CS & IT ENGINEERING

Connectivity in Graphs part 2

Lecture No. 7

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TOPICS TO BE COVERED

6

01 Annalysis In Connectivity

02 Various definition in Connectivity

03 Edge Connectivity

04 Vertex Connectivity

05 Largest inequality theorem

Connectivity in Graphs

what will be min. no. of edges $n=7$ $k=2$?

$n=7 \quad k=2$

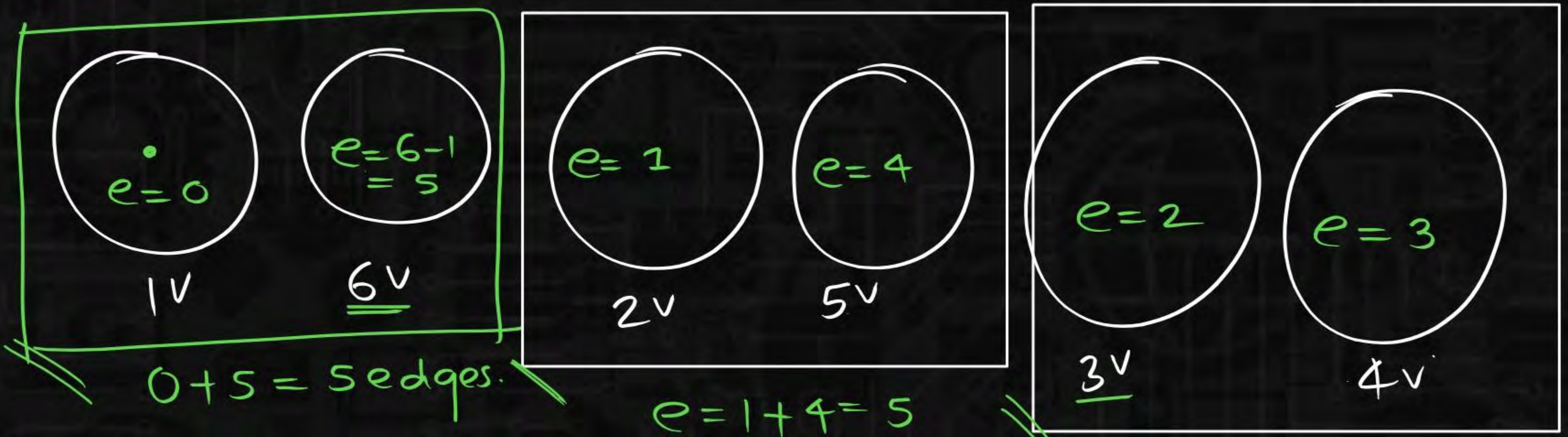
$e = n-k = 7-2 = 5.$

$$\begin{array}{cc|c} 7 & 2 & \\ 1 & 6 & \\ 2 & 5 & e=5. \\ 3 & 4 & \end{array}$$

Connectivity in Graphs

$n = 7 \quad k = 2$

$e = n - k = 7 - 2 = 5$



$0 + 5 = 5$ edges.

$e = 1 + 4 = 5$

$e = 2 + 3 = 5e$

Connectivity in Graphs

$n = 7 \quad k = 2 \quad e = (n-k)(n-k+1)/2 = 5 \cdot 6/2 = 15$

K_6	K_2	K_5	K_3	K_4
$e = 0$	$e = \frac{6 \cdot 5}{2} = 15$	$e = 1$	$e = \frac{5 \cdot 4}{2} = 10$	$e = 3$	$e = 6$
1v	6v	2v	5v	3v	4v

$0 + 15 = 15e$

$1 + 10 = 11e$

$e = 3 + 6 = 9$

Connectivity in Graphs

$n = 7 \quad k = 2 \qquad e = (n-k)(n-k+1)/2$

1v 6v

K_1 K_6

$e = 15$

2v 5v

K_2 K_5

3v 4v

K_3 K_4.

