CS & IT ENGINEERING

COMPUTER NETWORKS

Error Control

Lecture No-3

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TOPICS TO BE COVERED

1. min. Hamming distance For Error Correction
2. Simple Parity
3. 2D Parity

Min. Hamming Distance for Error Correction :

min.
H.D = 2

Valid codeword $\xrightarrow{\text{1 bit error}}$ Invalid codeword $\xrightarrow{\text{1 bit error}}$ Valid code word

can detect All one bit error

Valid

0000 $\xrightarrow{\text{1 bit error}}$ 0010 $\xrightarrow{\text{1bit error}}$ 0011

Ex1 :

Valid code word

1001
1010

min. Hamming distance = 2

① Sent

1001 → 1 bit error → 1011 Invalid codeword

It can detect one bit error but can't correct one bit error

1010 → 1 bit error → 1011 Invalid code word

It can detect one bit error but can't correct one bit error

Ex2 :

Valid code word

0000

0011

min. Hamming distance = 2

Sent: 0000 —1 bit error→ Rcvd: 0010

Invalid code word

It can detect one bit error but can't correct one bit error

0011 —1 bit error→ 0010

Invalid code word

It can detect one bit error but can't correct one bit error

Ex3 :

Valid code word

000 } min. Hamming
111 } distance = 3

Sol^n

$000 \xrightarrow[\text{error}]{\text{1 bit}} 100, 010, 001$ } Invalid code word

$111 \xrightarrow[\text{error}]{\text{1 bit}} 011, 101, 110$ } Invalid code word

① $\boxed{000}$ 1 bit error $\boxed{100}$ Invalid code word

It can detect and correct all one bit error

100
000
1 bit ✓

100
111
2 bit

2. $\boxed{1\,1\,1} \xrightarrow{\text{1 bit error}} \boxed{1\,1\,0}$

Invalid code word

It can detect and correct all one bit error

$$\begin{array}{ccc} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array} \qquad \begin{array}{ccc} 1 & 1 & 0 \\ 1 & 1 & 1 \end{array}$$

2 bit — 1 bit ✓

3. $\boxed{0\,0\,0} \xrightarrow{\text{2 bit error}} \boxed{0\,1\,1}$

Invalid code word

It can detect 2 bit error but can't correct 2 bit error

$$\begin{array}{ccc} 0 & 1 & 1 \\ 0 & 0 & 0 \end{array} \qquad \begin{array}{ccc} 0 & 1 & 1 \\ 1 & 1 & 1 \end{array}$$

2 bit — 1 bit ✓

Ex4 :

Valid code word

$$\left.\begin{array}{l}0000\\1111\end{array}\right\}\text{min. Hamming distance} = 4$$

① $\boxed{0000}$ 1 bit error $\boxed{0001}$

Invalid code word

It can detect and correct all one bit error

$$\begin{array}{r}000\boxed{1}\\000\boxed{0}\\\hline\end{array}$$

1 bit ✓

$$\begin{array}{r}\boxed{000}1\\\boxed{111}1\\\hline\end{array}$$

3 bit

② $\boxed{0000}$ 2 bit error $\boxed{0011}$

Invalid code word

It can detect 2 bit error but can't correct 2 bit error

$$\begin{array}{r}00\boxed{11}\\00\boxed{00}\\\hline\end{array}$$

2 bit

$$\begin{array}{r}\boxed{00}11\\\boxed{11}11\\\hline\end{array}$$

2 bit

Ex5 :

Valid code word

$$\left.\begin{array}{l}00000\\11111\end{array}\right\}\text{min Hamming distance}=5$$

sent

$\boxed{00000}$ 2 bit error $\boxed{00011}$ Invalid code word

It can detect and correct all 2 bit error

$$\begin{array}{l}000\,11\\000\,00\\\hline\text{2bit}\end{array}\qquad\begin{array}{l}000\,11\\111\,11\\\hline\text{3bit}\end{array}$$

✓

Note

1. To correct one bit error minimum Hamming distance required = 3 = 2*1 + 1
2. To correct two bit error minimum Hamming distance required = 5 = 2*2 + 1
3. To correct 'd' bit error minimum Hamming distance required = 2*d + 1

