CS & IT ENGINEERING

Types of Graphs

Lecture No. 3

By- SATISH YADAV SIR

01 Complete Graph

02 Cycle graph

03 Wheel graph

04 Bipartite graph

05 Line graph

Types of graph

Graph $e = 27$.

6 vertices → Degree 2.

3 vertices → Degree 4

Remaining n vertices → Degree 3

what will be total vertices ?

(GATE)

Total vertices

$= 6 + 3 + 10 = 19$ Ans

$\sum d(v_i) = 2e$

$6 \times 2 + 3 \times 4 + n \times 3 = 2 \times 27$

$12 + 12 + 3n = 54$

$24 + 3n = 54$

$3n = 54 - 24$

$= 30$

$3n = 30$

$n = 10$

Types of graph

Consider a graph having 15 edges & Degrees of all vertices are at least 3, then what will be maximum value of n?

$e = 15 \quad \delta(G) = 3$

$$\delta(G) \leq \frac{2e}{n} \leq \Delta(G)$$

$$\delta(G) \leq \frac{2e}{n}$$

$\delta(G) \leq \frac{2e}{n}$ $(e=15)$ $\delta(G)=3$

$3 \leq \frac{2\times 15}{n}$

$n \leq \frac{30}{3}$

$\boxed{n \leq 10}$

$n \leq 10$

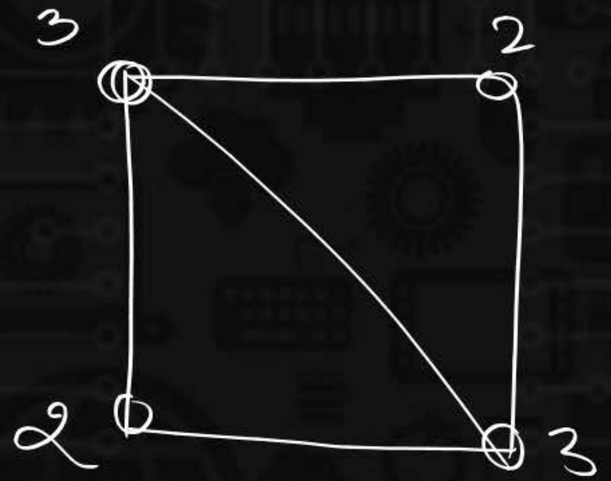
$(\ldots\ldots\ldots 10) \leq 10$

$\boxed{n=10}$

Inequalities thm :

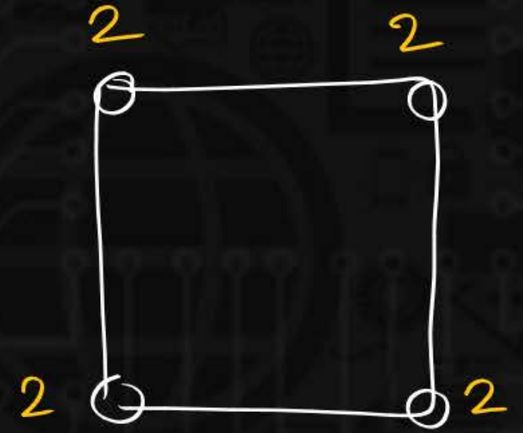
maximum degree ($\Delta(G)$)

minimum degree ($\delta(G)$)



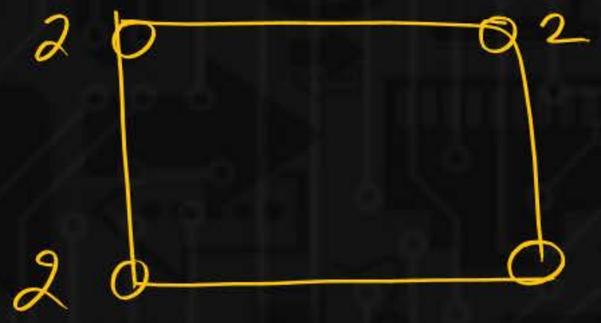
$\Delta(G) = 3$

$\delta(G) = 2$.



$\Delta(G) = 2$

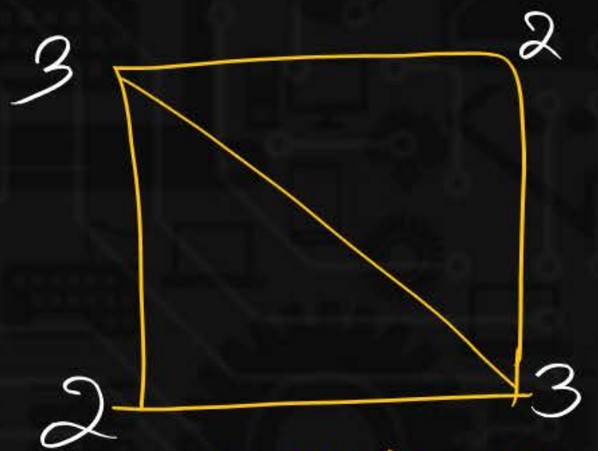
$\delta(G) = 2$



$\Delta(G) = 2$

$\delta(G) = 2$

$\frac{2e}{n} = 2$



$\Delta(G) = 3$

$\delta(G) = 2$.

