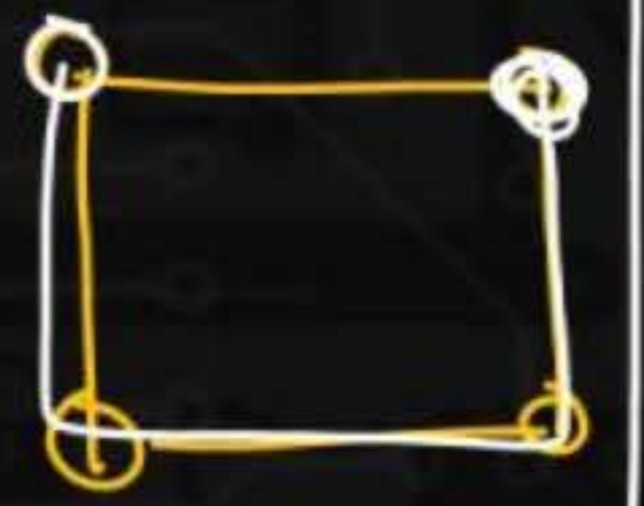
Connectivity in Graphs



* open path + covers all the vertices

Hamiltonian Path.

Every H.C contains H.P



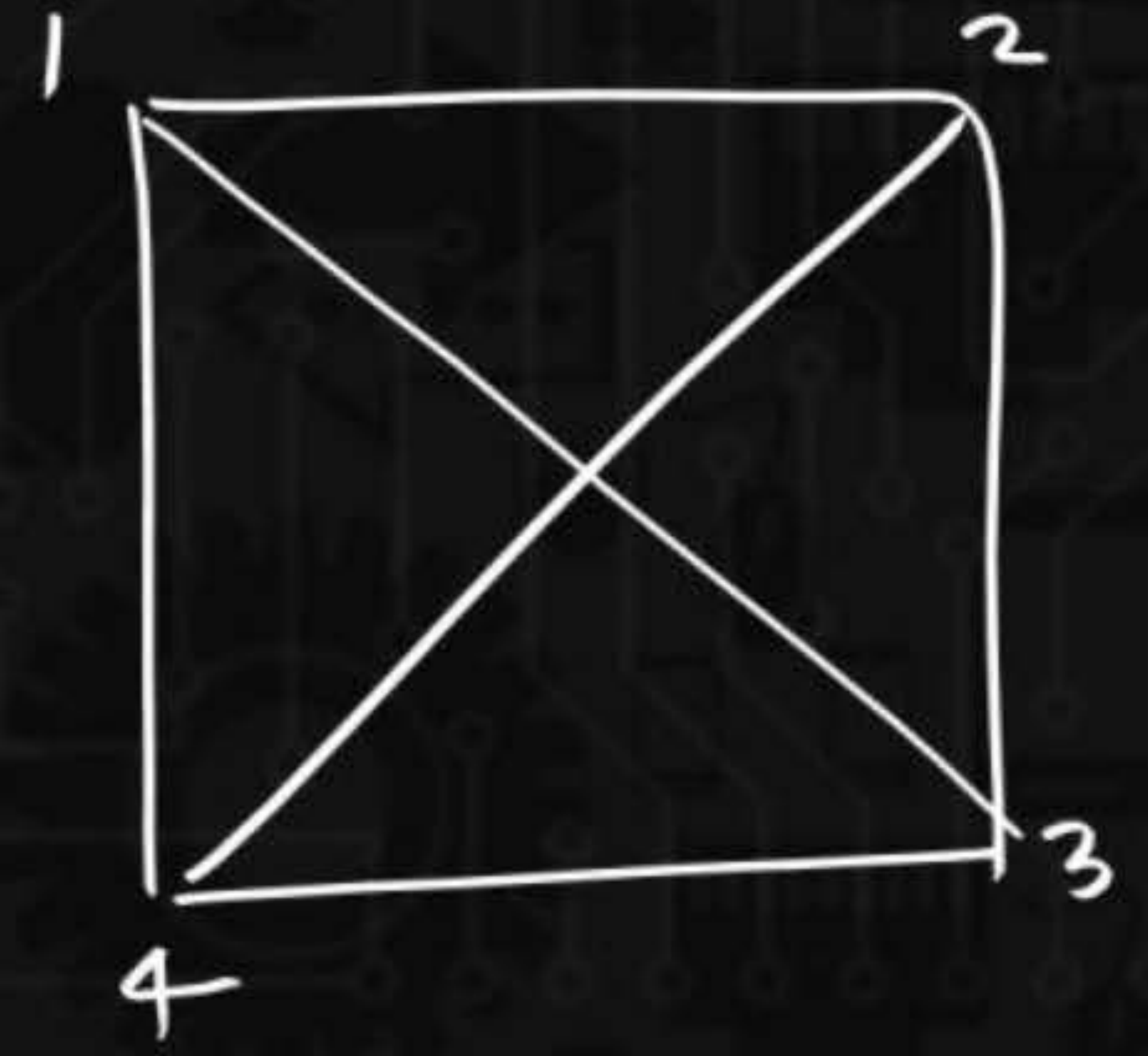
Every H.P contains H.C.

