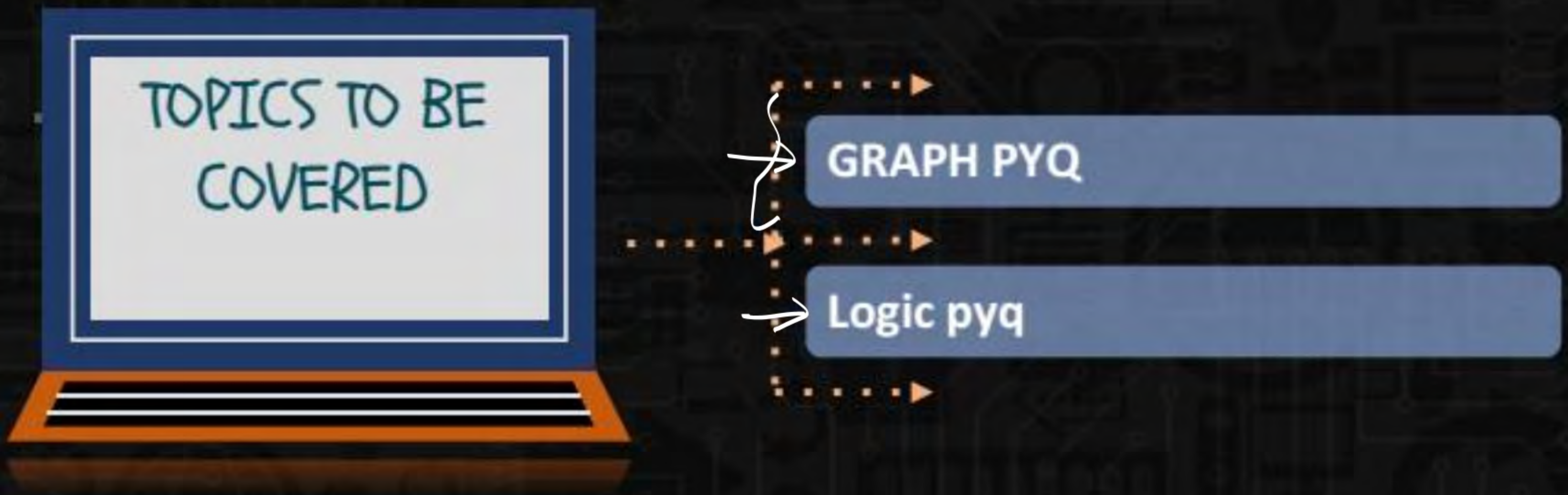
CS & IT ENGINEERING

GRAPH THEORY/ LOGIC

Lecture No: 15

Satish Sir


TOPICS TO BE COVERED
GRAPH PYQ
Logic pyq

Q.1 A non-planar graph with minimum number of vertices has

(GATE - 92)

planar $\begin{cases} n=1 \\ n=2 \\ n=3 \\ n=4 \end{cases}$

min no. of vertices = 5

min no edges = 9

A. 9 edges, 6 vertices ($K_{3,3}$)

B. 6 edges, 4 vertices ✗

C. 10 edges, 5 vertices (non planar) ✓

D. 9 edges, 5 vertices (planar)

nonplanar

K_5

$K_{3,3}$

vertices	edges
$\underline{\underline{5}}$	10
6	9

Q.2 The number of non-isomorphic simple graphs up to three nodes is

(GATE - 94)

vertices

A. 15

B. 10

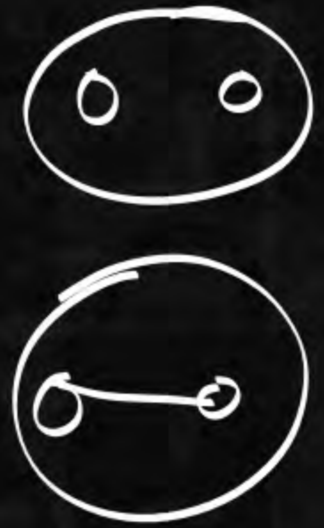
C. 7 Ans.

D. 9

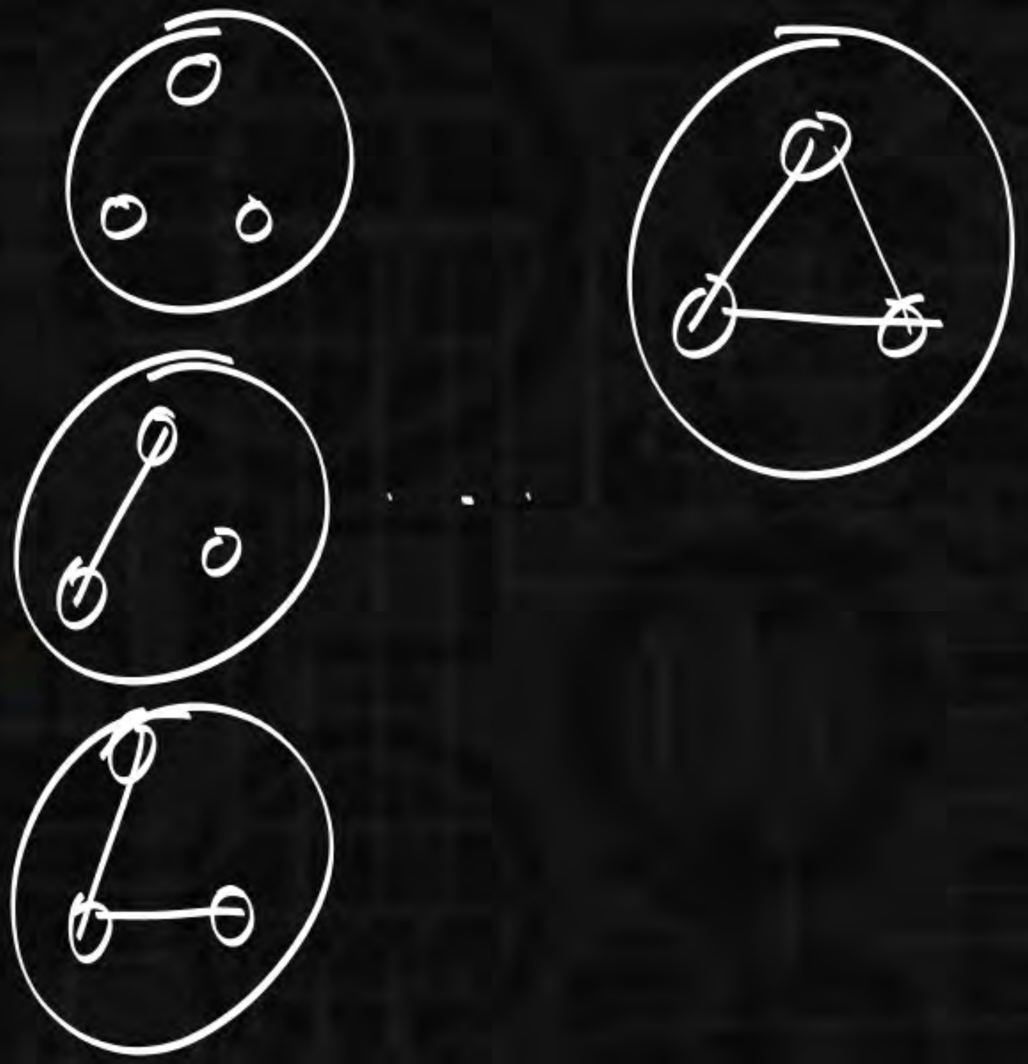
$n = 1$

1

$n = 2$



$n = 3$



$1 + 2 + 4$

$= 7$

Q.3 Maximum number of edges in a n - node undirected graph without self loops is (GATE - 02)

