CS & IT ENGINEERING

Connectivity in Graphs

Lecture No. 6

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TOPICS TO BE COVERED

01 Definition In Connectivity

02 Connected vs Disconnected

03 Range of Edges

04 Concepts of tree

05 Connectivity theorem

Walk: (R.V | R.E)

alternate sequence of vertices & edges.

1 e_1 3 e_3 2 e_3 3 e_5 4.

R.E.

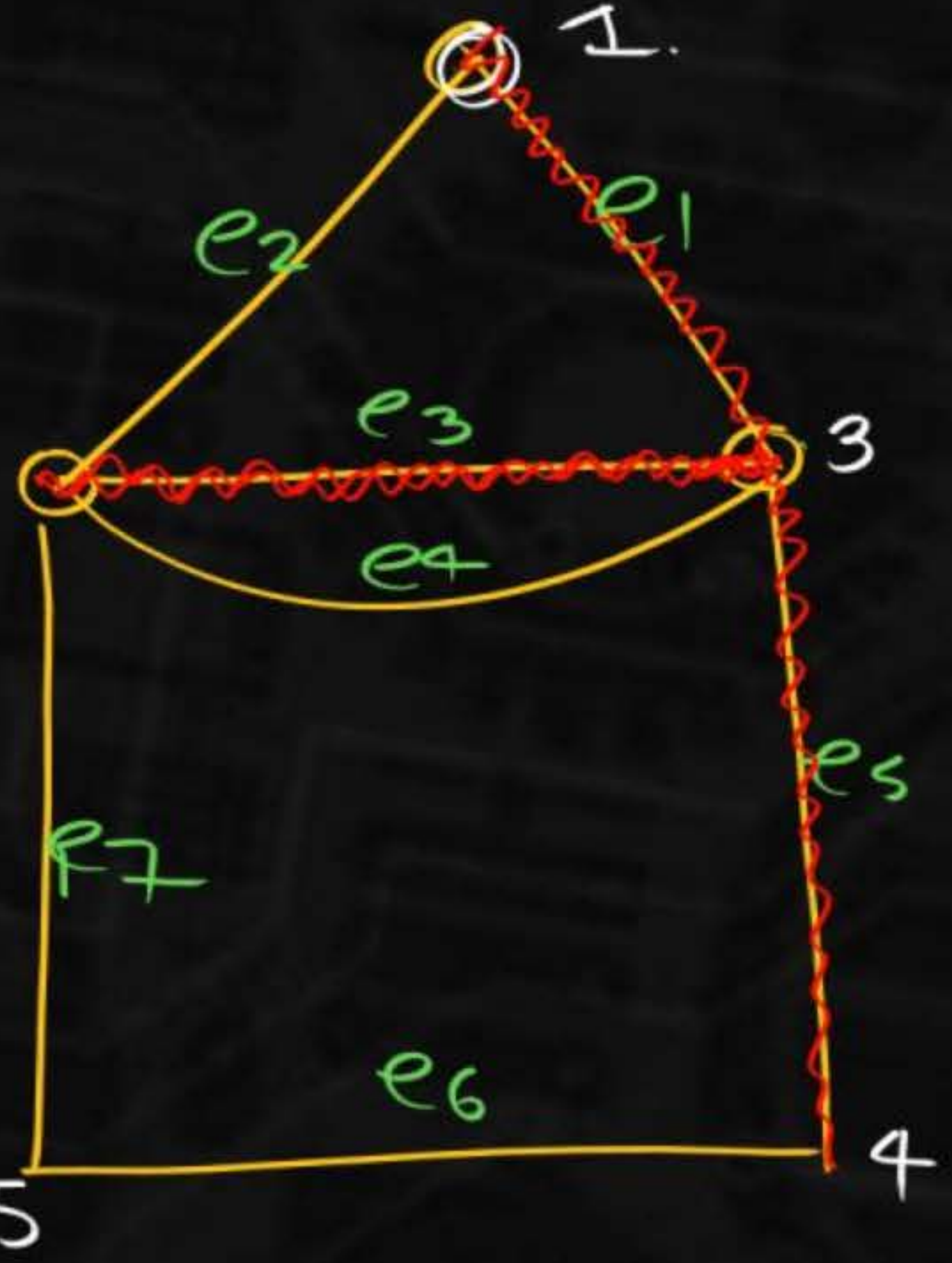
Trail:

alternate sequence vertices & E

(R.V | RXE) 1 e_1 3 e_3 2 e_4 3 e_5 4

R.V

Path: alternate sequence of vertices & edges. (RXV | RXE)