Problem Solving on Hamming Distance

Q.1 Consider a binary code that consists of only four valid code words as given below:

a b c d

00000, 01011, 10101, 11110

Let the minimum Hamming distance of the code be p and the maximum number of erroneous bits that can be corrected by the code be q. Then the values of p and q are

GATE 2017 (2m)

A. $p = 3$ and $q = 1$ ✓

B. $p = 3$ and $q = 2$

C. $p = 4$ and $q = 1$

D. $p = 4$ and $q = 2$

$$\left.\begin{aligned} d(a,b) &= 3 \\ d(a,c) &= 3 \\ d(a,d) &= 4 \\ d(b,c) &= 4 \\ d(b,d) &= 3 \\ d(c,d) &= 3 \end{aligned}\right\} \text{min. Hamming distance} = 3 \ (p)$$

min. Hamming distance required to correct 'd' bit error = 2d + 1

$2d + 1 = 3$

$2d = 3 - 1$

$2d = 2$

$\boxed{d = 1}$ (q)

Q.2 What is the distance of the following code

$\overset{a}{000000}$, $\overset{b}{010101}$, $\overset{c}{000111}$, $\overset{d}{011001}$, $\overset{e}{111111}$?

GATE 1995

A. 2

B. 3

C. 4

D. 1

$d(a,b) = 3$
$d(a,c) = 3$
$d(a,d) = 3$
$d(a,e) = 6$
$d(b,c) = 2$
$d(b,d) = 2$
$d(b,e) = 3$
$d(c,d) = 4$
$d(c,e) = 3$
$d(d,e) = 3$

Minimum Hamming distance = 2

$$\begin{array}{r} 010101 \\ 000111 \\ \hline 010010 \end{array} \qquad \begin{array}{r} 010101 \\ 011001 \\ \hline 001100 \end{array}$$

Q.3 An error correcting code has the following code words:

00000000, 00001111, 01010101, 10101010, 11110000.

a b c d e

What is the maximum number of bit errors that can be corrected ?

GATE 2007

A. 0

B. 1

C. 2

D. 3

(b) 0000 1111

(c) 01010101

01011010 → No. of 1's = 4 (Hamming distance)

Minimum Hamming distance = 4

'd' bit (d=2)

2d+1

2*2+1=5

Minimum Hamming distance required to correct 'd' bit error = 2d+1

$2d+1=4$

$2d=4-1$

$2d=3$

$d=\frac{3}{2}$

$d=\lfloor 1.5 \rfloor = 1$ bit

Q.4 The minimum Hamming distance to correct upto 5 bit error successfully is <u>11</u>

NIELIT 2020

min. Hamming distance required to correct 'd' bit error $= 2d+1$

$= 2*5+1$

$= 11$

Q.5 The minimum Hamming distance to detect upto 10 bit error successfully is __11__

NIELIT 2020

min. Hamming distance required to detect 'd' bit error = 'd+1'

= 10+1

= 11

Error control

Error detection	Error correction
① Simple Parity	① Hamming code
② 2D-Parity	
③ CRC	
④ checksum	

Simple Parity Check Code

Simple parity :

- In the Simple parity concept one extra bit (parity bit) is added to each dataword.
- Simple parity check can detect all single bit error .
- Simple parity check can not detect an even number of errors.
- Simple parity check can detect an odd number of errors .

Simple Parity

Even Parity	Odd Parity
No. of 1's must be even in each code word Including the Parity bit	No. of 1's must be odd in each Code word Including the Parity bit

Dataword

00 | 01 | 10 | 11

dataword

$K = 2, r = 1$

Dataword	Codeword
00	000
01	011
10	101
11	110

(even Parity)

000 — 1 bit error — 100

No. of 1's = odd

Receiver can detect one bit error

Sent: 000 — 2 bit error — Rcvd: 110

No. of 1's = even

Receiver can't detect 2 bit error

3. $\boxed{000}$ $\xrightarrow{\text{3 bit error}}$ $\boxed{111}$

No. of 1's = odd

Receiver can detect 3 bit error

2D Parity Check Code

2D parity :

- Two dimensional parity check can detect and correct all single bit error and detect two or three bit error that occur any where in the matrix
- However only some pattern with four or more Error can be detected.
- In a 2D-parity check code, the information bits are organized in a matrix consisting of row and columns.
- For each row and each column one parity check bits is calculated.

