

CS & IT ENGINEERING

Operating Systems

Memory Management



Lecture No. 7



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TOPICS TO BE
COVERED

Hashed Paging

Optimal Page Size

Multi-Level Paging

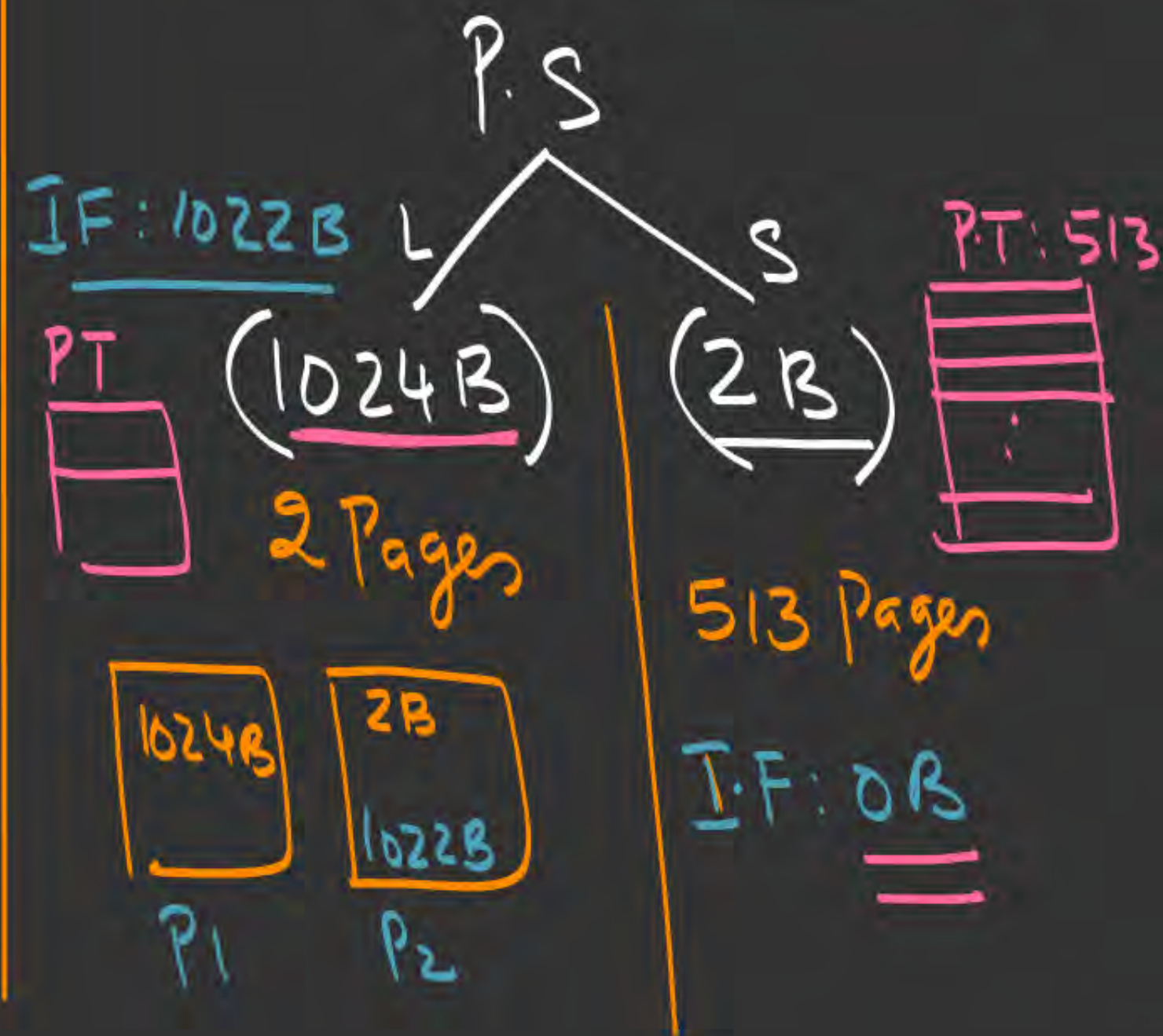
Associate a Smaller P.T with Process / Reduce P.T. Size;

$$L.A = 32 \text{ bits}; P.S = 4 \text{ KB}; N = 2^{32} / 2^{12} = \underline{2^{20}}$$

$$P.T.S \propto N \propto \frac{1}{P.S}$$

$$P.T.S \propto \frac{1}{P.S}$$

(i) Increasing Page Size (P.S);



Optimal Page Size?