Case 2:

$$\delta(G) < \frac{2e}{n} < \Delta(G)$$

Avg. degrees $= \frac{2+2+2+2}{\text{Total no. of vertices.}}$

Case 1

$= \frac{2+2+2+2}{4} = 2.$

$\delta(G) = \frac{2e}{n} = \Delta(G) = \frac{2e}{n}$ $\left(\Sigma d(v_i) = 2e\right)$

$$\text{avg. degree} = \frac{\text{Degrees of all}}{\text{Total vertices}}$$

$$= \frac{2e}{n}$$

Types of graph

Thm:

$$\delta(G) \leq \frac{2e}{n} \leq \Delta(G)$$

Case1: $\delta(G) = \frac{2e}{n} = \Delta(G)$

Case2: $\delta(G) < \frac{2e}{n} < \Delta(G)$

$2 < 2.5 < 3$.

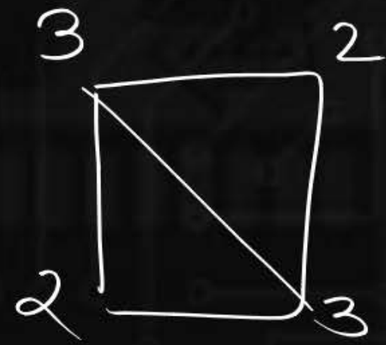
$\therefore \delta(G) = 2$

$\Delta(G) = 3$

avg degree

$= \frac{2e}{n} = \frac{3+2+2+3}{4}$

$= 2.5$



Types of graph

$$\delta(G) \leq \frac{2e}{n} \leq \Delta(G) \leq n-1.$$

atmost Degree

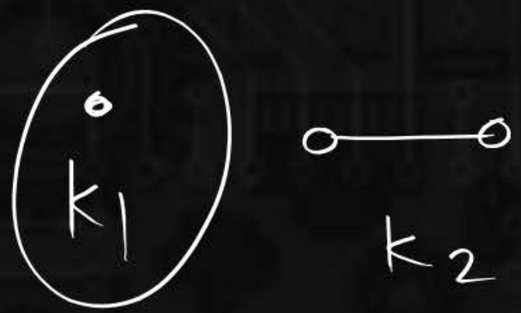
$$\frac{2e}{n} \leq \Delta(G)$$

atleast Degree $\left(\delta(G) \leq \frac{2e}{n}\right)$

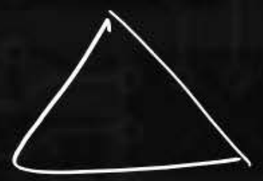
Types of graph

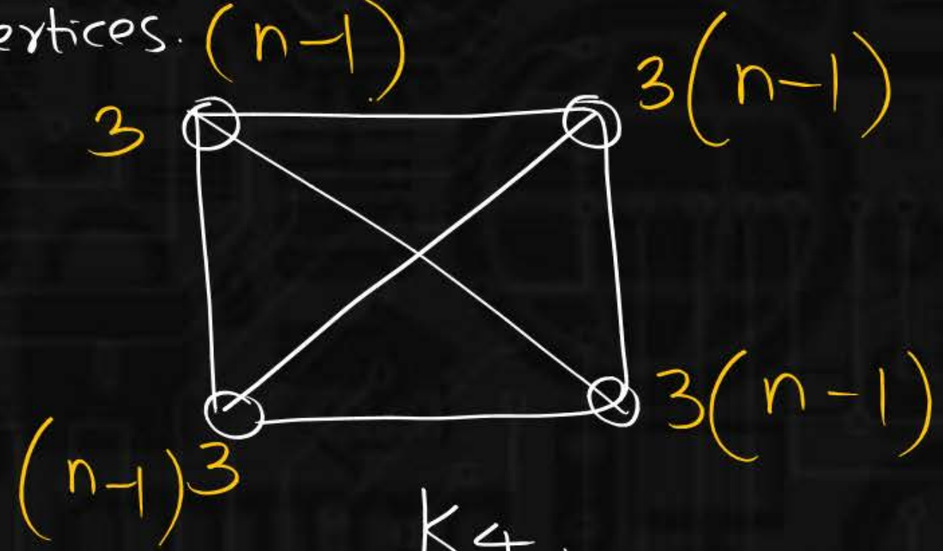
Complete graph (K_n) $(n \geq 1)$

↳ Total vertices.



Trivial Graph.





Types of graph

In K_n.

1. Degrees of all vertices $\longrightarrow (n-1)$

$$e = \frac{n(n-1)}{2}$$

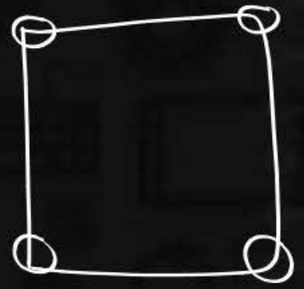
2. $\delta(G) = \frac{2e}{n} = \Delta(G) = n-1.$

Types of graph

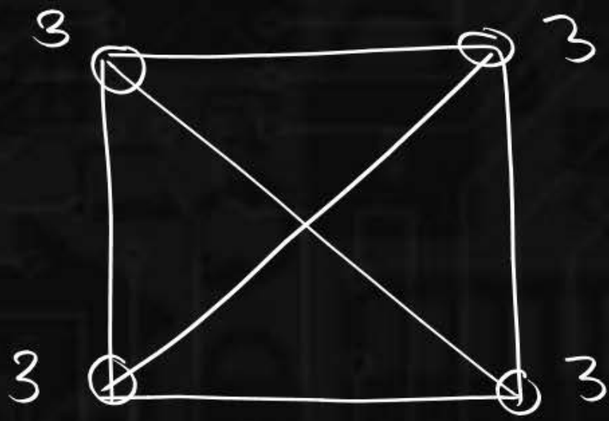
2. Regular Graph ($\delta(G) = \frac{2e}{n} = \Delta(G)$)

Degrees of all vertices are same.

eg.