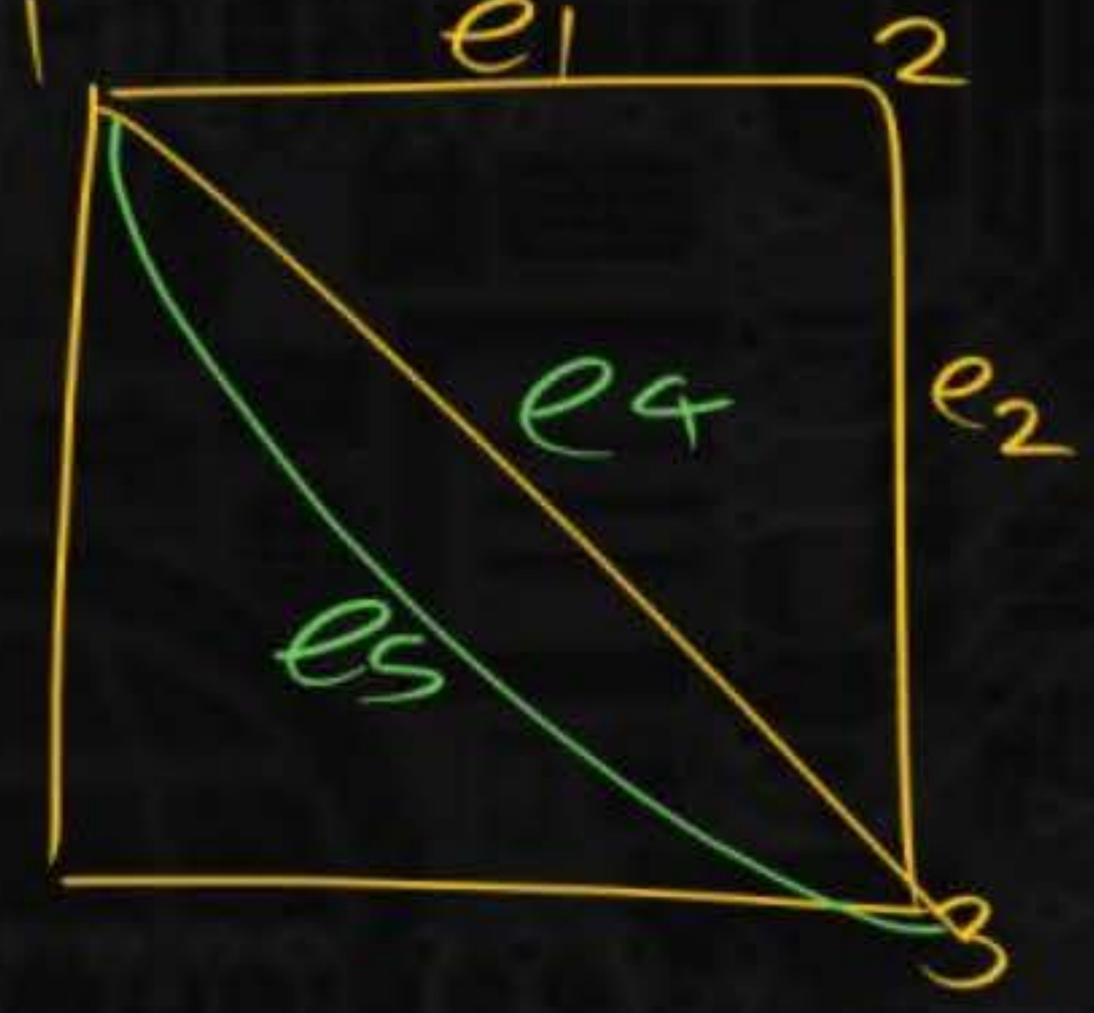


Connectivity in Graphs

Walk R.V | R.E $1\, e_1\, 2\, e_2\, 3\, e_2\, 2$ R.V R.E

Trail R.V | $R\cancel{\cdot}E$ | $1\, e_4\, 3\, e_5\, 1$ R.V

Path $R\cancel{\cdot}V$ | $R\cancel{\cdot}E$ $1\, e_1\, 2\, e_2\, 3$

