CS & IT ENGINEERING

Predicate logic

Lecture No. 04

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TOPICS TO BE COVERED

01 open statements

02 predicate variables

03 Universe of Discourse

04 Quantifier

05 Theorems on Quantifier

Truth value → Simple propositional.
T/F

Compound

p	q	$p \wedge q$

predicate variable.

$P(x)$: $\boxed{\cancel{x} \text{ is even no.}}$ $\longrightarrow$ open statement

↳ we can not define truth value but once we put input into this it changes simple stmt

$P(0)$: $\cancel{0}$ is even no. (T)

$P(1)$: $\cancel{1}$ is even no $\rightarrow$ (F)

$P(2)$: $\cancel{2}$ is even no (T)

$x + y \leq 0$

Domain of discourse

universe of discourse

<u>Domain</u> :

0

1

3

⋮

10

open stmt

$x \leq 0$

⋮

Simple stmt : p

Simple stmt : q

<u>Domain</u>: set of possible choices for open stmt.

$\rightarrow D : \{1, 2\}$

$P(x) : x \leq 3$

$P(1) : 1 \leq 3 \ (T)$

$P(2) : 2 \leq 3 \ (T)$

1, 2

T T

1, 2 10

1 .... 1L

1 .... 1K

Quantity : ....

Truth value
↓
in terms of quantity

ALL ✓
Some ✓

Quantifier:

Tool in terms to ask the truth value in quantity

Quantity:

↳ check all elements are True or not

$\forall x\, P(x)$

Universal quantifier $(\forall x)$ → check all elements are True or not in $P(x)$

$\forall x\, P(x) \rightarrow$ True

→ for all of x such that $P(x)$

for each of x such that $P(x)$

for every element of x such that $P(x)$

$\forall x P(x) \rightarrow$ True

T T
T T

$\rightarrow$ all elements in the domain must be True.

$\forall x P(x) \rightarrow$ false

F $\rightarrow$ atleast 1 false

$D : \{1, 2, 3\}$

$P(x) : x^2 \leq 9$

$P(1) : 1^2 \leq 9\ (T)$

$P(2) : 2^2 \leq 9\ (T)$

$P(3) : 3^2 \leq 9\ (T)$

$\underline{\forall x\, P(x)} \rightarrow$ True. OR $\begin{cases} D : \{1, 2, 3\} \\ \underline{\forall x}\,(x^2 \leq 9) \rightarrow \text{True} \end{cases}$

1. 2
3 $\rightarrow$ all elements are True

$D : \{1, 2, 3\}$

$p(x) : x^2 = 4$

$p(1) : 1^2 = 4 \ (F)$

$p(2) : 2^2 = 4 \ (T)$

$p(3) : 3^2 = 4 \ (F)$

$D : \{1, 2, 3\}$

$\forall x \, p(x) \rightarrow \text{false}$

1(F)
2(T)
3(F) → at least 1

$D : \{1, 2, 3\}$

$\forall x (x^2 = 4) \rightarrow$ false.

1. $D : \mathbb{Z}$

$P(x) : x^2 \geq 0$

$\forall x\, P(x)$

$\forall x\, P(x) \to$ True.

2. $D : \mathbb{Z}$

$P(x) : x^2 > 0$

$\forall x\, P(x)$

$\forall x\, P(x) \to$ false.

$0 \to F$.

Doubts:

1. $D : \mathbb{Z}$

$P(x) : x^2 > 0$

$\forall x\, P(x) \to$ False.

2. $D : \mathbb{Z}^+$

$P(x) : x^2 > 0$

$\forall x\, P(x) \to$ True.

$D: Z.$ $\qquad$ $D: Z.$

$P(x): x^2 \geq 0$ OR $P(m): m^2 \geq 0$

$\forall x\, P(x)$ $\qquad$ $\forall m\, P(m)$

check some of the elements
are True or not

Existential quantifier : $(\exists x)$

$\exists x\, P(x)$

- some value of x such that $P(x)$
- at least 1 value of x such that $P(x)$
- there exist x such that $P(x)$

$\exists x\, P(x)$ → True

→ at least 1 True.

$\exists x\, P(x) \rightarrow$ false

→ when all are false

$\forall x\, p(x) \rightarrow$ True

T T
T T

→ when all elements are True

$\forall x\, p(x) \rightarrow$ false

F → atleast 1 false

$\exists x\, p(x) \rightarrow$ True.

T → at least 1 True

$\exists x\, p(x) \rightarrow$ false

→ all false

$D : \{1, 2, 3\}$

$\exists x \, P(x)$

$P(x) : x^2 = 9$

$P(1) : 1^2 = 9 \; (F)$

$P(2) : 2^2 = 9 \; (F)$

$P(3) : 3^2 = 9 \; (T)$

1

2

3 (T) $\rightarrow$ atleast 1 True.