2-Regular



3-Regular

Degree → K

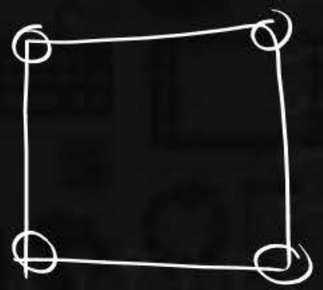
K-Regular Graph.

Types of graph

all kn are regular Graph (True)

(n-1)
Regular

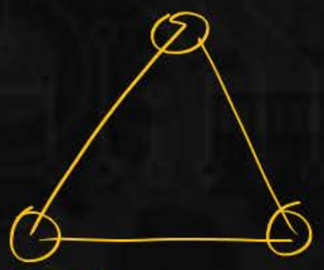
all Regular Graphs are kn (false)



Types of graph

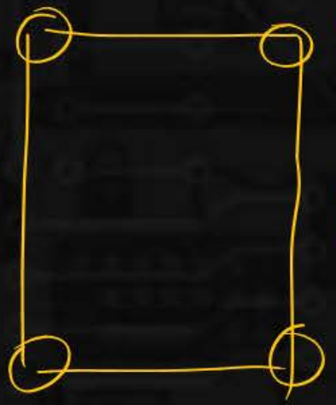
Cycle Graph (C_n) ($n \geq 3$)

n → Total vertices



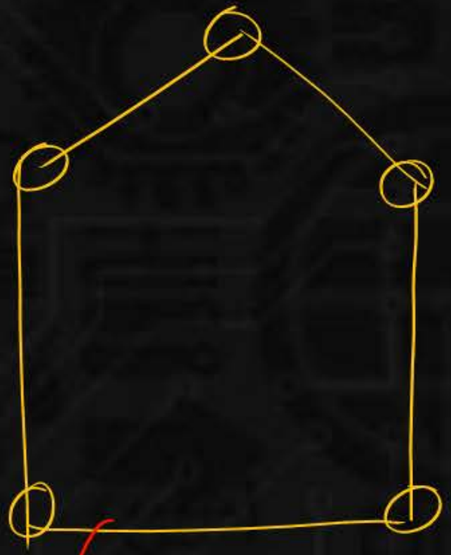
C_3

odd length cycle



C_4

(even length cycle)

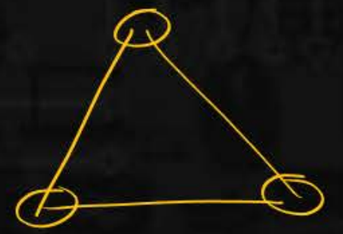


C_5

odd length cycle

Types of graph

1. if graph is C_n → Degrees of all vertices are 2.



$n = 3$
$e = 3$

2. if graph is $C_n \longrightarrow n = e$ (T)

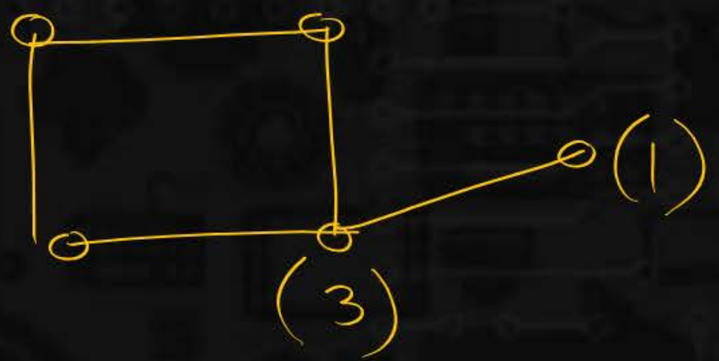
$$\begin{cases} n \times 2 = 2e \\ n = e \end{cases}$$

3. if $n = e$ then graph is C_n.
(false)

Types of graph

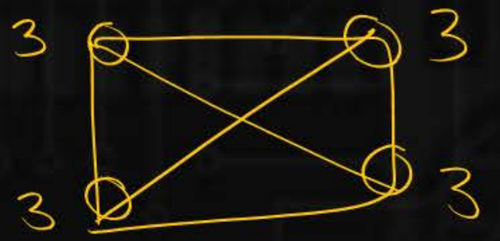
if $n = e$ then C_n (false)

$n = 5 \quad e = 5$



if Graph is C_n then it is Regular Graph. (T)