A. n^2

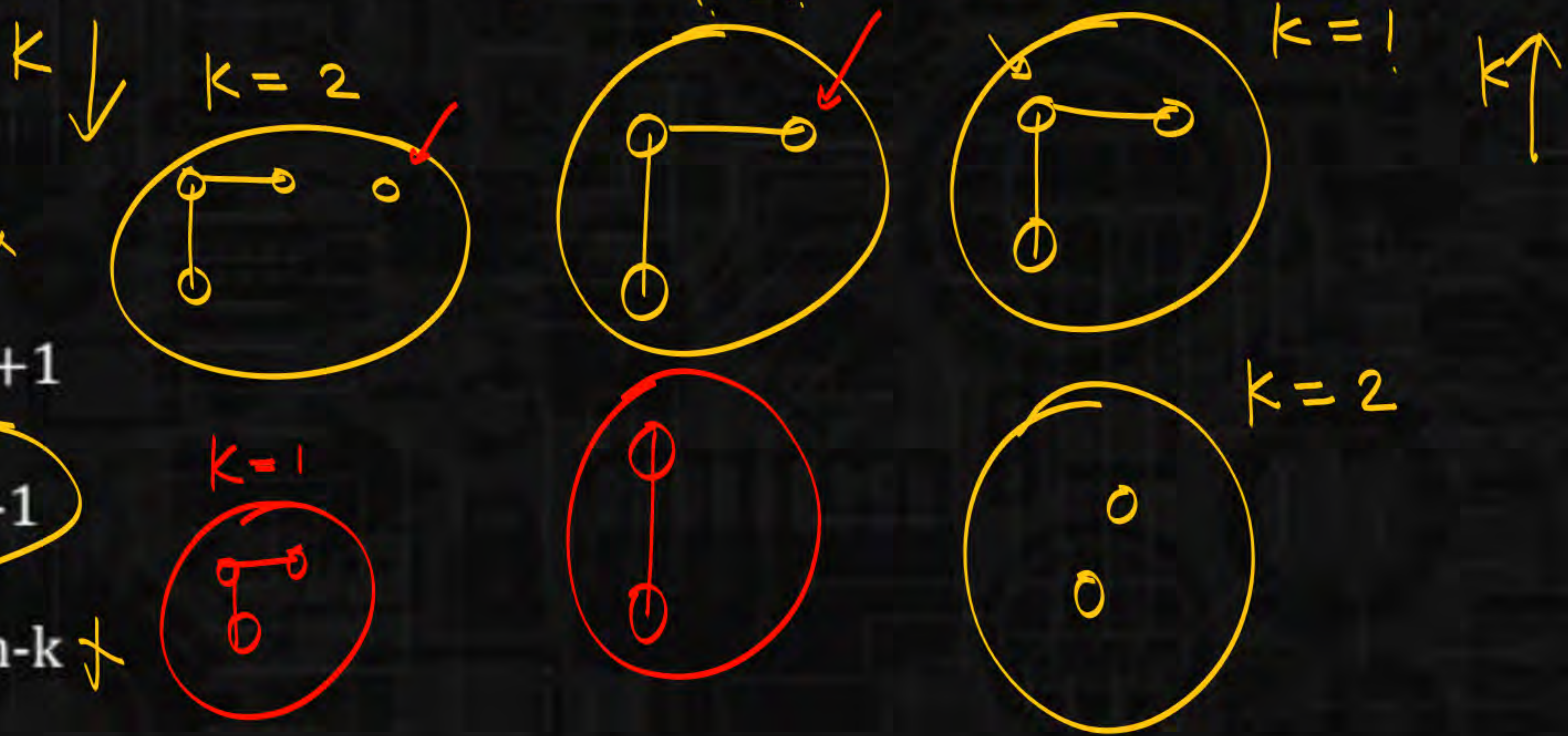
B. $\frac{n(n-1)}{2}$ ✓

C. $n-1$

D. $\frac{(n+1)(n)}{2}$

Q.4 Let G be an arbitrary graph with n nodes and k components. If a vertex is removed from G, the number of components in the resultant graph must necessarily lie between **(GATE - 03)**

K ↔



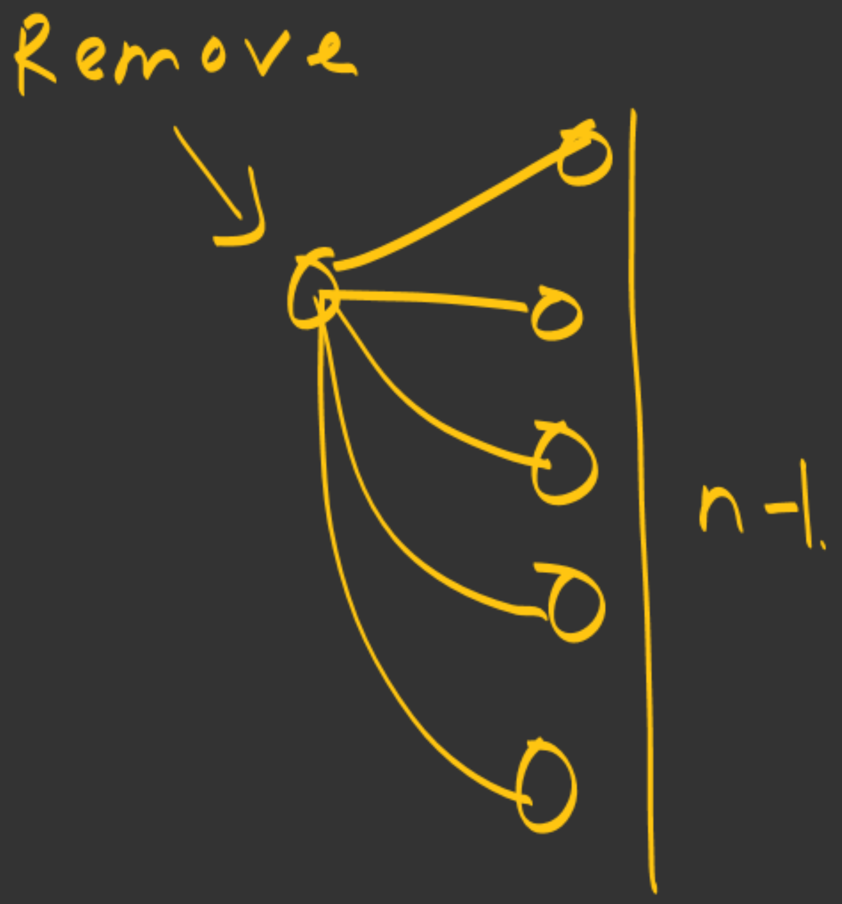
A. k and n

B. k-1 and k+1

C. k-1 and n-1

D. k+1 and n-k

$K_{1,n-1}$



$k = n-1$

Q.5 If all the edge weights of an undirected graph are positive, then any subset of edges that connects all the vertices and has minimum total weight is a

connected.

(GATE - 06)

A. Hamiltonian cycle

B. grid

C. hypercube

D. tree

Q.6 Let G be the non-planar graph with minimum possible number of edges. Then G has

(GATE - 07)

A. 9 edges and 5 vertices

B. 9 edges and 6 vertices

C. 10 edges and 5 vertices

D. 10 edges and 6 vertices

Q.7 Consider a weighted undirected graph with positive edge weights and let uv be an edge in the graph. It is known that the shortest path from the source vertex s to u has weight 53 and the shortest path from s to v has weight 65. Which one of the following statements is always true?