→ Let V.A.S = 'S' Bytes

→ Let PTE = 'e' Bytes

→ Let Page Size = 'P' Bytes

Optimal (P) = ?

$$\text{I. P.T.S} = \left[\left(\frac{S}{P} \right) * e \right] \text{ Bytes}$$

+

$$\text{II. I.F} = \frac{P}{2} \text{ Bytes}$$

$$\text{Total ovhd} = \left[\left(\frac{S}{P} \right) e + \frac{P}{2} \right] \text{ Should be Minimum.} \quad \textcircled{1}$$

diff. Eq. $\textcircled{1}$ w.r.to P ~ 0

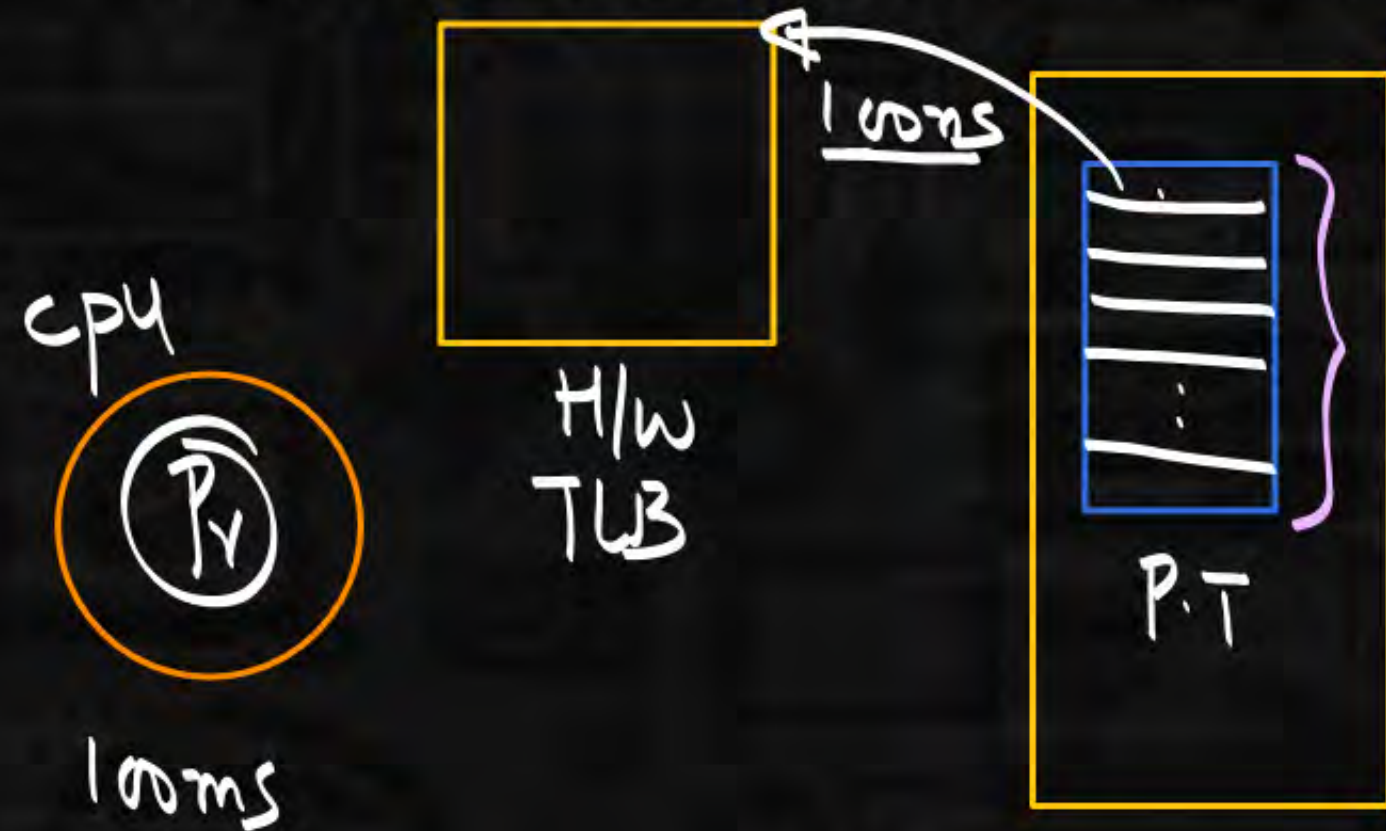
$$\left[-\frac{1}{P^2} S e + \frac{1}{2} \right] = 0$$