$\exists x \, P(x) \rightarrow \text{True}$

$\exists x \, (x^2 = 9)$

$D : \{1, 2, 3\}$

$P(x) : x^2 = 16$

$\boxed{\exists x \, (x^2 = 16)} \rightarrow \text{false}$

$P(1) : 1^2 = 16 \; (F)$

$P(2) : 2^2 = 16 \; (F)$

$P(3) : 3^2 = 16 \; (F)$

$D: Z$

$p(x): x^2 \geq 0$

$\underline{\forall x\, p(x)} \rightarrow$ True

$D: Z$

$p(x): x^2 \geq 0$

$\boxed{\exists x\, p(x)} \rightarrow$ True

$\underline{\underline{D: Z}}$

$$\underbrace{\forall x\, P(x)}_{\text{True}} \rightarrow \underline{\exists x\, P(x)}$$

negation of quantifier :

$$\neg \boxed{\forall x\, p(x)} \equiv \exists x \neg p(x)$$

$$\neg \{\exists x\, p(x)\} \equiv \forall x \neg p(x)$$

$$\neg [\forall x\, p(x)]$$

$$\neg\{\forall x\, P(x)\} \equiv \neg\{P(1) \wedge P(2) \wedge P(3)\}$$

$$\equiv \boxed{\neg P(1)} \vee \boxed{\neg P(2)} \vee \boxed{\neg P(3)}$$

$$\{1, 2\}$$

$$\neg\{\forall x\, P(x)\} \equiv \neg\{P(1) \wedge P(2)\}$$

$$\equiv \neg P(1) \vee \neg P(2)$$

$$\equiv \exists x \neg P(x)$$

$D: \{1, 2, 3\}$

$P(x): x+1=4$

$\boxed{\exists x\, P(x)} \rightarrow$ True

$x=3$

$P(1): 1+1=4\ (F)$

$P(2): 2+1=4\ (F)$

$P(3): 3+1=4\ (T)$

$D: \{1, 2, 3\}$

$Q(x): 2x+1=7$

$\exists x\, Q(x)$

$x=3$

$Q(1): 2(1)+1=7\ (F)$

$Q(2): 2(2)+1=7\ (F)$

$Q(3): 2(3)+1=7\ (T)$

$x=3$ $\boxed{P(x): x+1=4}$ $\boxed{Q(x): 2x+1=7}$ — $x=3$.

$D: \{1, 2, 3\}$

$\exists x\,[P(x) \wedge Q(x)] \longrightarrow \exists x\,P(x) \wedge \exists x\,Q(x)$

