

# CS & IT ENGINEERING



Matching no. and  
covering no.

**Lecture No. 11**



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# TOPICS TO BE COVERED

01 Matching set

02 Maximal matching set

03 Matching no.

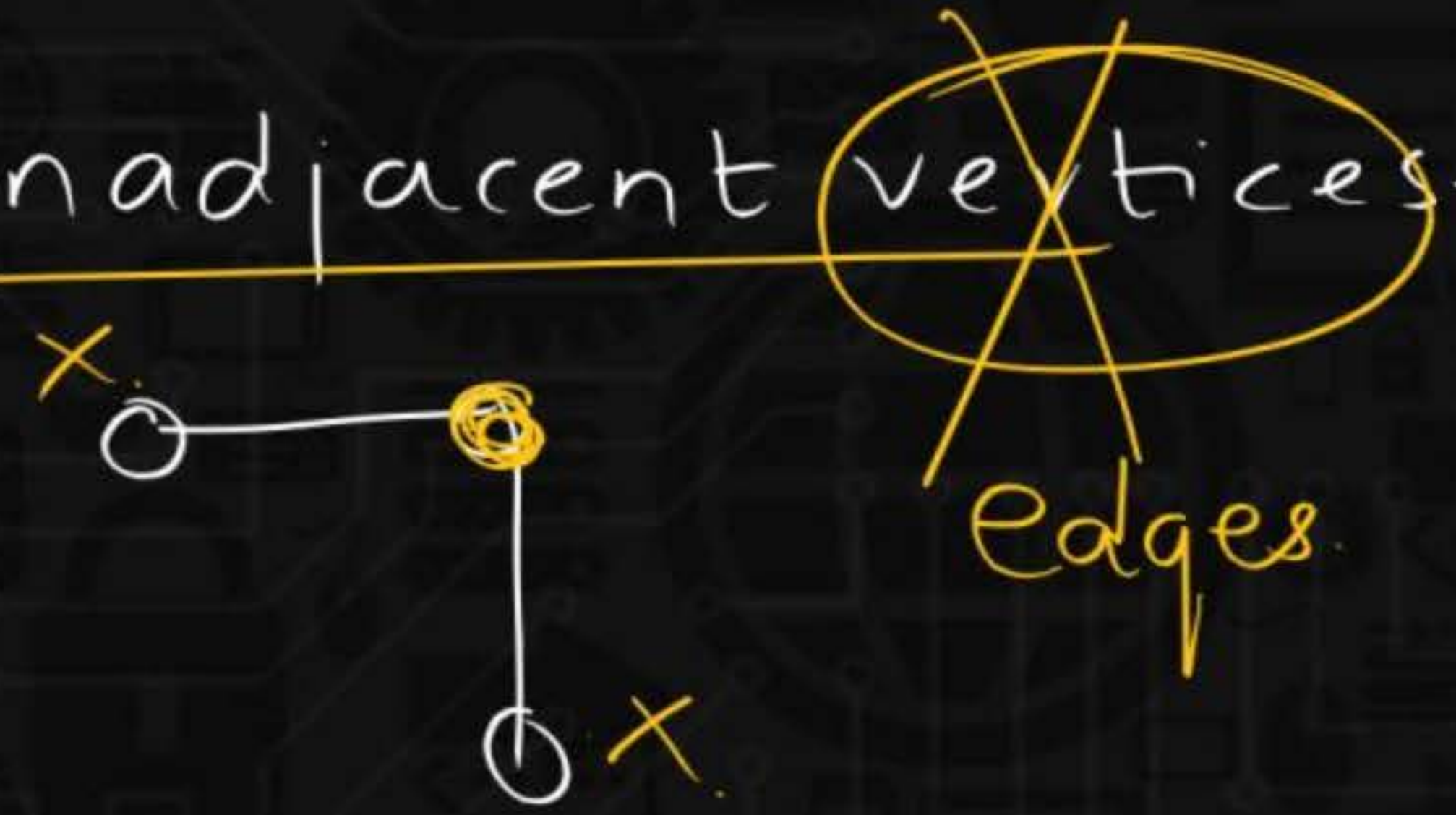
04 Covering set

05 Covering number



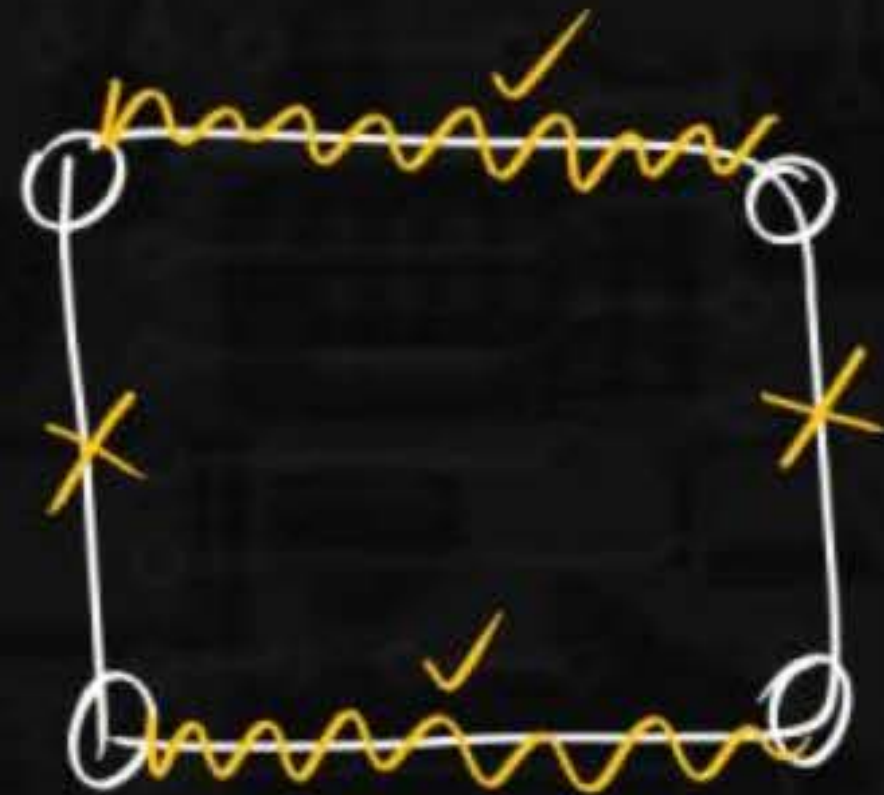
Independent set:

Set of nonadjacent vertices



Independent edge set :

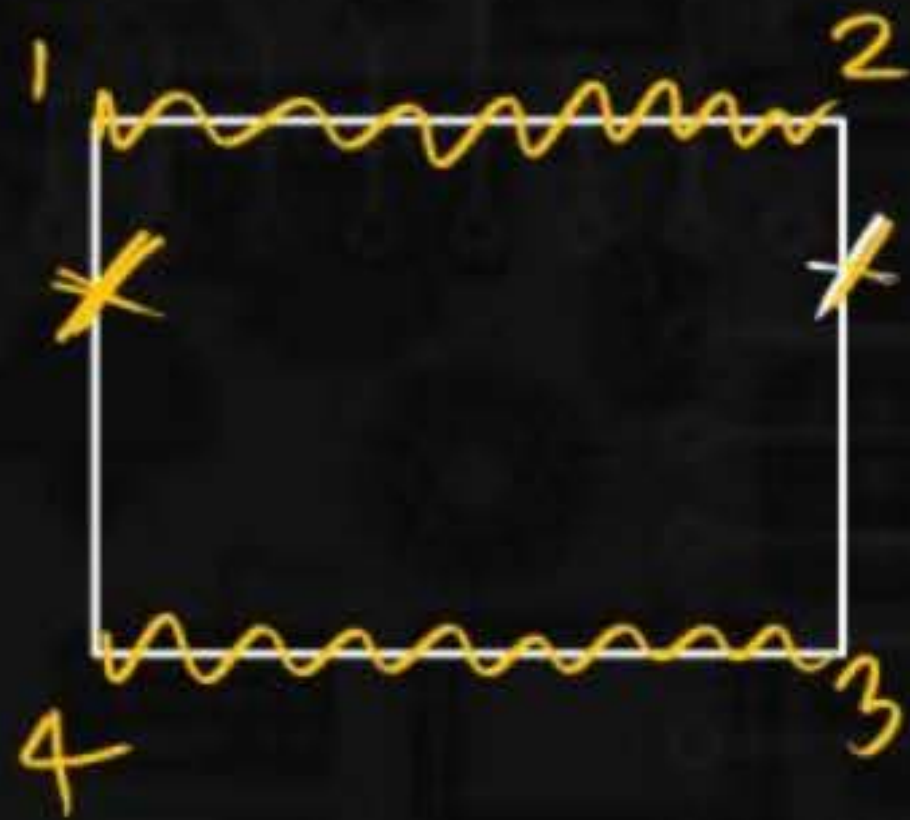
set of non adjacent edges





(Independent edge set)  
matching set :

Set of non adjacent edges.

