CS & IT ENGINEERING

Independence no. and dominance no.

Lecture No.10

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Coloring

TOPICS TO BE COVERED

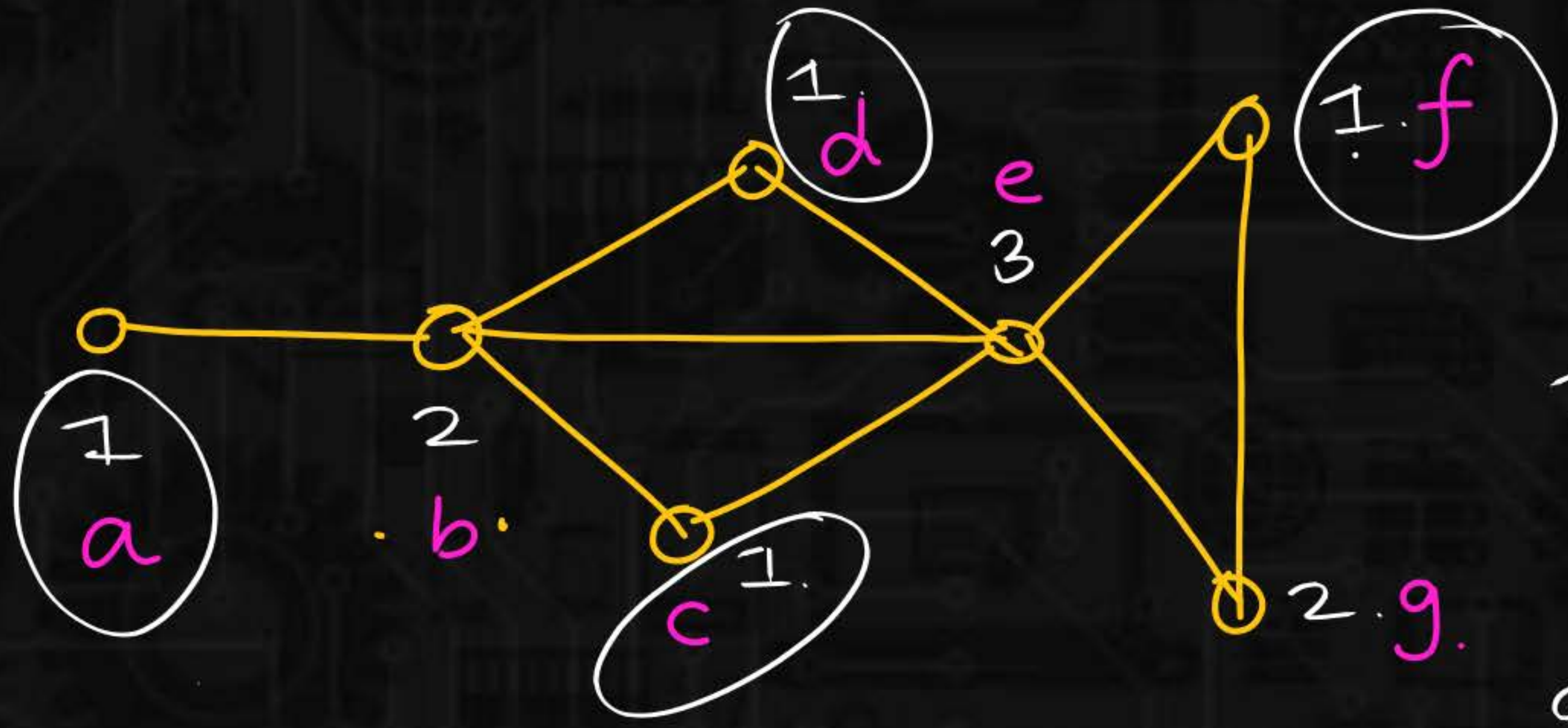
01 Independent set

02 Maximal Independent set

03 Dominance set

04 Minimal dominating set

05 Domination number