$$\frac{S e}{P^2} = \frac{1}{2} \Rightarrow P^2 = 2 S e$$

$$P = \sqrt{2 S e}$$



A Machine has a ^{VA}32-bit Address Space and an 8KB Page. The Page Table is entirely in hardware, with one 32-bit word per entry. 4B
When a Process starts, the Page Table is copied to the hardware from memory, at one word every 100 nsec. If each Process runs for 100 msec (including the time to load the page table), what fraction of the CPU time is devoted to loading the Page Tables? Potential



Memory 2^{19}

$$N_{\text{pages}} = \frac{2^{32}}{2^{13}} = 2^{19}$$

$$\text{Time to Load P.T.} = 2^{19} \times 100 \text{ ns}$$

$$\begin{aligned} \text{P.T. Size in Bytes} &= N * e \\ &= 512K * 4B = \underline{\underline{2MB}} \end{aligned}$$

$$\% \text{ (fraction of cpu time)} = \frac{2^{19} \times 100 \text{ ns}}{100 \text{ ms}} = \left[\frac{2^{19} \times 100 \times 10^{-9} \text{ s}}{100 \times 10^{-3} \text{ s}} \right]$$

$\approx 50\%$



Consider a System using Paging technique with an Address Space of 65,536 Bytes. The Page Size in this System is 4096 Bytes. The Program Consists of Text, Data and Stack Sections as per the specifications given below:

4B

8192 : Text: 32,768 Bytes → $\frac{32768}{4096} = 8$
4097 : Data: 16,386 Bytes → $4.000488 \sim 5$
3968 : Stack: 15,870 Bytes → $3.87 = 4$
16,257 ✓

$$\frac{64KB}{4B} = 16K \checkmark = \underline{VAS = 65,536 = 64KB}$$

$$N = \frac{64KB}{4KB} = \frac{2^{16}}{2^{12}} = 2^4 = \underline{16}$$

Total: 17 ✓

Text
Data
Stack

A Page of the program contains portion of only one section i.e either Text or Data or Stack.

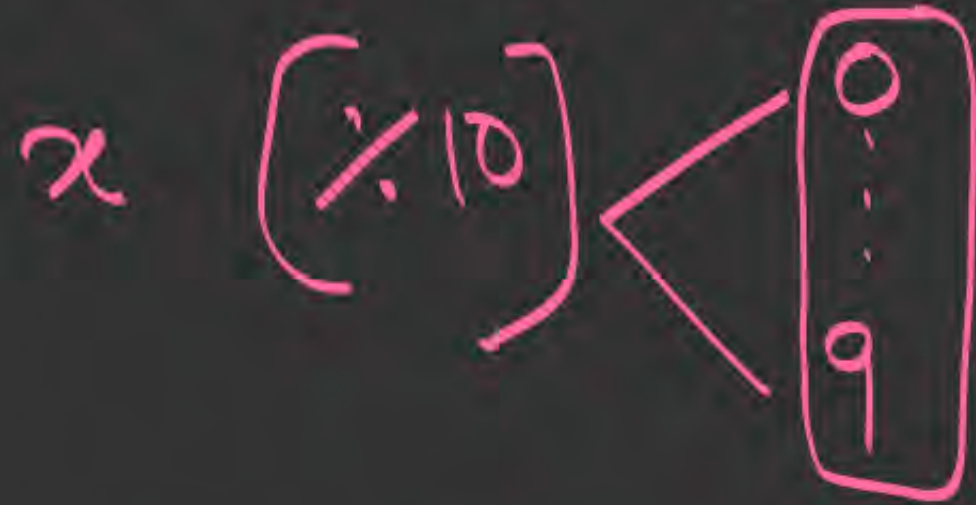
- ✓ (a) Does the Program Fit in the given Address Space? (Y/N) "No"
- ✓ (b) What is the Maximum Page Size in Bytes such that the Program Fits in the Given Address Space? gk P.S = 4B : "Fit"