(GATE - 07)

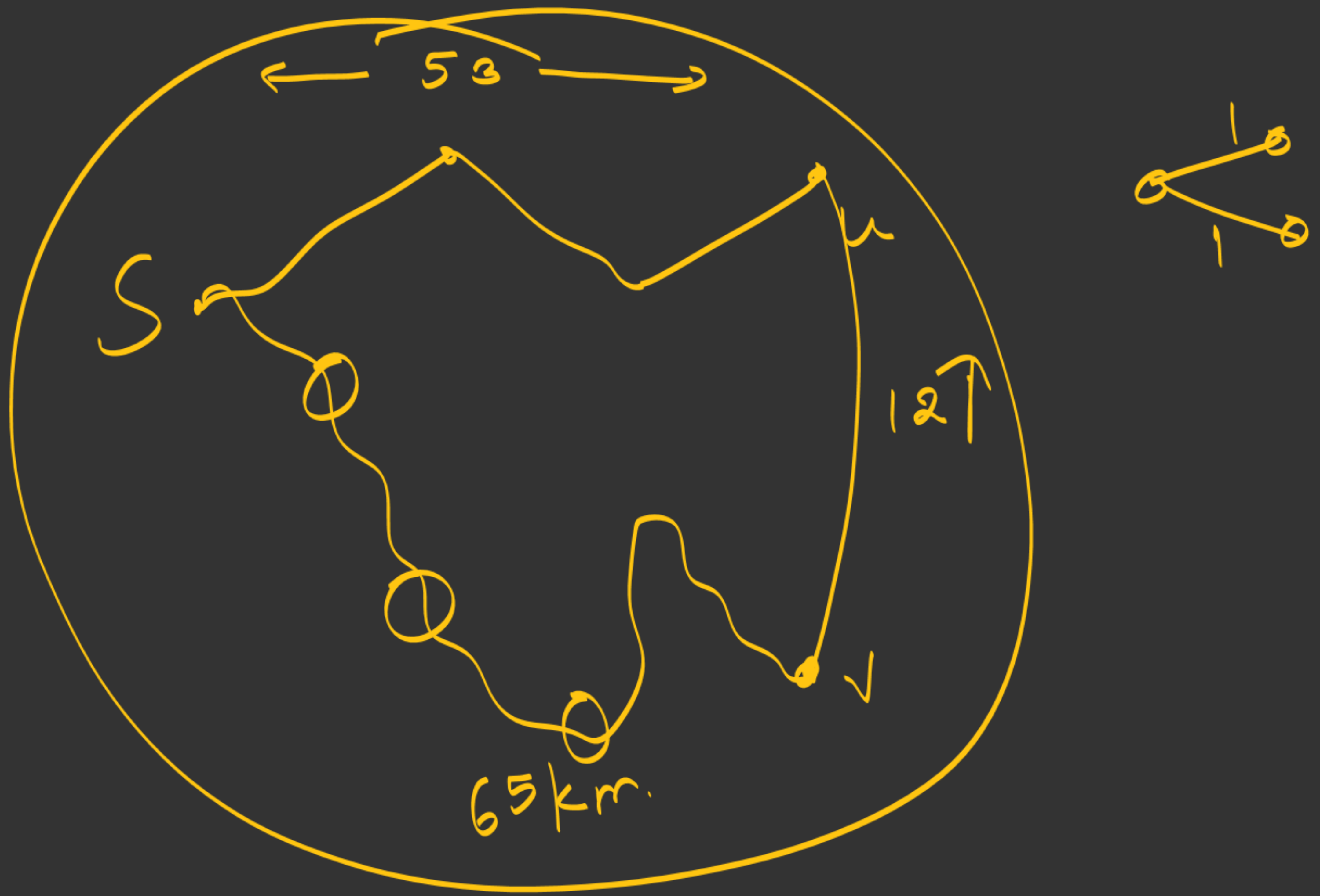
53 + 10 = 63

A. weight $(u,v) < 12$

B. weight$(u,v) \leq 12$

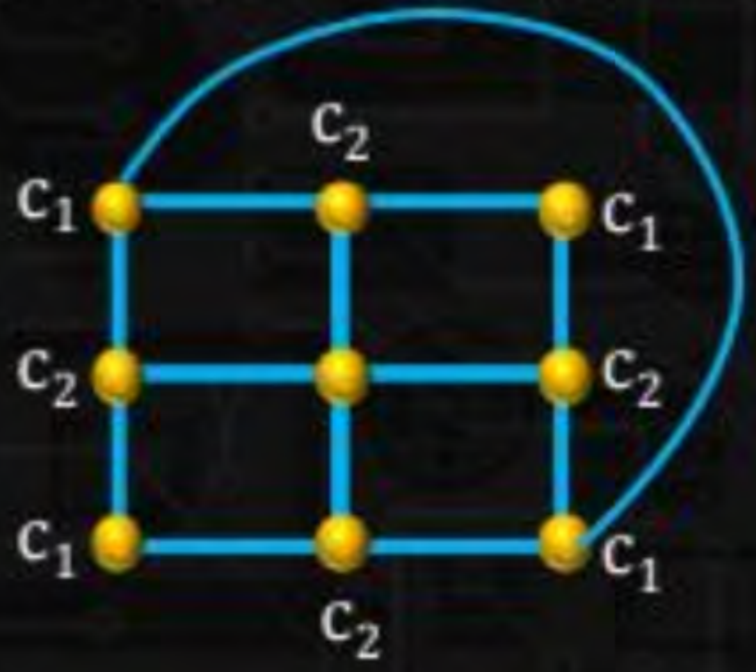
C. weight $(u,v) > 12$

D. weight$(u,v) \geq 12$ ✓


53
S
u
127
v
65km.
1
1

Q.8 What is the chromatic number of the following graph?

(GATE - 08)



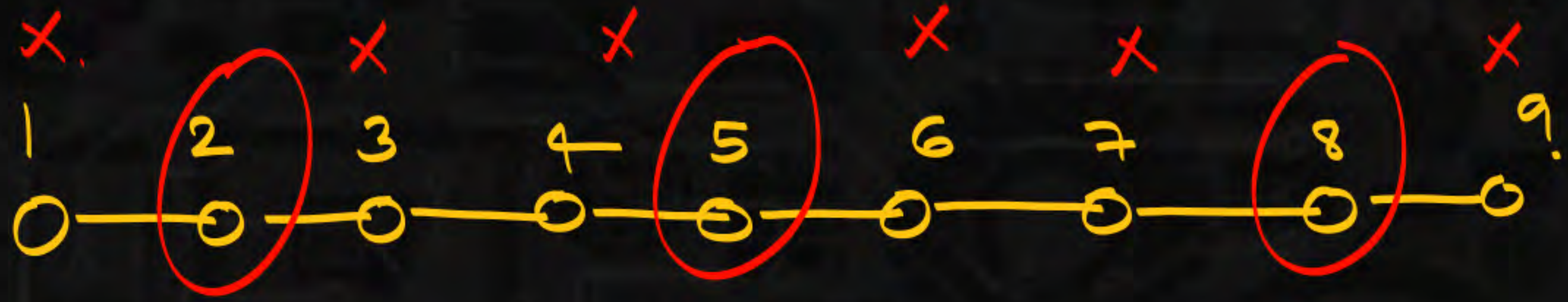
A. 2

B. 3 ✓

C. 4

D. 5

Q.9 What is the size of the smallest MIS (Maximal Independent Set) of a chain of nine nodes? (GATE - 08)



A. 5

B. 4

C. 3

D. 2

Q.10 What is the chromatic number of an n-vertex simple connected graph which does not contain any odd length cycle? Assume $n \geq 2$. (GATE - 09)

A. 2

B. 3

C. n-1

D. n

Q.11 K4 and Q3 are graphs with the following structures. Which one of the following statements is TRUE in relation to these graphs?

planar → $e \leq 3n-6$.