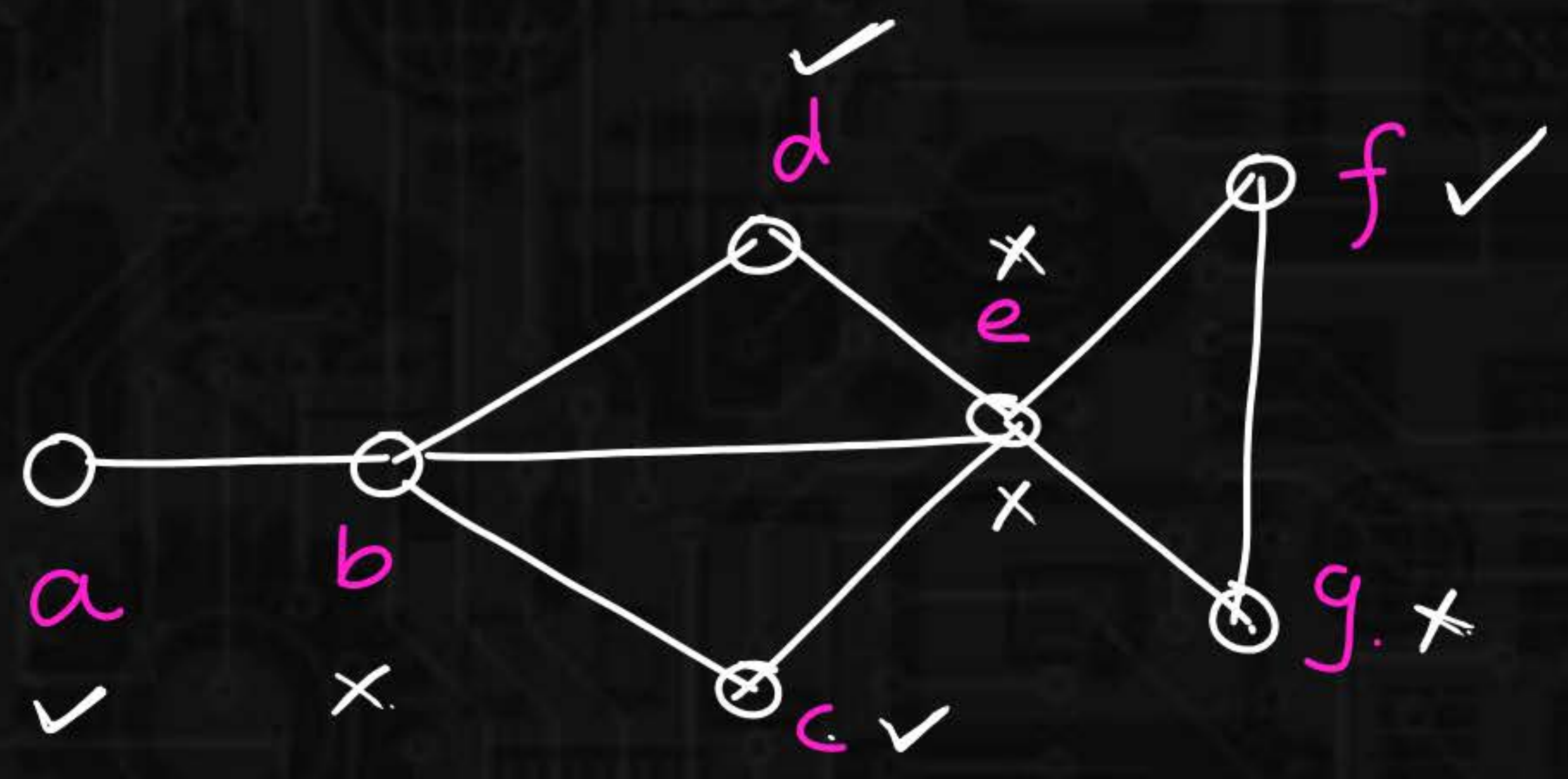
Set of non adjacent vertices

$1 \rightarrow \{\boxed{a, c, d, f}\}$

$2 \rightarrow \{b, g\}$

$3 \rightarrow \{e\}$

painting all
vertices with
min color
writing same color
vertices in set
called partitioning.

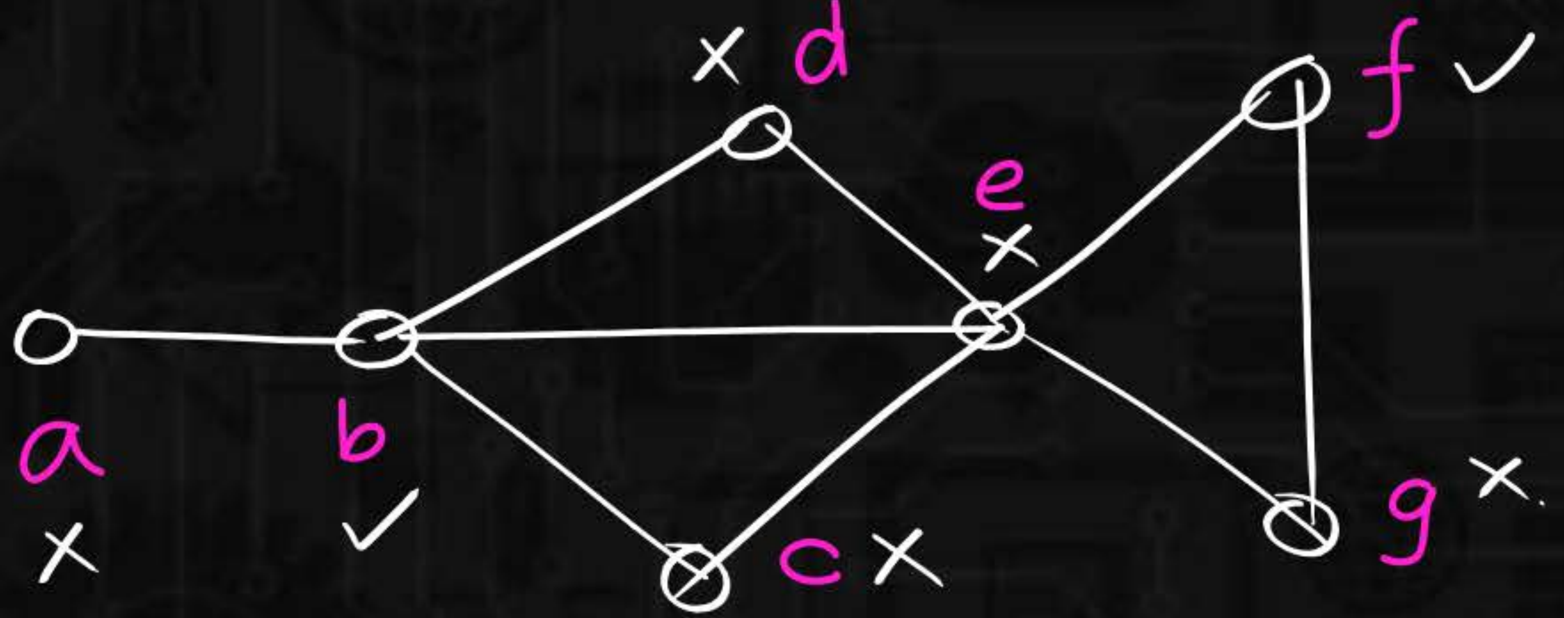


Independence set :

Set of non adjacent vertices.