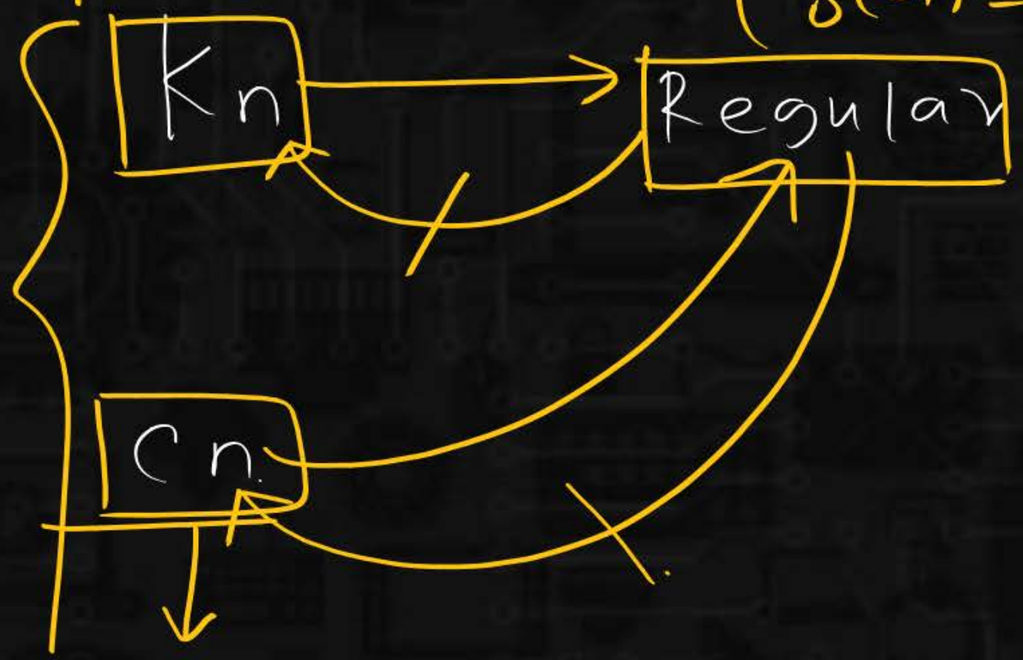
if Regular graph $\longrightarrow$ C_n (false)



Types of graph

$\delta(G) = \frac{2e}{n} = \Delta(G) = n-1.$

$\left(\delta(G) = \frac{2e}{n} = \Delta(G)\right)$

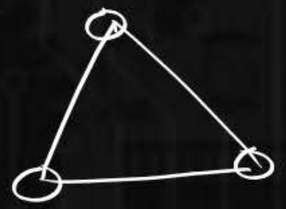


$\begin{cases} Cn \longrightarrow n = e \\ n = e \not\rightarrow Cn. \end{cases}$

$\delta(G) = \frac{2e}{n} = \Delta(G) = 2.$

Types of graph

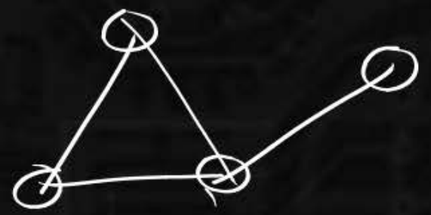
Cycle Graph.

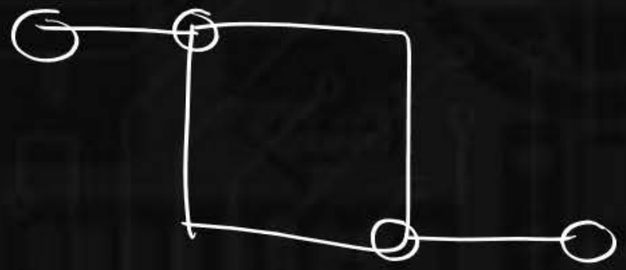


{ (2 2 2)