Original Data :

0 1 0 0 1 0 | 0 1 0 1 0 1 | 1 0 0 1 0 1 | 1 1 1 0 1 1 | 0 0 1 0 0 1

1st row | 2nd row | 3rd row | 4th row | 5th row

(By using even Parity)

						row Parity
0	1	0	0	1	0	0
0	1	0	1	0	1	1
1	0	0	1	0	1	1
1	1	1	0	1	1	1
0	0	1	0	0	1	0
0	1	0	0	0	0	1

column Parity

No. of rows = 5

No. of columns = 6

Transmitted data :

0100100	0101011	1001011	1110111	0010010	0100001
1st row	2nd row	3rd row	4th row	5th row	6th row

One Error :

0	1	0	0	1	0	0
0	1	0	1	0	1	1
1	0	0	1	0	1	1
1	0	1	0	1	1	1
0	0	1	0	0	1	0
0	1	0	0	0	0	1

1. one bit error detected as well as corrected
2. one bit will effect 2 parity bit

Two - Error :

$$
\begin{array}{cccccc|c}
0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & \boxed{0} & 0 & 1 & 1 \\
1 & 0 & 0 & 1 & 0 & 1 & 1 \\
1 & \boxed{0} & 1 & 0 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 \\
\hline
0 & 1 & 0 & 0 & 0 & 0 & 1
\end{array}
$$

1. It can detect 2 bit error but it can't correct two bit error.
2. 2 bit error will effect maximum 4 Parity bit
3. 2 bit error will effect minimum 2 Parity bit

Two - Error :

$$
\begin{array}{cccccc|c}
0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & \boxed{1} & \boxed{0} & 0 & 1 & 1 \\
1 & 0 & 0 & 1 & 0 & 1 & 1 \\
1 & 1 & 1 & 0 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 \\
\hline
0 & 1 & 0 & 0 & 0 & 0 & 1
\end{array}
$$

3 - Error :

$$
\begin{array}{cccccc|c}
0 & 1 & 0 & 0 & \boxed{0} & 0 & 0 \\
0 & \boxed{0} & 0 & 1 & 0 & 1 & 1 \\
1 & 0 & 0 & 1 & 0 & 1 & 1 \\
1 & 1 & \boxed{0} & 0 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 \\
\hline
0 & 1 & 0 & 0 & 0 & 0 &
\end{array}
$$

① 3 bit error detected but Not corrected

② 3 bit error will effect maximum 6 Parity bits

3 - Error :

0	1	0	0	1	0	0
0	$\boxed{0}$	0	$\boxed{0}$	0	1	1
1	0	0	1	0	1	1
1	$\boxed{0}$	1	0	1	1	1 ←
0	0	1	0	0	1	0
0	1	0	0 ↑	0	0	

3 bit error will eFFect minimum 2 Parity bit

4 - Error :

0	1	0	0	1	0	0
0	$\boxed{0}$	0	$\boxed{0}$	0	1	1
1	0	0	1	0	1	1
1	$\boxed{0}$	1	$\boxed{1}$	1	1	1
0	0	1	0	0	1	0
0	1	0	0	0	0	1