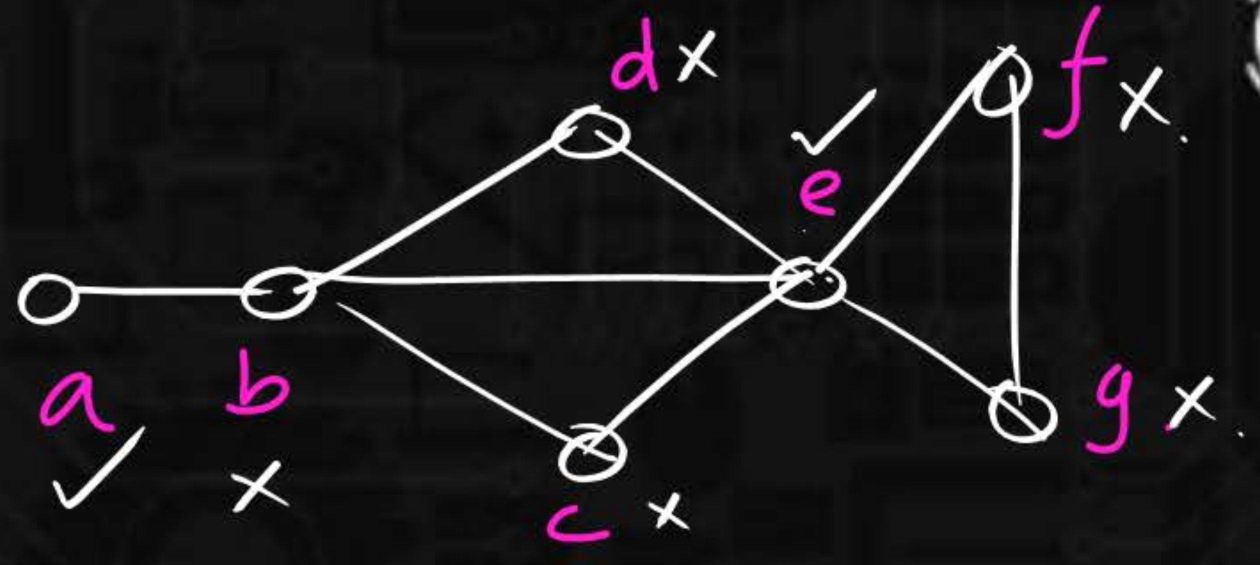
$\{a\} - IS$ $\quad \{a, c, d\} \to IS$

$\{a, c\} - IS$ $\quad \{\underline{a, c, d, f}\} - IS$ (MIS)



$\{b\}$ – IS

$\{b, f\}$ – (MIS)



$\{a\}$ – IS

$\{a, e\}$ (MIS)

Independent set:

Set of non adjacent vertices.

Maximal Independent set (MIS)

Independent set such that we can not add new element into this.

maximal

It is related to property.

we can not add

not related size

MIS:

$\boxed{\{a\,c\,d\,f\}}$ — 4 ✓

$\{a\,e\}$ — 2

$\{b\,g\}$ — 2

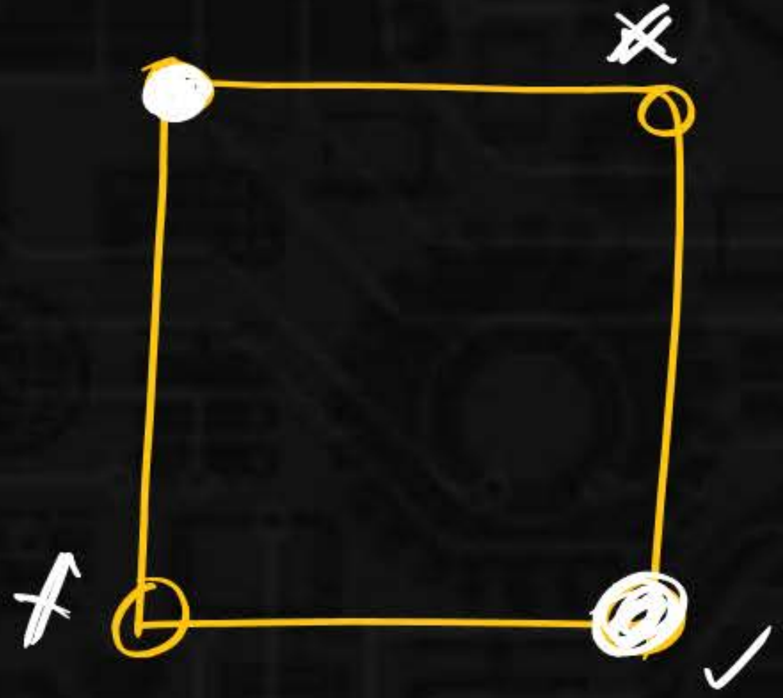
$\{b\,f\}$ — 2

Independence no ($\beta(G)$)

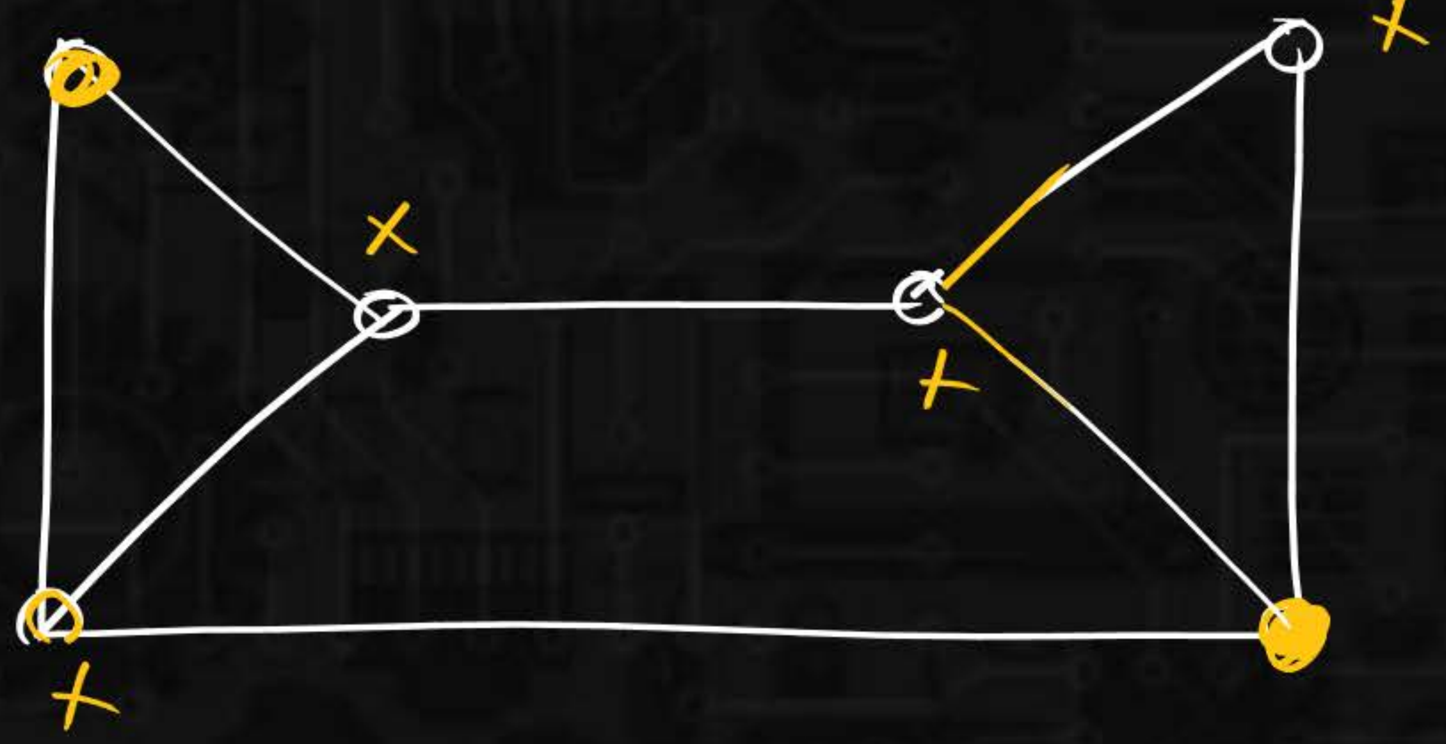
no. of vertices present in largest maximal Independent set