Connectivity in Graphs

$n = 11 \quad K = 3$ min no. of edges $= n-K = 11-3 = 8$

0e	0e	8e	= 8e.
1v	1v	9v	

1e	1e	6e	= 8e.
2v	2v	7v	

Connectivity in Graphs

$n = 11 \quad k = 3$

$$e = (11-3)(11-3+1)/2 = \frac{8 \cdot 9}{2} = 36e$$

$0e$ — $1v$

$0e$ — $1v$

$\frac{9 \times 8}{2} = 36$ — $9v$

$= 36e$

$1e$ — $2v$

$1e$ — $2v$

K_7, $e = \frac{7 \cdot 6}{2} = 21$ — $7v$

$= 23e$

$3e$ — $3v$

$\frac{5 \times 4}{2}$, $e = 10$ — $5v$

$e = 3$ — $3v$

$= 16$

Connectivity in Graphs

$n = 11 \quad k = 3$

1	1	9
2	2	7
3	6	2
3	3	5

$\vdots$

$K_1 \quad K_1 \quad K_9$

$n = 10 \quad k = 2$

$K_1 \quad K_9$

$n = 10 \quad k = 3$

$K_1 \quad K_1 \quad K_8$

Connectivity in Graphs

$n = 11$, $k = 3$

what will be max no. of edges.

$$e = \frac{(n-k)(n-k+1)}{2}$$

What will be max no. of edges in G, having $k=3$

$n = 100$

$k = 3$

1st → 20V

2nd component : 50V

3nd component : 30V

Connectivity in Graphs

$n = 100 \quad k = 3$

$$e = \frac{(n-k)(n-k+1)}{2}$$

K_{20} $\quad$ K_{50} $\quad$ K_{30}

$\frac{20 \times 19}{2}$ $\quad$ $\frac{50 \times 49}{2}$ $\quad$ $\frac{30 \times 29}{2}$

20V $\quad$ 50V $\quad$ 30V

Connectivity in Graphs

What will be maximum no. of edges in disconnected $(K \geq 2)$ graph of 10V? $\rightarrow$ K_1 K_9

$e = (n-k)(n-k+1)/2$

$k = 2$ $\quad$ $k = 3$ $\quad$ $k = 4$

$K_1 \quad K_9$ $\quad$ $K_1 \quad K_1 \quad K_8$ $\quad$ $K_1 \quad K_1 \quad K_1 \quad K_7$

$e = 36$ $\quad$ $e = 28$ $\quad$ $e = 21$

Connectivity in Graphs

Graph is disconnected having maximum no. of edges $n=10$ $K_1 \quad K_9 \longrightarrow \underline{K_{1,9}}$

Star

tak complement of above graph.

Connectivity in Graphs

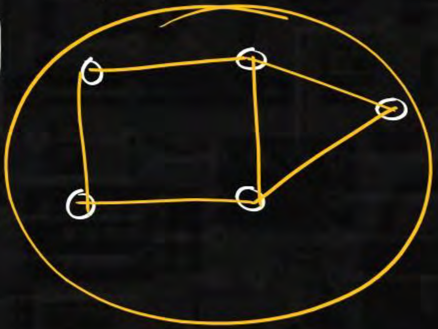
$$\underline{n = 5}$$

$$e = \frac{(n-1)(n-2)}{2} + 1$$

$$e = \frac{(5-1)(5-2)}{2}$$

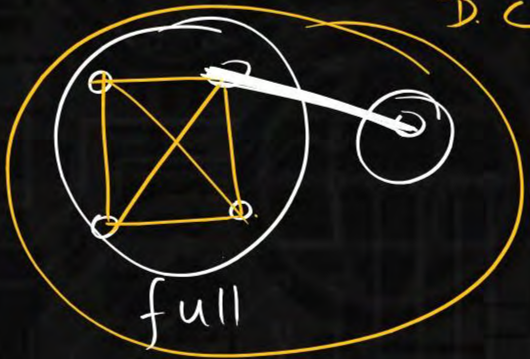
$$= \frac{4 \cdot 3}{2} = \underline{\underline{6e.}}$$

$n = 5 \quad e = 6.$



C.

