CS & IT ENGINEERING

Nested Quantifier with Inference Rule

Lecture No.08

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TOPICS TO BE COVERED

01 Nested Quantifier with Inference Rule

02 Universal Specialisation

03 Universal Generalisation

04 Existential specialisation / Gen

05 Type 6

Universal specification :

Domain :

$\forall x P(x)$

$$\forall x P(x) \rightarrow P(a)$$

True.

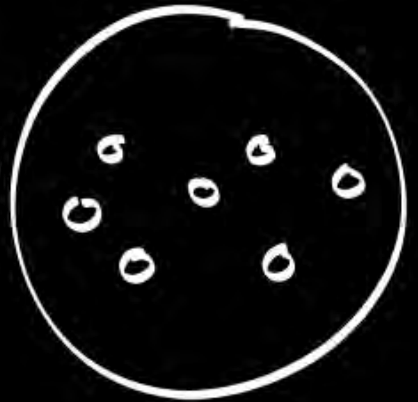
a is for all.

eg :

$$P(x) : x^2 \geq 0$$

$$\forall x P(x) \rightarrow P(a)$$

$\forall x\, P(x)$



D: Z. $P(x) : x^2 \geq 0$

$\forall x\, P(x) \longrightarrow P(a)$

a is all elements.

$\underline{\forall x\,(x^2 \geq 0)} \longleftrightarrow$

$P(0) \equiv 0^2 \geq 0$

$P(1) \equiv 1^2 \geq 0$

$P(2) \equiv 2^2 \geq 0$

True

Thm use

Universal specifications :

$\forall x P(x) \longrightarrow P(a)$

a is for all elements.

eg:

D: Z $\quad P(x) : x^2 \geq 0$

$\forall x P(x) \longrightarrow P(a)$

a is for all.

$\underbrace{\forall x (x^2 \geq 0)}_{\text{True}} \longrightarrow$

P(a) a is for all elements

P(0). ✓

P(1). ✓

P(2). ✓

P(3). ✓

P(4) ✓

Universal Generalization :

Thm. proof

$$P(a) \longrightarrow \forall x\, P(x)$$

a is for all elements.

$D : \mathbb{Z}$

$P(a)$
a is for all elements in the Domain.

$$\left\{\begin{array}{l} 1^2 \geq 0 \\ 2^2 \geq 0 \\ 3^2 \geq 0 \\ 4^2 \geq 0 \\ \vdots \end{array}\right. \longrightarrow \forall x\, P(x)$$

Existential specification :

$\exists x\, P(x) \longrightarrow P(a)$

a is fixed.

$P(x) : x^2 = 4.$ $\quad P(2) : 2^2 = 4$

$D : Z^+$

$a = 2$

$\underset{T}{\underline{\exists x\, P(x)}} \longrightarrow P(a)$

a is fixed

$\longrightarrow P(2)$

Existential Generalisation :

$$\left\{\begin{array}{l} P(a) \\ a \text{ is fixed} \end{array}\right. \longrightarrow \exists x P(x)$$

$D : Z^+$

$P(x) : x^2 = 9.$

$P(3) : 3^2 = 9$

$P(3) \longrightarrow \exists x P(x)$

$\forall x\, p(x) \rightarrow p(a)$ (U.S)

a is for all

$\exists x\, p(x) \rightarrow p(a)$ (E.S)

a is fixed.

$p(a) \longrightarrow \forall x\, p(x)$ (U.G)

a is for all

$p(a) \longrightarrow \exists x\, p(x)$ (E.G)

a is fixed

I.R. :

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array} \quad \text{(modus ponens)}$$

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

$$\begin{array}{l} p \to q \\ q \to R \\ \hline p \to R \end{array}$$

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

Simplification :

$$\begin{array}{l} p \wedge q \\ \hline \therefore p \end{array} \quad \text{OR} \quad \begin{array}{l} p \wedge q \\ \hline q \end{array}$$

$P(x): x+1=4$ $\quad\quad$ $Q(x): 2x+1=7$

$x=3$ $\quad\quad$ $x=3$

$$\exists x\,[P(x) \wedge Q(x)] \rightarrow \exists x\, P(x) \wedge \exists x\, Q(x)$$ (valid)

$$\boxed{P(1) \wedge Q(1)}$$

$$\vee$$

$$\boxed{P(2) \wedge Q(2)}$$

$$\vee$$

$x=3$

$$\boxed{P(3) \wedge Q(3)}$$

$\exists x(P(x) \wedge Q(x)) \longrightarrow \exists x P(x) \wedge \exists x Q(x)$

True 1) $\exists x[P(x) \wedge Q(x)]$ (premises) $\qquad p \wedge q \rightarrow p$

$\downarrow$

True 2) $P(a) \wedge Q(a)$ (E.S / $a = 3$
a is fixed)

$\downarrow$

valid 3) $P(a)$ (2, simpl)(a is fixed) $\qquad$ 3A)

$\downarrow$

valid 4) $\exists x P(x)$ (E.G)

Q.2: $C(x)$: students in the class.
$IIT(x)$: students will goto IIT.
$DPP(x)$: student is solving the DPP.