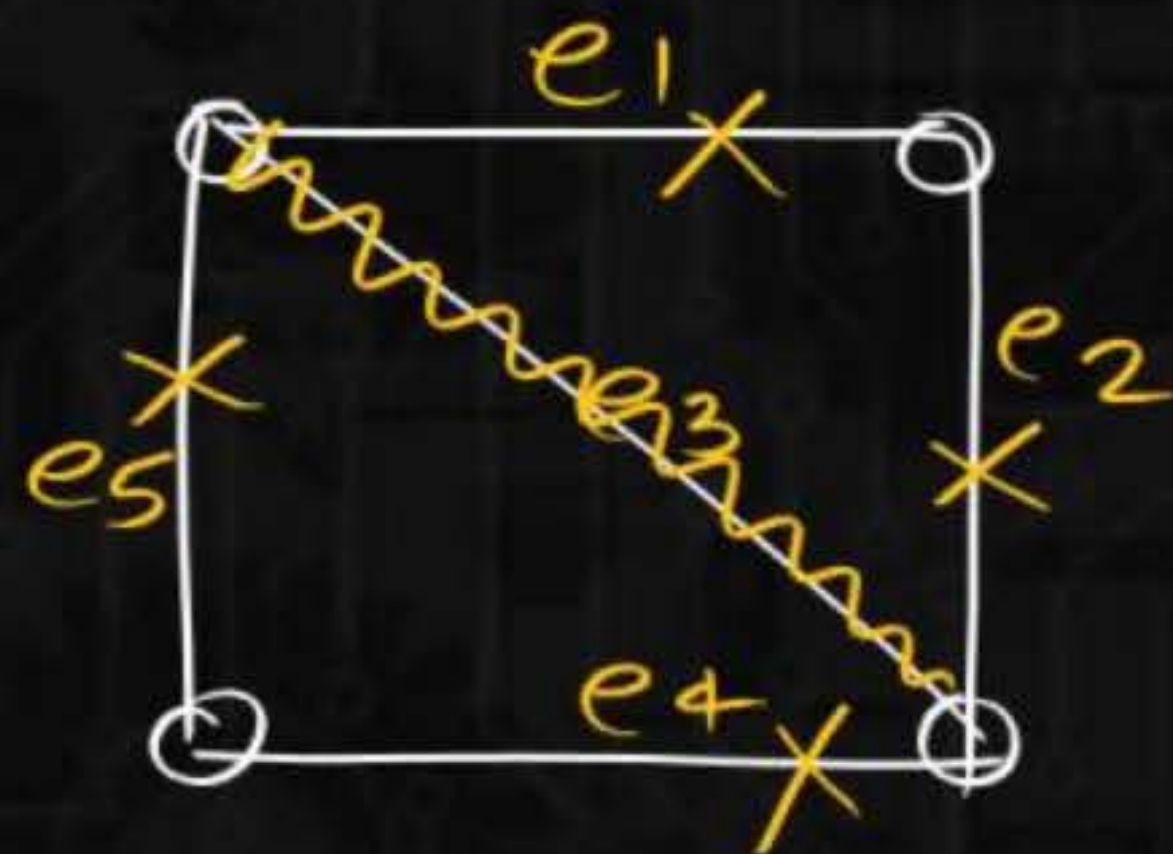


✓ { 1 2 } → matching set ✓

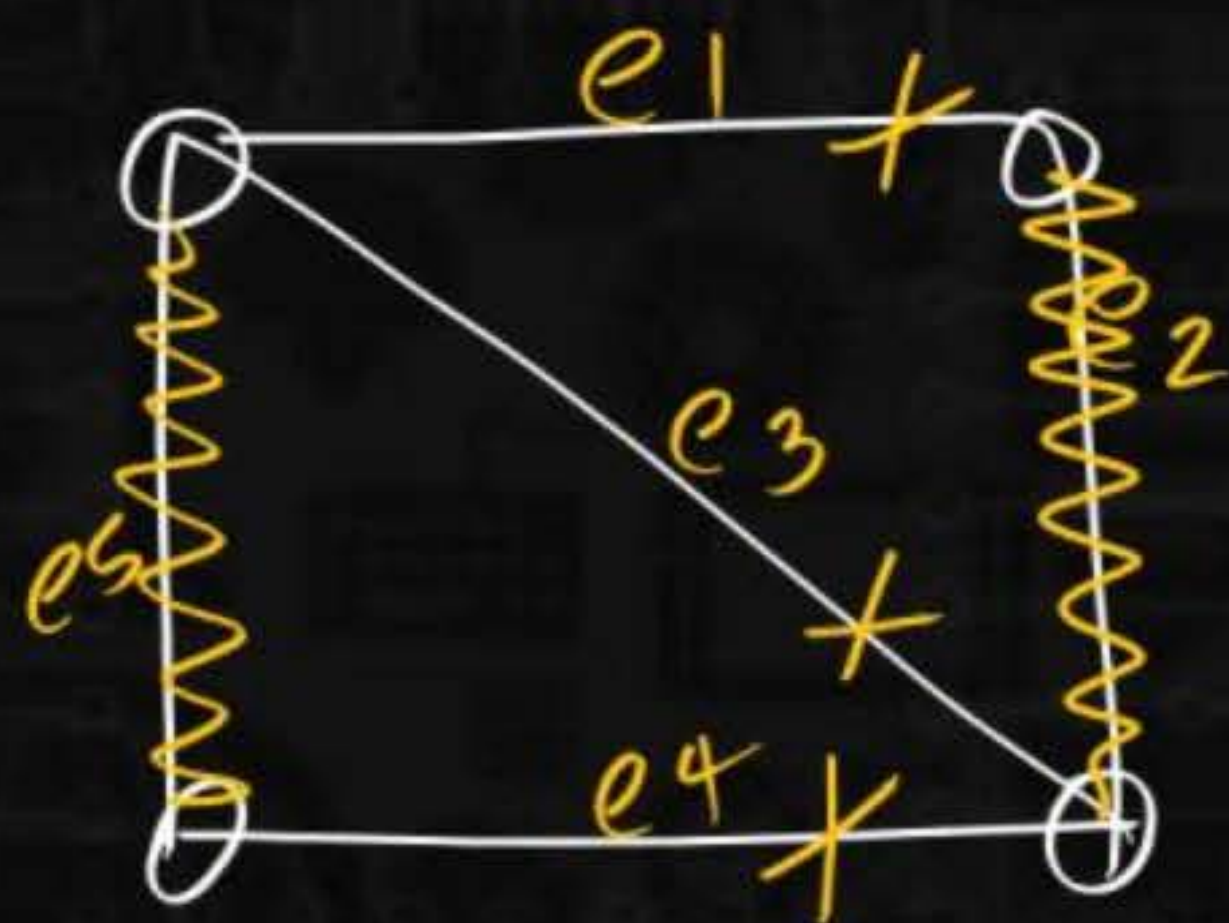
✓ { 1 2, 3 4 } → matching set ✓



$$m(G) = 2$$

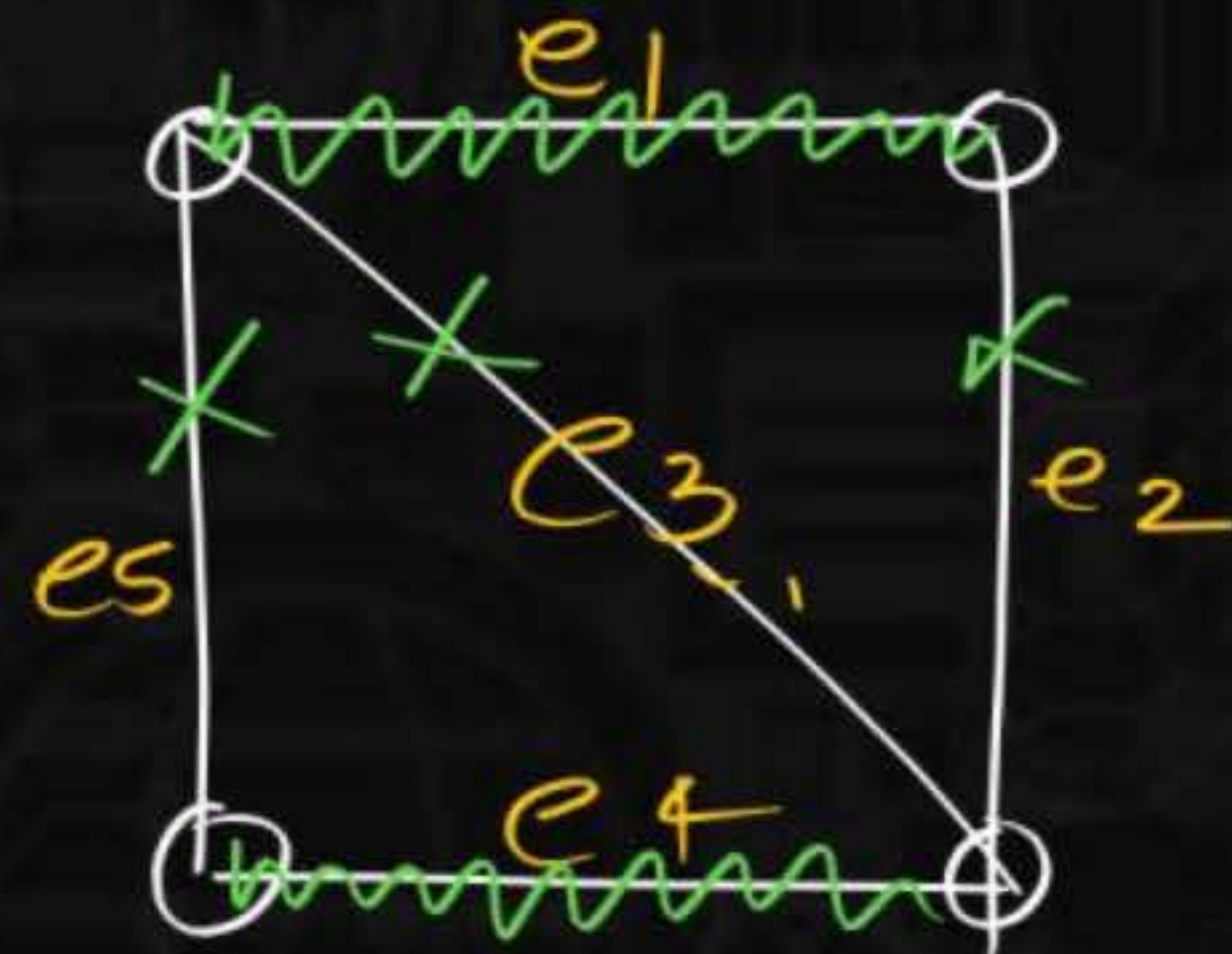


$\{e_3\}$  maximal matching ✓  
set



$\{e_2\}$  ms ✓

$\{e_2, e_5\}$  mms ✓



$\{e_1\}$  — ms

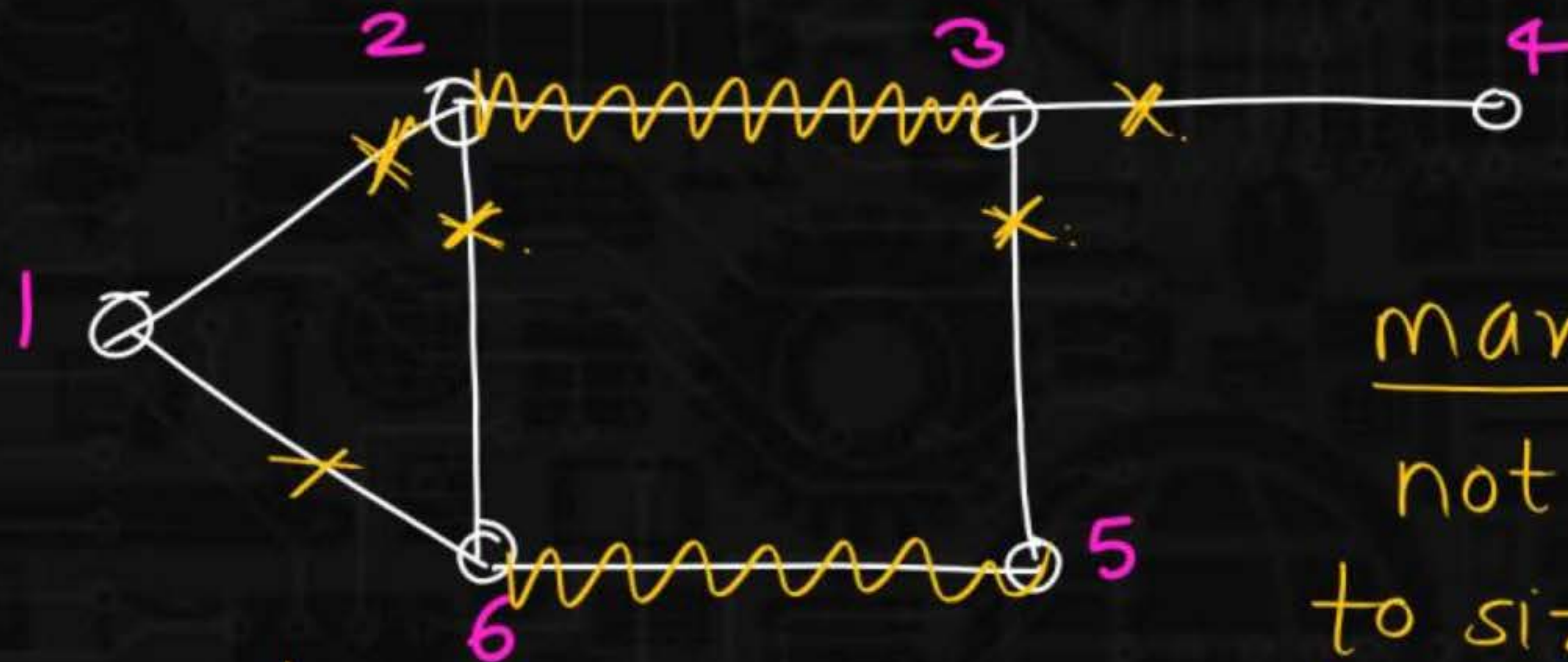
$\{e_1, e_4\}$  mms ✓



$\{2, 3\}$  ms

$\{2, 3, 6, 5\}$

maximal  
matching  
set

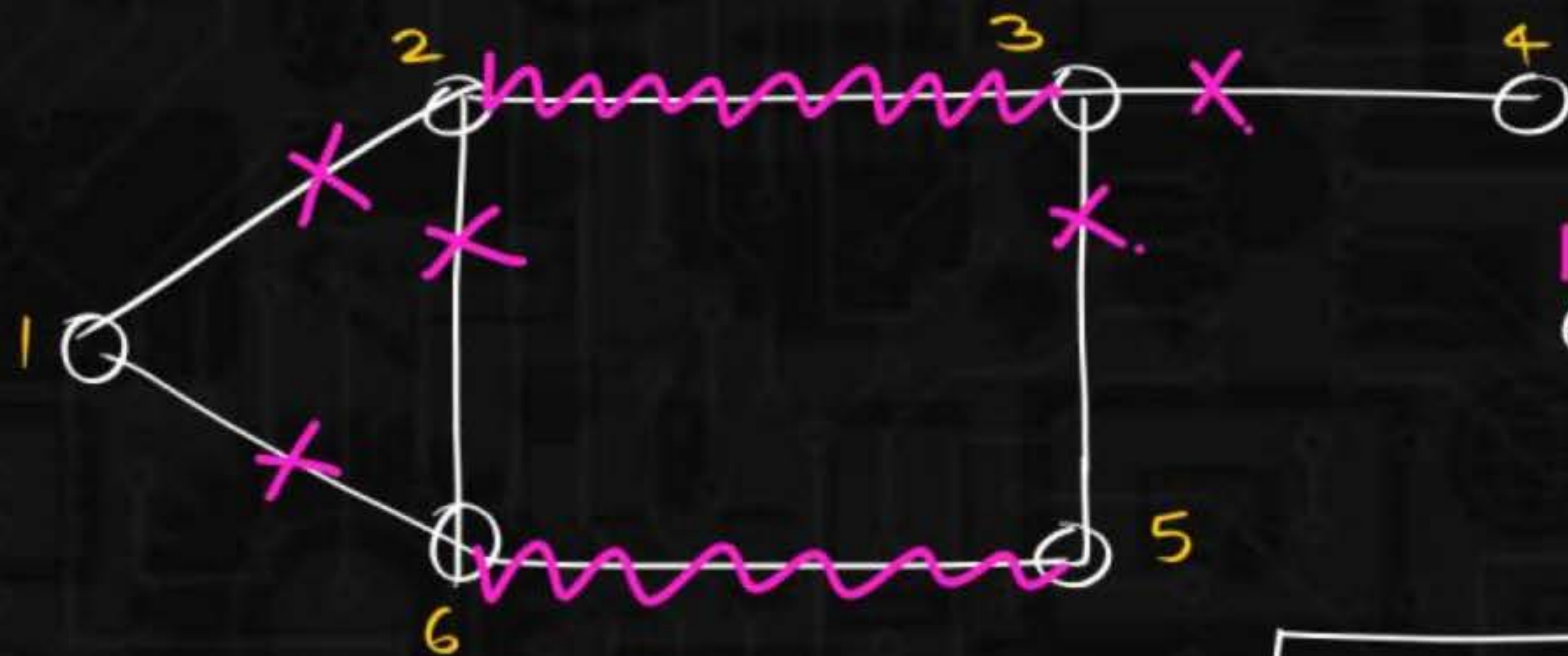


maximal  
not related  
to size  
but property  
can not add

maximal matching set

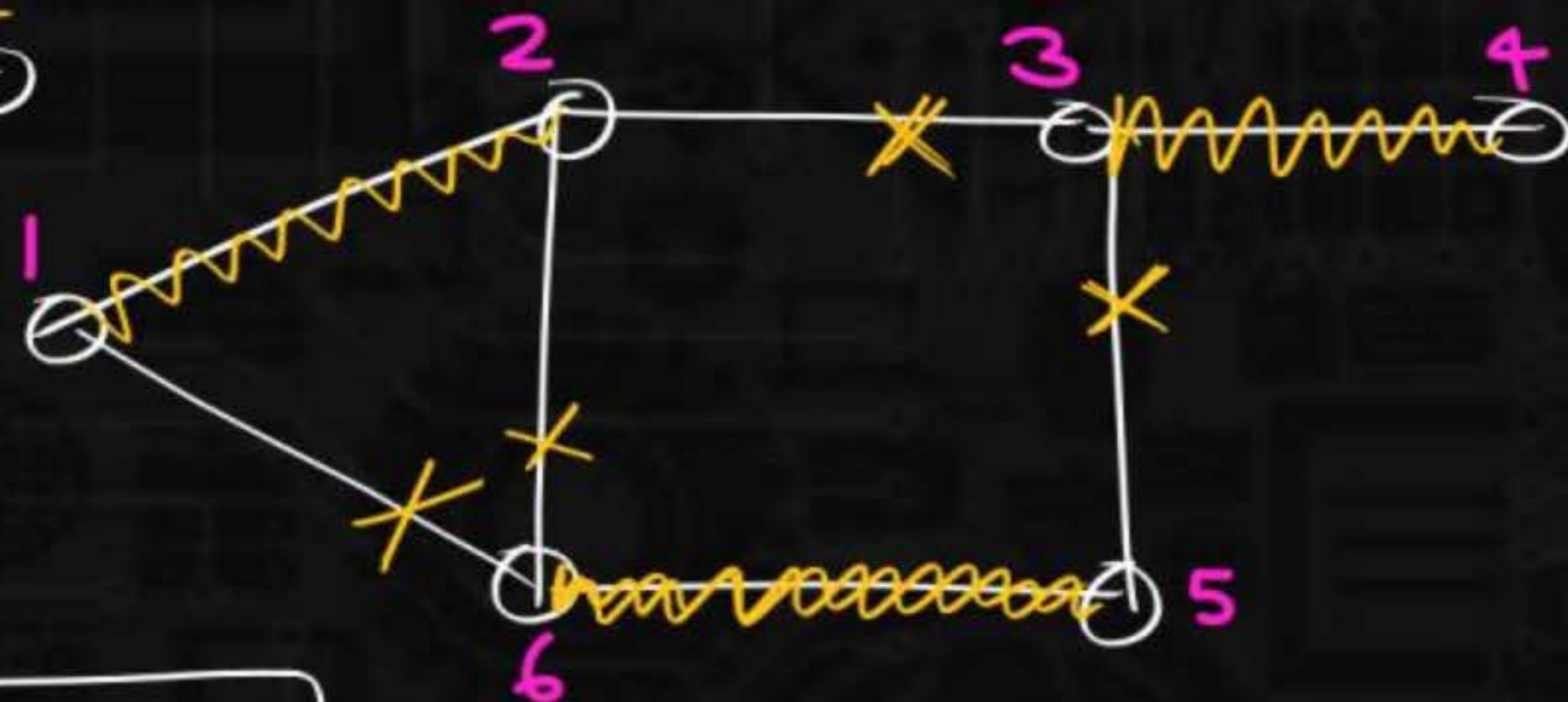
matching set  
such that we can  
not add new edge  
into this.