$\boxed{\beta(G) = 4}$ ✓

$\beta(G) = 1$



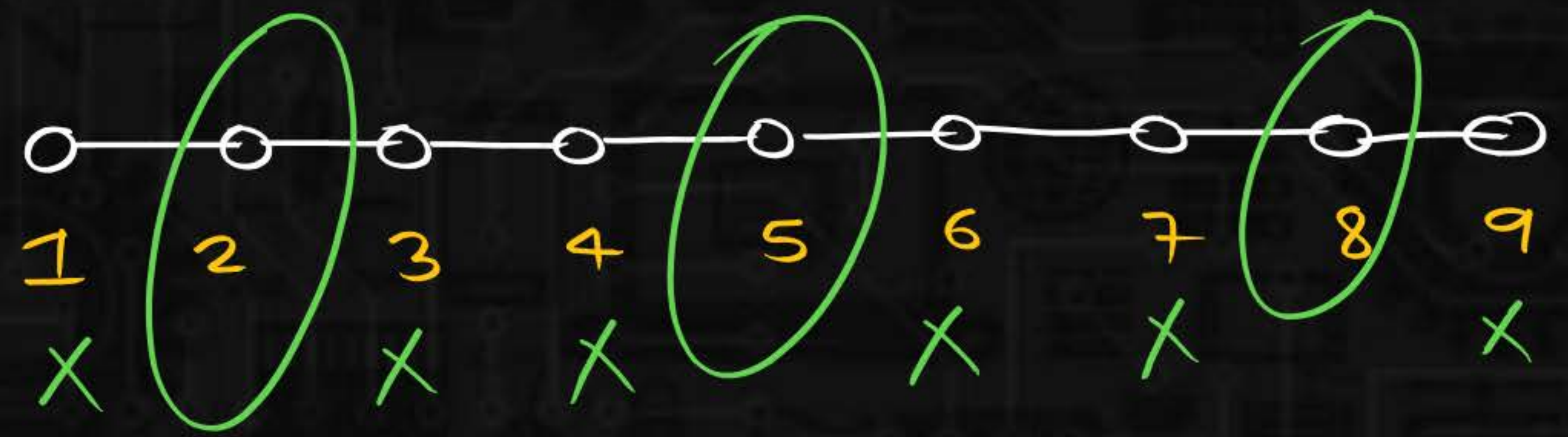
$\beta(G) = 2.$



$\beta(G) = 4.$

maximal Independent set

What is the size of smallest MIS? (GATE)