(GATE - 11)

planar → $e \leq 3n-6$

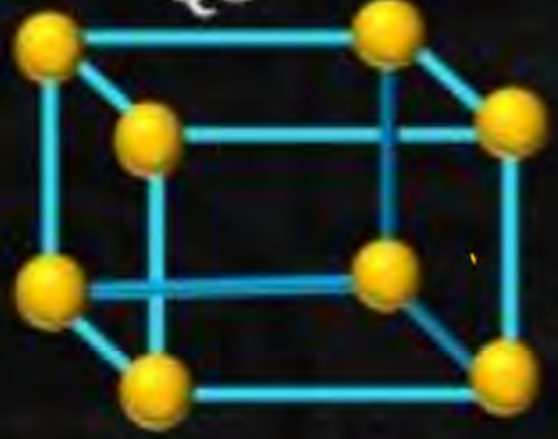
OR

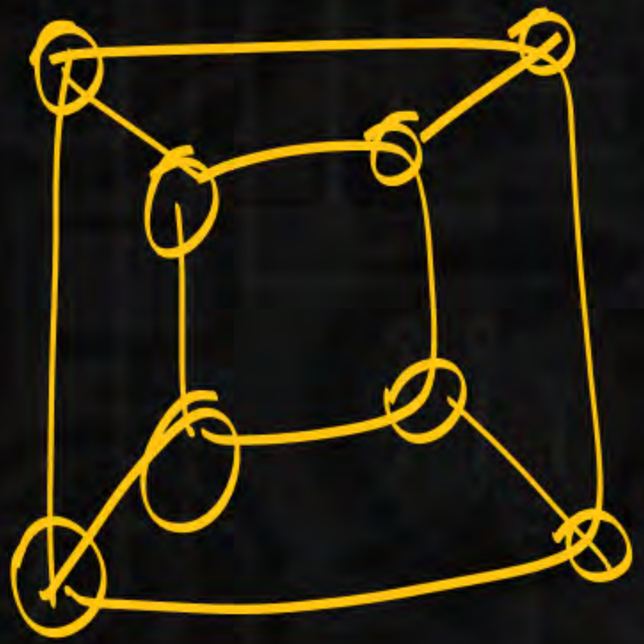
T. $e > 3n-6$ → nonplanar

T.

K4

Q3





A. K4 is planar while Q3 is not

B. Both K4 and Q3 are planar

C. Q3 is planar while K3 is not

D. Neither K4 nor Q3 is planar

Q.12 Let G be a simple undirected planar graph on 10 vertices with 15 edges. If G is a connected graph, then the number of bounded faces in any embedding of G on the plane is equal to

(GATE - 12)

A. 3

B. 4

C. 5

D. 6

Q.13 Which of the following statements is/are TRUE for undirected graphs?

P: Number of odd degree vertices is even.

Q: Sum of degrees of all vertices is even. (GATE - 13)

A. P only

B. Q only

C. Both P and Q

D. Neither P nor Q

Q.14 The maximum number of edges in a bipartite graph on 12 vertices is

GATE-14-Set2

$K_{6,6} \quad e = 6 \times 6 = 36.$

Q.15 Let G be an undirected complete graph on n vertices, where n>2. Then, the number of different Hamiltonian cycles in G is equal to

(GATE - 19)

A. 1

B. $\frac{(n-1)!}{2}$

C. n!

D. (n-1)!

Q.17 The minimum number of colours required to colour the following graph, such that no two adjacent vertices are assigned the same colour, is

(GATE - 04)

A. 2

B. 3

C. 4

D. 5

Q.19 Consider the undirected graph G defined as follows. The vertices of G are bit strings of length n. We have an edge between vertex u and vertex v if and only if u and v differ in exactly one bit position (in other words, v can be obtained from u by flipping a single bit). The ratio of the chromatic number of G to the diameter of G is

(GATE - 04)

Hypercube.

B.P $\longrightarrow$ $\chi(B.P) = 2$

A. $1/2^{n-1}$

Diam(Q2) = 2

Diam(Qn) = n

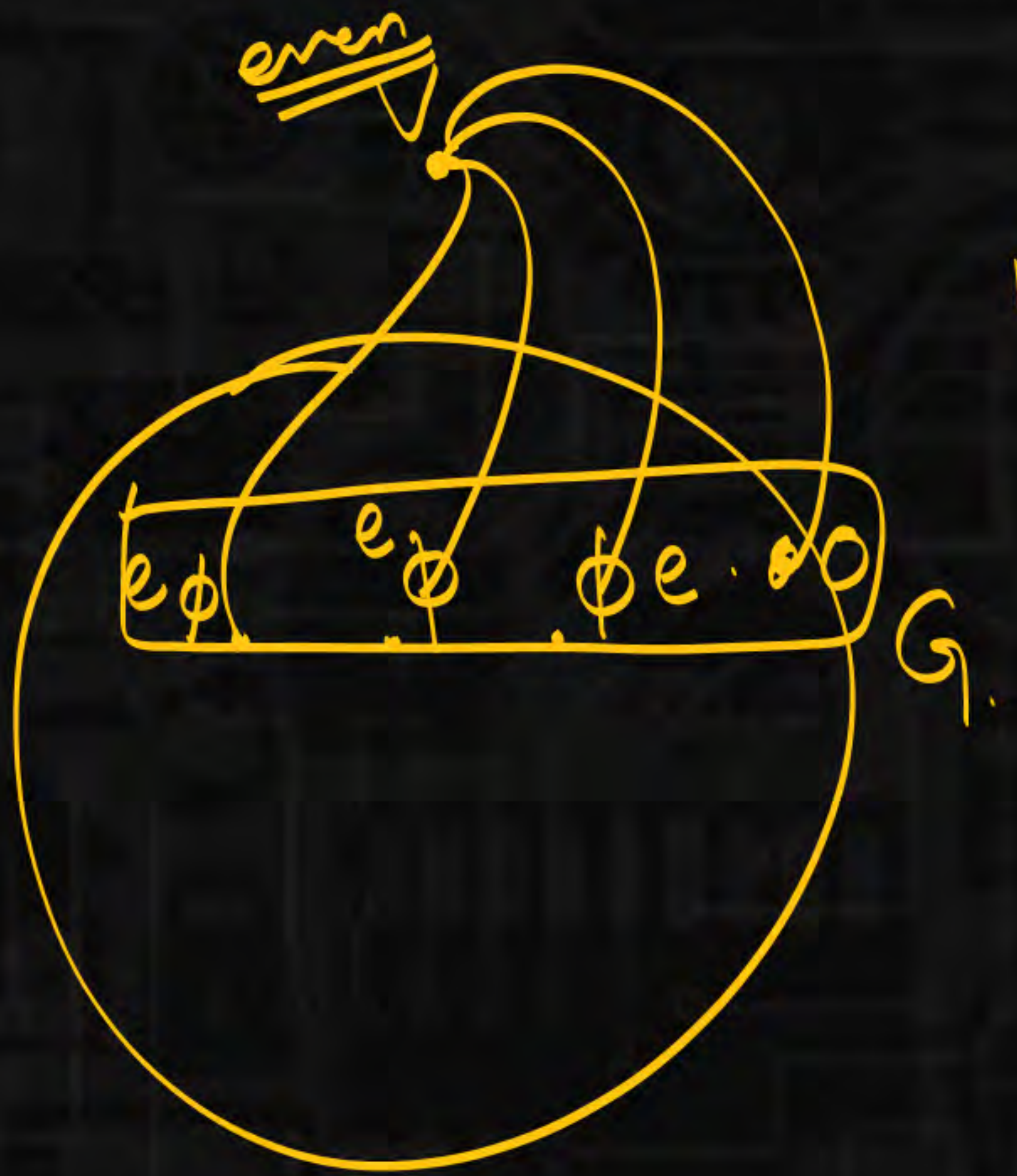
B. $1/n$

C. $2/n$

D. $3/n$

Q.20 G is a simple undirected graph. Some vertices of G are of odd degree. Add a node v to G and make it adjacent to each odd degree vertex of G. The resultant graph is sure to be

(GATE - 08)



no. of odd degr

A. Regular

B. complete

C. Hamiltonian

D. Euler

Q.21 The degree sequence of a simple graph is the sequence of the degrees of the nodes in the graph in decreasing order. Which of the following sequences cannot be the degree sequence of any graph?

(GATE - 10)

I. 7,6,5,4,4,3,2,1

II. 6,6,6,6,3,3,2,2

III. 7,6,6,4,4,3,2,2

IV. 8,7,7,6,4,2,1,1

A. I and II

B. III and IV

C. IV only

D. II and IV

Q.22 Which of the following graph is isomorphic to (GATE - 12)

cycle length =

A. X

B.