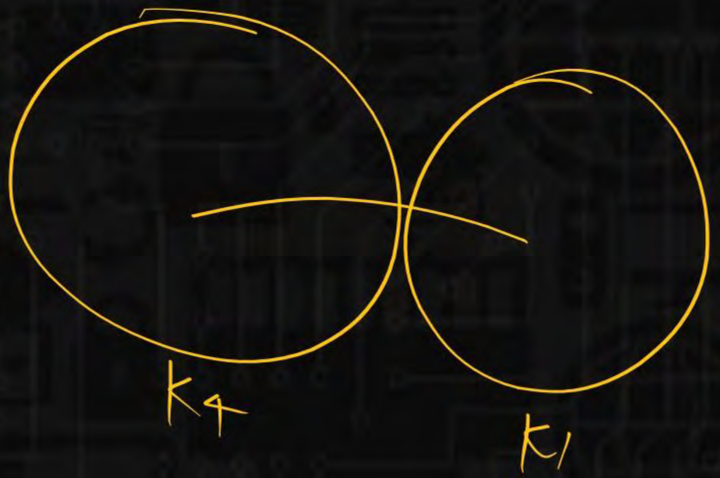
$n = 5 \quad e = 6.$ D. C



Connectivity in Graphs

Thm.

if graph contains more than $\frac{(n-1)(n-2)}{2}$ edges it will be connected.