$$m(G) = 3$$

mm  
 $\{23, 65\}$



$\{34\} - ms$

$\{34, 65\} - ms$

$\{34, 65, 12\}$  mm





In Graph we can get  
Different mms of  
diff. sizes

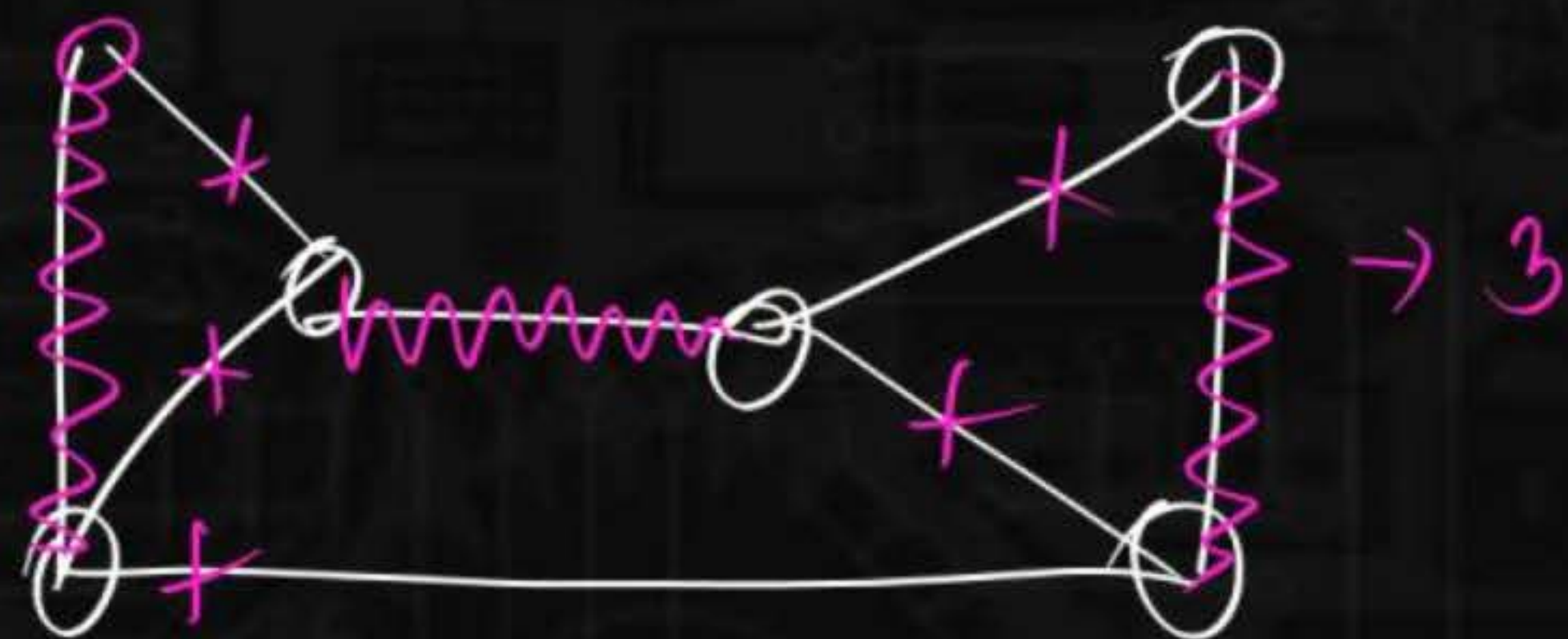
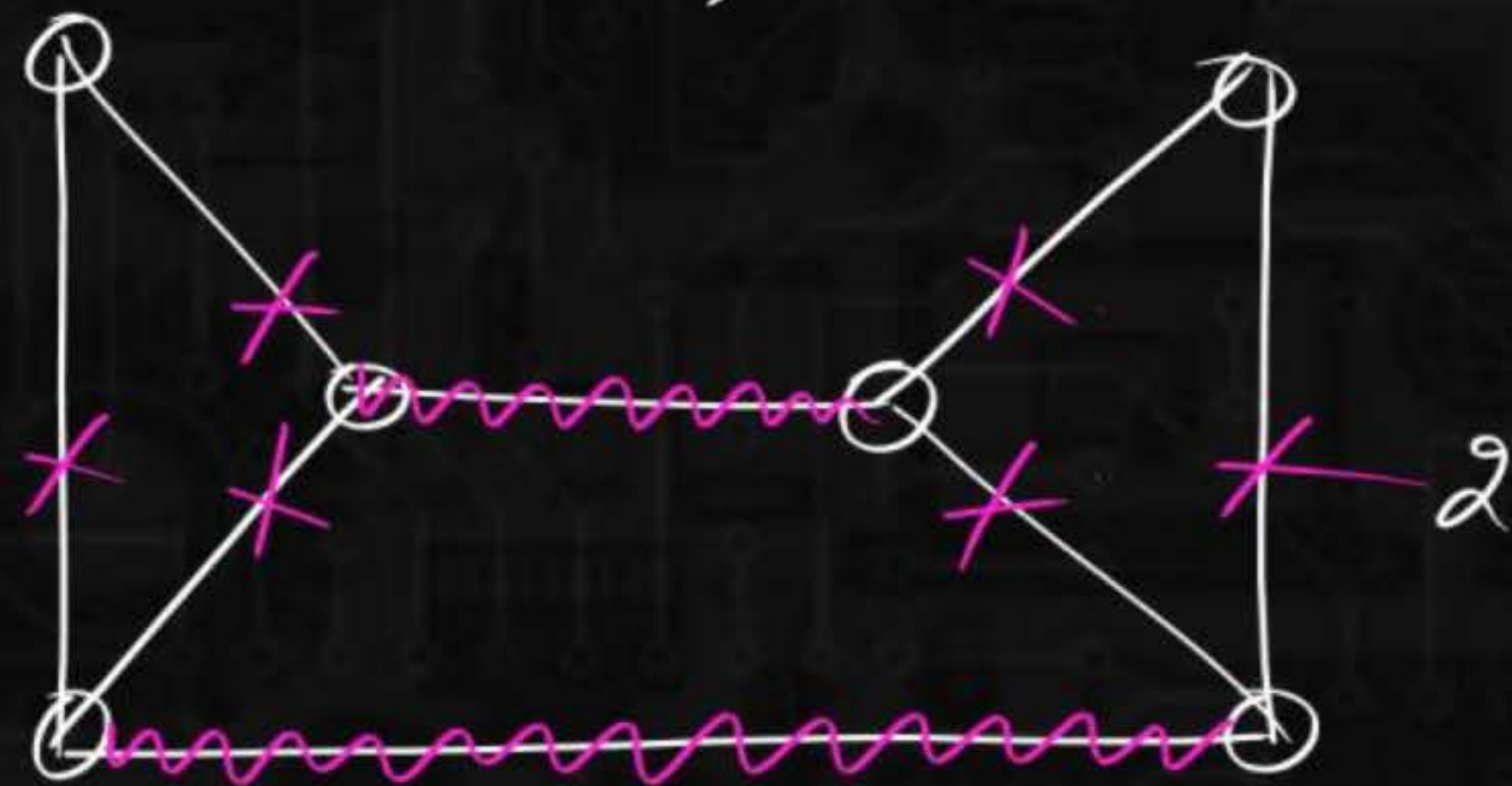
→ largest one