C. X

D. X

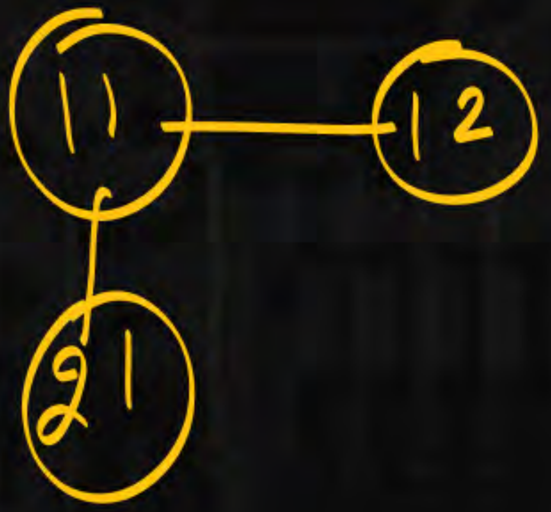
Q.23 Consider an unidirectional graph G where self loops are not allowed. The vertex set of G is $\{(i, j): 1 \le i \le 12, 1 \le j \le 12\}$. There is an edge between (a, b) and (c, d) if $|a-c| \le 1$ $\boxed{\text{and}}$ $|b-d| \le 1$. The number of edges in this graph is

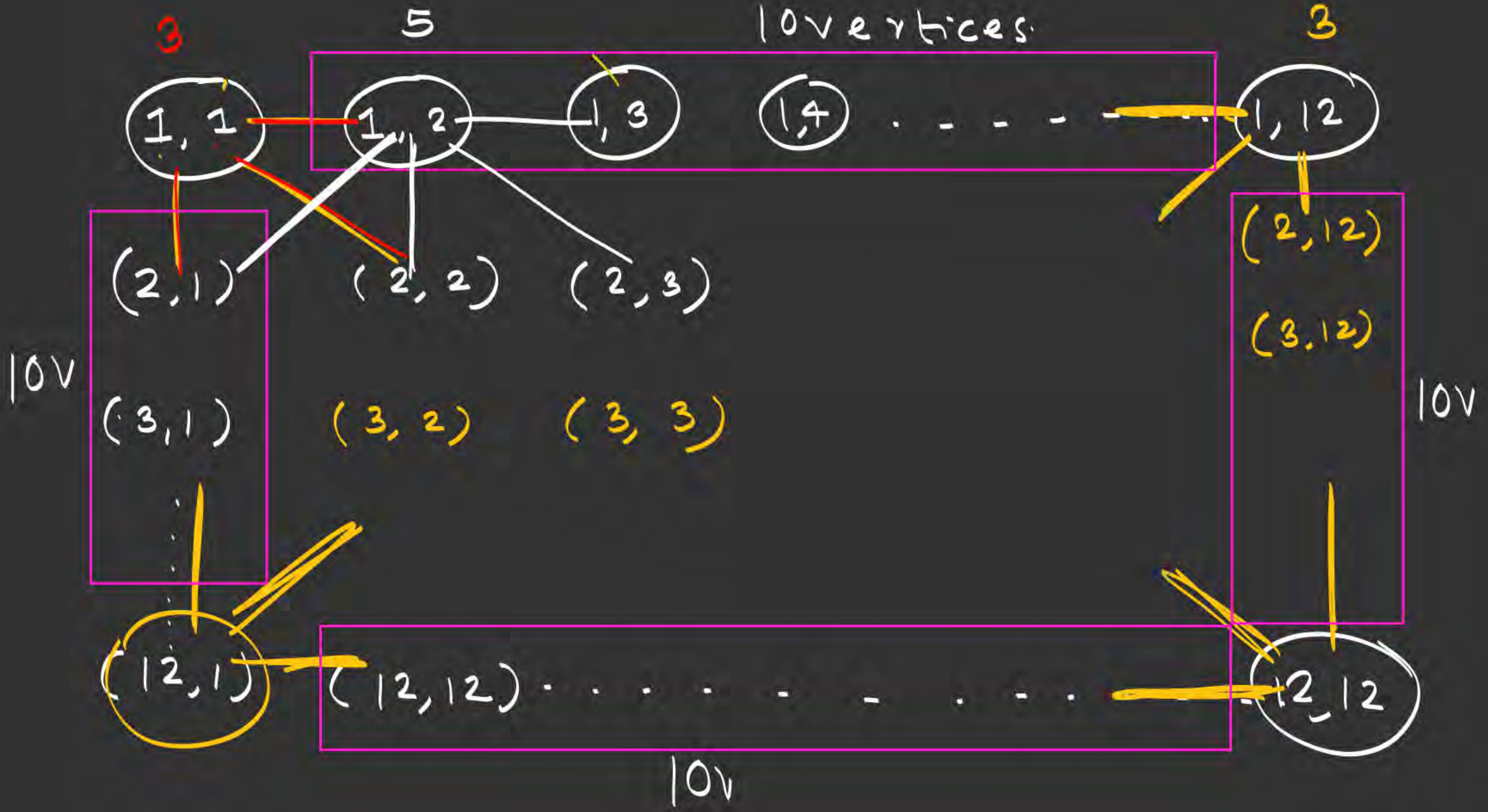
(GATE-14-Set 1)