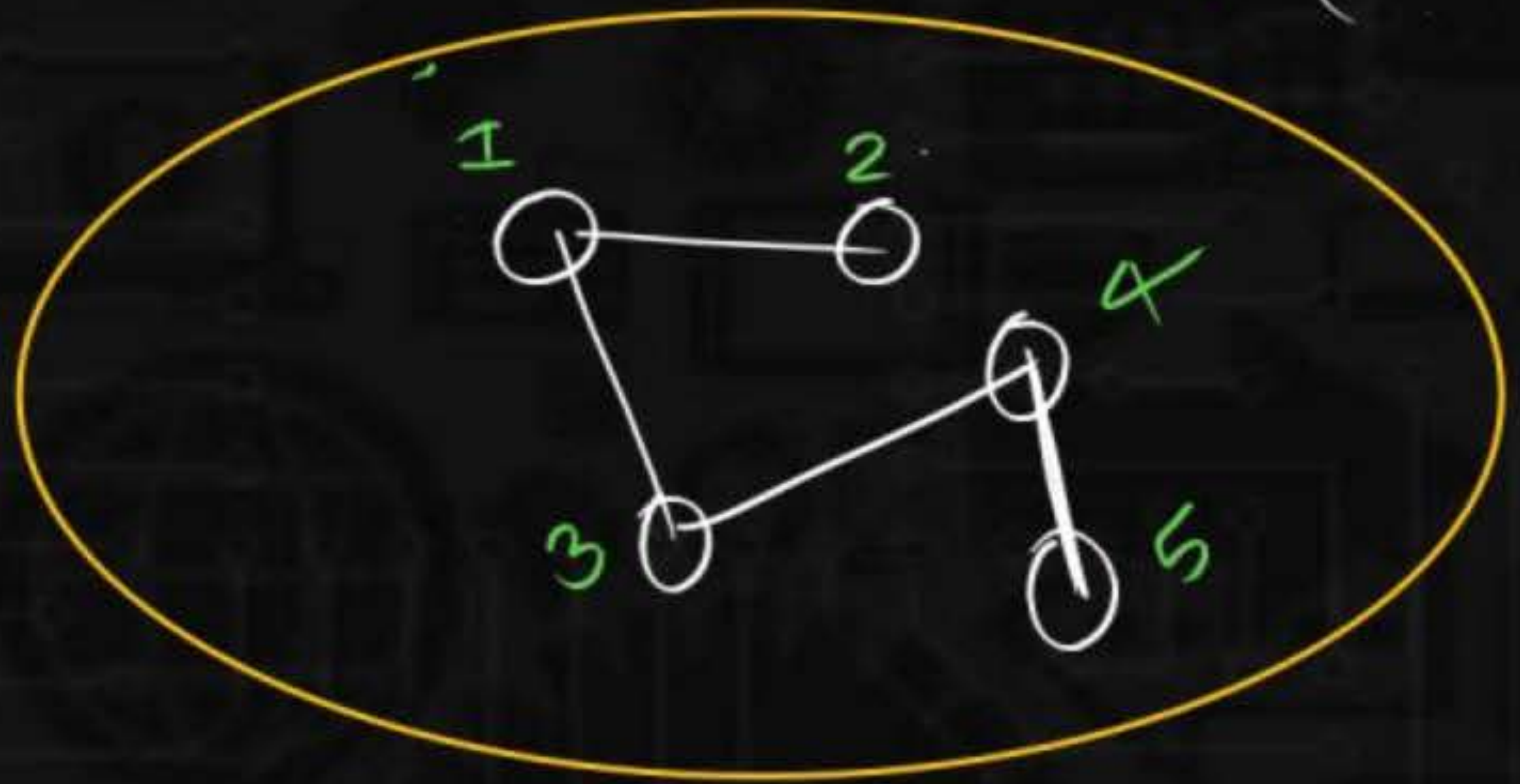


Connectivity in Graphs

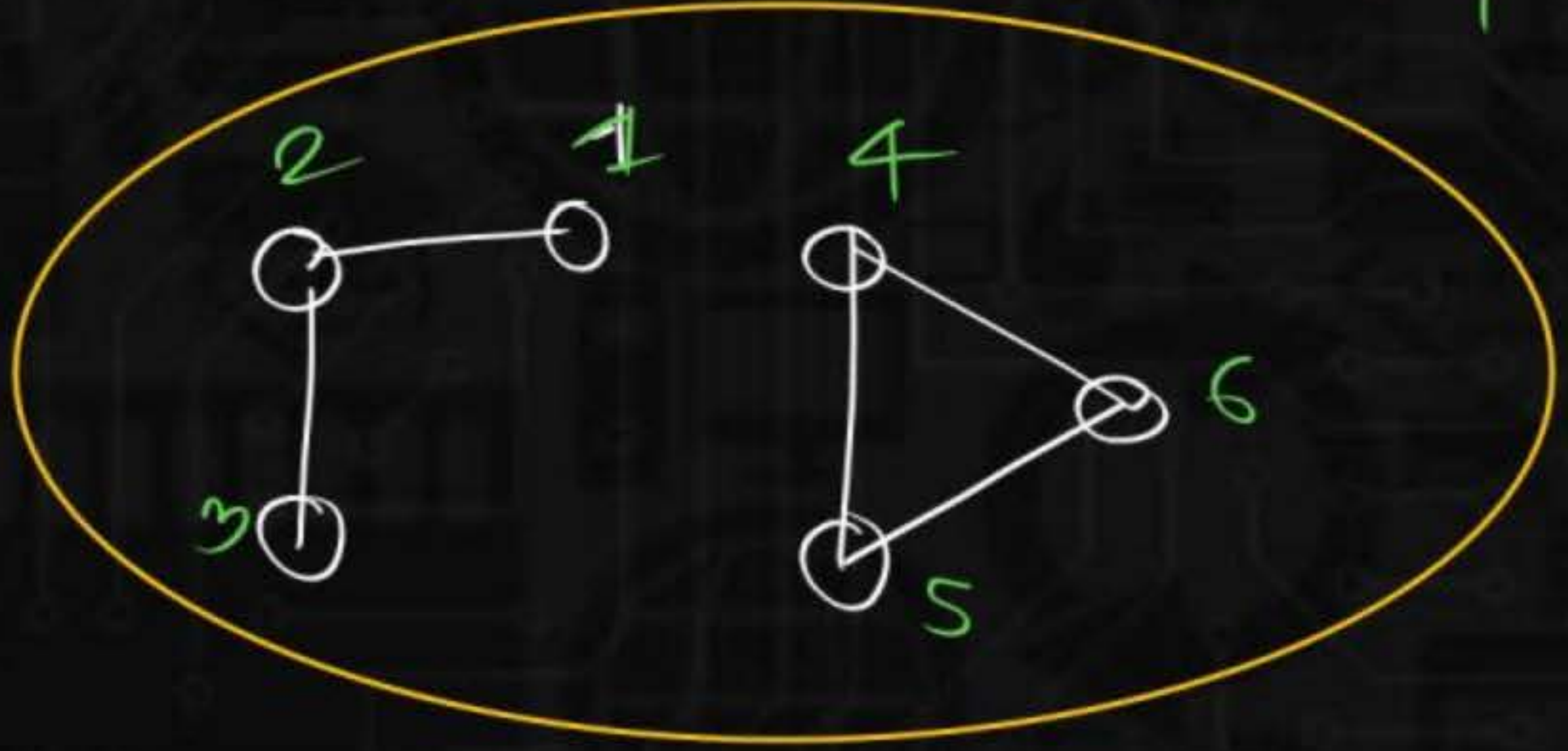
Graph

- Path is available betⁿ all pair of vertices → Connected Graph ($K=1$)
- for atleast 1 pair "Path is not available" → Disconnected Graph ($K \geq 2$)