$$Max(P.S) = ?$$

2) Hashed Paging / Paging with Hashing [to Assoc. a smaller P.T with Process]

→ Element (x) $\sim h(f(x)) \sim 'i'$

→ we will design a P.T using Hashing



Hashed P.T

To Resolve Collision ~ chaining is used

Collision

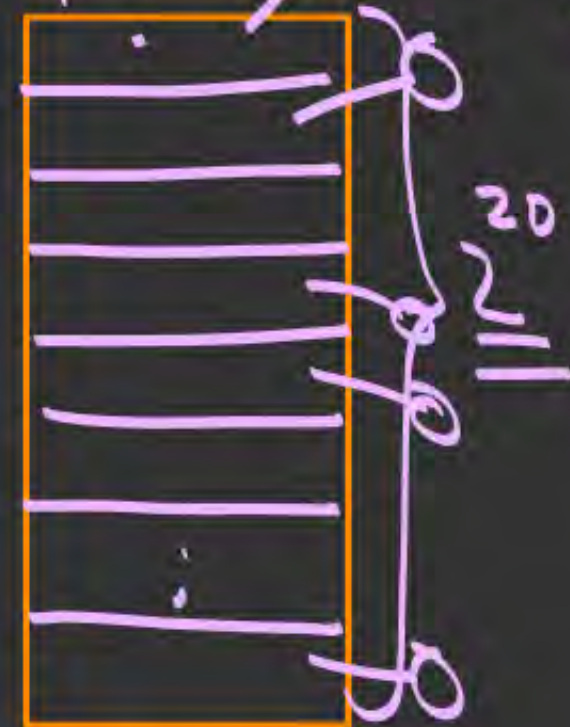
Let I_1 & I_2 be two distinct elements

$$f(I_1) = f(I_2) = i$$



V.A: 32 bits; P.S = 4KB

$$N = \frac{2^{32}}{2^{12}} = 2^{20} = 1M$$



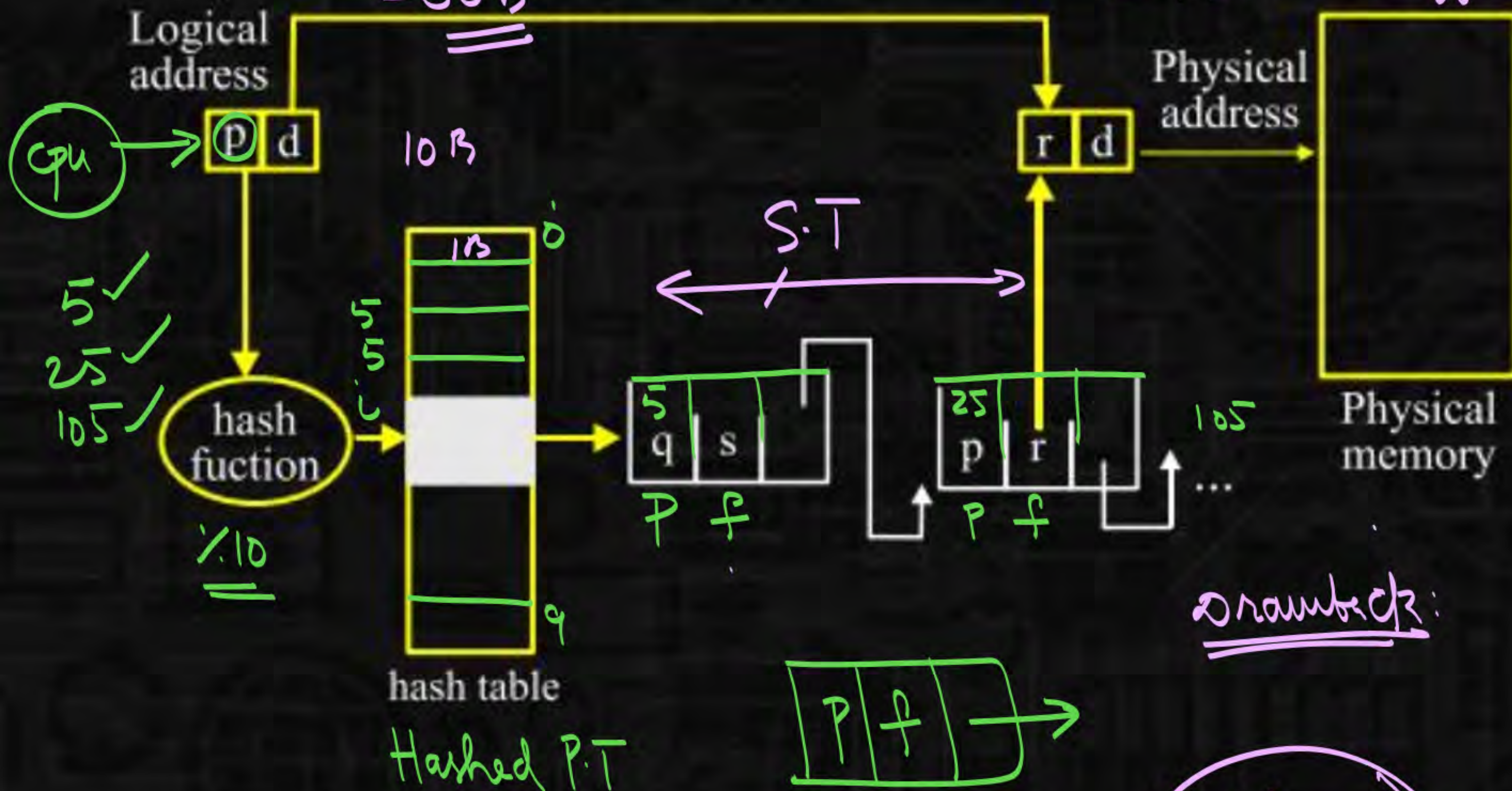
Process is using 5 Pages



Hashed Page Table

Space optimiz.
Time ineff.