$(a, b) \quad (c, d) \qquad |a-c| \le 1 \wedge |b-d| \le 1.$

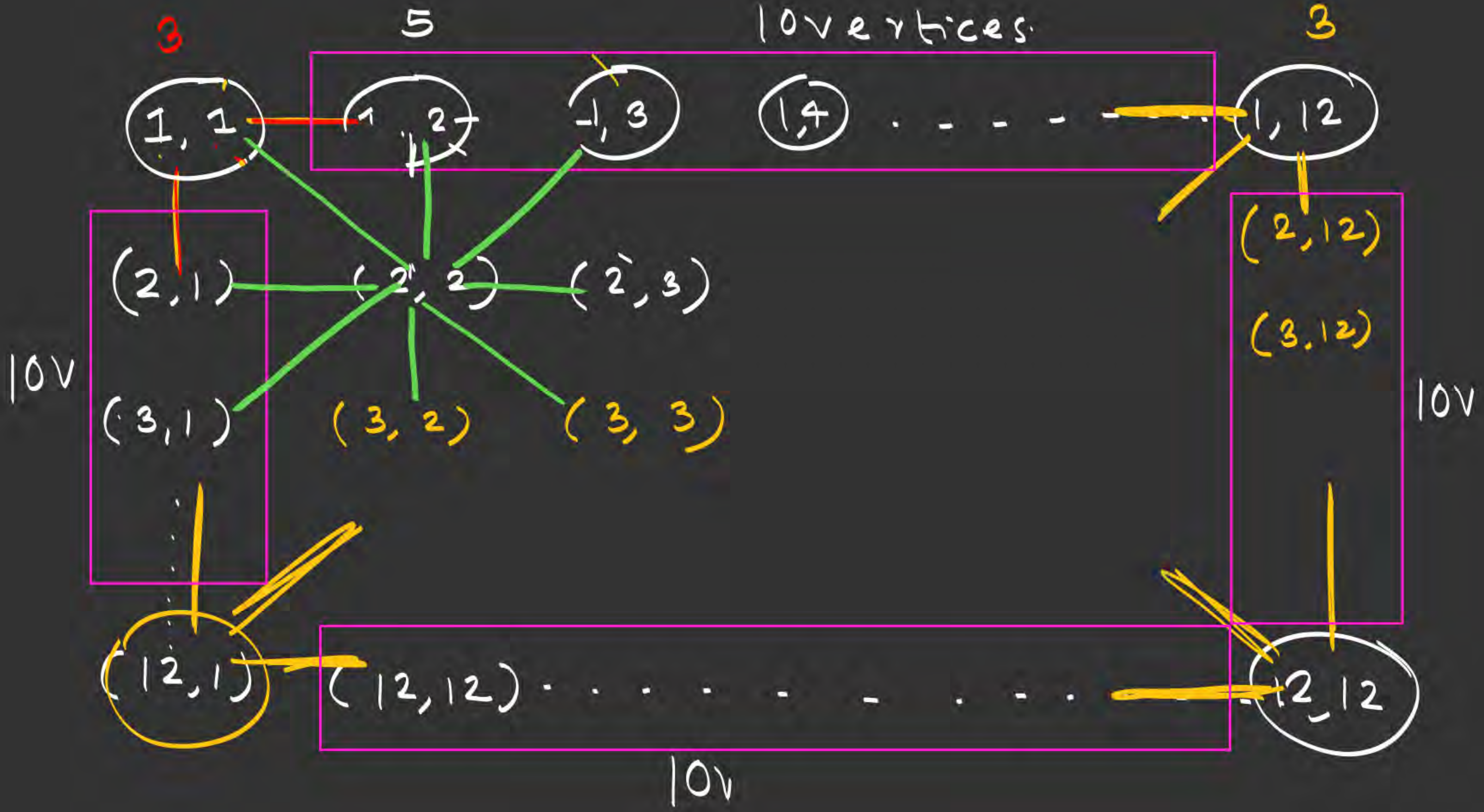
$(1, 1) \quad (1, 2)$

$|1-1| \le 1 \quad |1-2| \le 1.$





3
5
10 vertices
3
1, 1
1, 2
1, 3
1,4
1, 12
(2, 1)
(2, 2)
(2, 3)
(2, 12)
(3, 12)
10V
(3, 1)
(3, 2)
(3, 3)
10V
(12, 1)
(12, 12)
12, 12
10V


3
5
10 vertices
3
1, 1
1, 2
1, 3
1,4
1, 12
(2, 1)
(2, 2)
(2, 3)
(2, 12)
(3, 12)
10V
(3, 1)
(3, 2)
(3, 3)
10V
(12, 1)
(12, 12)
12, 12
10V

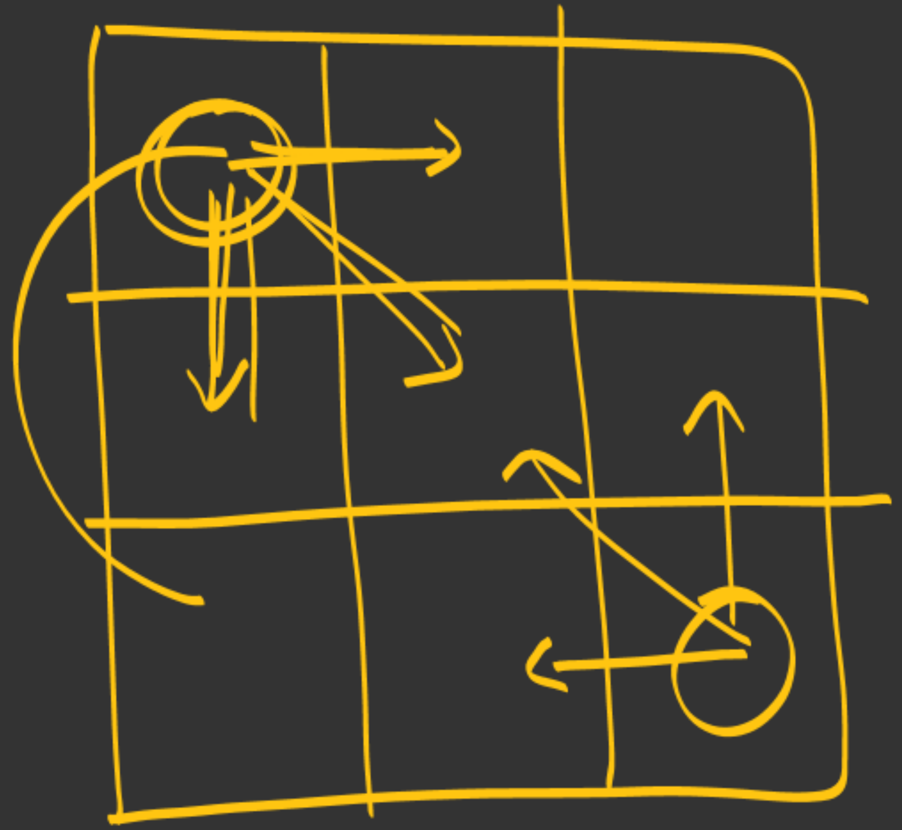
$$4v \times 3 + 40v \times 5 + 100 \times 8 = 2e$$

corner

$$e = 506.$$

Total vertices $= 144.$

100 v



$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$|a-c| \leq 1.$$

Q.24 An ordered n-tuple $(d_1, d_2, \ldots, d_n)$ with $d_1 \geq d_2 \geq \ldots \geq d_n$ is called graphic if there exists a simple undirected graph with n vertices having degrees $d_1 \geq d_2 \geq \ldots \geq d_n$ respectively. Which of the following 6-tuples is NOT graphic?

(GATE-14-Set 1)

A. (1,1,1,1,1,1)

B. (2,2,2,2,2,2)

C. (3,3,3,1,0,0)

D. (3,2,1,1,1,0)

Q.25 Let G be a graph with 100 ! Vertices, with each vertex labeled by a distinct permutation of the numbers 1,2,....,100. There is an edge between vertices u and v if and only if the label of u can be obtained by swapping two adjacent numbers in the label of v. Let y denote the degree of a vertex in G, and z denote the number of connected components in G. Then, y+10z=

(GATE-18-CSIT)

$y = 99$.

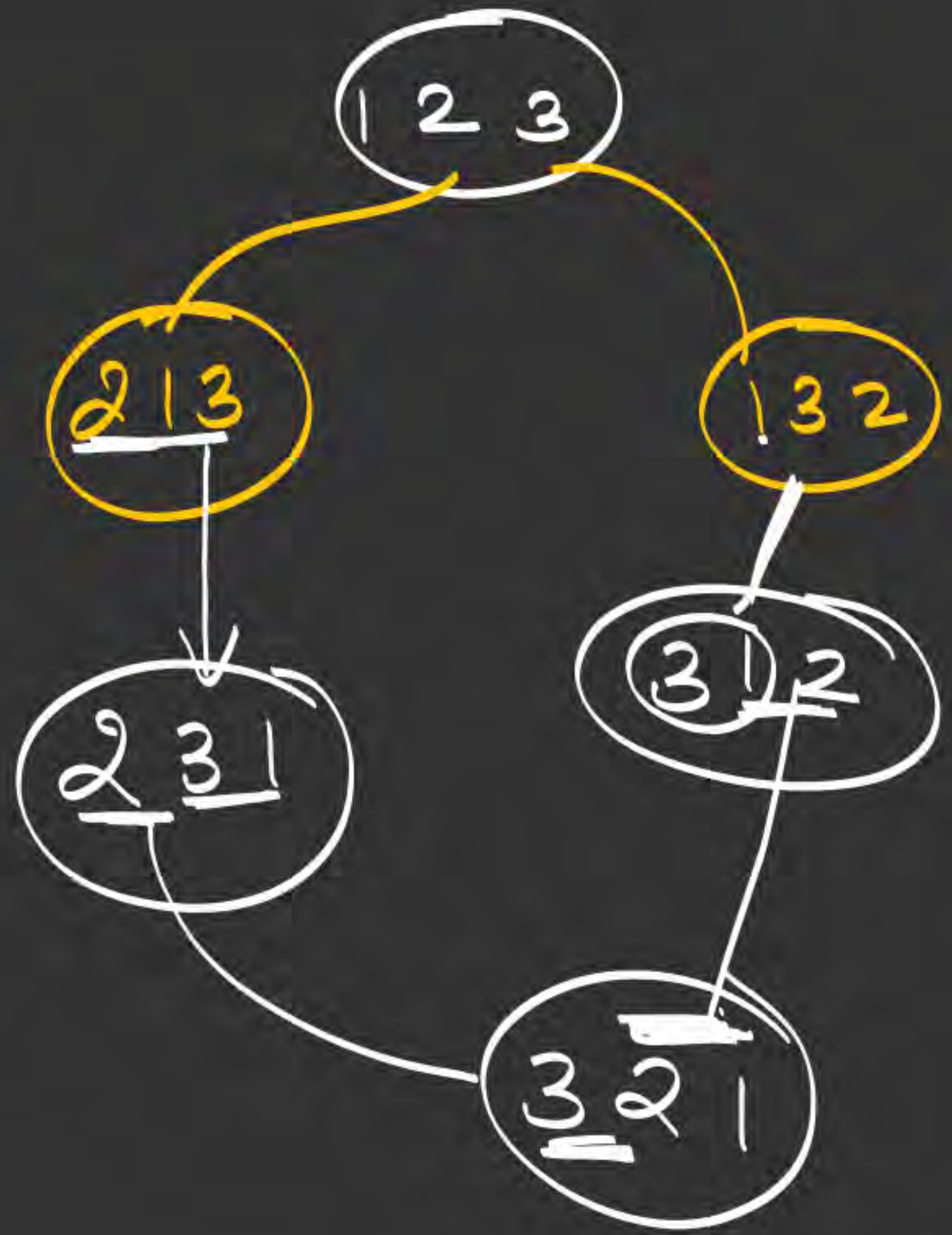
$z = 1$.