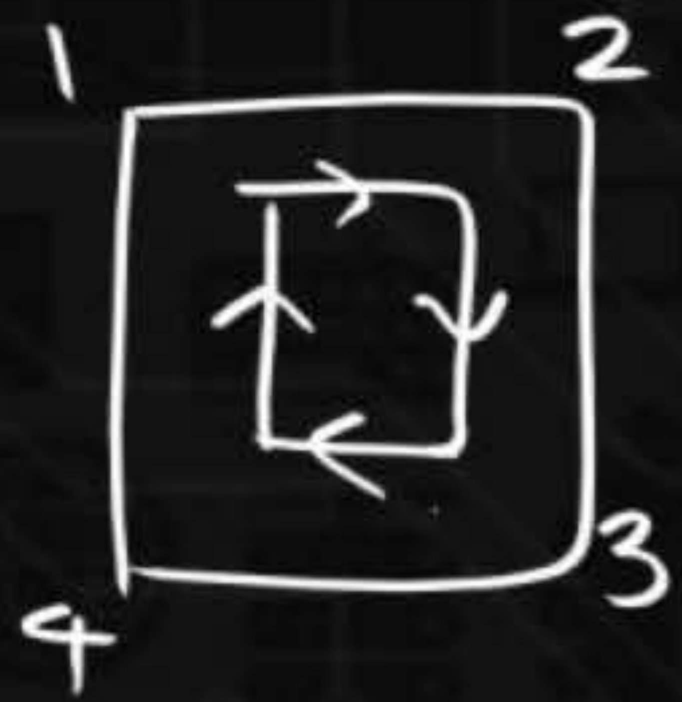
↓ st.