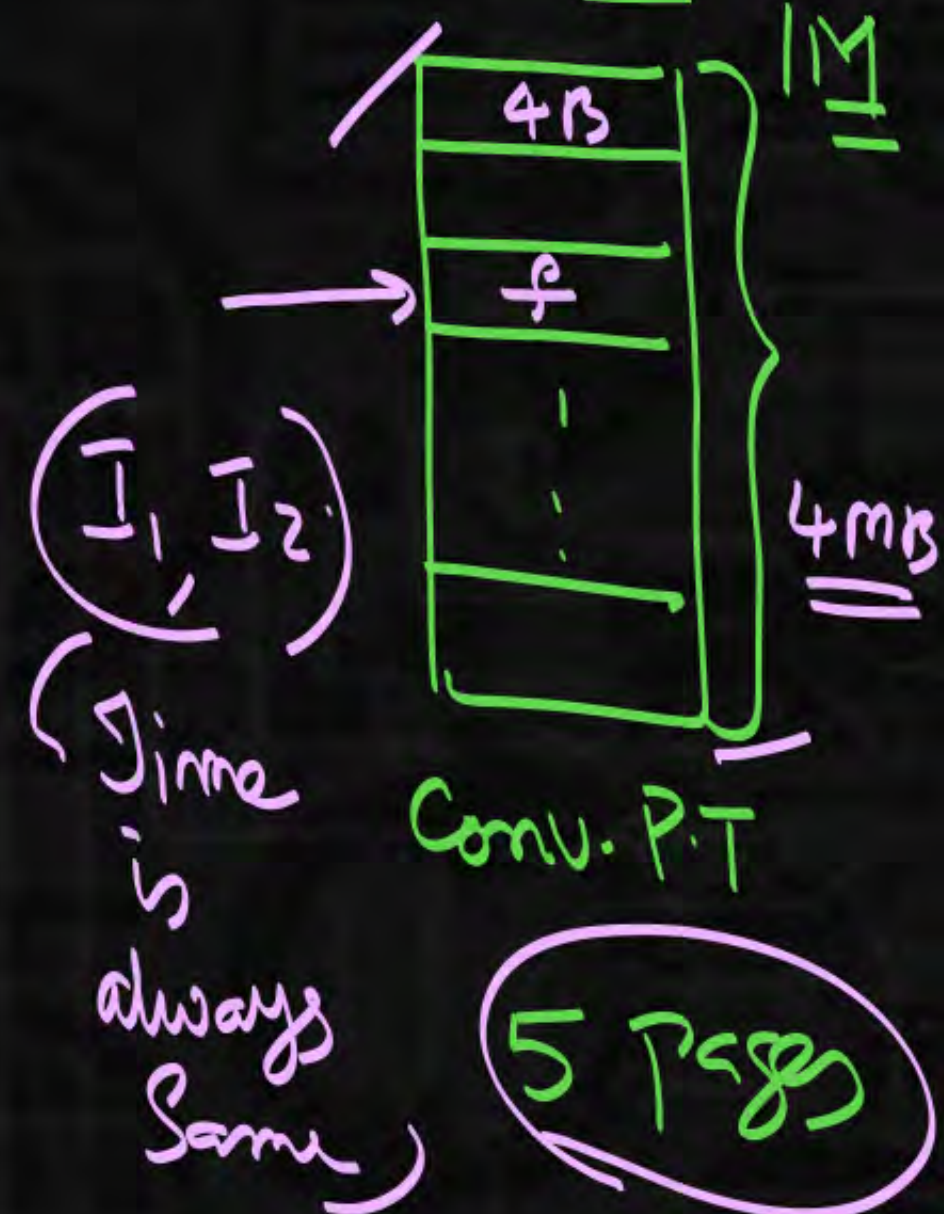
$$(10B + 50B) = 60B$$



Drawback:

$$1 \text{ Node} = 10B$$

$$VA = 32 \text{ bits}$$
$$P.S = 4KB$$
$$N = 1M$$



5 Pages

3) Multi-Level Paging | Hierarchical Paging (Recursive Paging)

< Paging on P.T >

(Paging) as a Concept:
Involves 3-Steps

1. Divide the address space into Pages (chunks)
2. Storing the Pages in PAS (chunk)
3. Access the Pages (chunks) of A.S in PAS, thru Page Table

To reduce P.T-Size out of

Assoc. Smaller P.T's with Processes;

When do we say
P.T. is small

(If the P.T. fits in one frame of mem)