matching no  $m(G)$

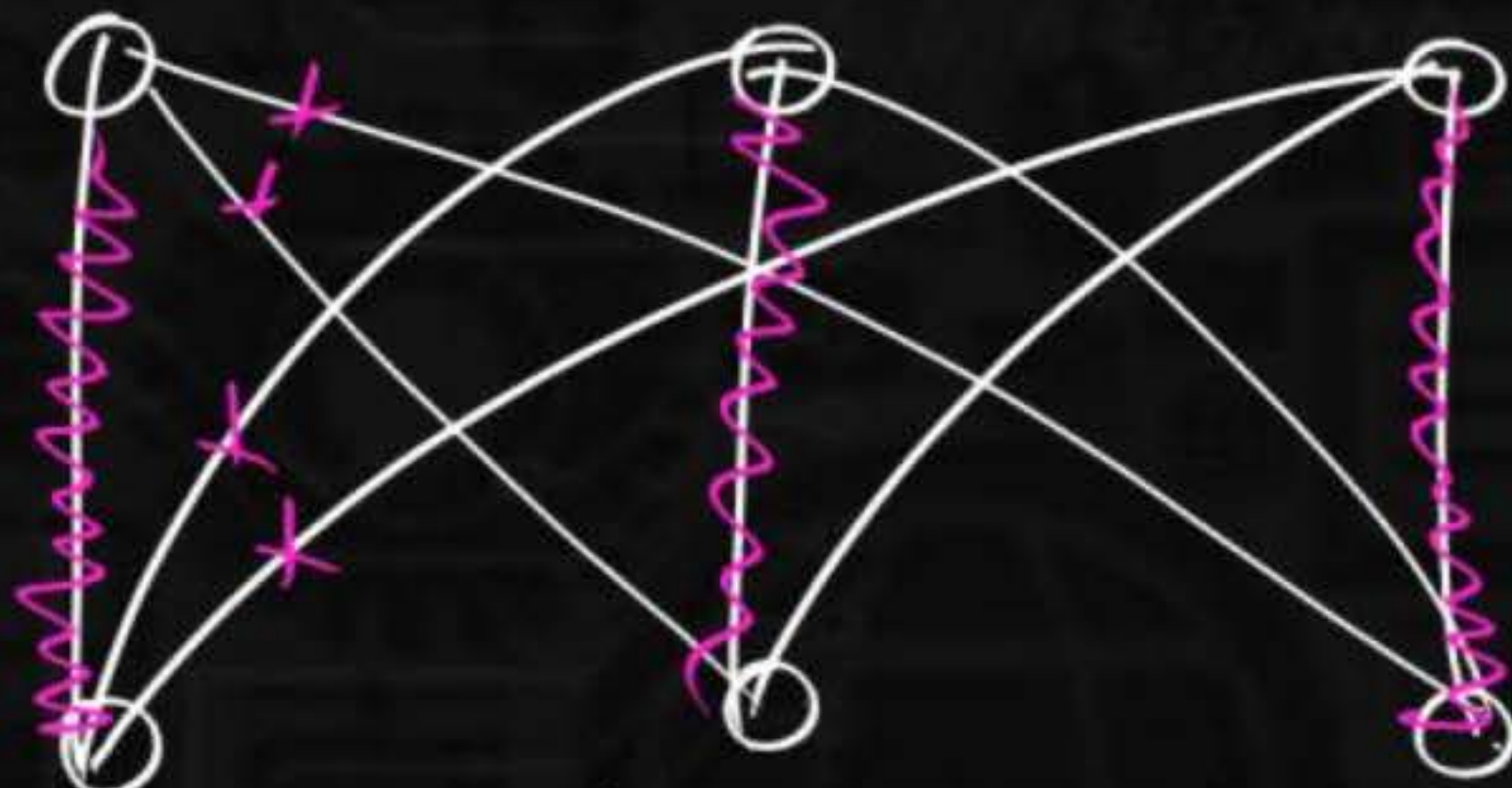
largest  
maximal  
matching  
set



$$m(G_1) = 3$$



$G_2$



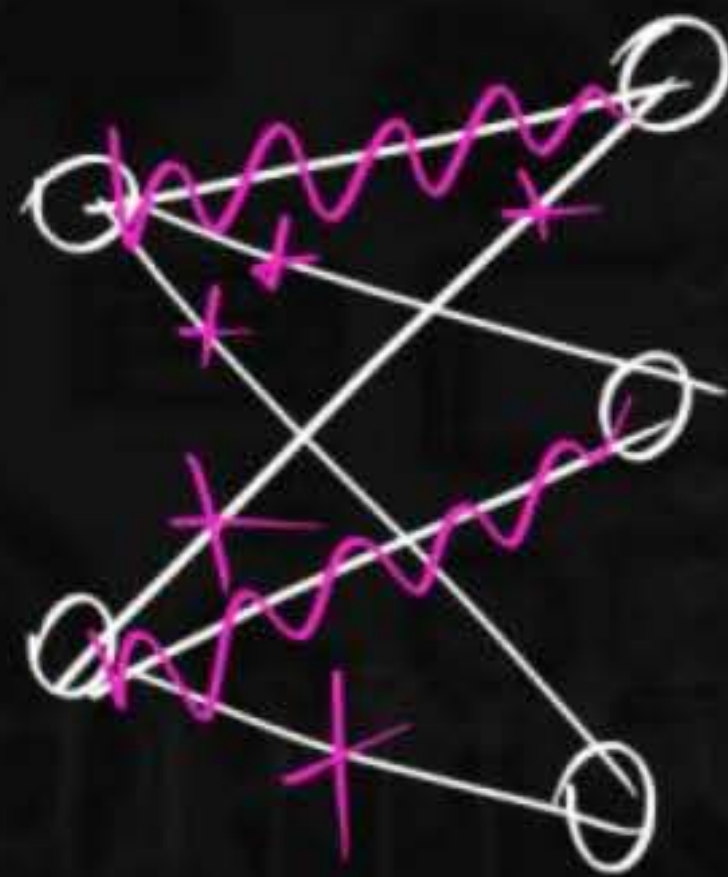
$$m(K_{3,3}) = 3$$



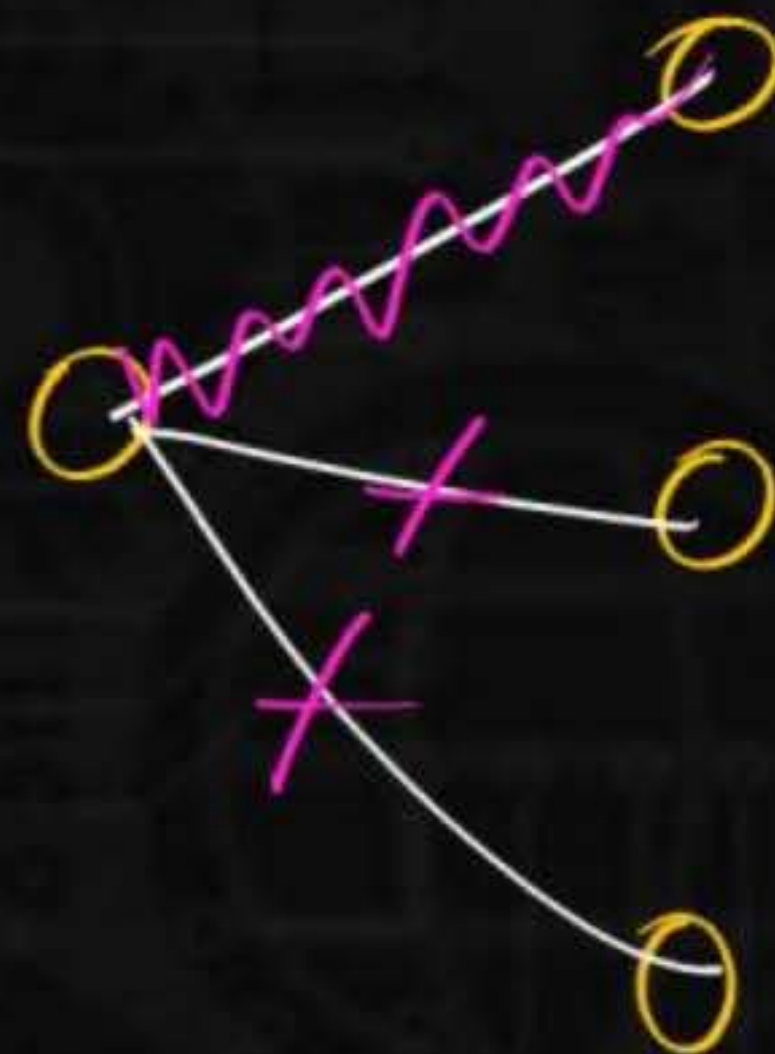
$k_{m,n}$

$m(k_{m,n}) =$

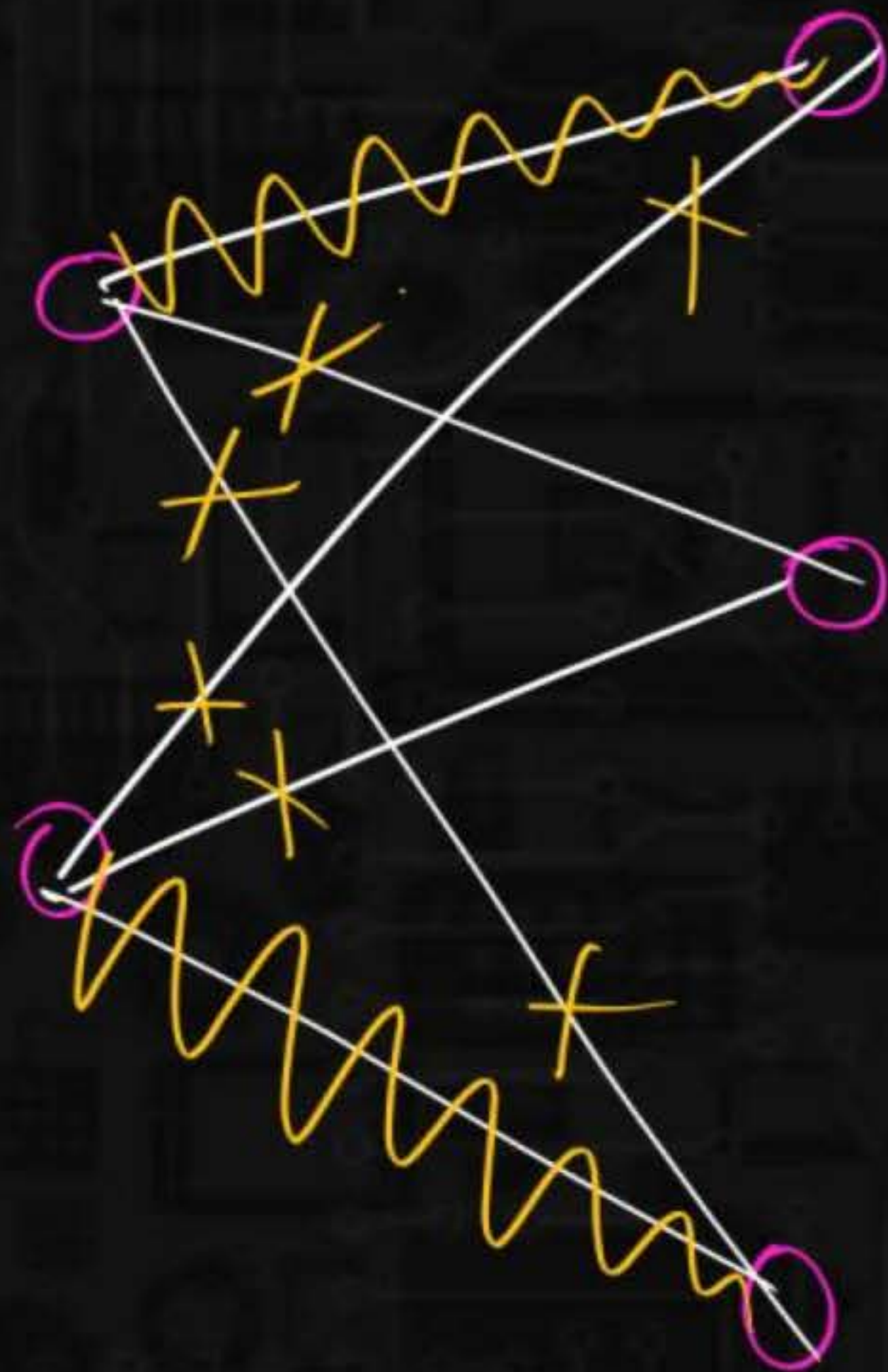
eg  $m(k_{2,3}) = 2$   $k_{3,3}$



$m(k_{1,3}) = 1$







$$m(K_{2,3}) = 2$$

$$m(K_{3,3}) = 3$$

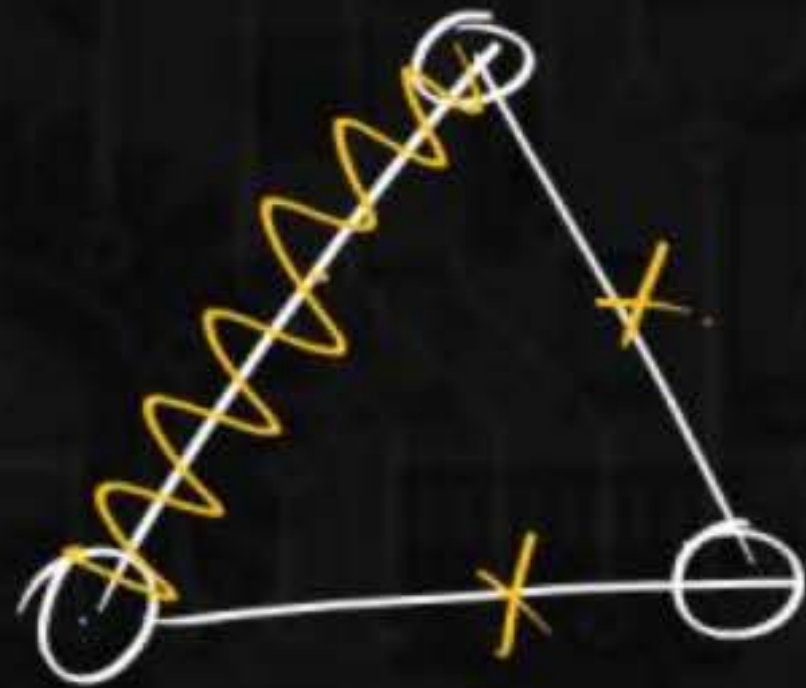
$$m(K_{m,n}) = \min(m, n)$$

$$m(K_{1,4}) = \min(1, 4) = 1.$$

$$m(2, 3) = \min(2, 3) = 2.$$

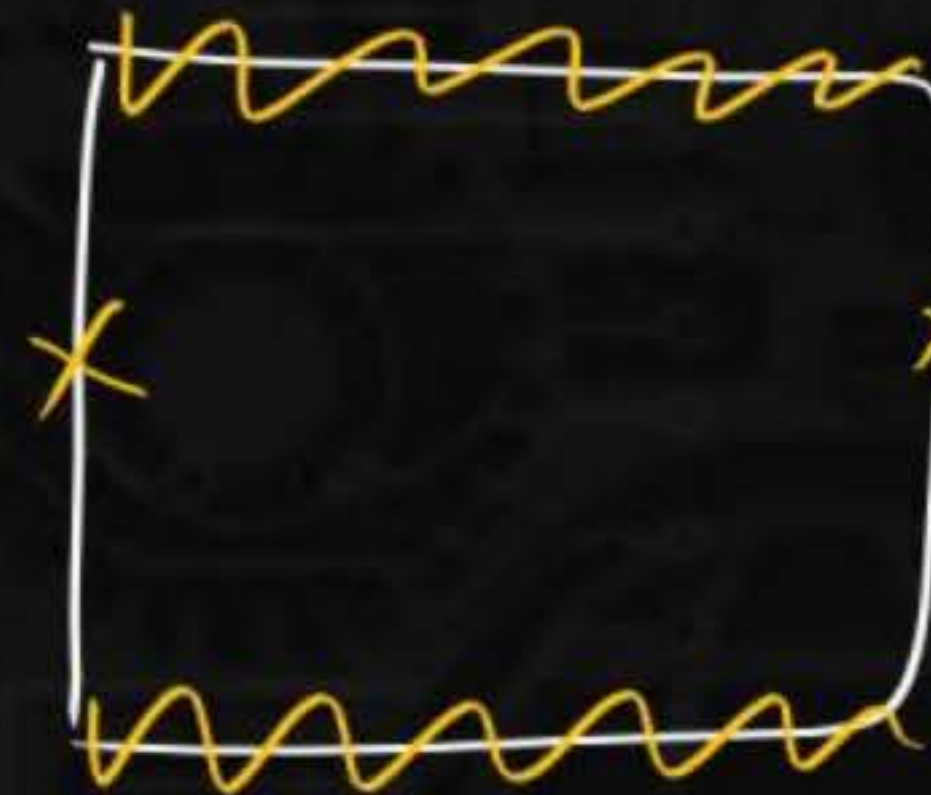


Cycle Graph.  $(C_n) (n \geq 3)$



$$m(C_3) = \underline{\underline{1}}$$

$$\frac{3}{2} = \lfloor 1.5 \rfloor$$

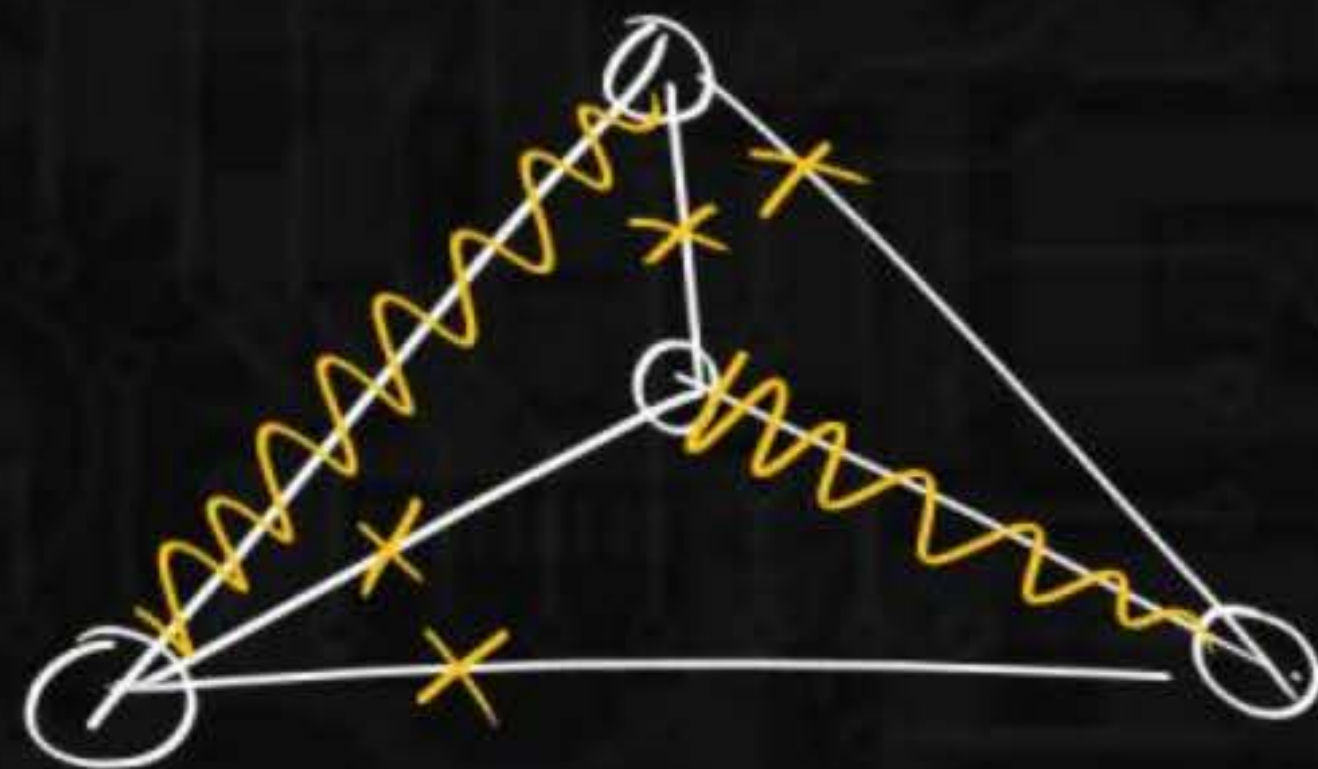


$$m(C_4) = \lfloor 2 \rfloor = 2$$

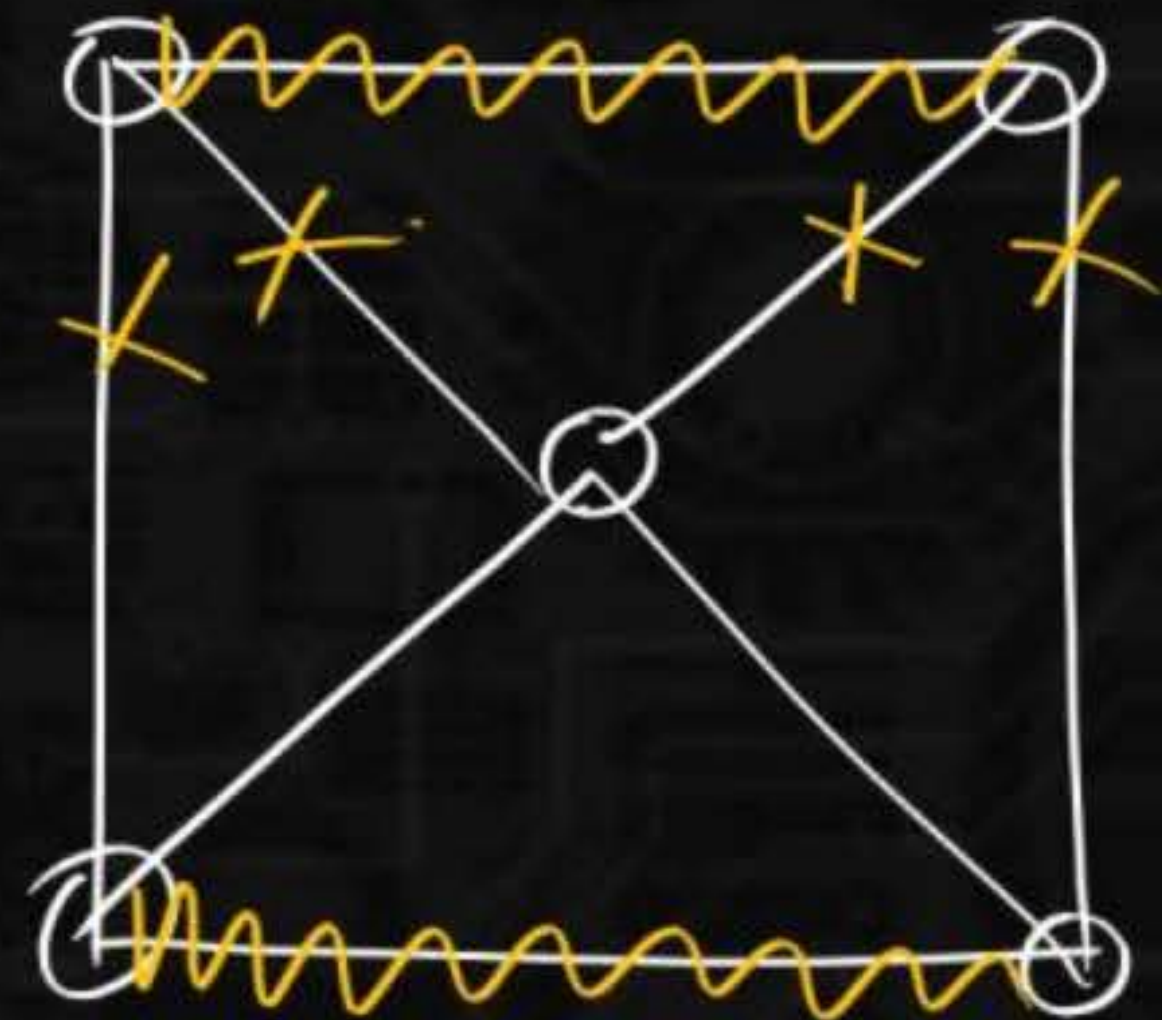
$$m(C_n) = \left\lfloor \frac{n}{2} \right\rfloor$$