Connected Graph ($K=1$)



Disconnected Graph ($K \geq 2$)



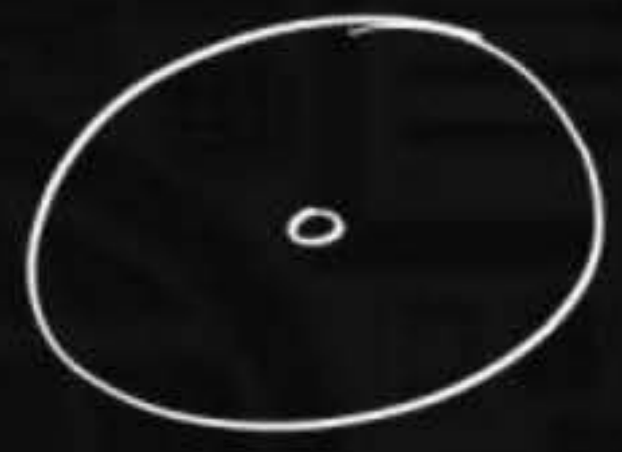
Connectivity in Graphs

Disconnected graph

Contains connected subparts

Component (K)

Connected graph



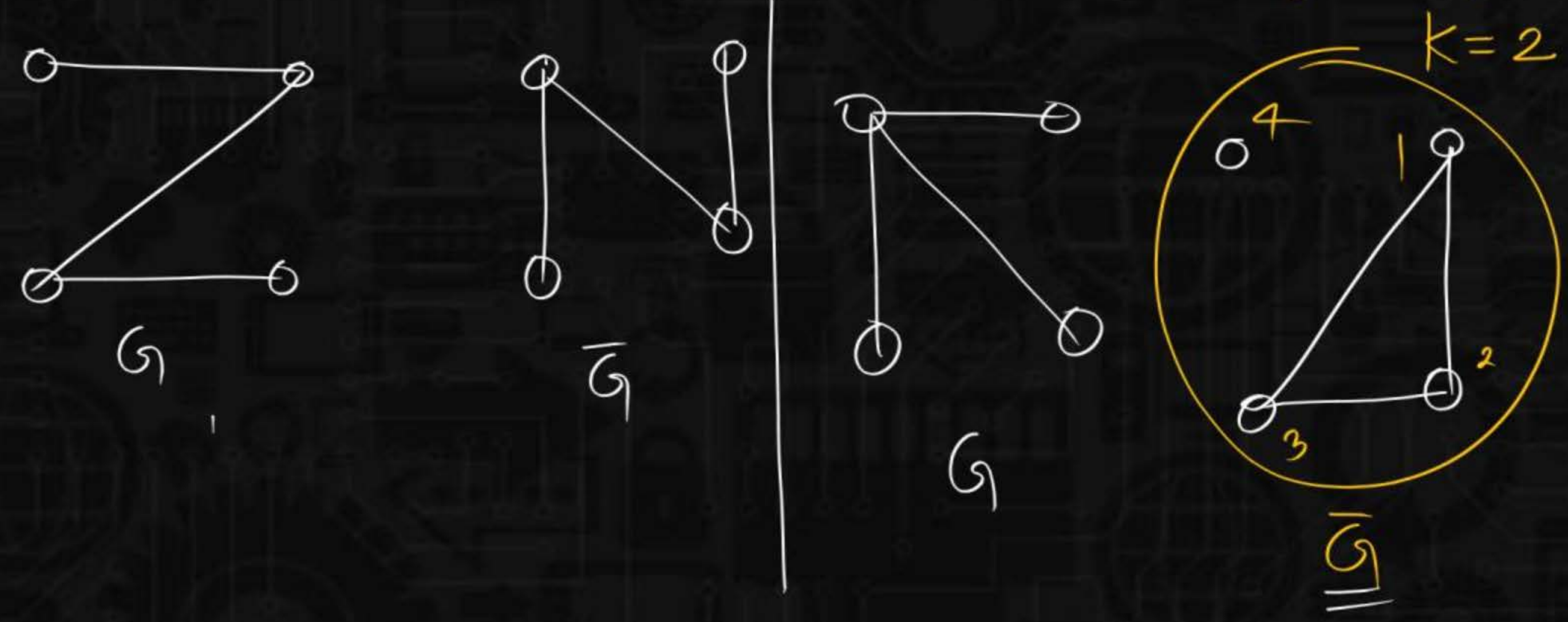
k = 1

Disconnected

k = 2

Connectivity in Graphs

if G is connected then $\overline{G}$ is connected (false)