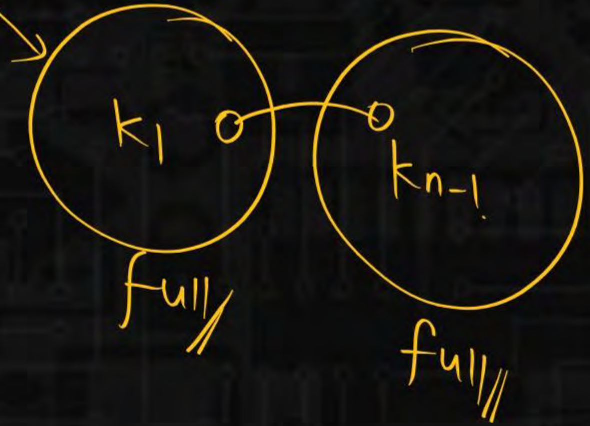


Connectivity in Graphs

$$e = \frac{(n-k)(n-k+1)}{2}$$

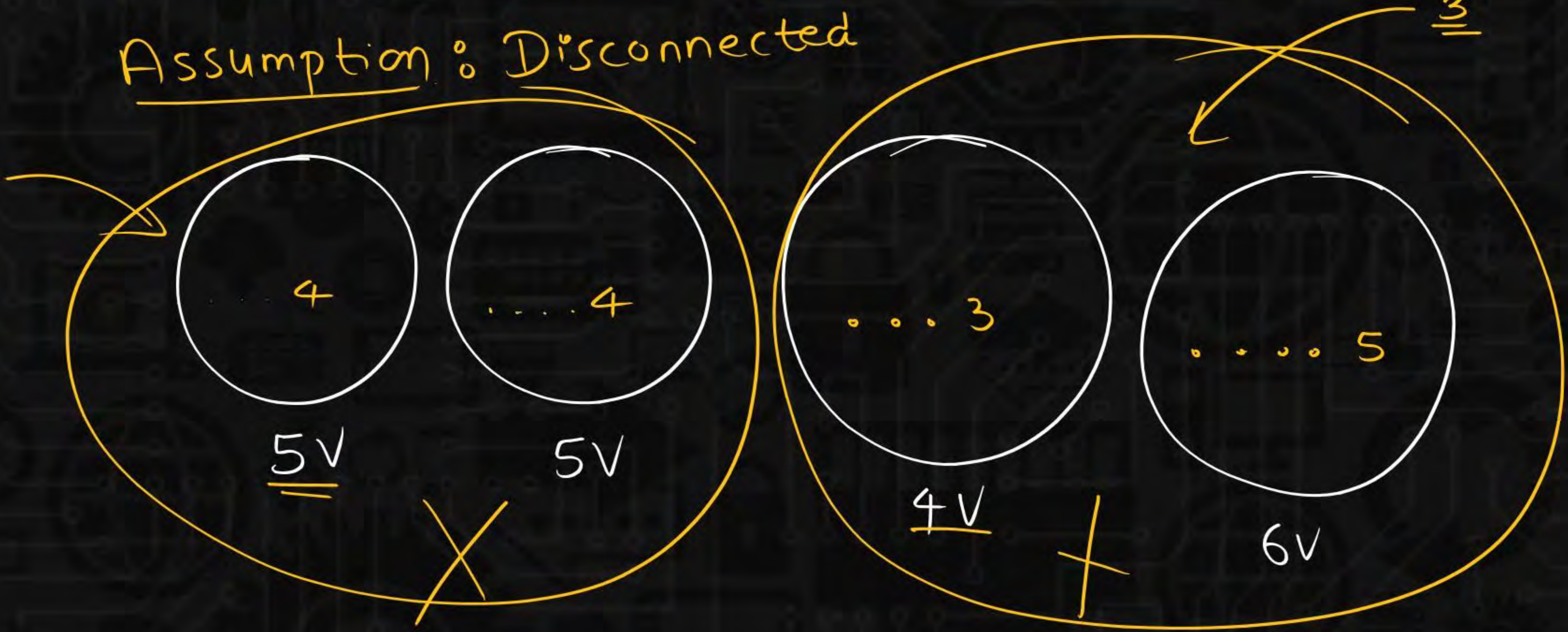
$\underline{k = 2}$

$$e = \frac{(n-2)(n-1)}{2}$$



if $\boxed{\delta(G) \geq 5}$ having $n=10$ then it is connected.

Assumption : Disconnected

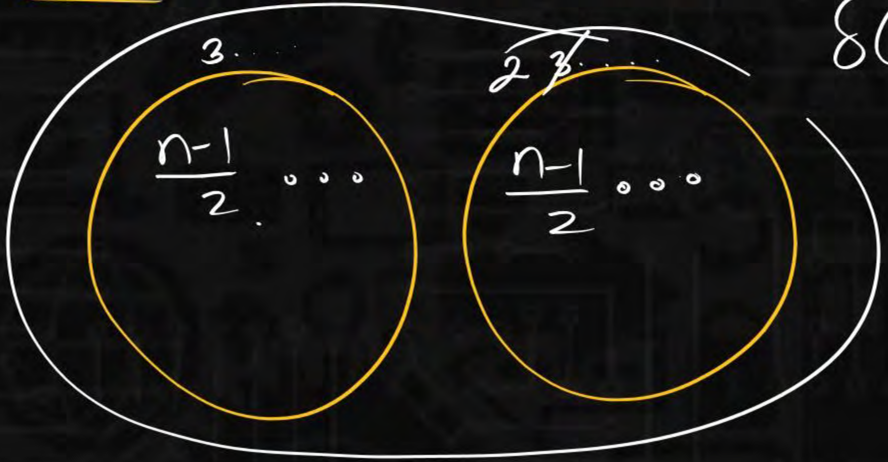


Connectivity in Graphs

Thm:

if $\delta(G) \geq \frac{n-1}{2}$ then it is connected

Assm: Disconnected.