# Wheel Graph $(w_n)(n \geq 4)$



$$m(w_4) = 2$$



$w_5$

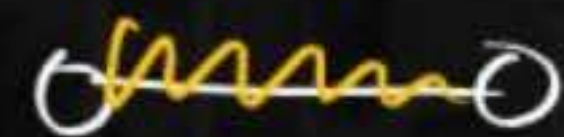
$$m(w_5) = 2$$

$$m(w_n) = \lfloor n/2 \rfloor$$

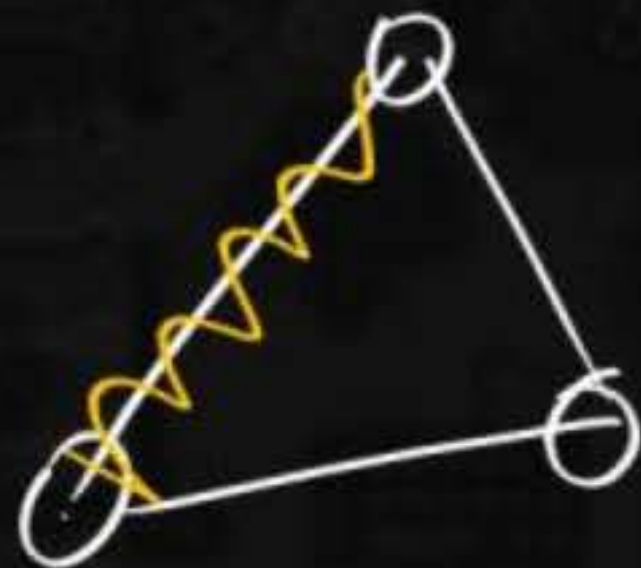


# Complete Graph $(K_n)$ $(n \geq 1)$

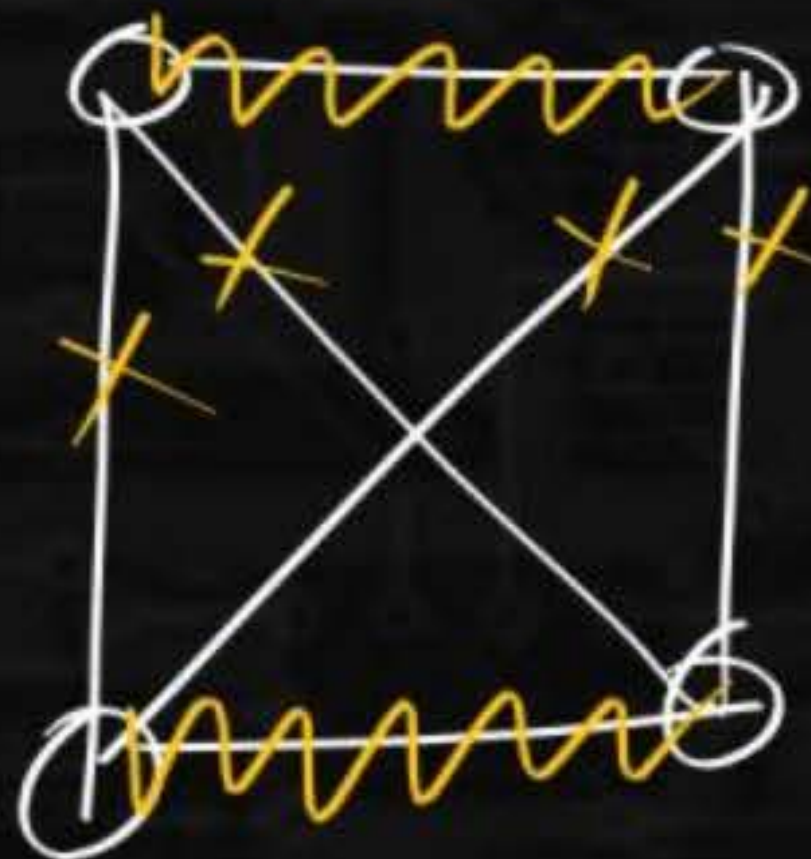
$$m(K_n) = \lfloor n/2 \rfloor$$



$$m(K_2) = 1$$



$$m(K_3) = 1$$



$$m(K_4) = 2.$$

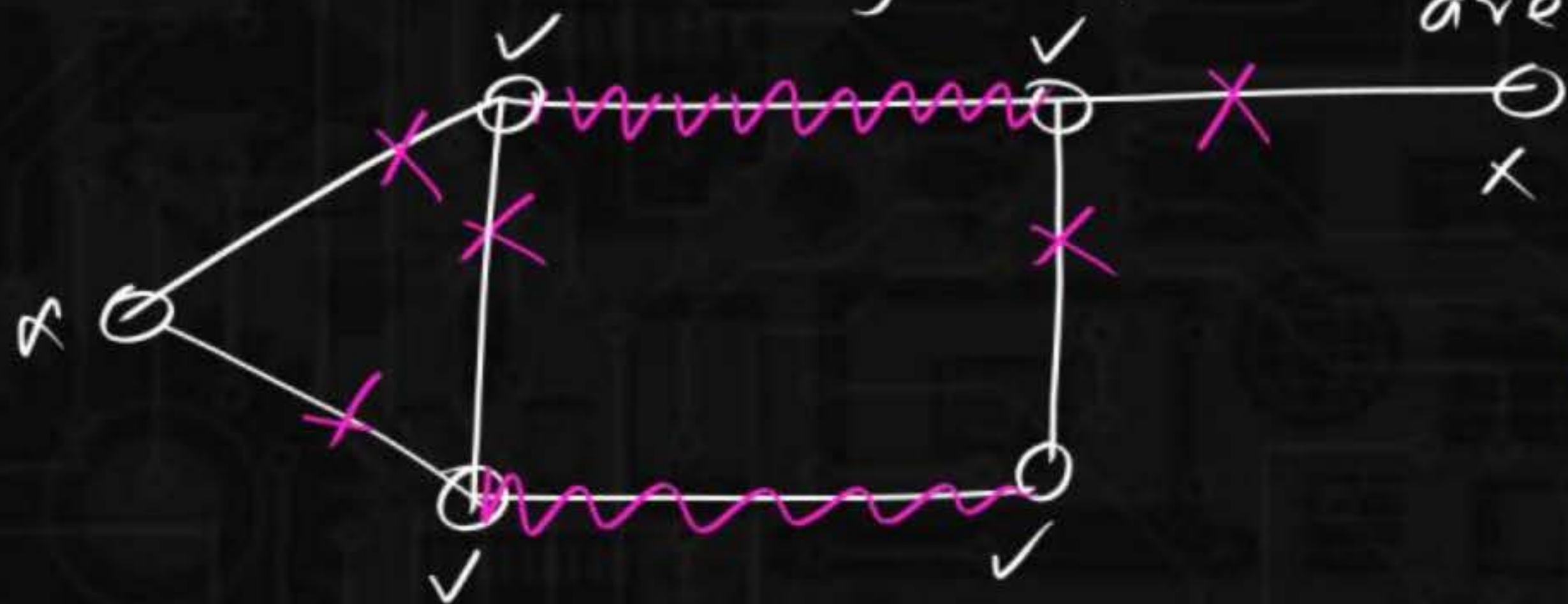


$$m(k_n) = m(c_n) = m(w_n) = \lfloor n/2 \rfloor$$