True

$x=1 \rightarrow$ $P(1)$ F $\wedge$ $Q(1)$ F — F

$\vee$

$x=2 \rightarrow$ $P(2)$ F $\wedge$ $Q(2)$ F $\rightarrow$ F

$\vee$

$x=3 \rightarrow$ $P(3)$ T $\wedge$ $Q(3)$ T — T

$P(x): x+1=4$ — T

$\exists x\,P(x)$

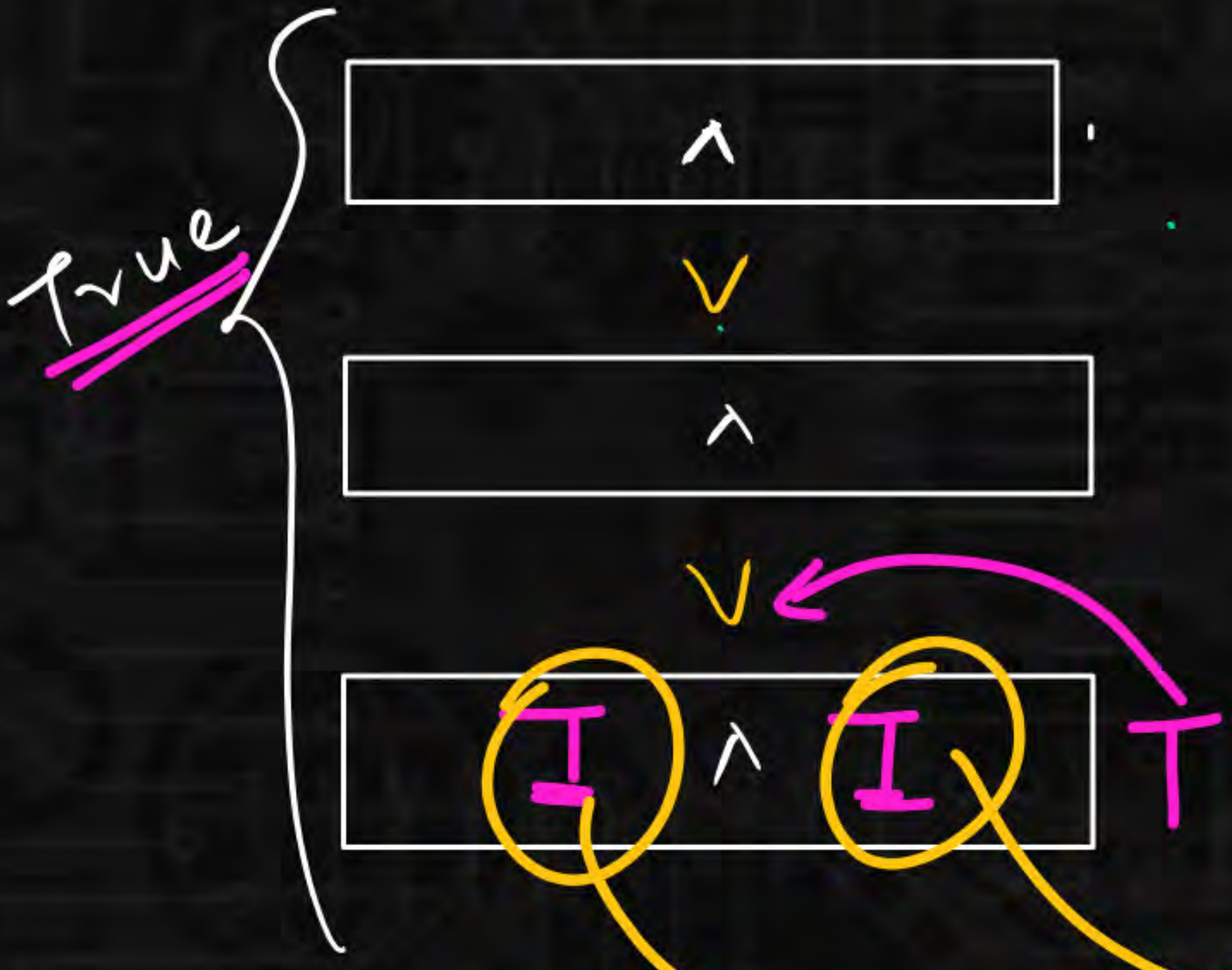
$Q(x): 2x+1=7$ — T

$\exists x\,Q(x)$

$P(1)$ — F		$Q(1)$ — F
$\vee$		$\vee$
$P(2)$ — F	$\wedge$	$Q(2)$ — F
$\vee$		$\vee$
$P(3)$ — T		$Q(3)$ — T

$$\underbrace{\exists x\,(\underline{P(x) \wedge Q(x)})}_{\text{common element}}^{T} \longrightarrow \exists x P(x) \wedge \exists x Q(x)$$

$$\boxed{T \quad \wedge \quad T} \rightarrow \underline{\underline{\text{True.}}}$$



$$\overset{T}{\overline{\exists x\,(P(x) \wedge Q(x))}} \longrightarrow \overset{\text{Check}}{\exists x P(x) \wedge \exists x Q(x)}$$

$$T \wedge T \equiv T$$

$T.$ $\left\{ \quad \boxed{} \quad \vee \quad \boxed{T \wedge T} \right.$ $T.$

$\vee \qquad \wedge \qquad \vee$

$T. \qquad\qquad T.$

$$\overset{T}{\overline{\exists x P(x) \land \exists x Q(x)}} \not\to \underline{\underline{\exists x}}(P(x) \land Q(x))$$

$T \qquad T$

$$\begin{array}{ccc} T & & F \\ \lor & \land & \lor \\ F & & T \end{array}$$

$$\underbrace{T \land T}_{T}$$

$$\left.\begin{array}{l} \boxed{T \land F} \to F \\ \quad \lor \\ \boxed{F \land T} \to F \end{array}\right\} \text{false}$$

$$\exists x\,[P(x) \wedge Q(x)] \rightarrow \exists x P(x) \wedge \exists x Q(x)$$

H.W.

$$\exists x\,[P(x) \vee Q(x)] \qquad \exists x P(x) \vee \exists x Q(x)$$

$$\forall x\,[P(x) \wedge Q(x)] \qquad \forall x P(x) \wedge \forall x Q(x)$$

$$\forall x\,[P(x) \vee Q(x)] \qquad \forall x P(x) \vee \forall x Q(x)$$

$$\forall x\,[P(x) \rightarrow Q(x)] \qquad \forall x P(x) \rightarrow \forall x Q(x)$$

$$\forall x\,[P(x) \leftrightarrow Q(x)] \qquad \forall x P(x) \leftrightarrow \forall x Q(x)$$

$\rightarrow$

$\leftarrow$

$\leftrightarrow$


THANK YOU GW
SOLDIERS !