$\delta(G) = \frac{n-1}{2}$

Connectivity in Graphs

$$\delta(G) = \frac{n-1}{2} \qquad \delta(G) = \frac{n-1}{2}$$

$$n_1 = \frac{n-1}{2} + 1 \qquad n_2 = \frac{n-1}{2} +$$

$$n = n_1 + n_2$$

$$= \frac{n-1}{2} + 1 + \frac{n-1}{2} + 1$$

$$= \frac{n-1}{2} + \frac{n-1}{2} + 2$$

$$= \frac{n-1+n-1}{2} + 2$$

$$= \frac{2n-2+4}{2} = \frac{2n+2}{2} = n+1$$

Connectivity in Graphs

$\delta(G) \geq \frac{n-1}{2}$

Assm : Disconnected

$\delta(G) = \frac{n-1}{2}$

$\delta(G) = \frac{n-1}{2}$

$\frac{n-1}{2} + 1$

$\frac{n-1}{2} + 1$

$\delta(G) = 2$

2

Total vertices

$= 2 + 1$

Connectivity in Graphs

$$\frac{n-1}{2}+1 \quad \frac{n-1}{2}+1$$

$$= \frac{n-1}{2}+\frac{n-1}{2}+2$$

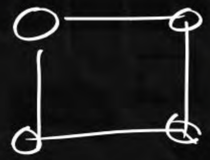
$$= \frac{2n-2}{2}+2$$

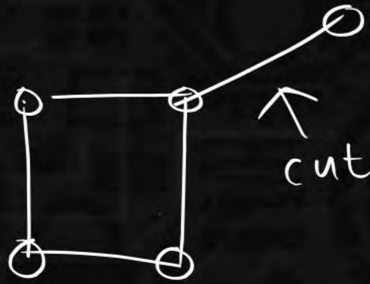
$$= \frac{2n-2+4}{2} = \frac{2n+2}{2} = n+1$$

Connectivity in Graphs

cut edge : bridge.

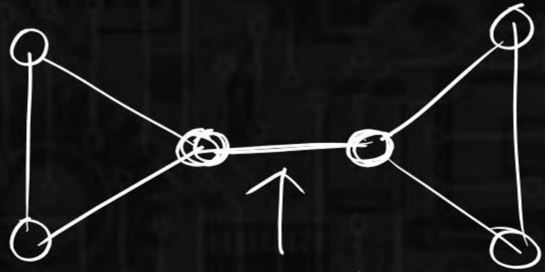
Removal of single edge from a graph will make graph as a disconnected graph.



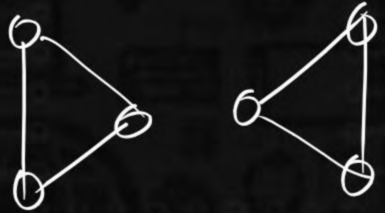


* if cut edge exist it does not belongs to cycle.

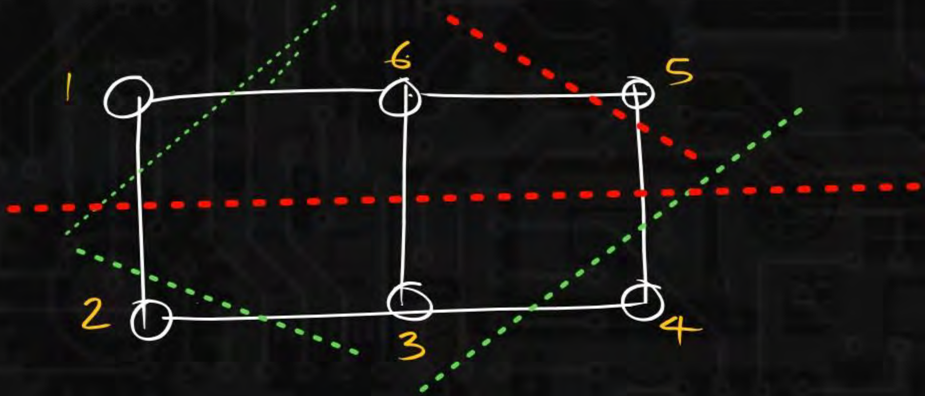
Connectivity in Graphs



cutedge.