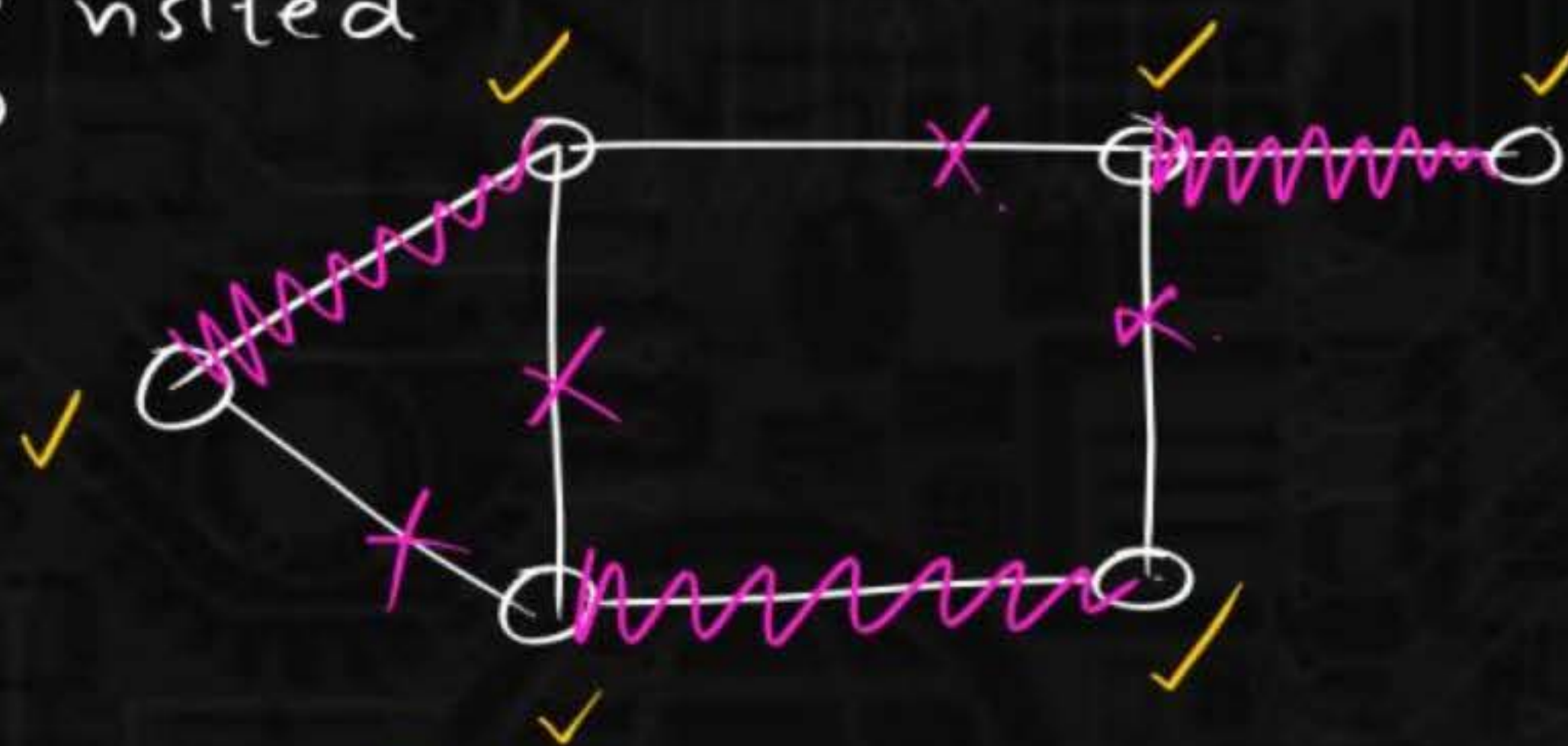
$$m(k_m, n) = \min(m, n)$$



collection of edges  $\rightarrow$  all vertices  
are visited

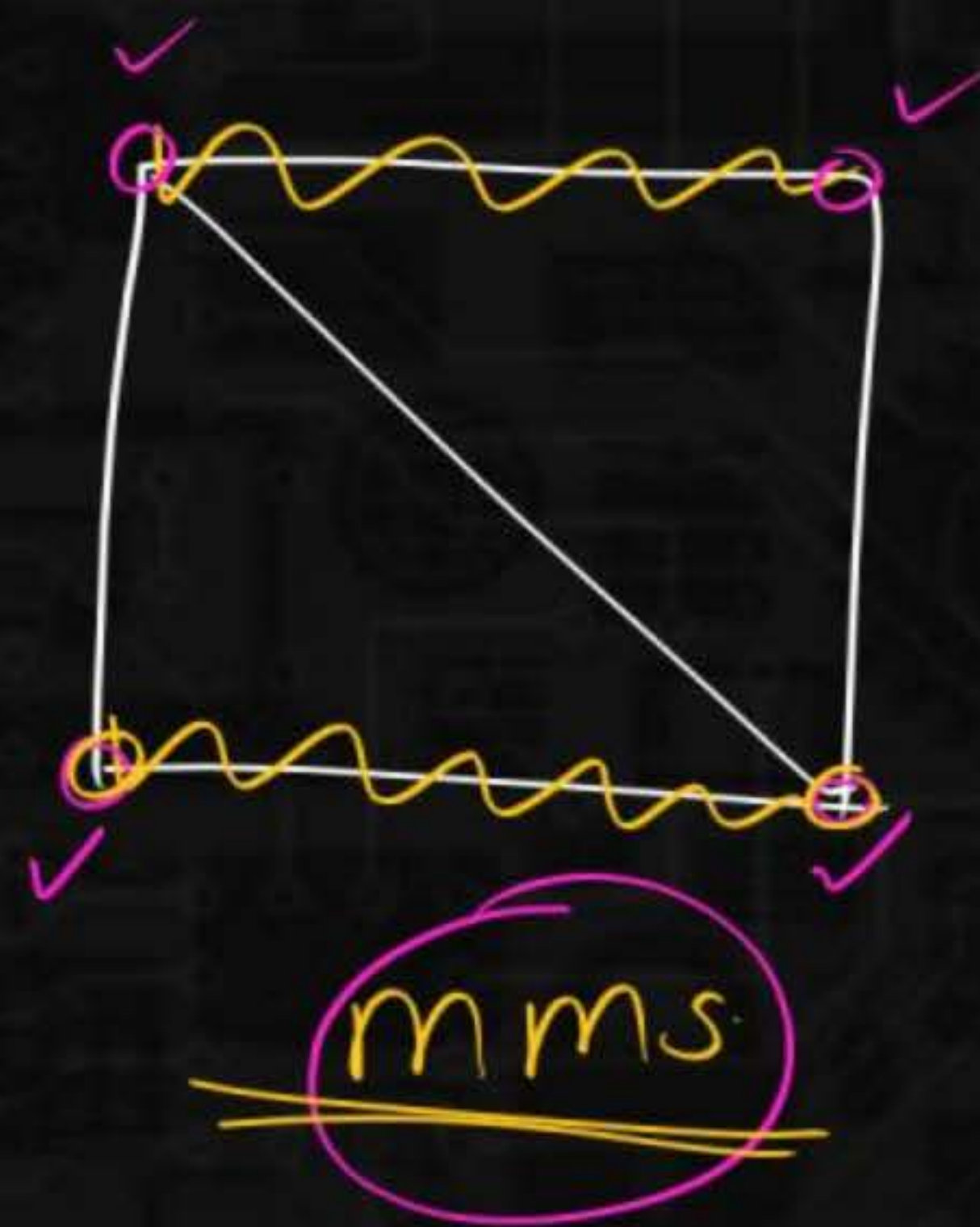
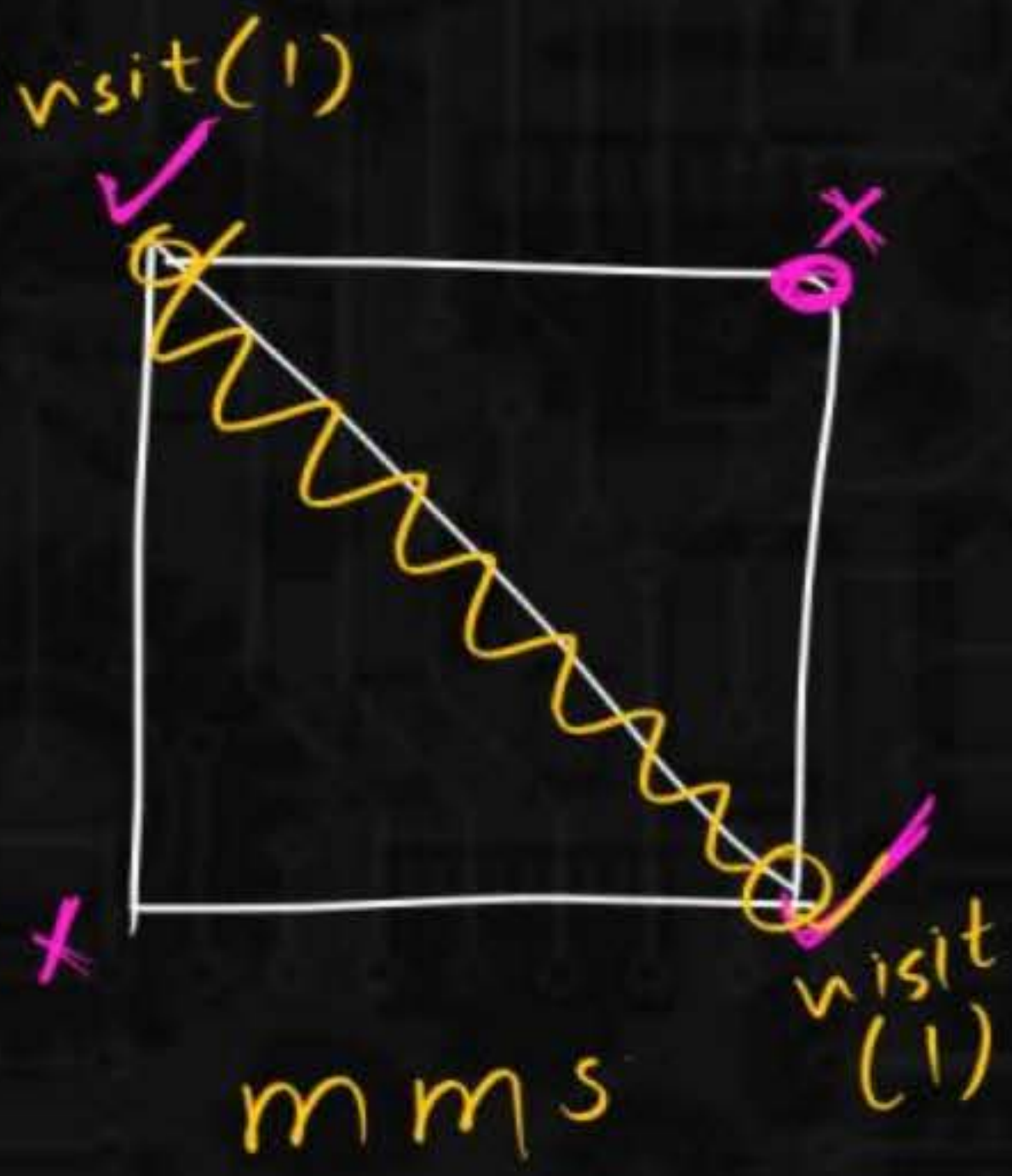


maximal  
matching set



{ maximal  
matching set





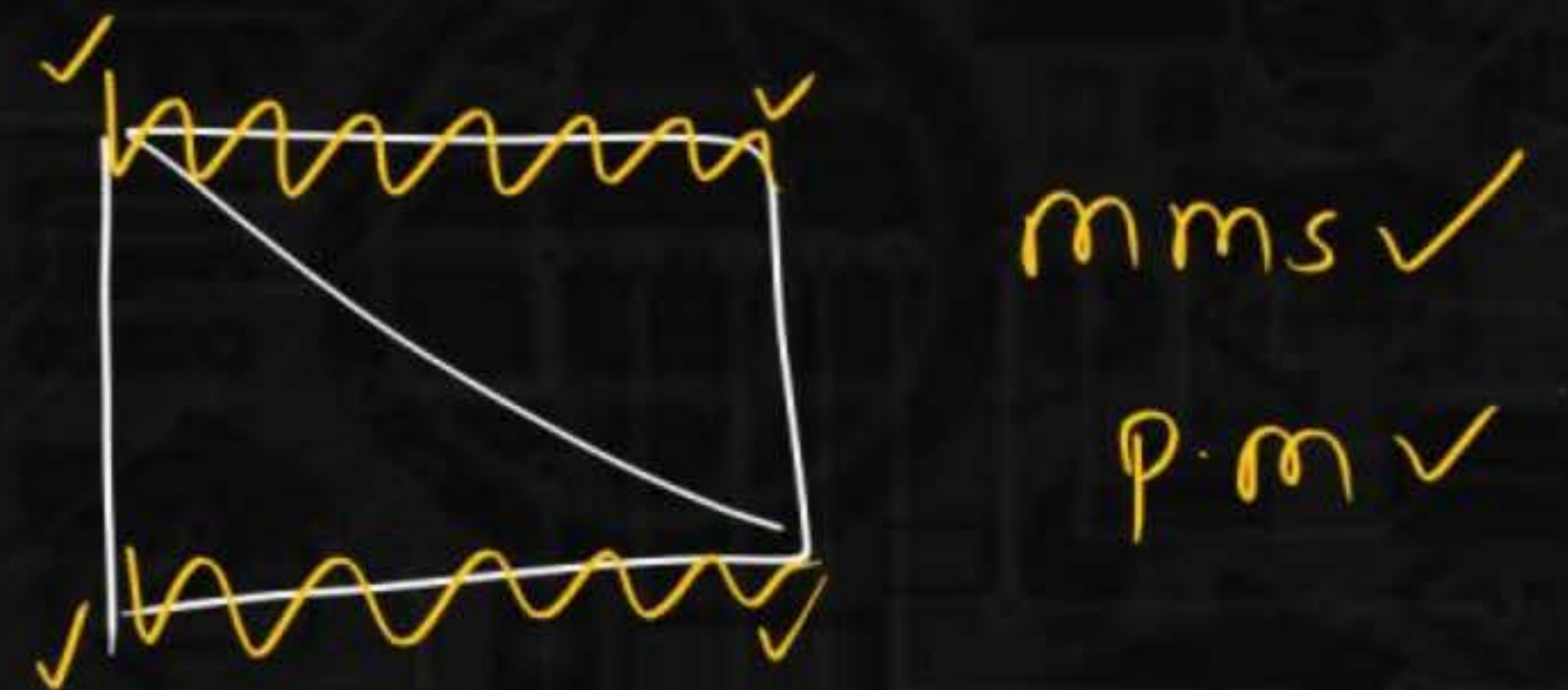
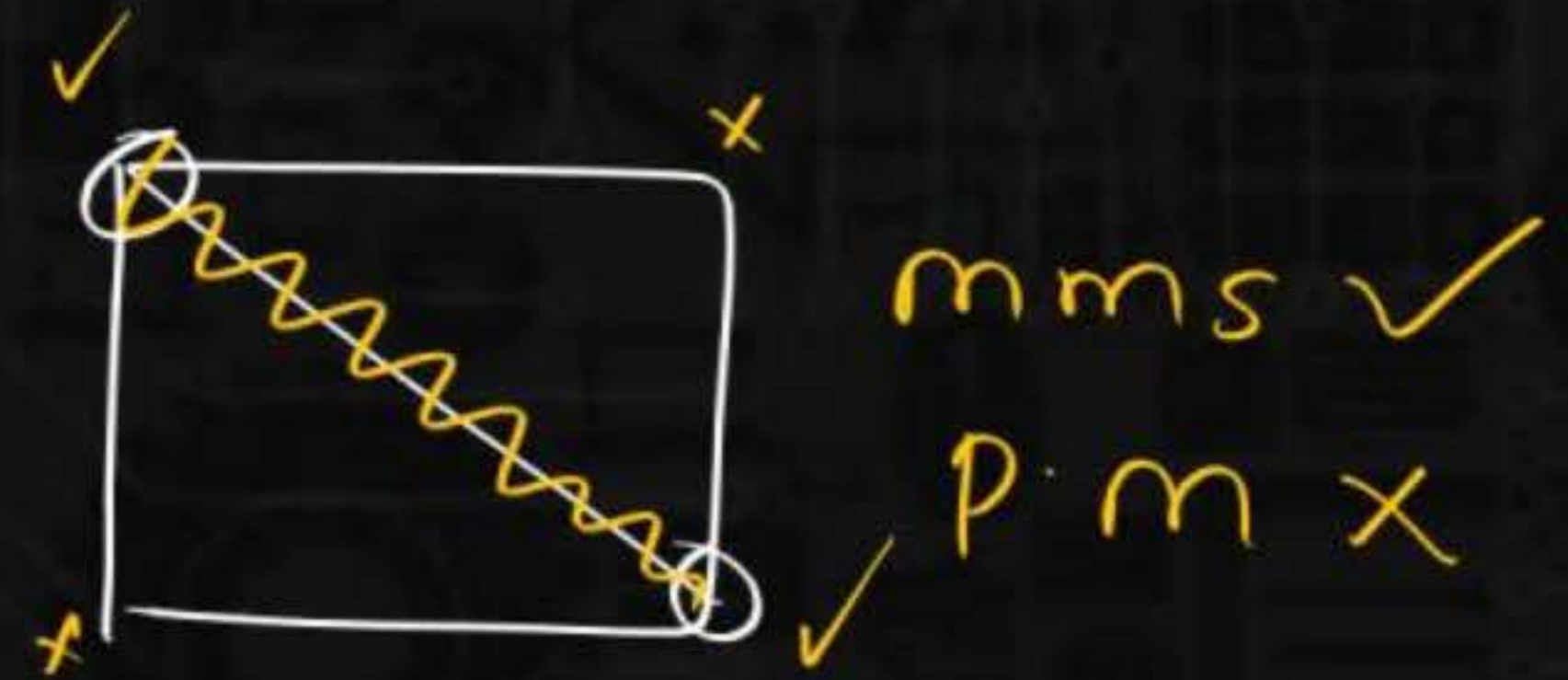
mms pack

↓

all vertices  
must be visited  
by collection of  
edges.