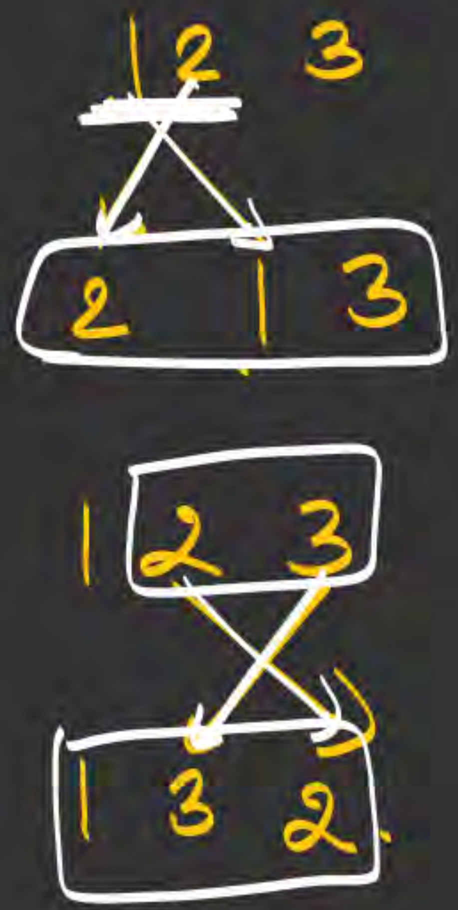
Degree of each vertex will be 99.

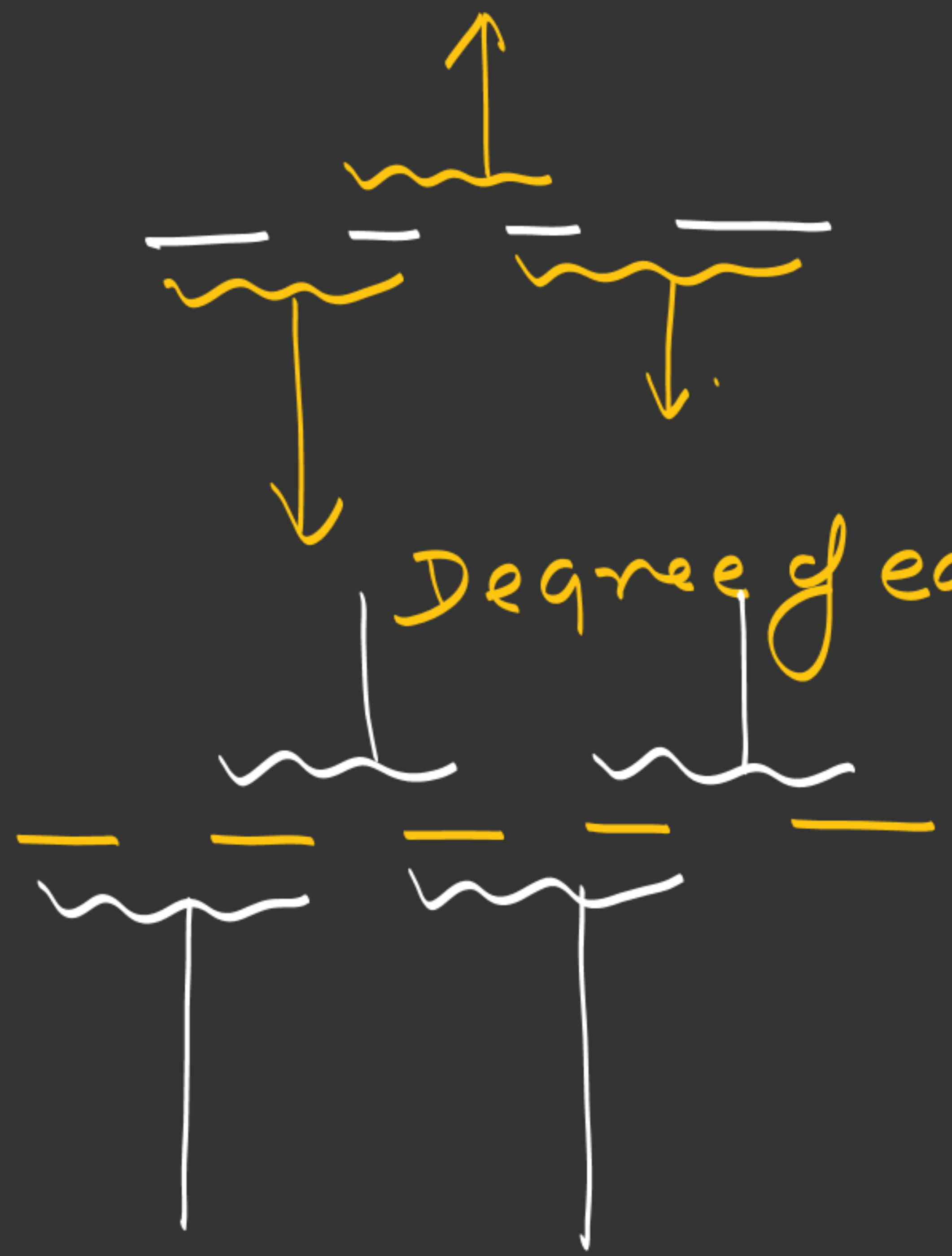
adjacent swap.

$\{1, 2, 3\}$

$3! \left\{ \begin{array}{l} 1\ 2\ 3 \\ 1\ 3\ 2 \\ 2\ 1\ 3 \\ 2\ 3\ 1 \\ 3\ 1\ 2 \\ 3\ 2\ 1 \end{array} \right.$







4!.

Degree of each vertex will be 3.

Q.16 Let p,q,r,s represent the following propositions.

p: x∈{8,9,10,11,12}

F F

q:x is a composite number 4, 6, 8

r. x is a perfect square 8 is perfect.

s:x is a prime number 8 is prime (GATE - 16 - set 1)

The integer x≥2 which satisfies

$x = 9$ $\sim((p \Rightarrow q) \land (\sim r \lor \sim s))$ is

$$\neg\left[(P \rightarrow q) \land (\neg R \lor \neg S)\right) \quad \neg[F] \equiv T$$

$$(T \rightarrow T) \land (T \lor T)$$

Q.17 Let p,q and r be propositions and the expression $(p \to q) \to r$ be a contradiction.