2 2 2 2 - - - -

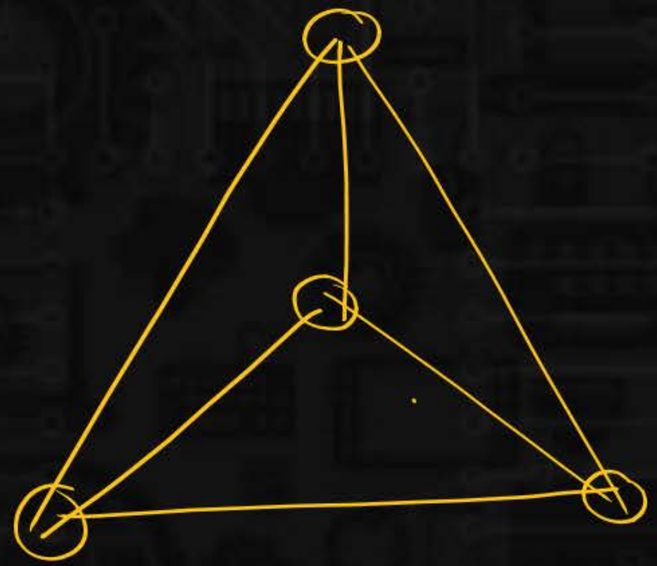
Graph containing Cycle



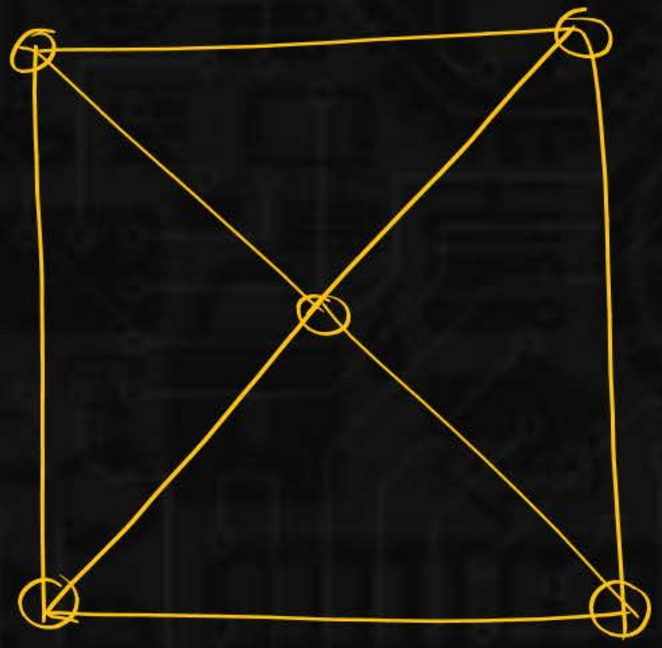


Types of graph

Wheel Graph. $(W_n)(n \geq 4)$

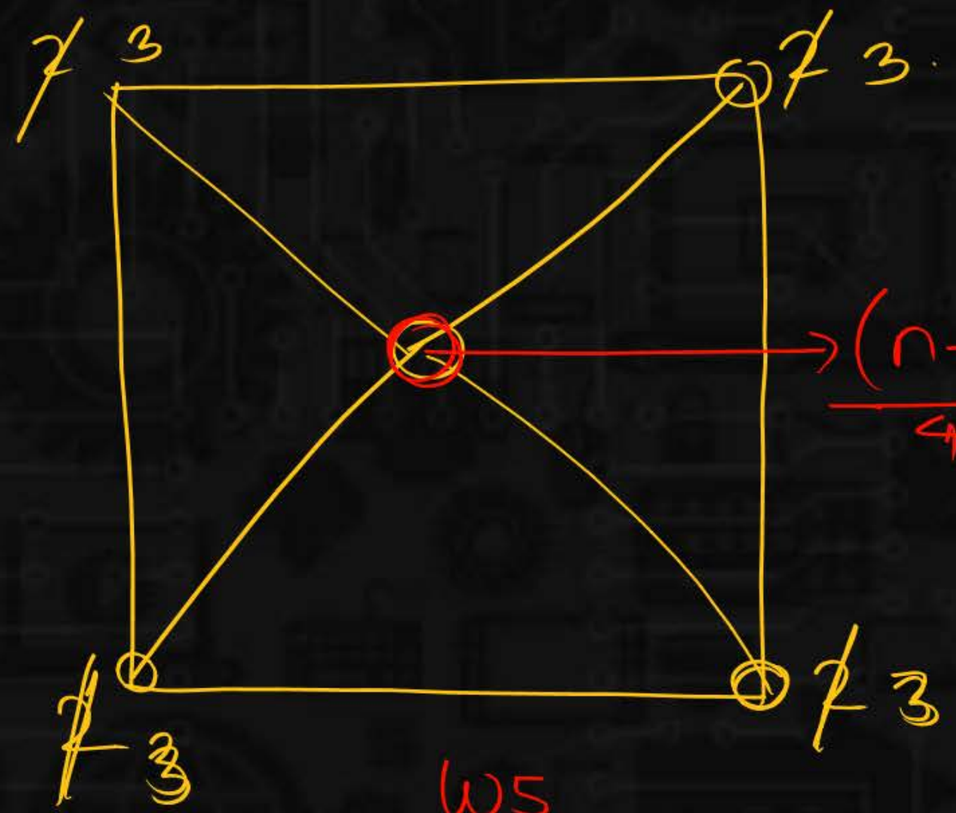


W_4.



W_5.

Types of graph



n = 6

5 3 3 3 3 3

middle

corner vertices

(n-1)

(3 ..... 3)

Types of graph

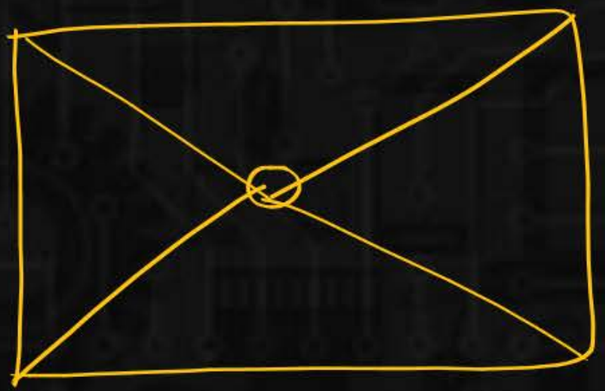
$n = 100$

99, 3 3 3 3 3 3 3

99 vertices

99 → middle vertex

Types of graph



W_5

edges.

$C_4 \longrightarrow 4\,(n-1)$

$\bullet \longrightarrow 4\,(n-1)$

$n-1+3\times(n-1)=2e.$

$4(n-1)=2e$

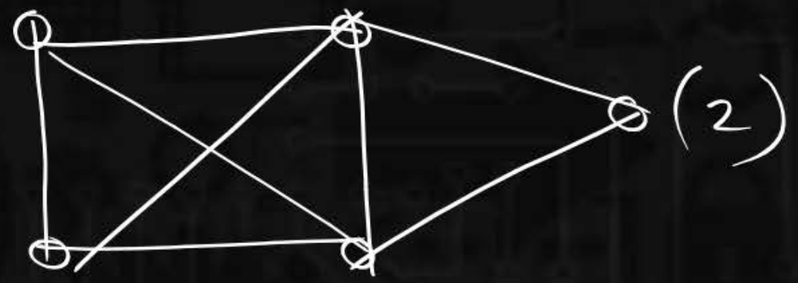
$e=2(n-1)$

Types of graph

1. if Graph is ω_n then no of edges $= 2(n-1)$

2. if $e = 2(n-1)$ then Graph is ω_n (false)

$\underline{n=5}$ $e = 2\times(4) = 8$



(2)