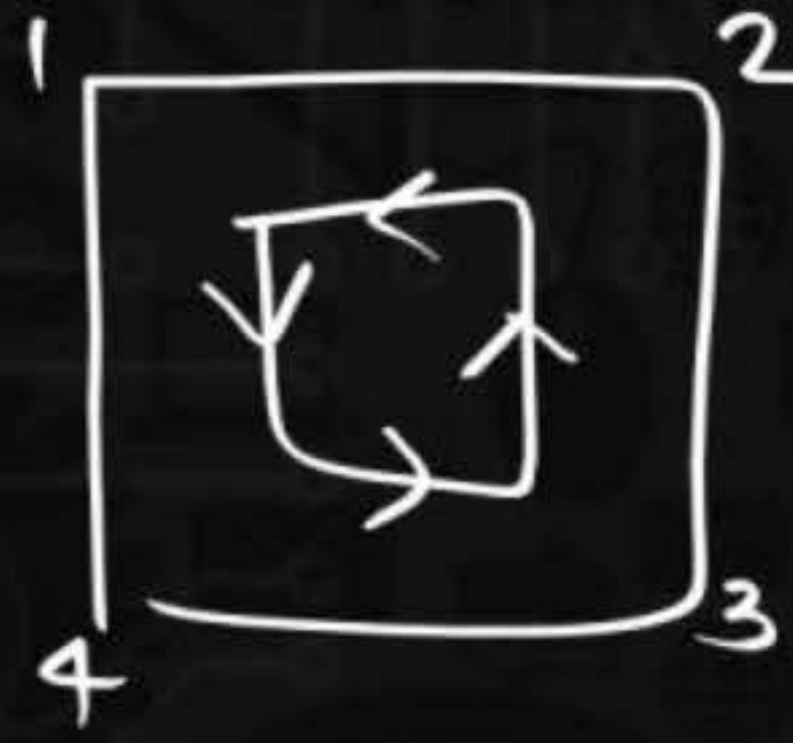
covering

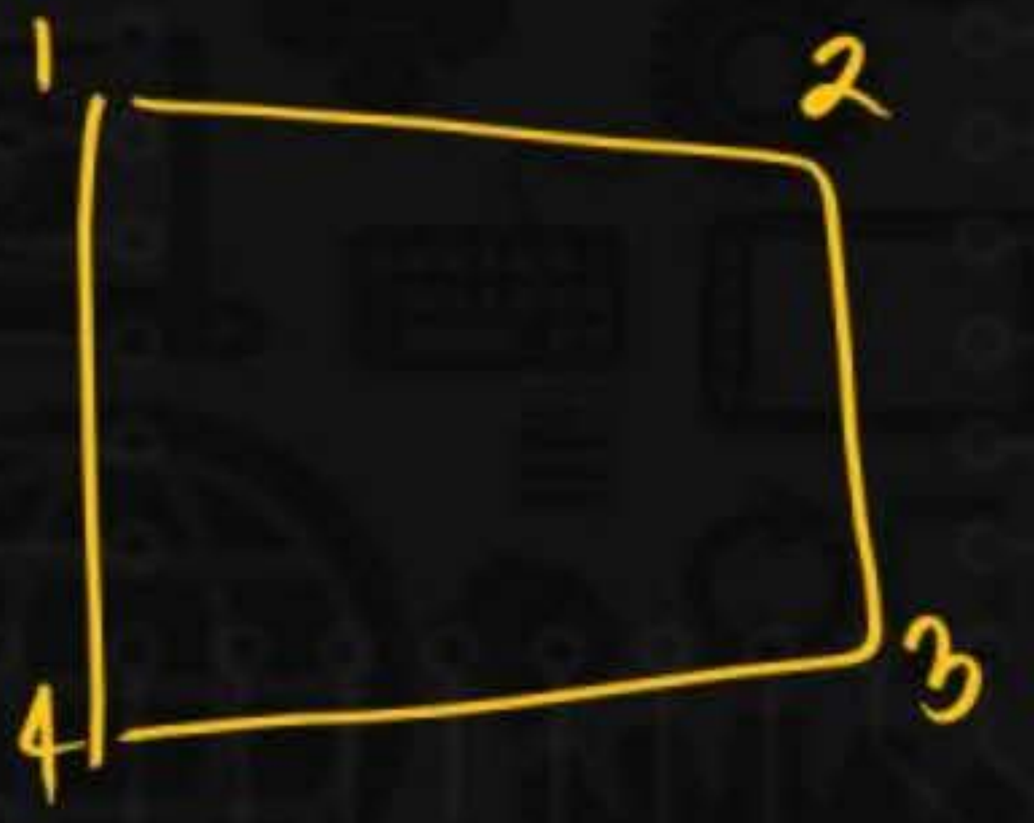
- all edges.
  - Closed Trail.
    - $\boxed{\text{euler cycle}}$.
    - E.G ✓
  - open Trail
    - Euler path.
- all vertices.
  - closed path
    - $\boxed{\text{Hamiltonian cycle}}$
    - H.G ✓
  - open path
    - Hamiltonian path.