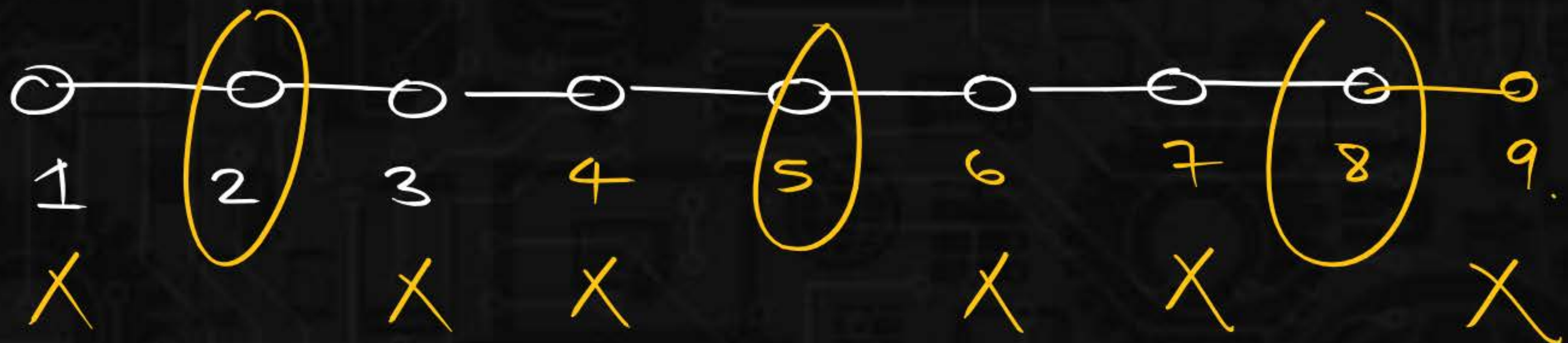


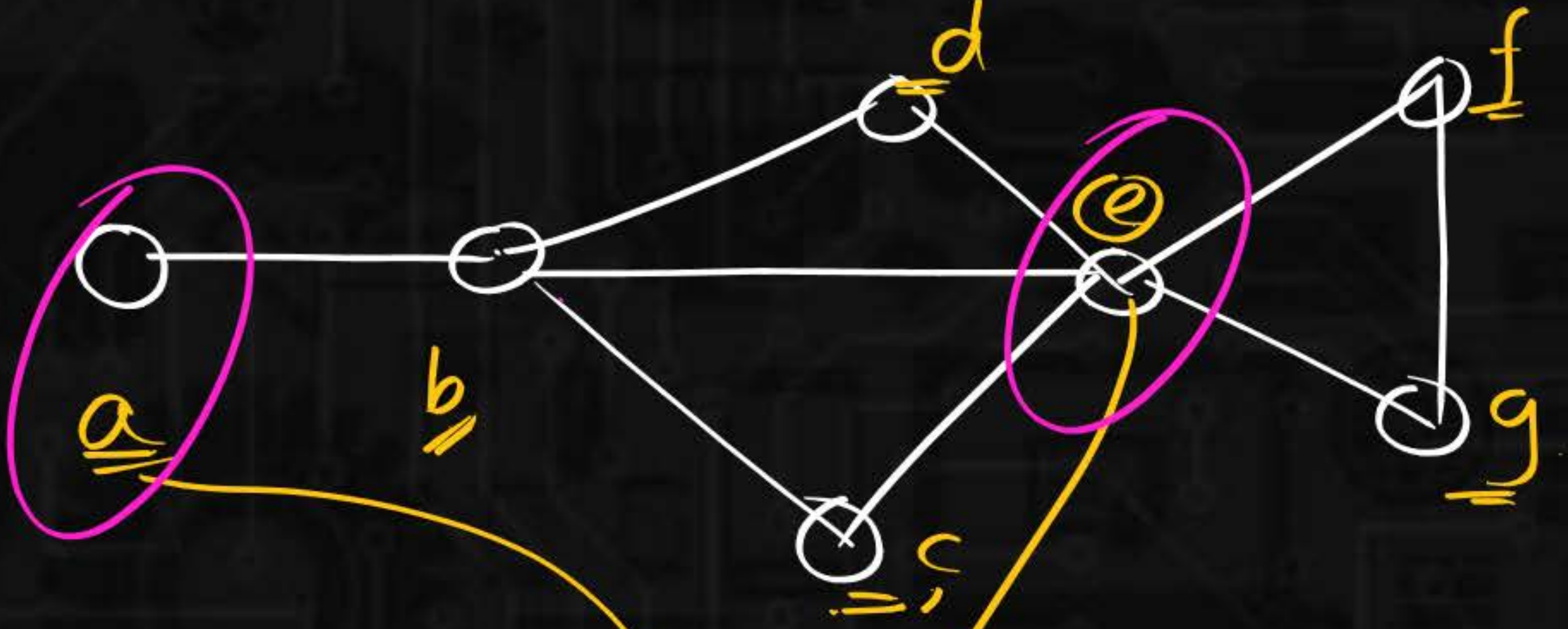
MIS

$\{1, 3, 5, 7, 9\}$. MIS.

$\{2, 5, 8\}$ MIS.

$\{2, 4, 6, 8\}$ - MIS.


1
2
3
4
5
6
7
8
9.
X X X X X X



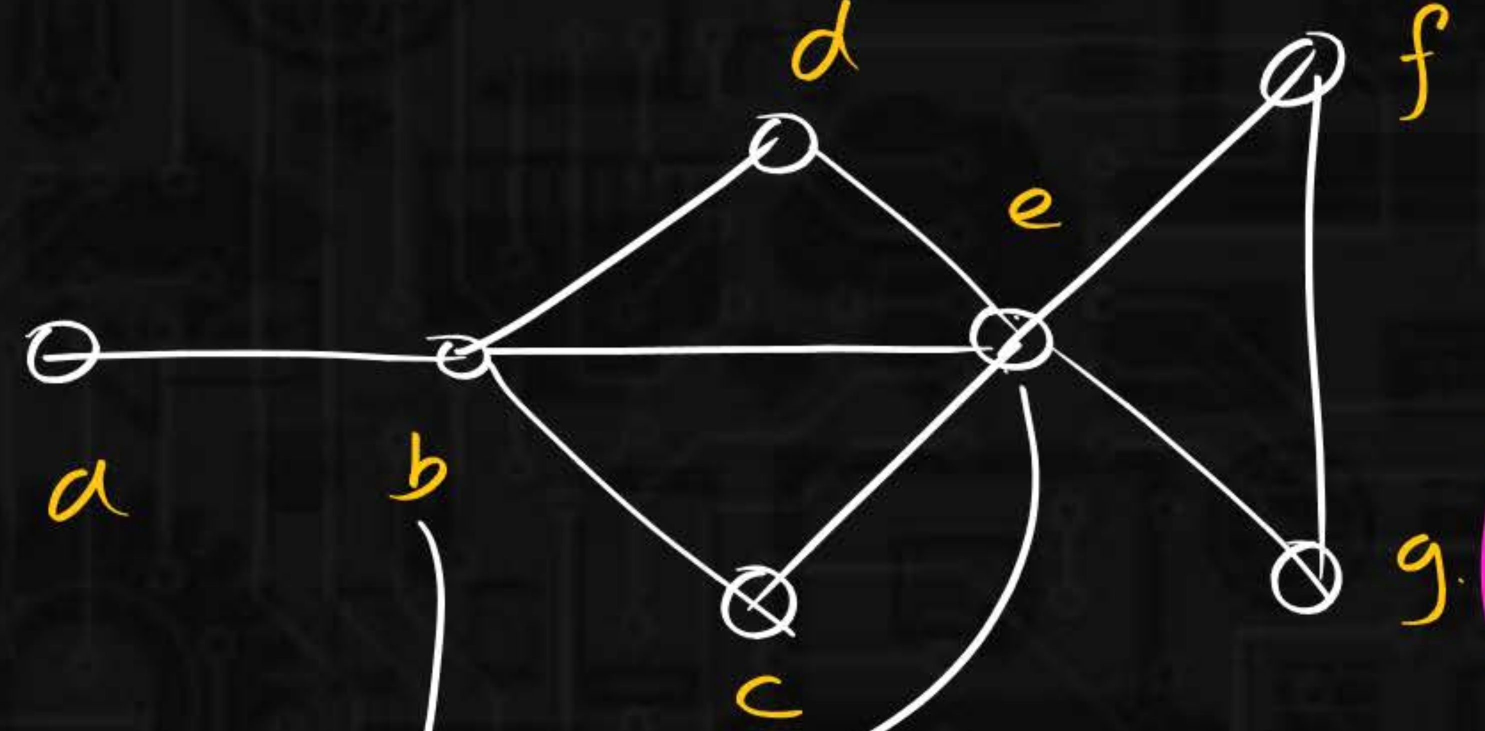
$\rightarrow \{a, e\}$ | D.S

Dominating set (D)