Types of graph

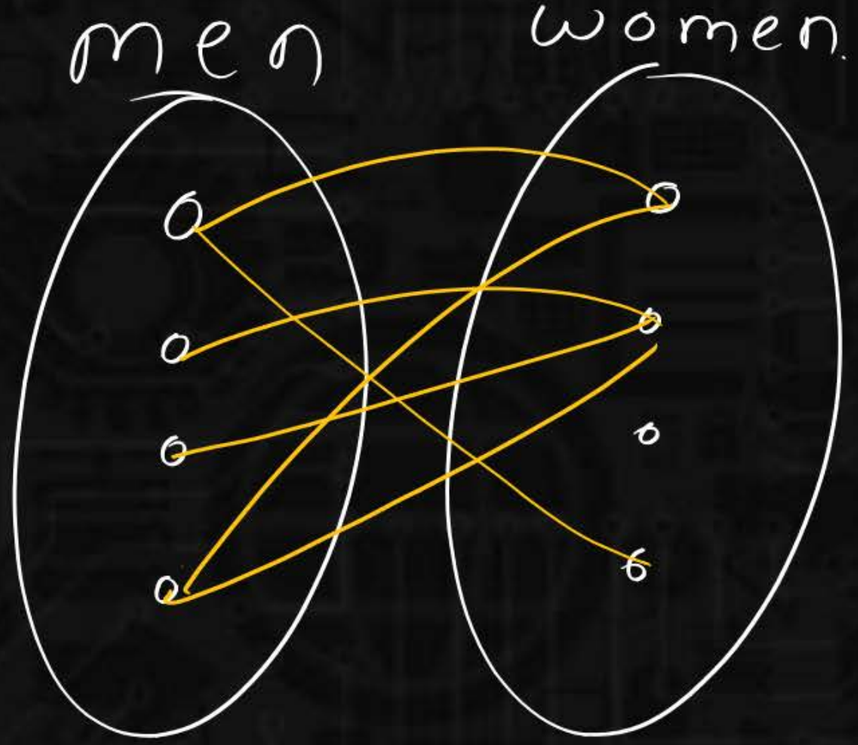
Bipartite Graph :

$G = (V, E)$

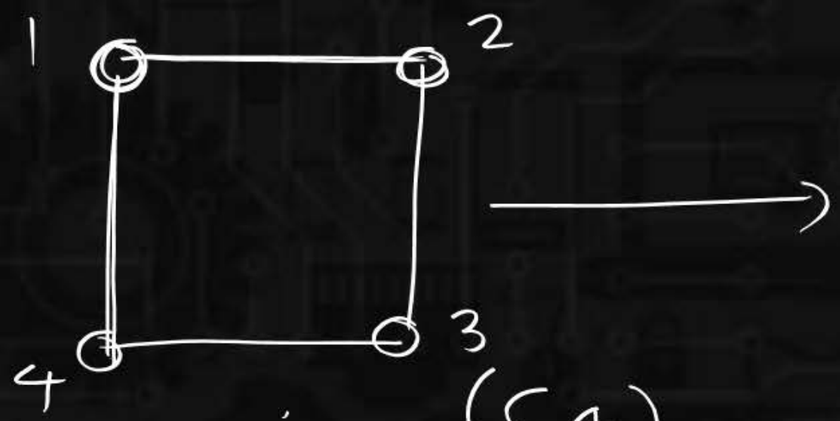
V can be divided into

V1, V2.