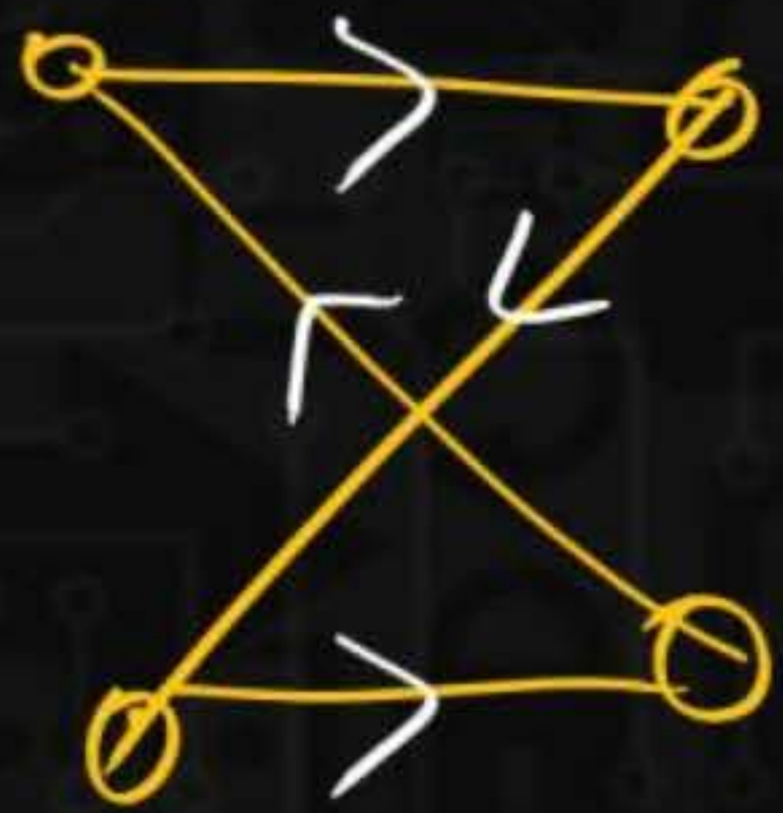
K4

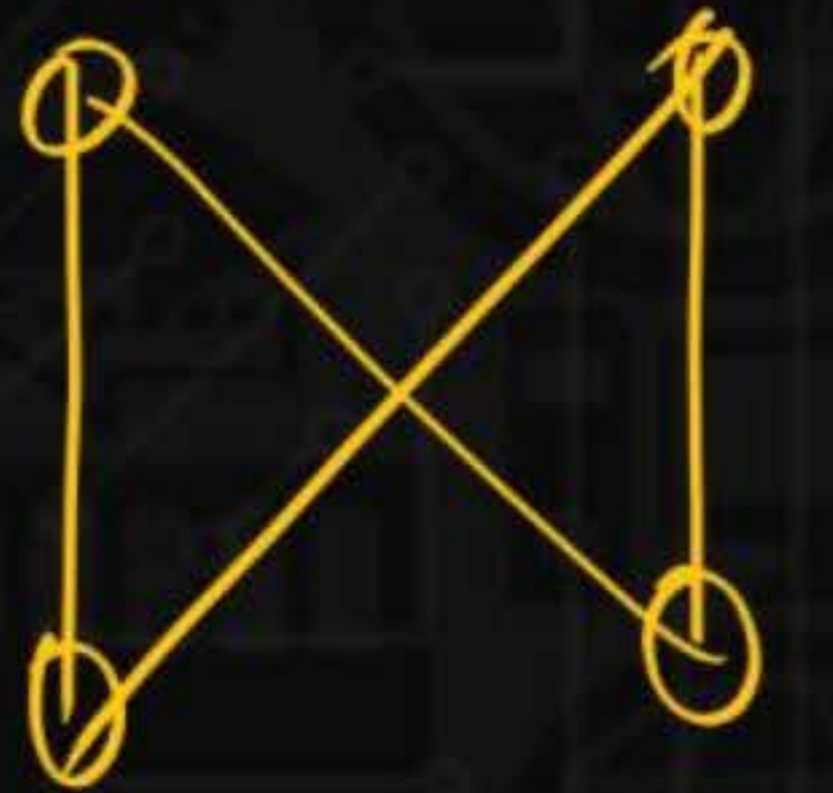




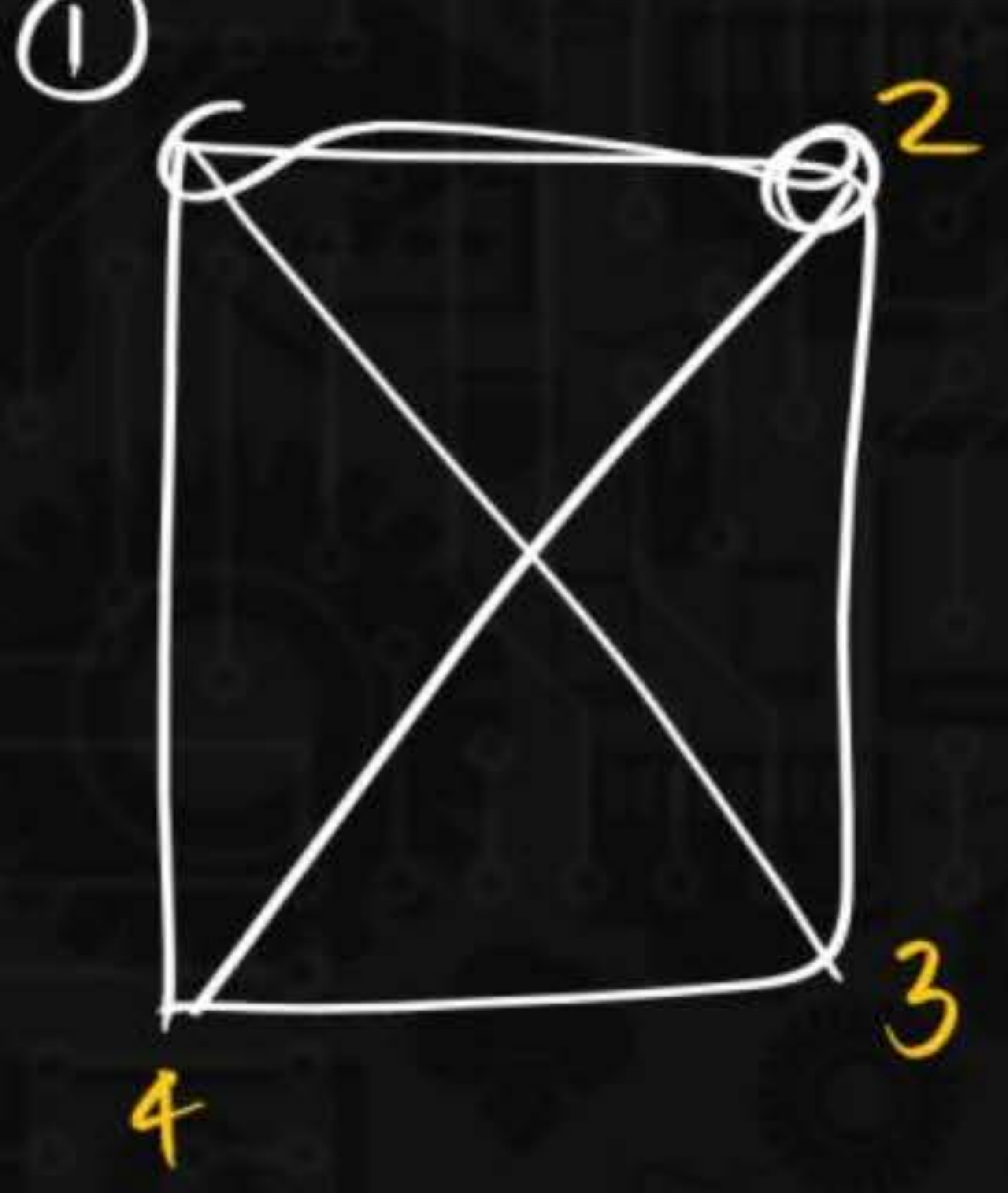








$n = 4.$



3 ways $\times$ 2 ways

$(n-1)$ ways $\times$ $(n-2)$ ways ....

$= (n-1)!$

Total distinc H.C in K_n $= \frac{(n-1)!}{2}$ $(n \geq 3)$

H.W

$K_{n,n}$ $(n \geq 2)$

How many distinct Hamiltonian cycle ?

K_n

$\downarrow$

$L(K_n)$

Degree of each vertex in $L(K_n)$

$= 2(n-2)$


THANK YOU GW
SOLDIERS !