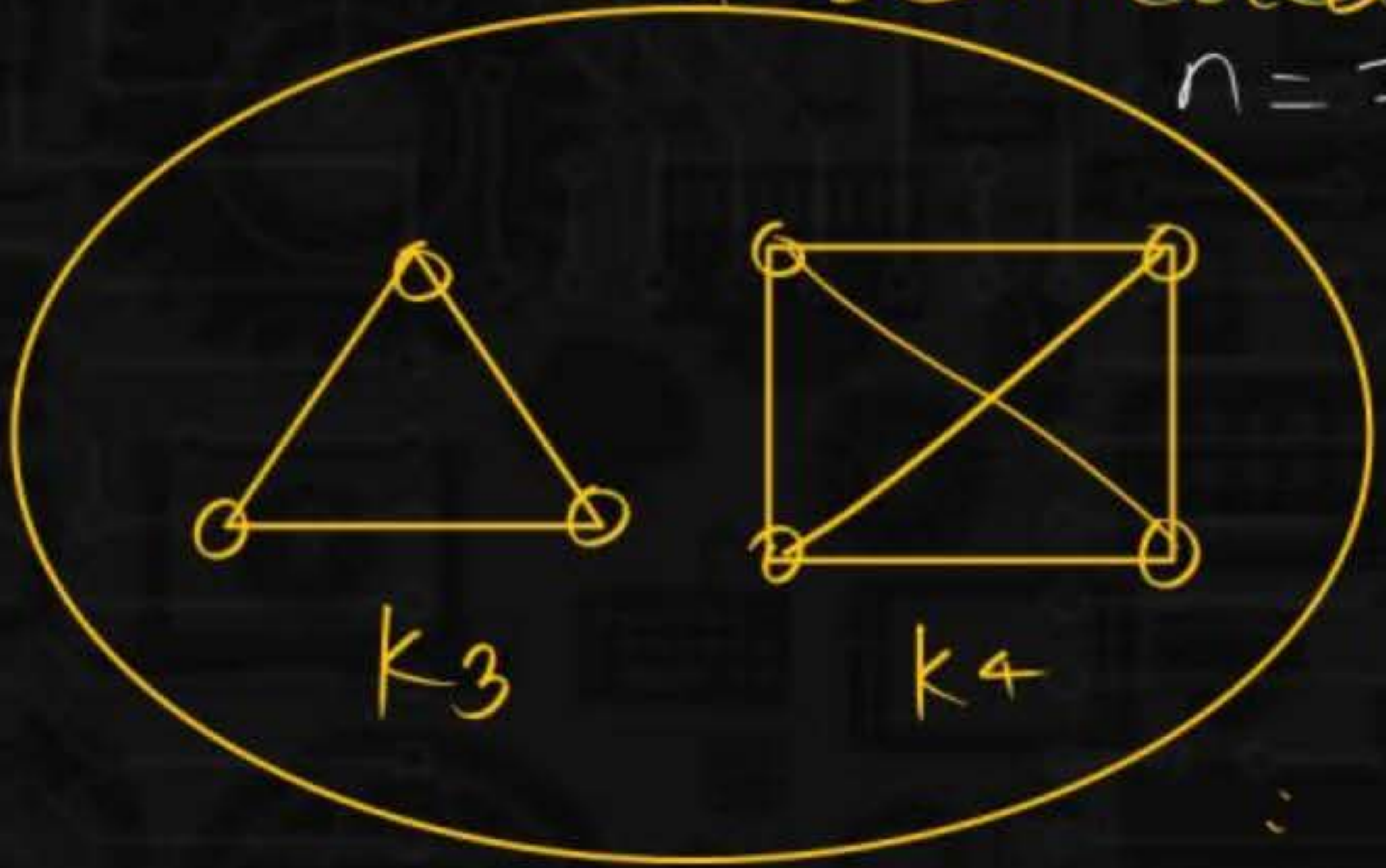


Connectivity in Graphs

if G is Disconnected then $\overline{G}$ is connected (True)

Disconnected

$n=7$



$\underline{K=1}$

connected Graph

Connectivity in Graphs

Range of edges $(K=1)$ (Connected Graph)

Tree

1. $n-1 \leq e \leq n(n-1)/2$

   + connected

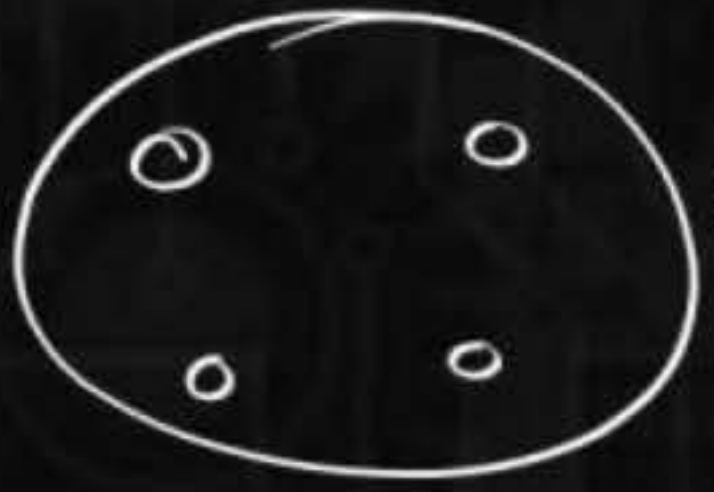
2. unique path is available betⁿ all pair of vertices