K=2



Cut edge set :

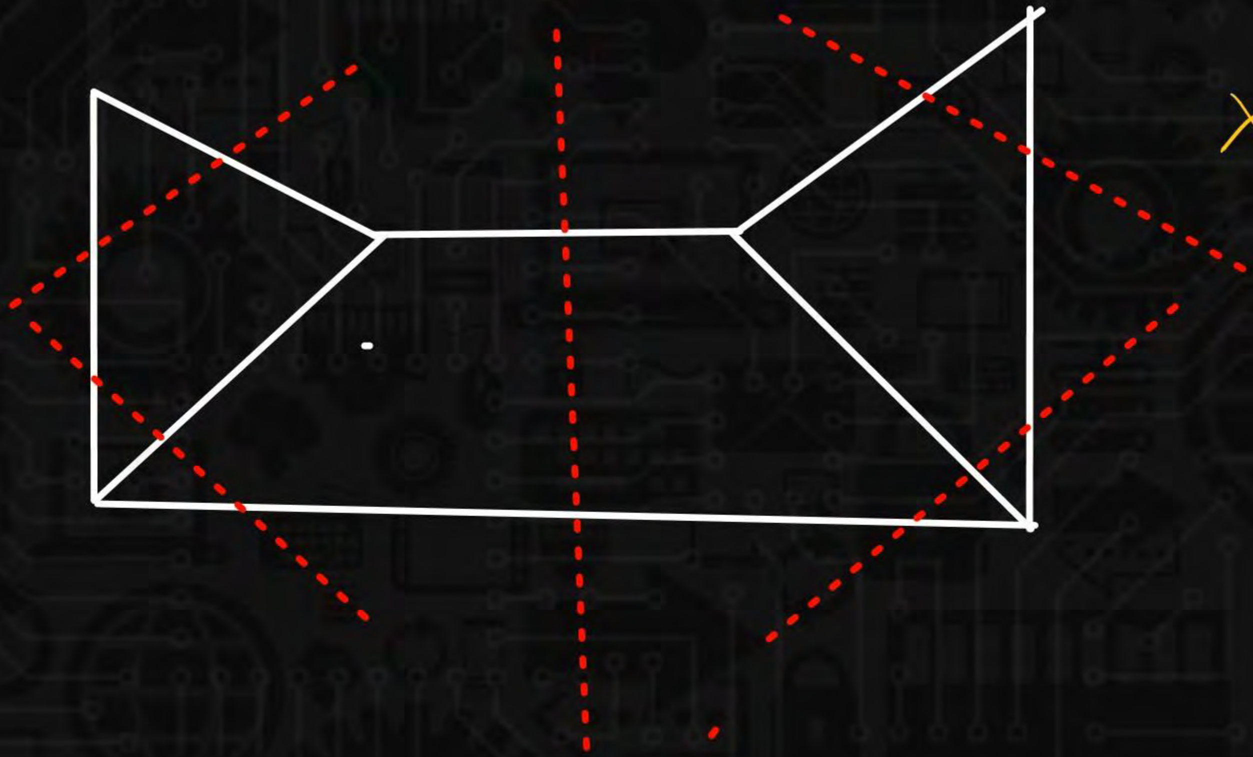
Removal of $\boxed{\text{set of edges}}$ from a graph will make graph as a disconnected graph.

in graph we will get multiple cut edge set of various sizes.

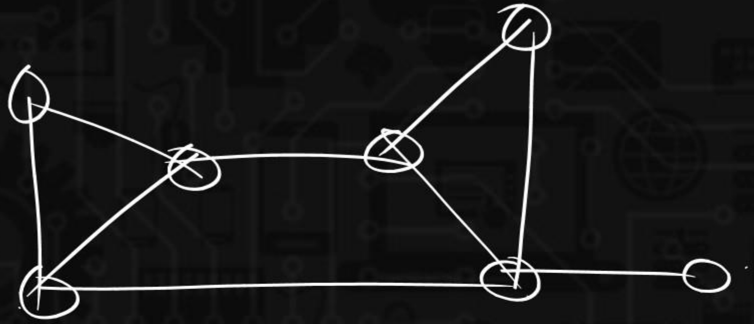
{12, 16}

edge connectivity $(\lambda(G))$

Removal of $\boxed{\text{min. no. of}}$ edges from a graph will make graph as a disconnected graph.