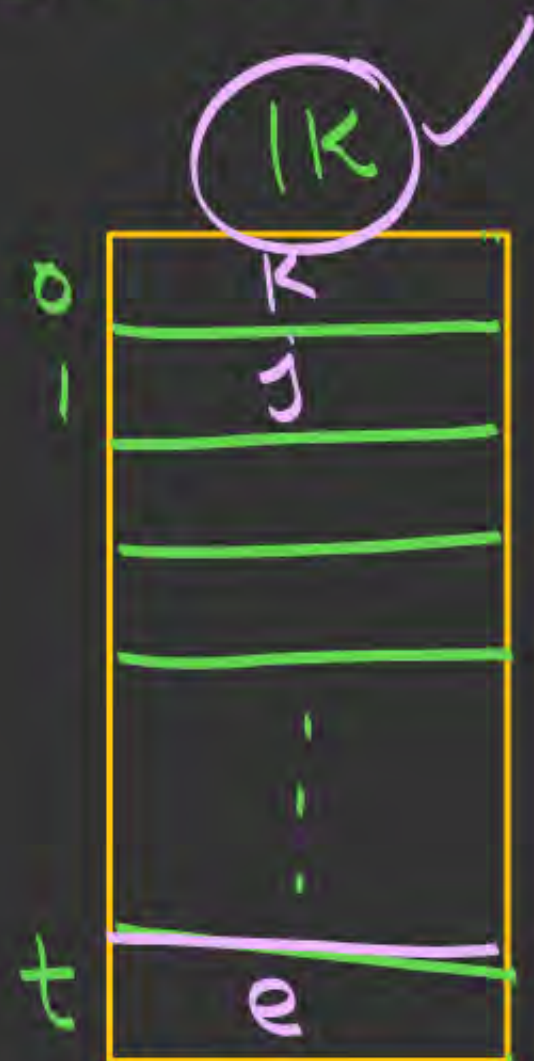
$$\begin{aligned} VA &= 32 \text{ bits;} \\ P.S &= \underline{4 \text{ KB}} \\ e &= 4 \text{ B} \\ P.T.S &= 1 \text{ M} \times 4 \text{ B} \\ &= \underline{\underline{4 \text{ MB}}} \end{aligned}$$



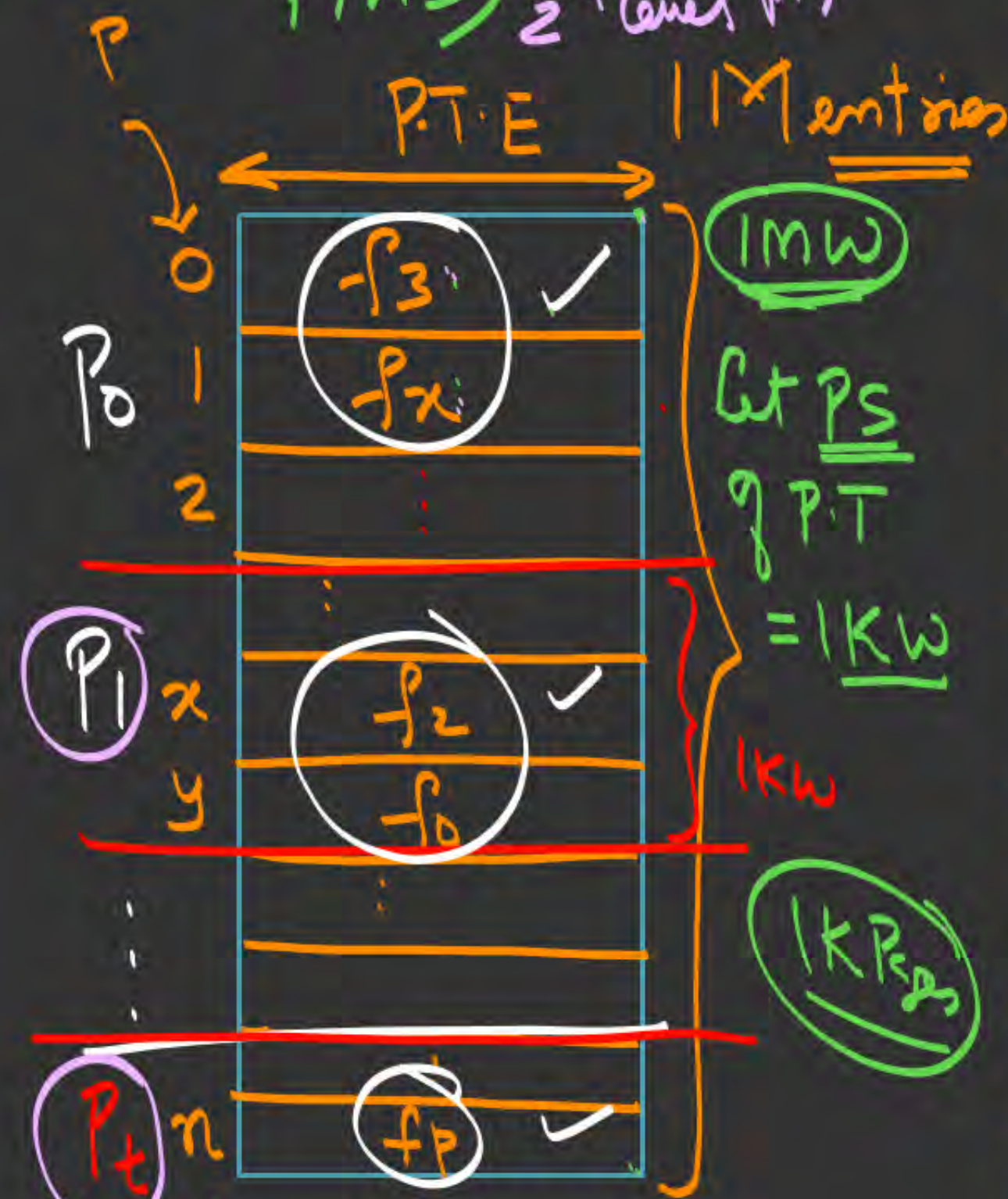
V.A = 32 bits ; P.S = 4KB ; PTS = ~~4MB~~