3. Graph does not contains cycle.

4. G is minimally connected.

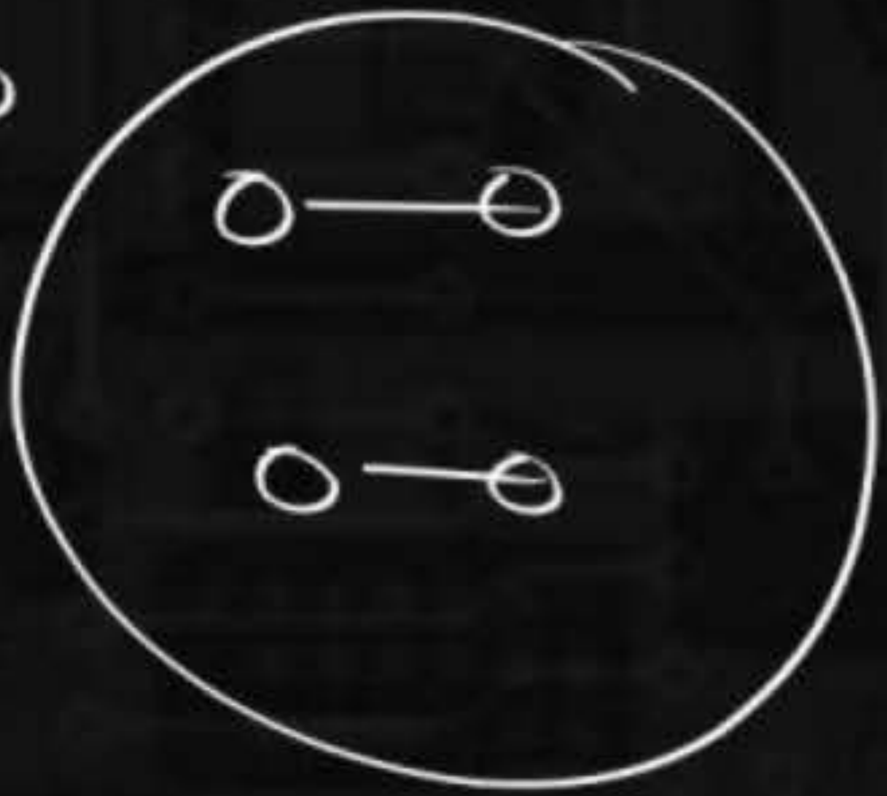
Connectivity in Graphs

$n = 4$.