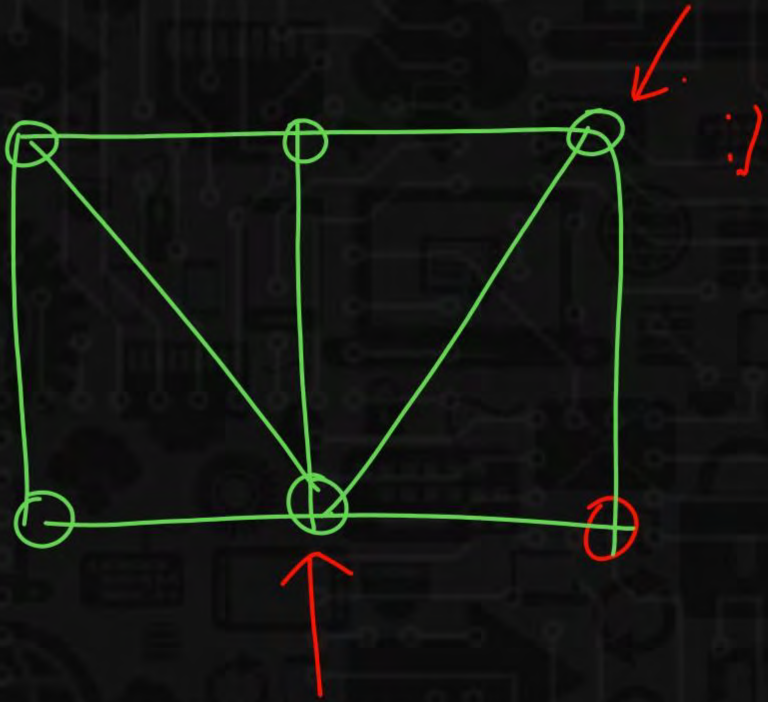
$\chi(G) = 2$



$\lambda(G) = 1$

Cut vertex set:

Removal of set of vertices from a graph will make graph as a disconnected graph.



Vertex connectivity ($K(G)$)

Removal of min. no. of vertices from a graph will make graph as a disconnected graph.

cut edge → 1 edge

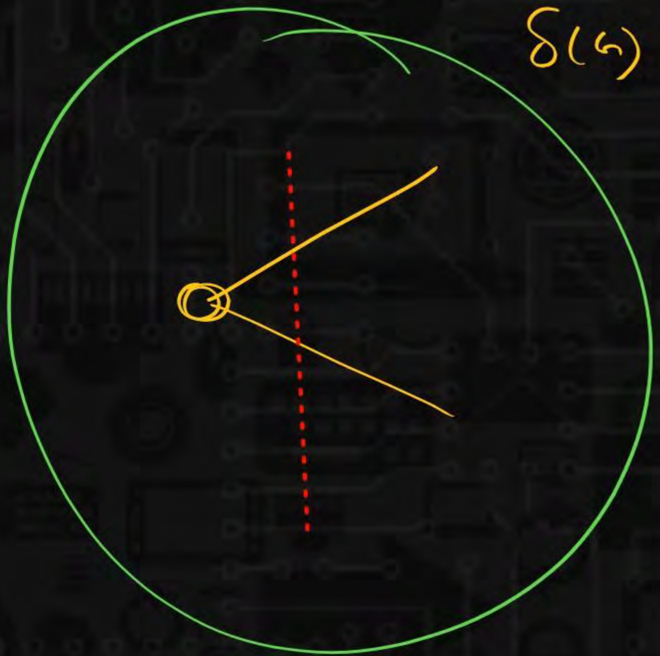
cut edge set → set of edges.

edge connectivity
↳ min no. of edges.

cut vertex → 1 vertex.