each edge will be from one set to another but not in same set

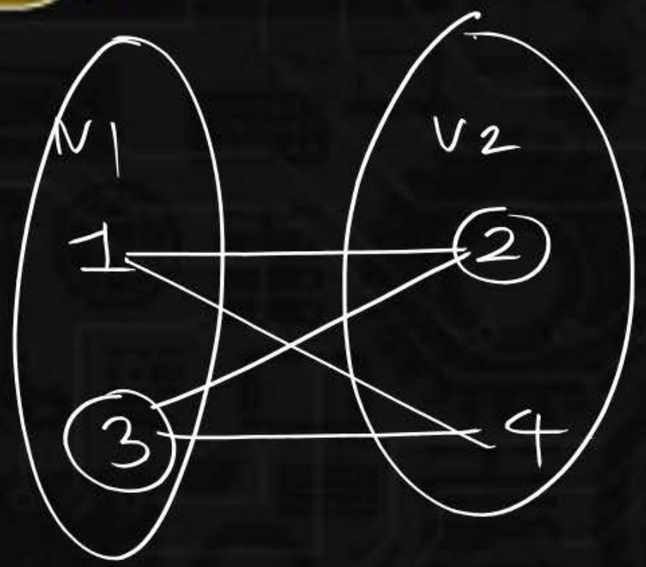


Types of graph

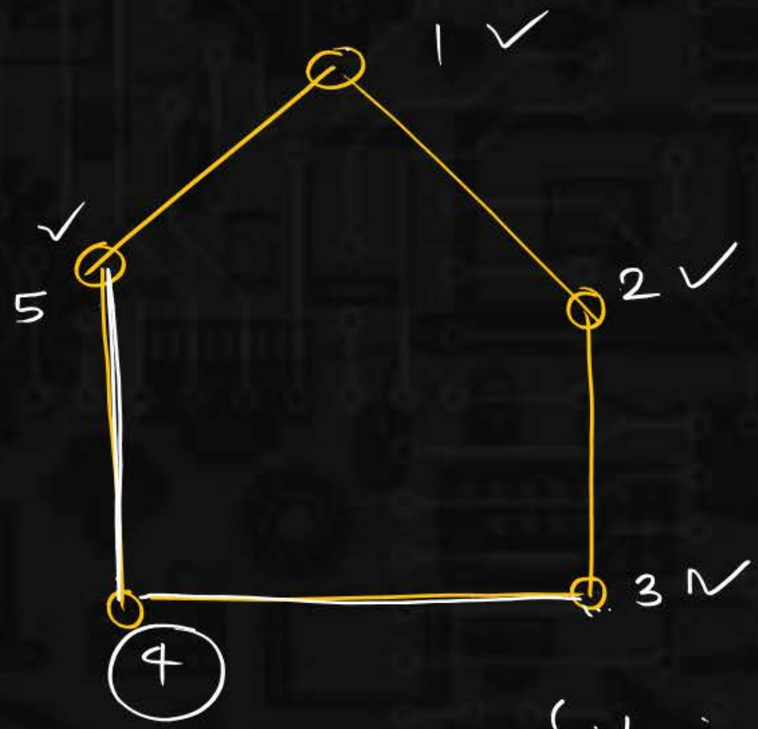


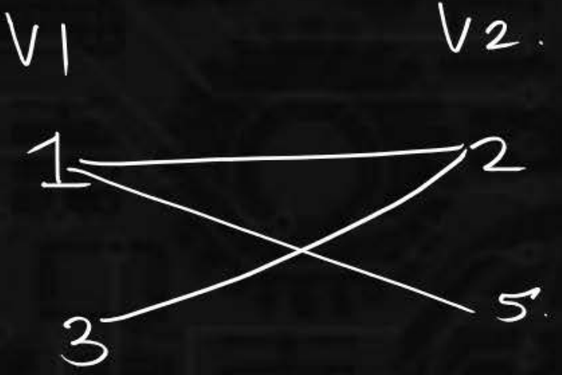
(C_4)

$G_1 = (V, E)$



Types of graph



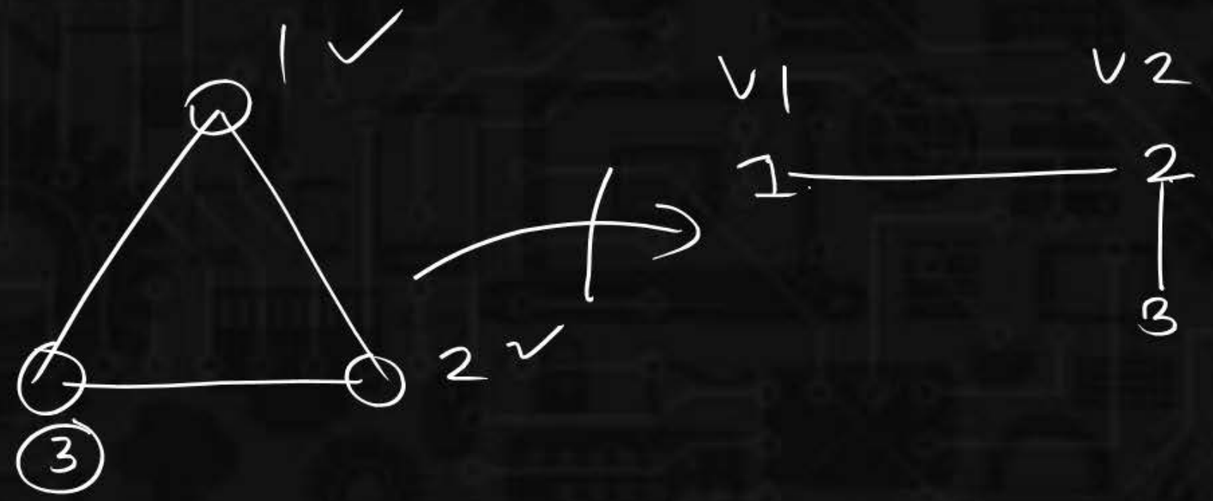


4.
X

4.
X

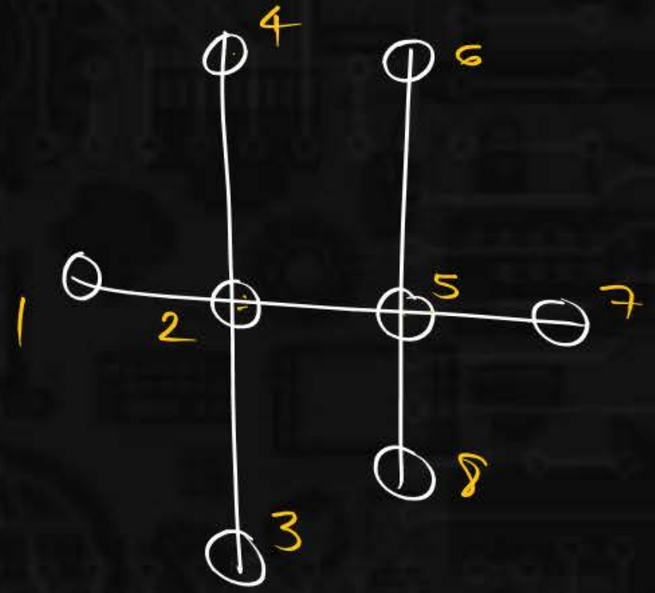
{it is not
bipartite graph.