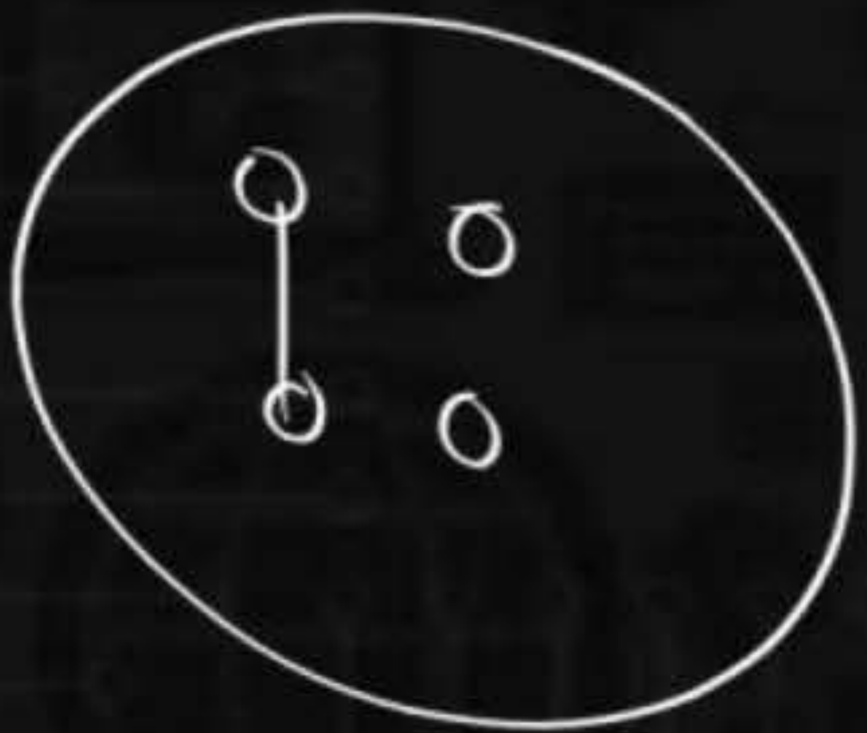


$e = 0$

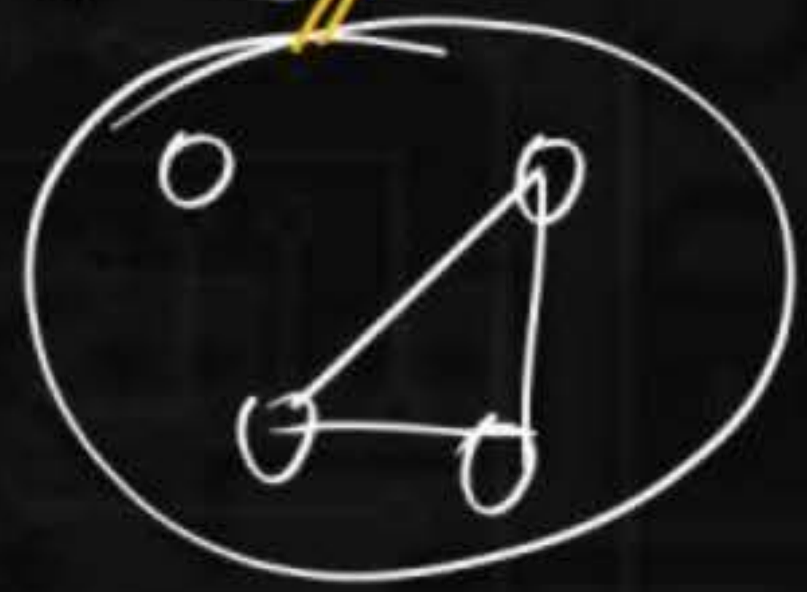
$e = 2$

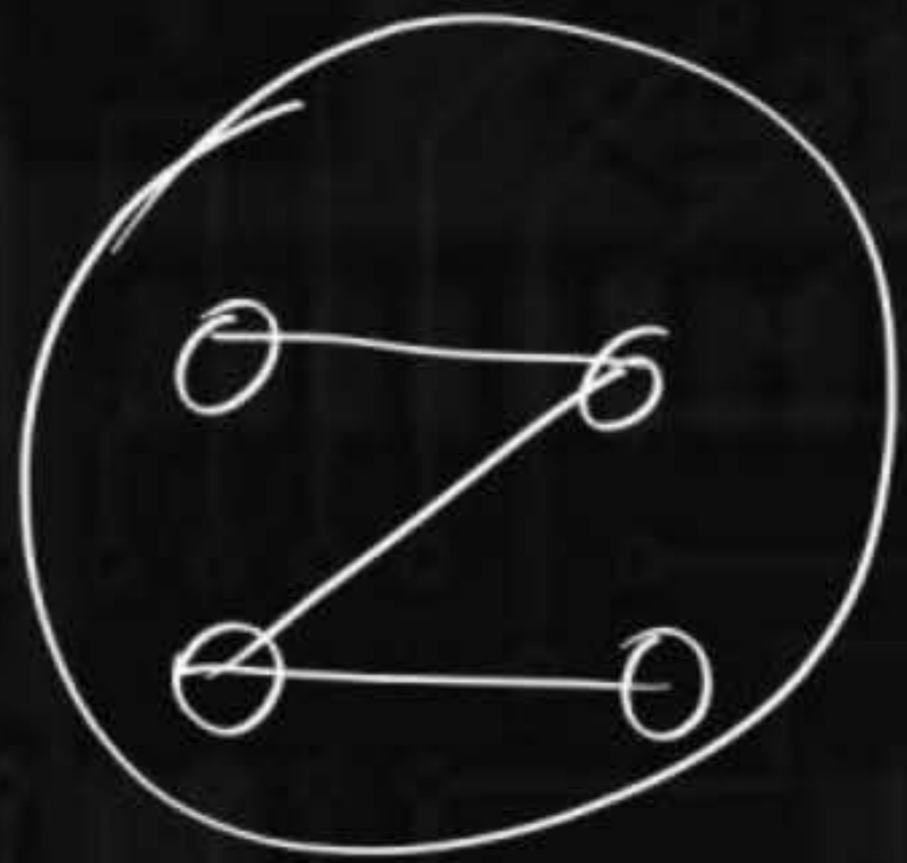


$e = 1$



$e = 3$





connected
+
min-no. of edges.

Connectivity in Graphs

1. Graph is minimally connected then G is Tree.
2. Graph contains unique path betⁿ all pair of vertices then G is Tree
3. Graph does not contains cycle then G is Tree
4. Graph connected & $(n-1)$ edges then G is Tree

Connectivity in Graphs

Range of edges ($k \geq 2$) (disconnected)

forest

$$\underbrace{n-k}_{\text{min no. of edges}} \leq e \leq \underbrace{\frac{(n-k)(n-k+1)}{2}}_{\text{max no. of edges}}$$

(k is component)

Collection of Trees.

Connectivity in Graphs

$$n-k \le e \le \frac{(n-k)(n-k+1)}{2}$$

$k=1$ (connected)