$G = (V, E)$

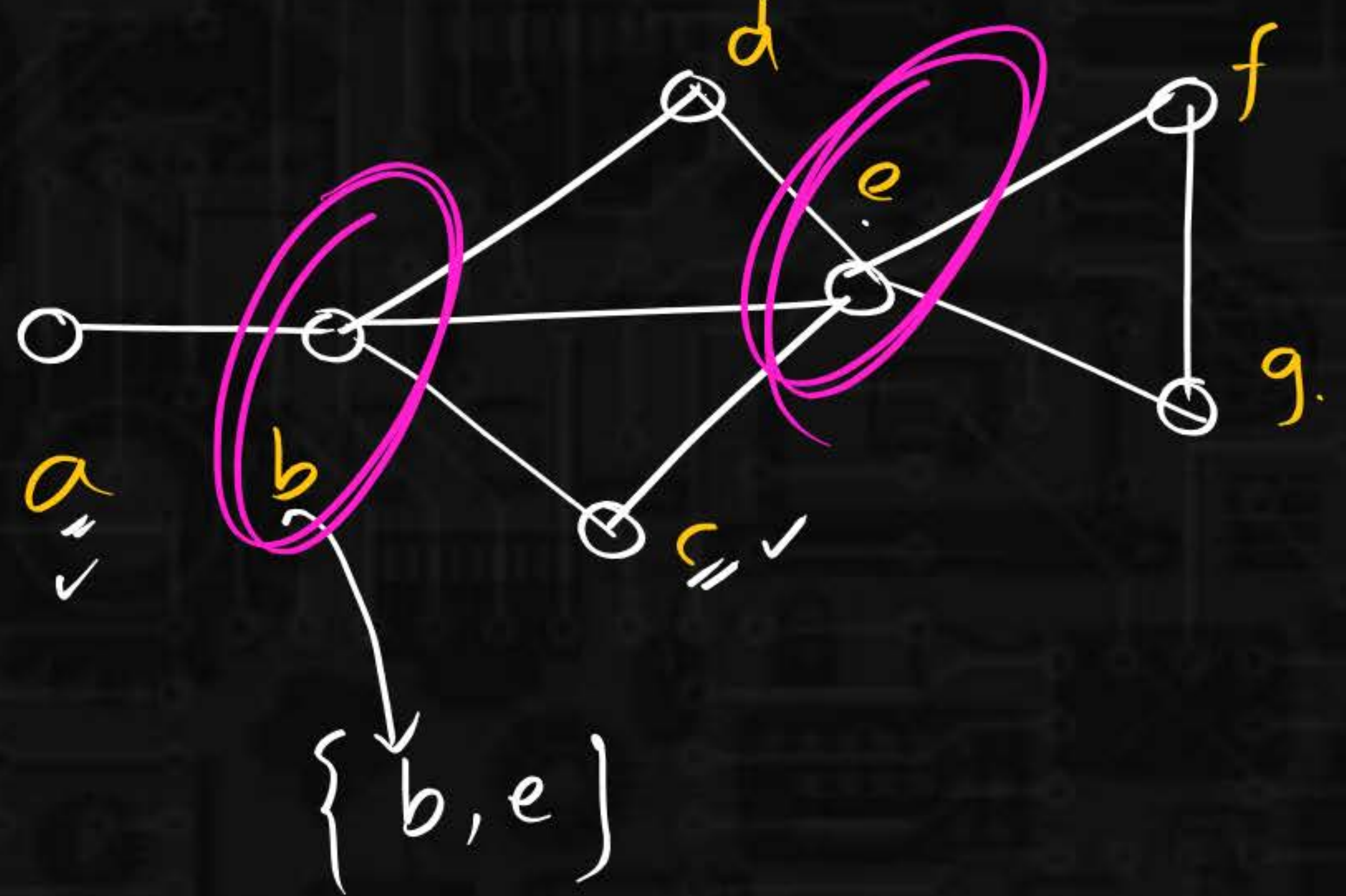
$D \subseteq V$

Such that vertex directly belongs to D or its adjacent belongs to D.



{be}

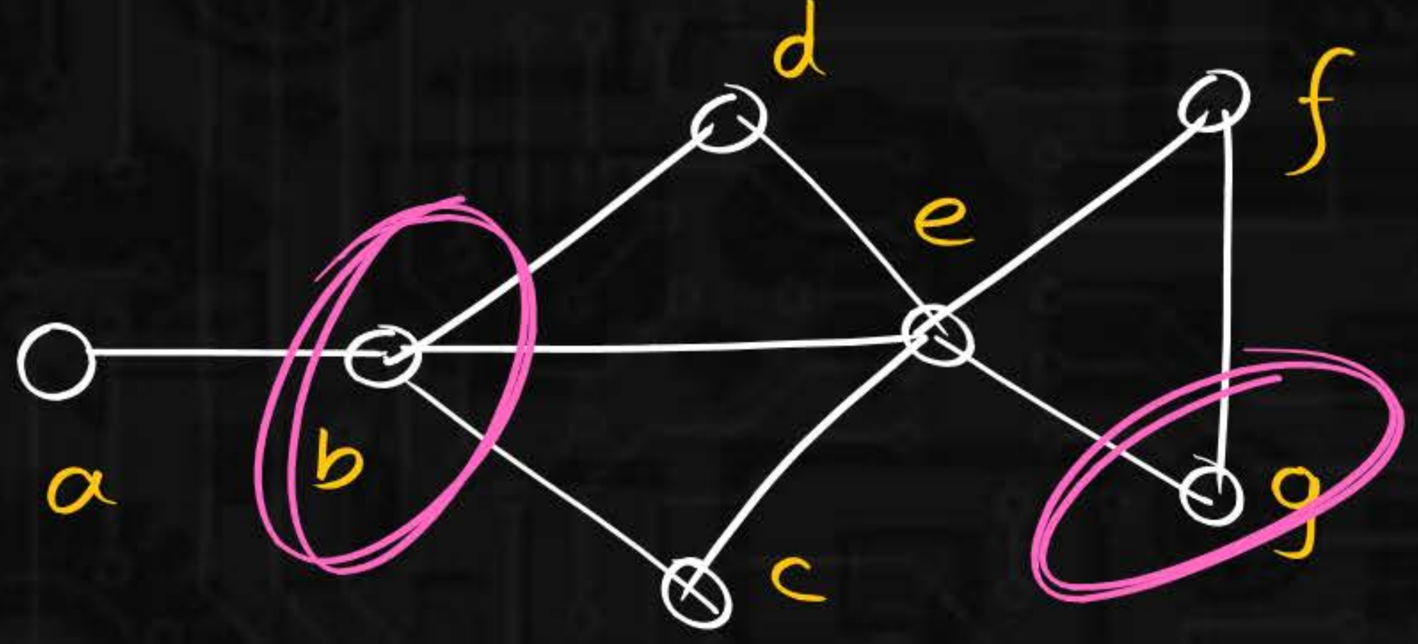
{a, c, d} : it is not dominating set.