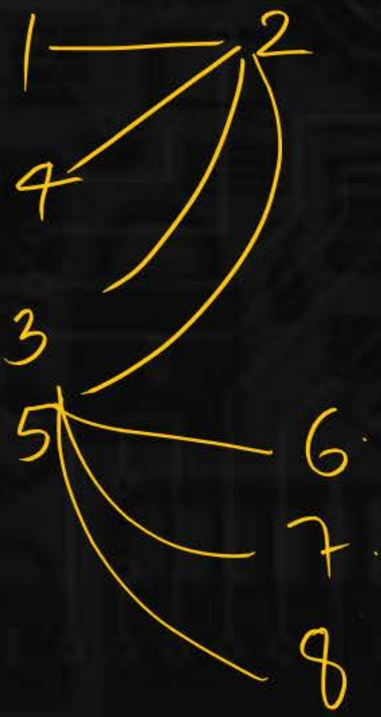
Types of graph



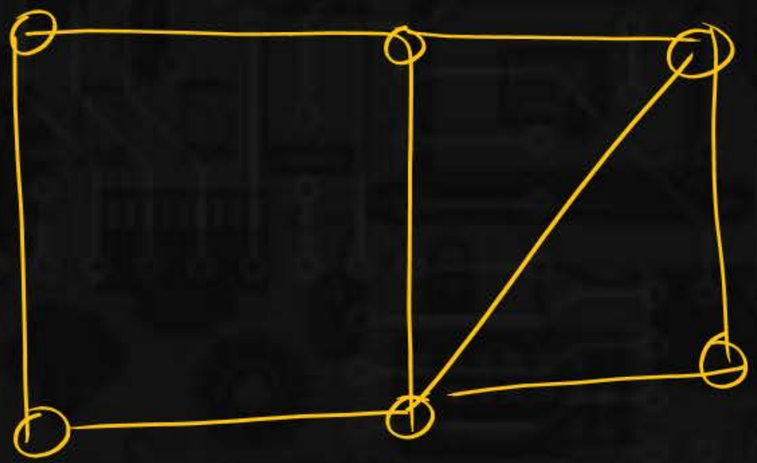
Types of graph

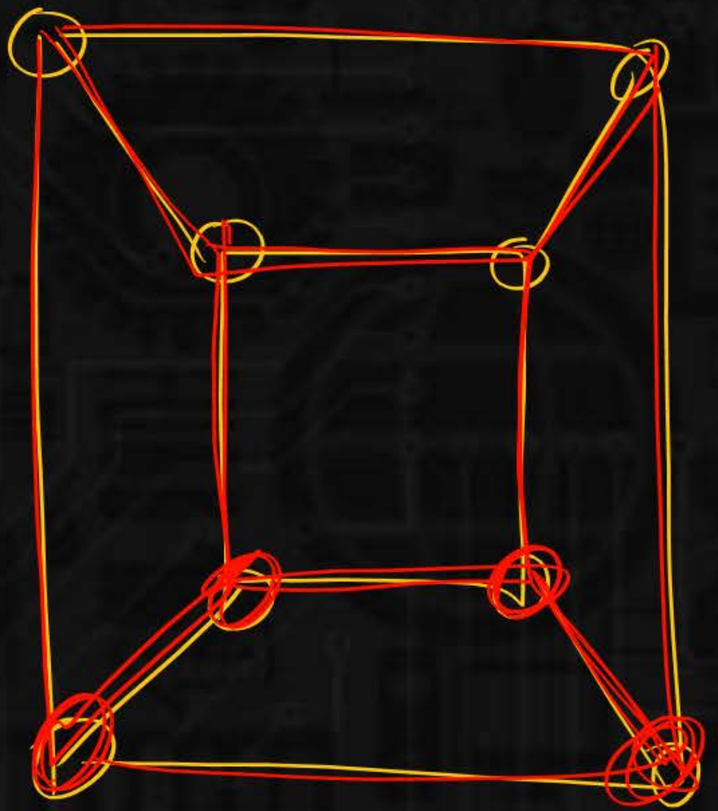
Thm: Bipartite graph does not odd length cycle





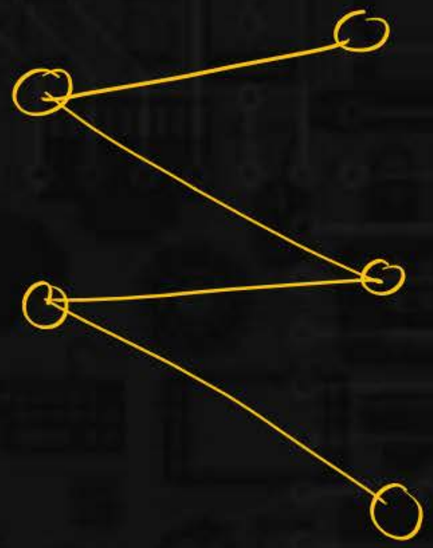
Types of graph





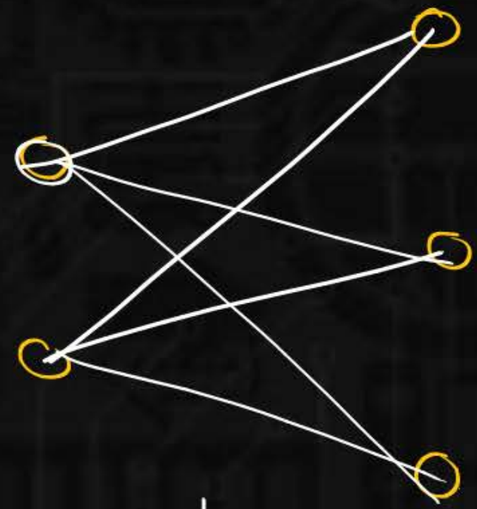
Types of graph

Bipartite Graph.



Complete Bipartite Graph.

$(K_{m,n})$



$K_{2,3}$.


THANK YOU GW
SOLDIERS !