P1: all students in the class will goto IIT. (True) ✓ $\forall x[C(x) \rightarrow IIT(x)]$

P2: Some student in the class are not solving DPP's. (T) ✓ $\exists x[C(x) \land \neg DPP(x)]$

Some students who are not solving DPP's will also goto IIT. $\exists x[\neg DPP(x) \land IIT(x)]$

$$\forall x[C(x) \rightarrow IIT(x)] \checkmark$$
$$\exists x[C(x) \land \neg DPP(x)] \checkmark$$
$$\overline{\exists x[\neg DPP(x) \land IIT(x)]} \text{ (valid)?}$$

$$\{\forall x[C(x)\rightarrow IIT(x)] \wedge \exists x[C(x)\wedge\neg DPP(x)]\} \rightarrow \exists x[\neg DPP(x)\wedge IIT(x)]$$

1) $\forall x[C(x)\rightarrow IIT(x)]$ (True) (premises)

2) $C(a)\rightarrow IIT(a)$ (a is for all. $a = 1, 2, 3, \ldots, 10$) (U.S)

3) $\exists x[C(x)\wedge\neg DPP(x)]$ (premises) (True)

4) $\boxed{C(a)\wedge\neg DPP(a)\ (\text{a is fixed } a=3)\ (E.s)}$

5) $C(a)$ (a is fixed) (Step 4, Simpl)

2) $C(a)\rightarrow IIT(a)$ (a is for all)

5) $C(a)$ (a is fixed)

(modus ponens)

6) $IIT(a)$ (a is fixed)

7) $\neg DPP(a)$ (a is fixed)

(Step 4, simpl)

6) $IIT(a)$ (a is fixed)

7) $\neg D\hat{P}P(a)$ (a is fixed)

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8) $\neg DPP(a) \wedge IIT(a)$ (conjunction)

$\exists x (\neg DPP(x) \wedge IIT(x))$ (8, EG)

$\forall x [C(x) \rightarrow IIT(x)]$

$\exists x [C(x) \wedge \neg DPP(x)]$

$\exists x [\neg DPP(x) \wedge IIT(x)]$ ✓

(we can use simplification 2 times)

$\forall x [C(x) \to IIT(x)] \longrightarrow C(a) \to IIT(a) \longrightarrow C(a) \to IIT(a)$ (a is fraw)

$\exists x [C(x) \wedge \neg DPP(x)] \longrightarrow C(a) \wedge \neg DPP(a) \longrightarrow C(a)$ (simpl) (a is fixed)

$IIT(a)$ (a is fixed)

$\neg DPP(a) \wedge IIT(a)$

$\exists x [\neg DPP(x) \wedge IIT(x)]$

$$\left(\forall x\, P(x) \wedge \forall x\, [P(x) \rightarrow Q(x)]\right) \longrightarrow \forall x\, Q(x)$$

1) $\forall x\, (P(x) \rightarrow Q(x))$

2) $P(a) \rightarrow Q(a)$ (a is for all)

3) $\forall x\, P(x)$ (premises) (given)

4) $P(a)$ (a is for all)

$P(a) \rightarrow Q(a)$ (all)

$P(a)$ (all)

$Q(a)$ (all)

$\downarrow$

$\forall x\, Q(x)$

$$T.\ \overbrace{(\forall x P(x) \wedge \forall x (P(x) \rightarrow Q(x)))}^{T.} \rightarrow \forall x Q(x)$$

$$\boxed{\begin{array}{c} P(1)(T) \\ \wedge \\ P(2)(T) \\ \wedge \\ P(3)(T) \\ | \end{array}} \wedge \begin{array}{c} T \rightarrow T \\ T\ \boxed{P(1) \rightarrow Q(1)} \\ \wedge \\ T\ \boxed{P(2) \rightarrow Q(2)} \\ \wedge \\ T\ \boxed{P(3) \rightarrow Q(3)} \end{array} \qquad \boxed{\begin{array}{c} Q(1)(T) \\ \wedge \\ Q(2)(T) \\ \wedge \\ Q(3)(T) \end{array}}$$

all

$P(1) \rightarrow Q(1)$
$P(1)$
$\therefore Q(1)$ ✓

$P(2) \rightarrow Q(2)$
$P(2)$
$Q(2)$ ✓

$P(3) \rightarrow Q(3)$
$P(3)$
$Q(3)$ ✓

$$\left(\forall x\,[P(x) \to Q(x)] \wedge \exists x\, P(x)\right) \to \forall x\, Q(x) \quad \text{(valid)}$$

$$\left(\forall x\,(P(x) \to Q(x)) \wedge \exists x\, P(x)\right) \to \exists x\, Q(x) \quad \text{(valid?)}$$

$\exists x (P(x) \land Q(x)) \longrightarrow \exists x P(x) \land \exists x Q(x)$

1) $\exists x (P(x) \land Q(x))$

2) $P(a) \land Q(a)$ (1, E.S)

3) $P(a)$ (2, simpl)

4) $\exists x P(x)$ (step 3, E.G)

5) $Q(a)$ (step 2. simpl)

6) $\exists x Q(x)$ (step 5. E.G)

(conjunction)

$\exists x P(x) \land \exists x Q(x)$

$$\forall x\, p(x) \longrightarrow \exists x\, p(x)$$

valid

$$\begin{array}{l} \forall x\,[\neg p(x) \vee Q(x)] \\ \forall x\, p(x) \\ \forall x\,(Q(x) \rightarrow T(x)] \\ \hline \exists x\, T(x) \end{array}$$

$$\begin{array}{ll} \neg p(a) \vee Q(a) & (\text{all}) \\ p(a) & (\text{all}) \\ \hline Q(a) & (\text{all}) \end{array}$$

(Disjunctive syllogism)

$$\begin{array}{ll} Q(a) & (\text{all}) \\ Q(a) \rightarrow T(a) & (\text{all}) \\ \hline T(a) & (\text{all}) \end{array}$$

(modus ponens)

$$\downarrow$$

$$\forall x\, T(x) \longrightarrow \exists x\, T(x)$$


THANK YOU GW
SOLDIERS !