4 bit error can't detected

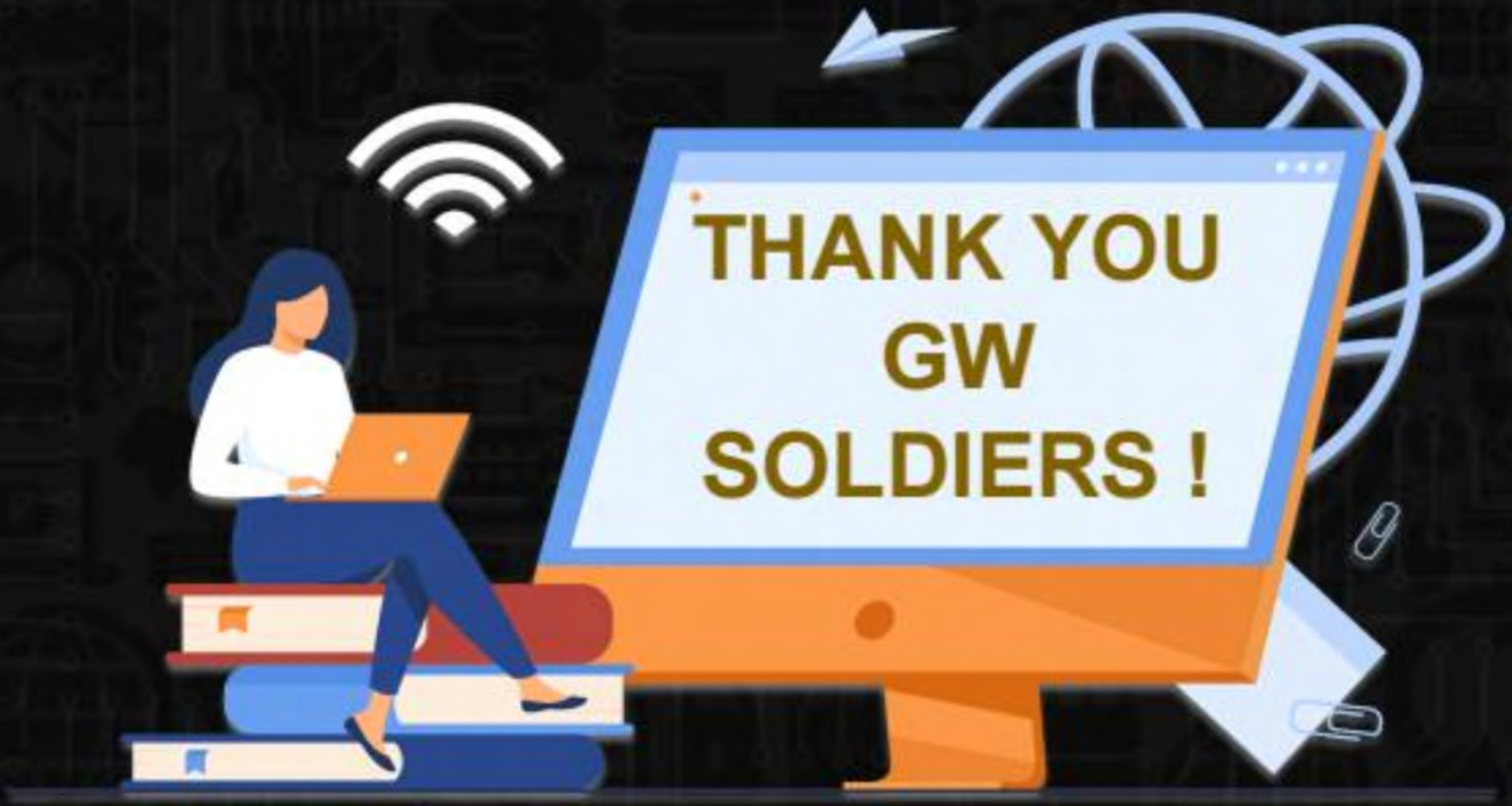
0	$\boxed{0}$	0	0	1	0	0
0	1	$\boxed{1}$	1	0	1	1
1	0	0	$\boxed{0}$	0	1	1
1	1	1	0	$\boxed{0}$	1	1
0	0	1	0	0	1	0
0	1	0	0	0	0	1

4 bit error detected

Disadvantage of 2D parity :

If we have a error in the parity then this scheme does not work fine

0	1	0	0	1	0	0
0	1	0	1	0	1	1
1	0	0	1	0	1	1
1	1	1	0	1	1	1
0	0	1	0	0	1	0
0	1	0	0	0	0	1


THANK YOU
GW
SOLDIERS !