Independent set

nonadjacent

Dominating set



$\{\cancel{a}bcdefg\}$ – DS.

$\{b\cancel{c}defg\}$ – DS ✓

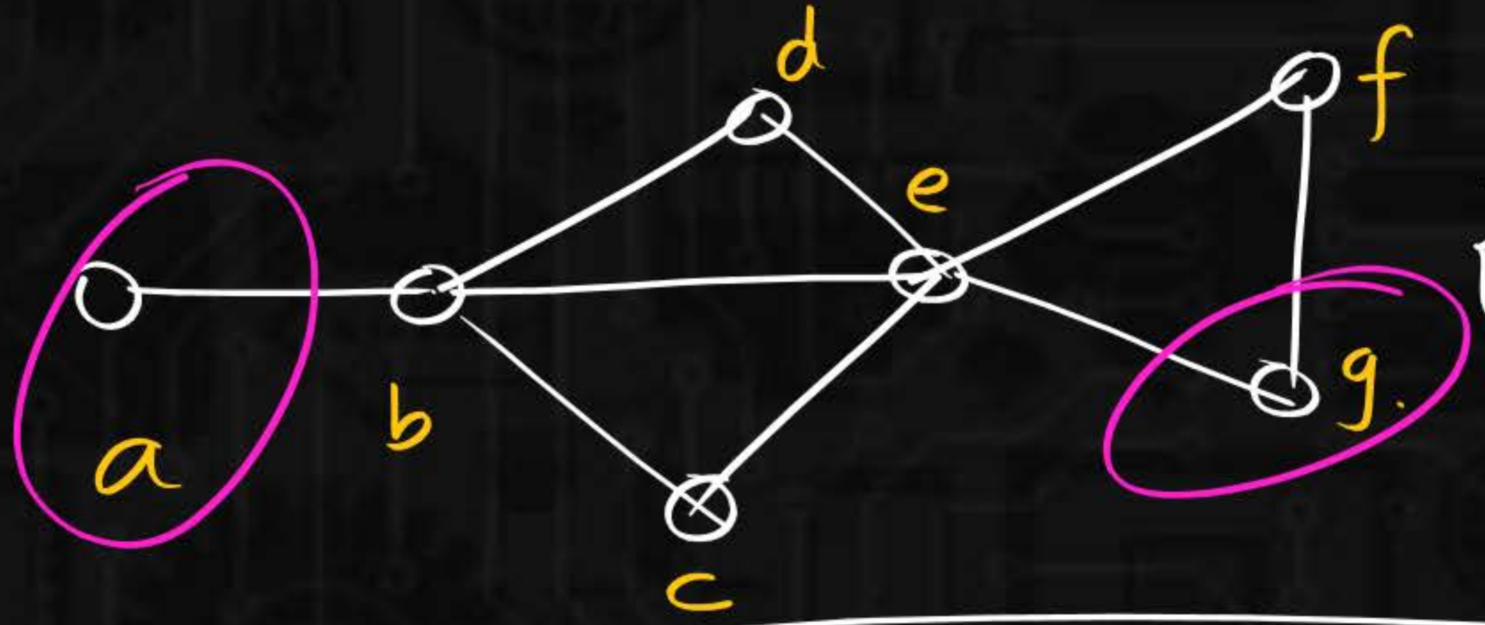
$\{b\cancel{d}efg\}$

$\{b\cancel{e}fg\}$

$\{b\cancel{f}g\} \rightarrow \{bg\}$ D.S

minimal dominating set

dominating set such that we can not remove new element from this.



{all vertices} – DS.

<u>minimal</u>

it is not related size. but property such that we can not remove any element

{be} – MDS ✓

{ae} MDS ✓

{bf} MDS ✓

{bg} MDS ✓

{acdf} MDS ✓

{ag}

Domination no($\alpha(G)$)

no. of vertices present in Smallest minimal dominating set

can not remove

$\alpha(G) = 2$.

MDS

$$\begin{cases} acdf \rightarrow 4 \\ be \rightarrow 2 \\ bf \rightarrow 2 \\ bg \rightarrow 2 \\ ae \rightarrow 2 \end{cases}$$

Indepenceset

{non adjacent vertices.

Dominating set

MIS. cannot add.