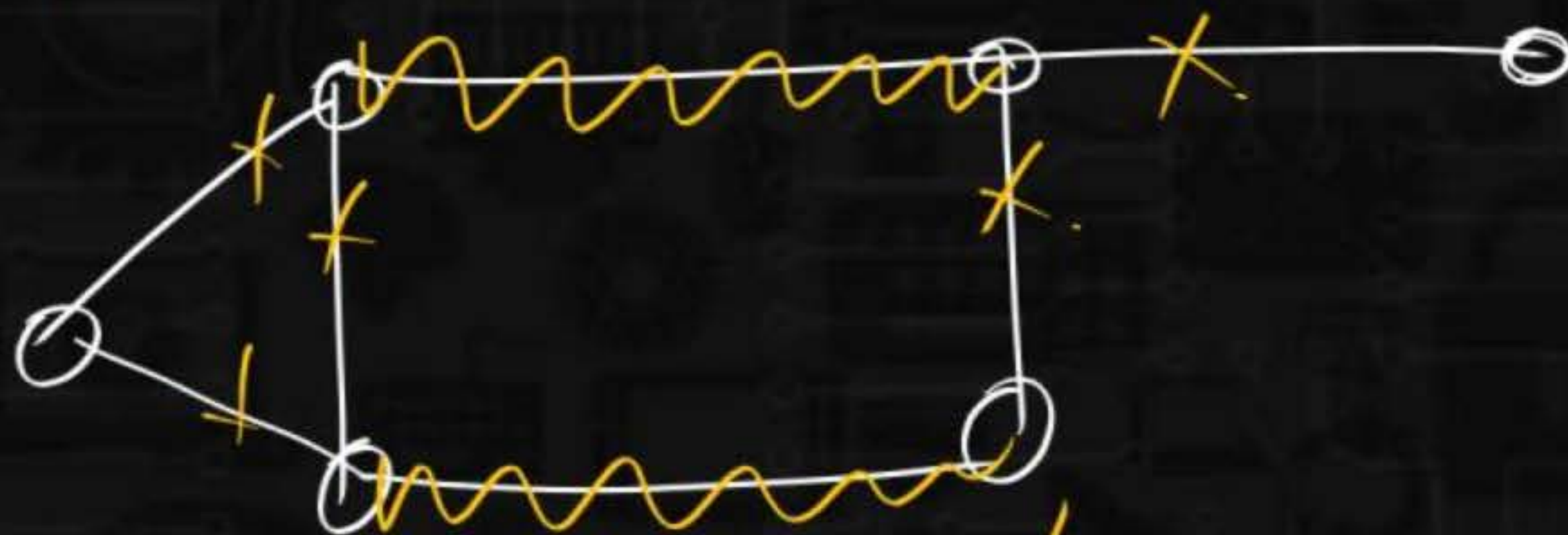


Perfect matching  
 maximal matching  
 set, such that  
 collection of edges  
 such that it should  
 visit all vertices

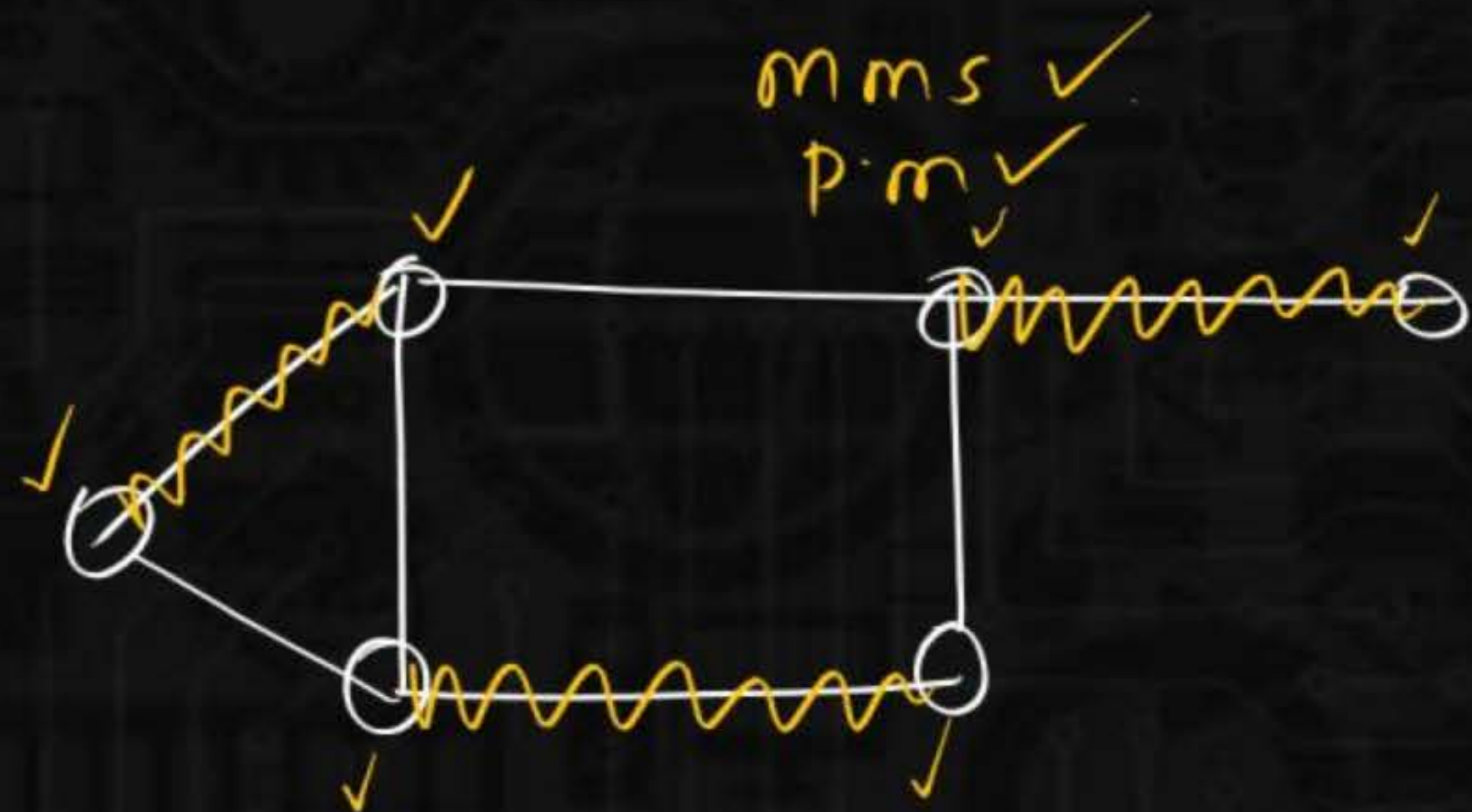




Every P.M is maximal matching set  
but viceversa is not true.



mms ✓  
p.m ✗

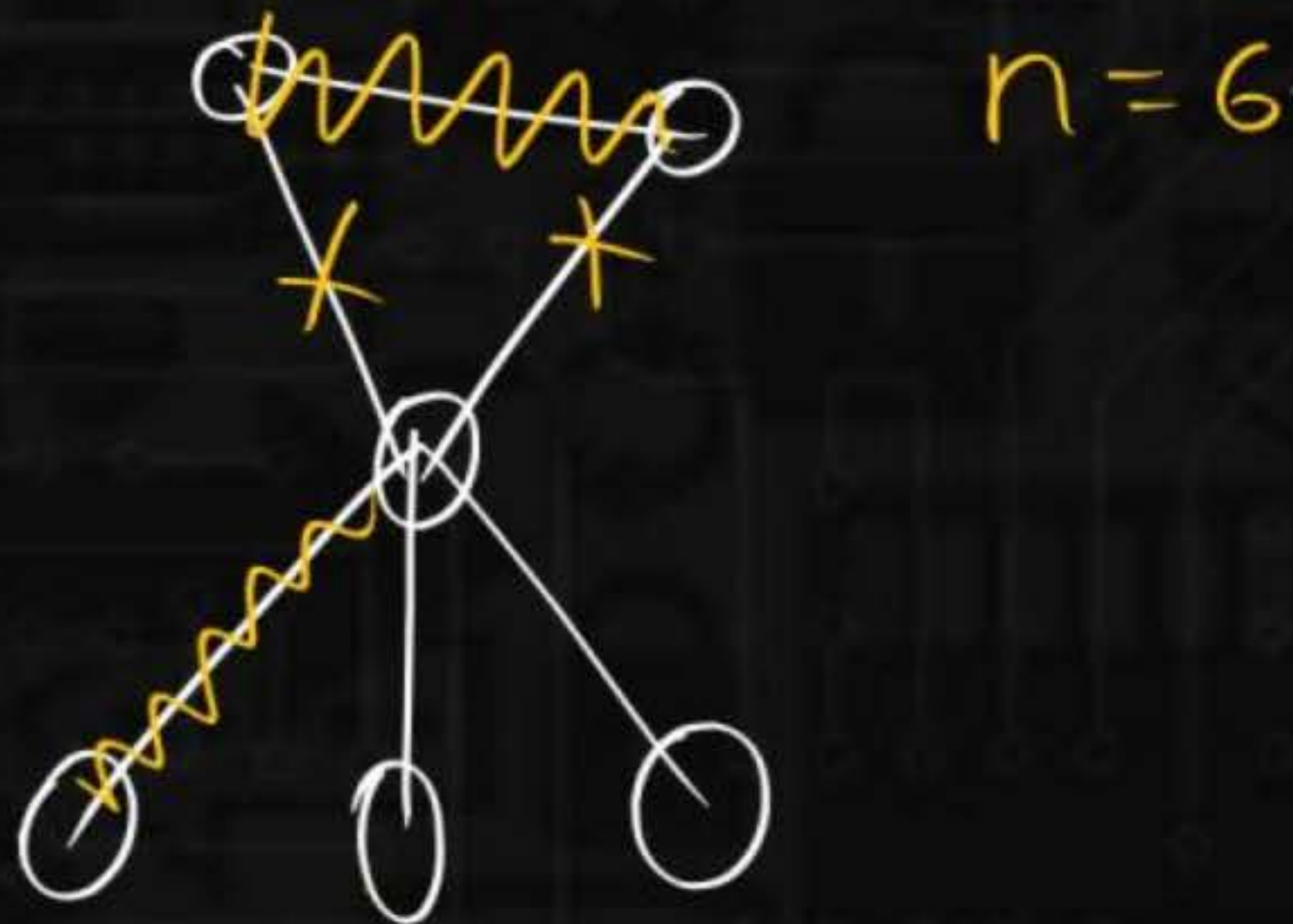


mms ✓  
p.m ✓



if p.m exist then **no. of** vertices will be even

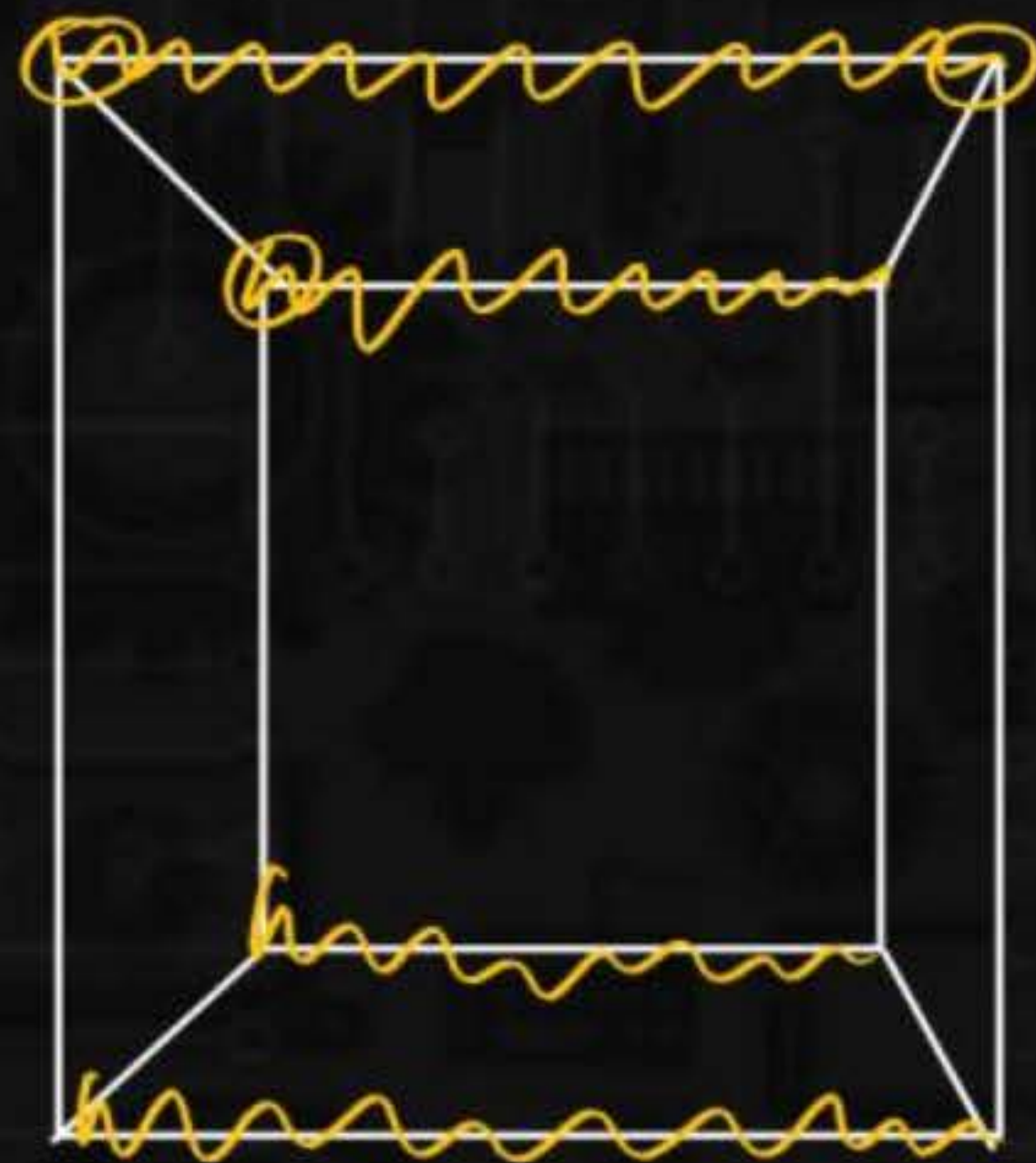
if  $G$  is having even no. of vertices then p.m exist.  
(false)