$$n-1 \le e \le \frac{(n-1)(n)}{2}$$

Connectivity in Graphs

$G = (V, E)$

Total vertices $= n$

4 components

$K = 4$

1st component $\rightarrow n_1$ vertices

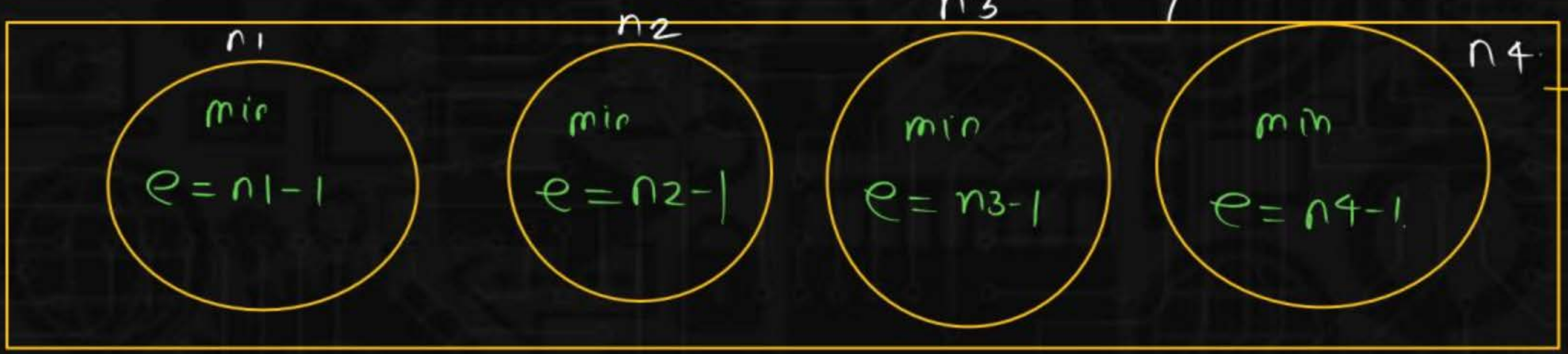
$\rightarrow n_2$

$\rightarrow n_3$

$\rightarrow n_4$

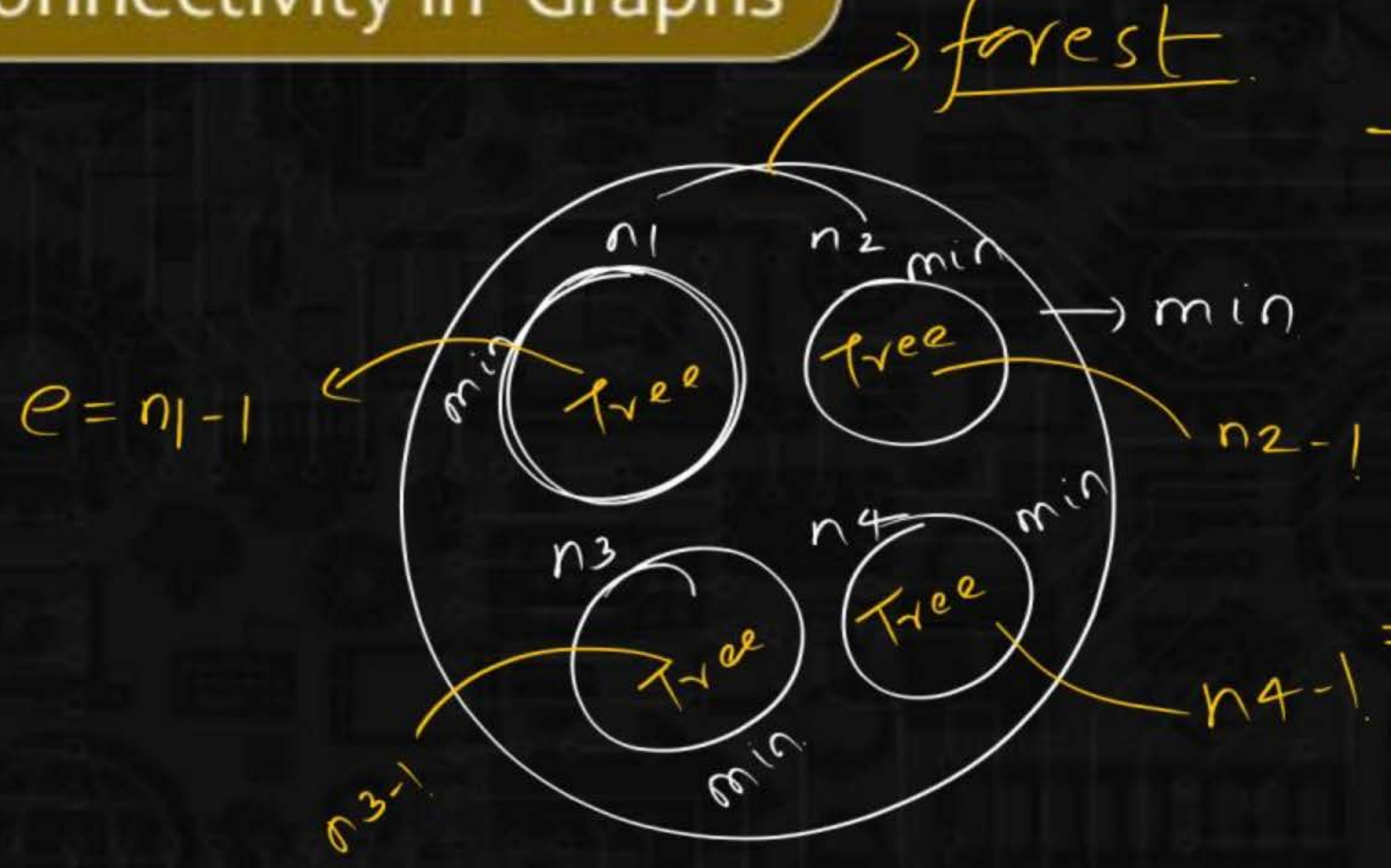
$n_1 + n_2 + n_3 + n_4 = n$

Total vertices $= n$



— min

Connectivity in Graphs



Total min no. of edges

$= n_1 - 1 + n_2 - 1 + n_3 - 1 + n_4 - 1$

$= \underbrace{(n_1 + n_2 + n_3 + n_4)}_{n} - 4$

$n - 4$

$= n - k$

Connectivity in Graphs

Consider a graph having 4 components with 16 vertices, what will be min, & max no. of edges?

$$\min = n-k = 16-4 = 12$$

$$e = \frac{(n-k)(n-k+1)}{2}$$

$$n = 16$$
$$k = 4$$

$$= \frac{(16-4)(16-4+1)}{2}$$

$$= \frac{\cancel{12}^{6} \times 13}{\cancel{2}} = 13 \times 6 = (10+3) \times 6$$

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THANK YOU GW
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