Then, the expression $(r \to p) \to q$ is

(GATE-17-Set 1)

A. a tautology

B. a contradiction

C. always TRUE when p is FALSE

D. always TRUE when q is TRUE

Q.18 Let p,q,r denote the statements "It is raining", "It is cold" and "It is pleasant", respectively. Then the statement "It is not raining and it is pleasant, and it is not pleasant only if it is raining and it is cold" is represented by

(GATE-17-Set2)

A. $(\sim p \wedge r) \wedge (\sim r \rightarrow (p \wedge q))$

B. $(\sim p \wedge r) \wedge ((p \wedge q) \rightarrow \sim r)$

C. $(\sim p \wedge r) \vee ((p \wedge q) \rightarrow \sim 1)$ ✗

D. $(\sim p \wedge r) \vee (r \rightarrow (p \wedge q))$ ✗

Q.19 Which of the following predicate calculus statements is/are valid?

(GATE - 92)

$\forall x (p(x) \vee Q(x)) \Leftarrow \forall x P(x) \vee \forall x Q(x)$

A. $((\forall x)P(x) \vee (\forall x)Q(x)) \rightarrow (\forall x)\{P(x) \vee Q(x)\}$ ✓

B. $\{(\exists x)P(x) \wedge (\exists x)Q(x)\} \rightarrow (\exists x)\{P(x) \wedge Q(x)\}$ ✗

C. $(\forall x)\{P(x) \vee Q(x)\} \rightarrow \{(\forall x)P(x) \vee (\forall x)Q(x)\}$ ✗

D. $(\exists x)\{P(x) \vee Q(x)\} \rightarrow \sim\{(\forall x)P(x) \vee (\exists x)Q(x)\}$ ✗

Q.20 Identify the correct translation into logical notation of the following assertion.

Some boys in the class are taller than all the girls

Note: taller (x, y) is true if x is taller than y.

(GATE - 04)

A. $(\exists x)(boy(x) \rightarrow (\forall y)(girl(y) \wedge taller(x, y)))$

B. $(\exists x)(boy(x) \wedge (\forall y)(girl(y) \wedge taller(x, y)))$

C. $(\exists x)(boy(x) \rightarrow (\forall y)(girl(y) \rightarrow taller(x, y)))$

D. $(\exists x)(boy(x) \wedge (\forall y)(girl(y) \rightarrow taller(x, y)))$

Q.21 Let a(x,y), b(x,y) and c(x,y) be three statements with variables x and y chosen from some universe. Consider the following statement

$(\exists x)(\forall y)[(a(x,y) \wedge b(x,y)) \wedge \sim c(x,y)]$

Which one of the following is its equivalent?

$\exists x \forall (a \wedge b \wedge \neg c)$

(GATE - 04)

$\exists x \forall y ((a \wedge b) \wedge \neg c)$

$\neg \forall x \exists y (\neg(a \wedge b) \vee c)$

$\neg \forall x \exists y (a \wedge b \rightarrow c)$

$\exists y$

A. $(\forall x)(\exists y)[(a(x,y) \vee b(x,y)) \rightarrow c(x,y)]$ ✗

B. $(\exists x)(\forall y)[(a(x,y) \vee b(x,y)) \wedge \sim c(x,y)]$ ✗

C. $\sim[(\forall x)(\exists y)[(a(x,y) \wedge b(x,y)) \rightarrow c(x,y)]$

D. $\sim[(\forall x)(\exists y)[(a(x,y) \vee b(x,y)) \rightarrow c(x,y)]$

Q.22 Which one of the following is the most appropriate logical formula to represent the statement:

"Gold and silver ornaments are precious"

The following notations are used:

G(x):x is a gold ornament.

S(x):x is a silver ornament.

P(x):x is precious.

(GATE - 09)

$\forall x\,(G(x) \wedge S(x))$

A. (a) $\forall x(P(x) \rightarrow (G(x) \wedge S(x)))$ ✗

$x \rightarrow \boxed{T \wedge T}$

B. (b) $\forall x((G(x) \wedge S(x)) \rightarrow P(x))$

C. (c) $\exists x((G(x) \wedge S(x)) \rightarrow P(x))$ ✗

D. (d) $\forall x((G(x) \vee S(x)) \rightarrow P(x))$ ✓ (Ans)

Q.23 Consider the following well-formed formulae:

I. $\sim\forall x(P(x))$

II. $\sim\exists x(P(x))$

III. $\sim\exists x(\sim P(x))$

IV. $\exists x(\sim P(x))$

Which of the above are equivalent? (GATE - 09)

A. I and II

B. II and III

C. I and IV

D. II and IV

Q.24 Which one of the following options is correct given three positive integers x, y and z, and a predicate

$P(x): \sim(x=1) \wedge \forall y\{(\exists z(x=y * z) \Rightarrow (y=x) \vee (y=1))\}$

(GATE - 11)

A. P(x) being true means that x is a prime number

B. P(x) being true means that x is a number other than 1

C. P(x) is always true irrespective of the value of x

D. P(x) being true means that x has exactly two factors other than 1 and x

Q.25 Consider the statement:

"Not all that glitters is gold"

Predicate glitters (x) is true if x glitters and predicate gold(x) is true if x is gold.

Which one of the following logical formulae represents the above statement? **(GATE-14-Set1)**

A. $\forall x$: glitters $(x) \Rightarrow \sim$gold(x)

B. $\forall x$: gold $(x) \Rightarrow$ glitters(x)

C. $\exists x$: gold$(x) \wedge \sim$glitters(x)

D. $\exists x$: glitters $(x) \wedge \sim$gold(x)

Q.26 Which one of the following well-formed formulae in predicate calculus is NOT valid?

(GATE-16-Set 2)

A. $((\forall x\, p(x) \Rightarrow \forall x\, q(x)) \Rightarrow (\exists x \sim p(x) \vee \forall x\, q(x))$

B. $(\exists x\, p(x) \vee \exists x\, q(x)) \Rightarrow \exists x(p(x) \vee q(x))$

C. $(\exists x(p(x) \wedge q(x)) \Rightarrow (\exists x\, p(x) \wedge \exists x\, q(x))$

D. $\forall x(p(x) \vee q(x)) \Rightarrow (\forall x\, p(x) \vee \forall x\, q(x))$

Q.27 Consider the first-order logic sentence $F: \forall x(\exists y R(x, y))$. Assuming non-empty logical domain, which of the sentences below are implied by F ?

I. $\exists y(\exists x R(x,y))$

II. $\exists y(\forall x R(x,y))$

III. $\forall y(\exists x R(x,y))$

IV. $\sim x(\forall y \sim R(x,y))$

(GATE-17-Set1)

A. IV only

B. I and IV only

C. II only

D. II and III only

Q.29 Consider the first order predicate formula φ :

$$(\forall x[(\forall z\ z|x \Rightarrow ((z=x) \vee (z=1)))$$
$$\Rightarrow \exists w(w>x) \wedge (\forall z\ z|w \Rightarrow ((w=z) \vee (z=1)))])$$

Here ' a|b ' denotes that ' a divides b ', where a and b are integers.

Consider the following sets:

S1. {1,2,3......100} $x=100$

S2. Set of all positive integers ✓

S3. Set of all integers ✓

(GATE-19)

Which of the above sets satisfy φ ?

A. (a) S2 and S3

B. (b) S1, S2 and S3

C. (c) S1 and S2

D. (d) S1 and S3

(GATE-19)

always

$$(\forall x[(\forall z\ z|x \Rightarrow ((z=x) \vee (z=1))) \Rightarrow \exists w(w>x) \wedge (\forall z\ z|w \Rightarrow ((w=z) \vee (z=1)))])$$

$$1 \longrightarrow \exists w(w>x) \wedge T$$

$$T \longrightarrow \_\_ > 100 \wedge T$$


THANK YOU GW
SOLDIERS !