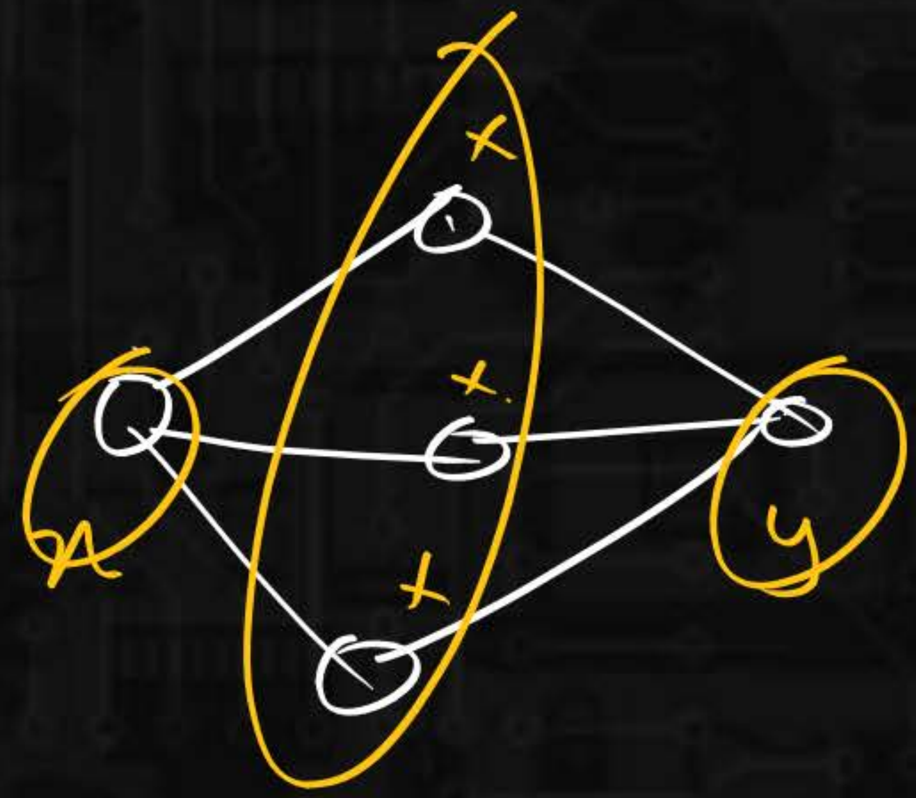
MDS. we can not remove.

$\beta(G)$

largest

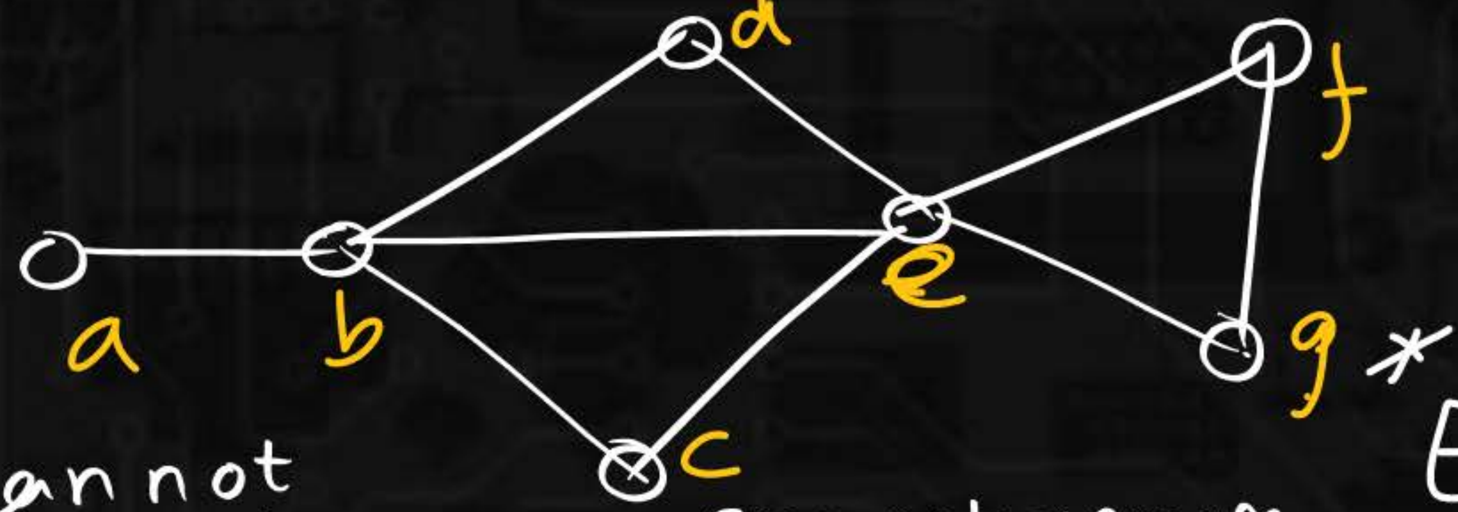
$\alpha(G)$

smallest



MIS $\rightarrow$ MDS

$\{x\,y\}$ $\{x\,y\}$



*

can not add

can not remove

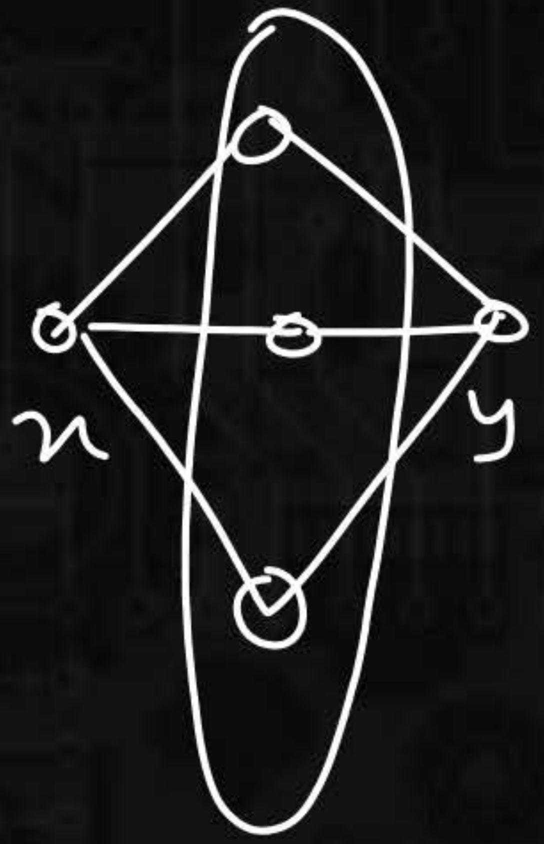
$\beta(G)$ =

MIS	MDS
acdf	acdf
ae	ae
bf	bf $\rightarrow \alpha(G)$
bg	bg
	be

Every MIS will always be MDS.

Every MDS need not be MIS.

$$\boxed{\alpha(G) \leq \beta(G)}$$



MIS: $\longrightarrow$ MDS:

can not add

$\{xy\}$ $\{xy\}$


THANK YOU GW
SOLDIERS !