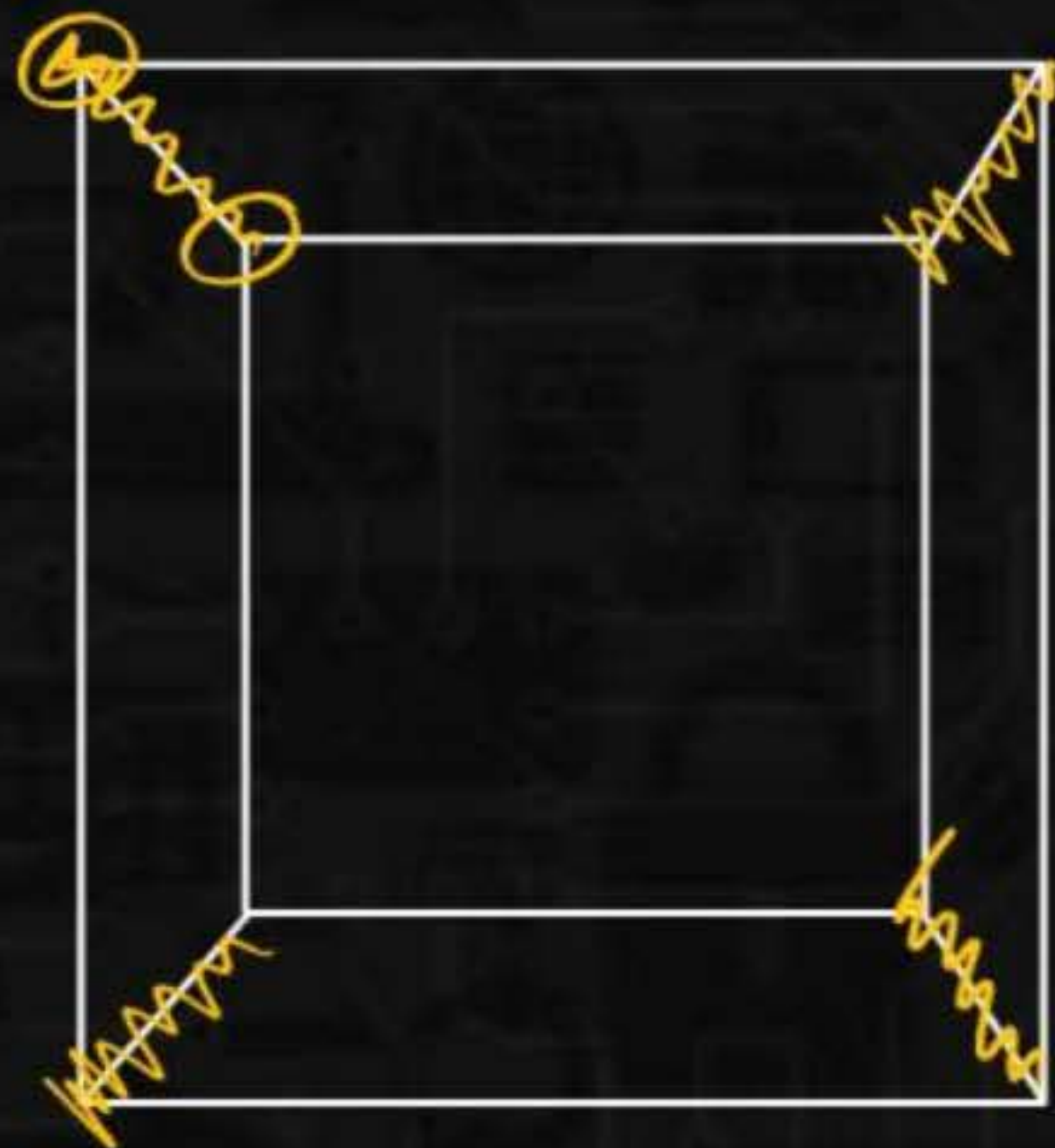


In Graph we can have different p.m

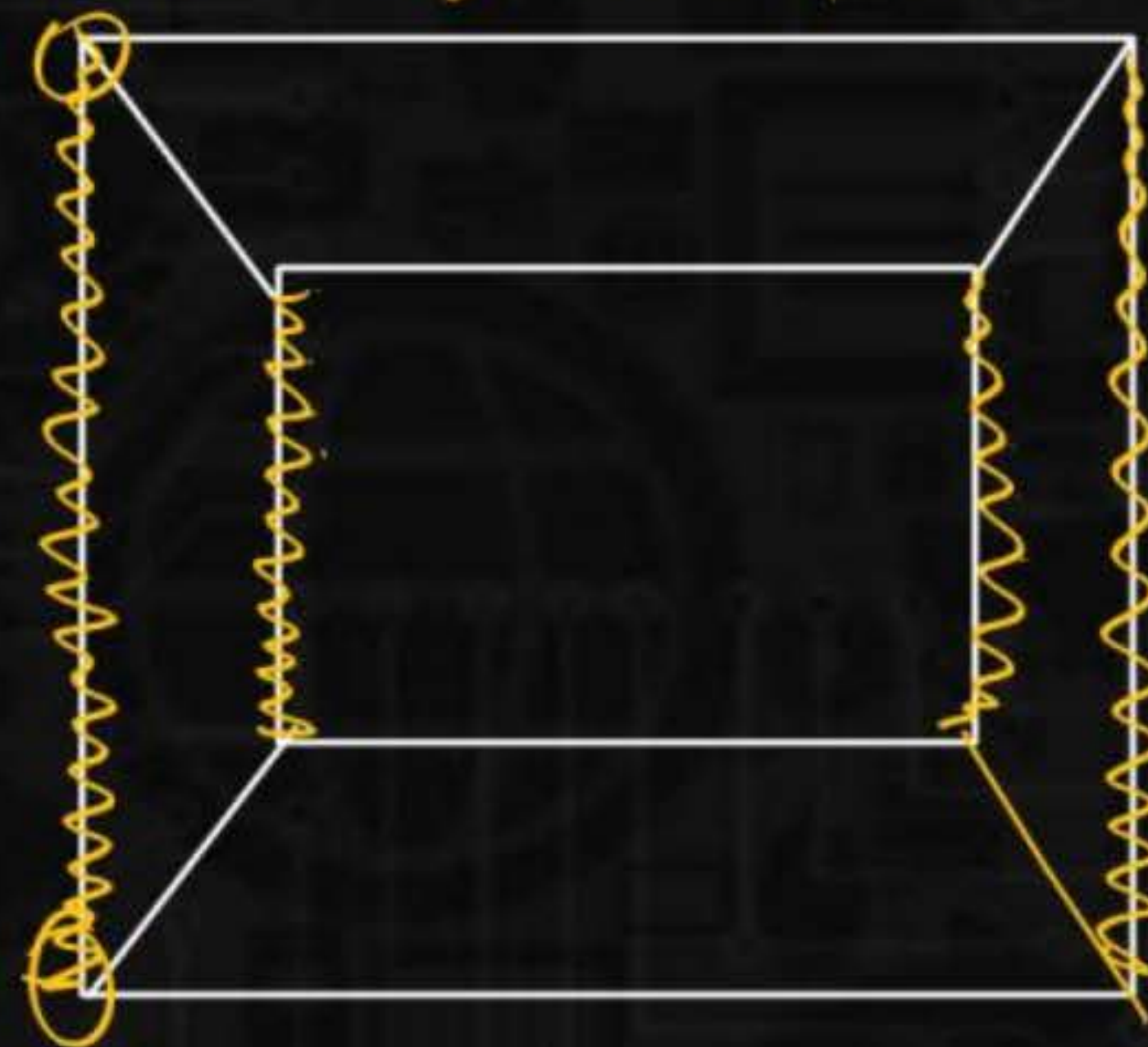
1 Type - pm



2 Type - pm



3 Type - pm

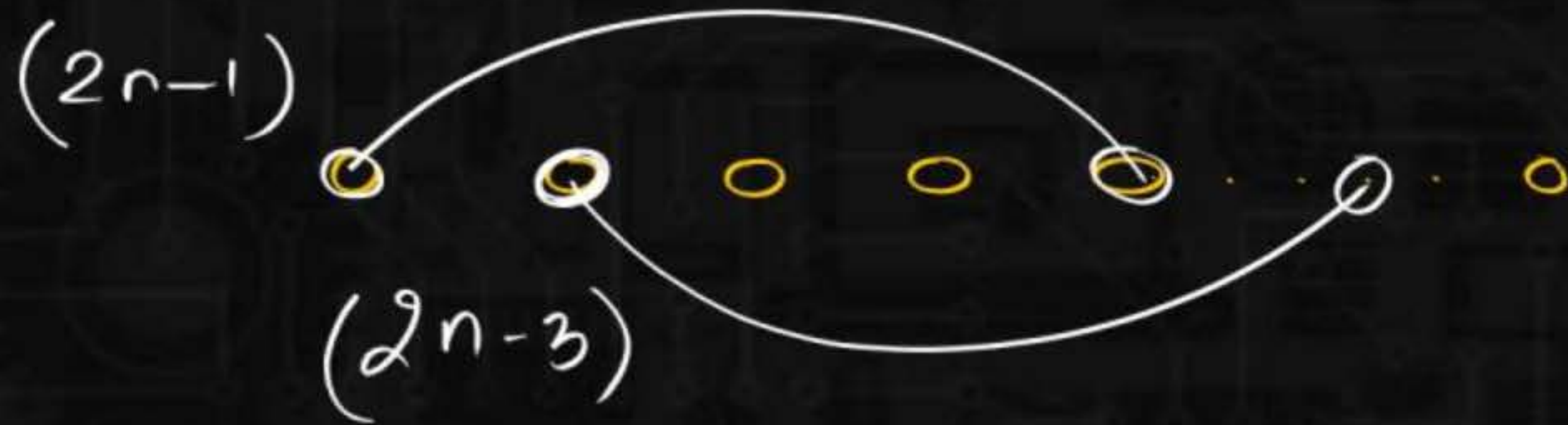




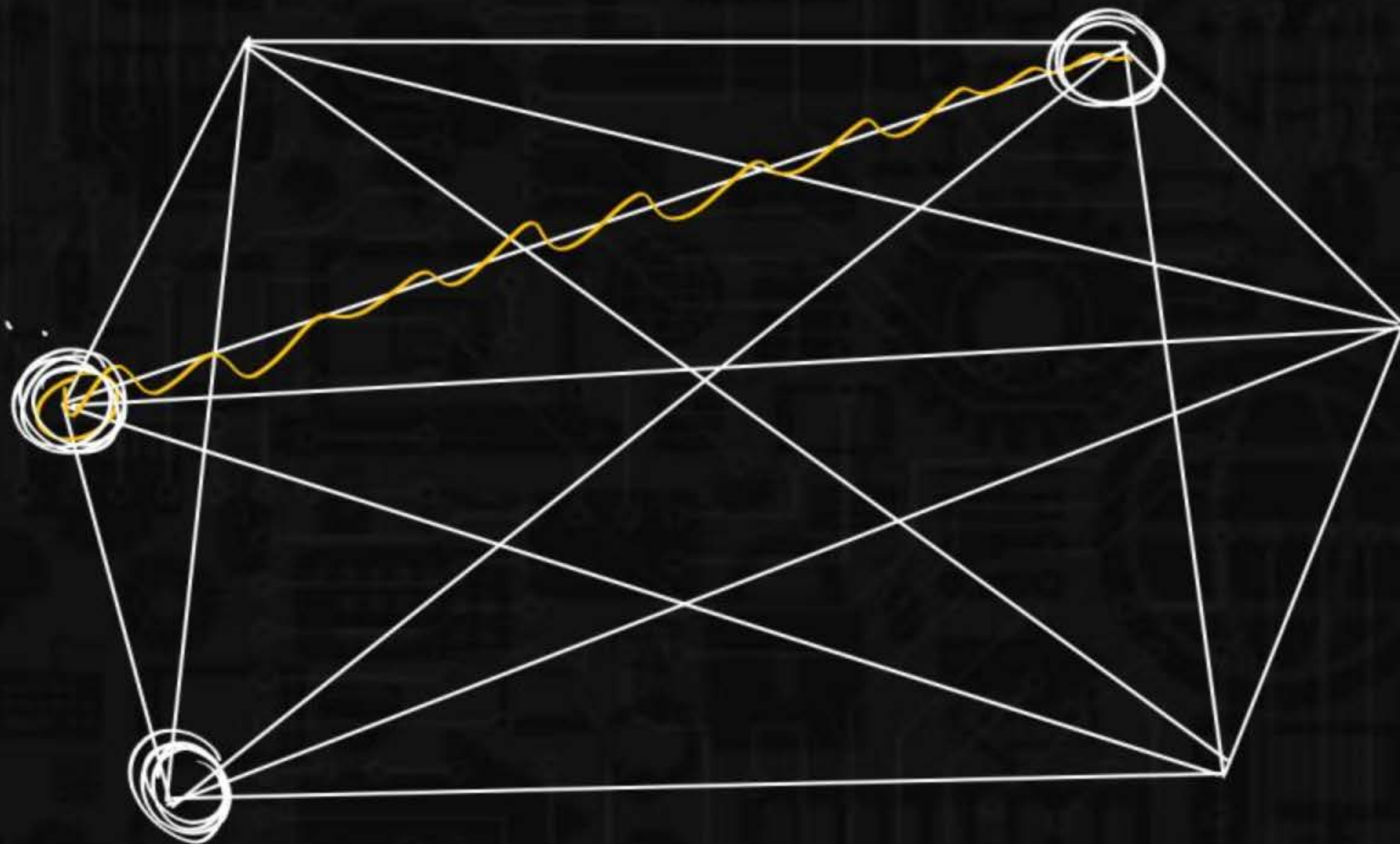
Total no. of p.m in Complete Graph of  $2n$  vertices.



Total vertices =  $2n$ .









$$(2n-1) \cdot (2n-3) \cdot (2n-5)$$

$$\frac{2n}{2n} \times (2n-1) \times \frac{(2n-2)}{(2n-2)} \times (2n-3) \times \frac{(2n-4)}{(2n-4)} \times (2n-5) \dots$$

$$= \frac{(2n)!}{2n \cdot (2n-2) \cdot (2n-4) \dots} = \frac{(2n)!}{2^n \times n \times (n-1) \times (n-2) \dots}$$

take 2 common

$$= \frac{(2n)!}{2^n \times n!}$$