Paging (concept on PTAS) ~~4MB = 4MB~~

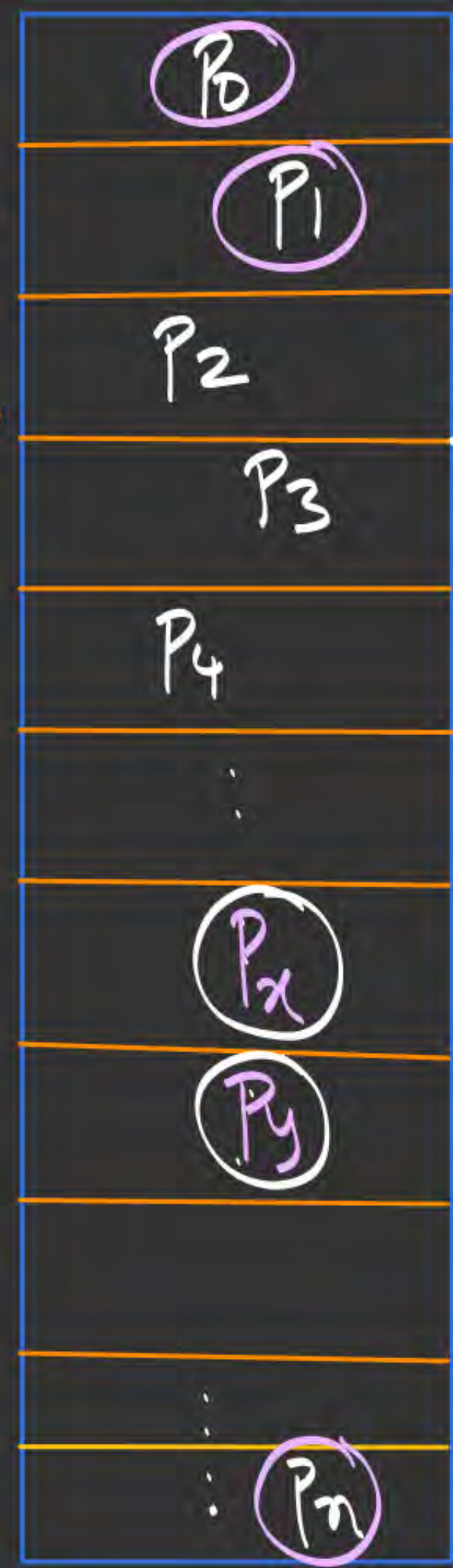
$1K + 3 \cdot 1K$
4K
1st Level P.T



outer P.T



inner P.T
 $N_{chunks} = \frac{1m}{1K} = 1K$



VAS ✓
 $= 2^{32}$
 $= 4GB$

4KB

$N = 2^{20}$
 $= 1M$