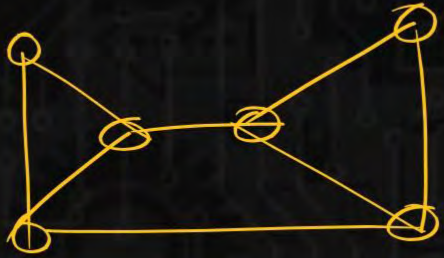
cut vertex set.
↳ set of vertices.

vertex connectivity
↳ min no. of vertices.



$\delta(G)$ min degree vertex = ~~min no. of edges~~

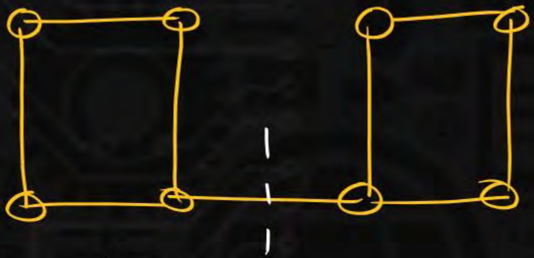
edge connectivity



$\delta(G) = 2$

$\lambda(G) = 2$.

$\boxed{\lambda(G) \leq \delta(G)}$

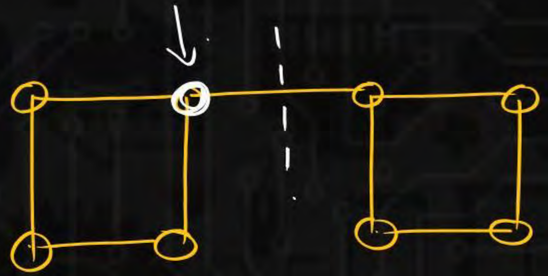


$\delta(G) = 2$

$\lambda(G) = 1$.

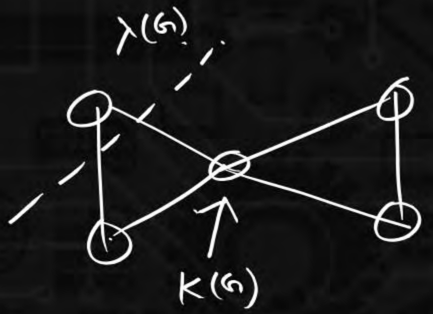
$\lambda(G) = \delta(G)$ ✓

$\lambda(G) < \delta(G)$



$\lambda(G) = 1$

$K(G) = 1.$



$\lambda(G) = 2$

$k(G) = 1.$

$K(G) = \lambda(G)$

$K(G) < \lambda(G)$

$$K(G) \leq \lambda(G)$$

$$\lambda(G) \le \delta(G)$$

$$k(G) \le \lambda(G)$$

$$\delta(G) \le \frac{2e}{n} \le \Delta(G) \le n-1$$

$$k(G) \le \lambda(G) \le \delta(G)$$

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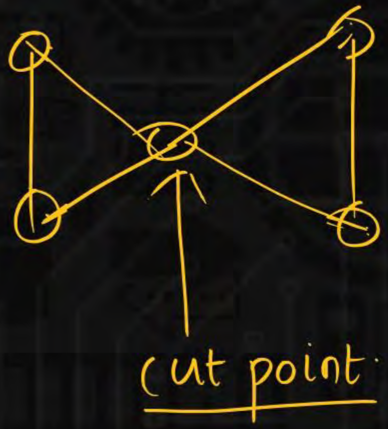
$$k(G) \le \lambda(G) \le \delta(G) \le \frac{2e}{n} \le \Delta(G) \le n-1.$$

Cut vertex/cut point/Articulation point:

Removal of single vertex from a graph will make graph as a disconnected graph.




THANK YOU GW
SOLDIERS !