CS & IT ENGINEERING

Nested Quantifier

Lecture No.07

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TOPICS TO BE COVERED

01 Types of Nested Quantifier

02 English statement to Logic Conversion

03 Problems on Nested Quantifier

04 Type 5

05 Cycle Method

$D: \mathbb{Z}$

$P(x, y): x + y = 10$

$\forall x \exists y$

for all of x, there exist y.

$\forall x \exists y \, (x + y = 10)$

$x = 1 \quad x + y = 10$

$y = 10 - 1 = 9.$

$\underline{x = 1} \quad \underline{y = 9.}$

$\underline{x = 2} \quad \underline{y = 8}$

$\underline{x = -3} \quad \underline{y = 13.}$

$D: \mathbb{Z}$

$\forall x \exists y (x+y=10) \rightarrow$ True

$$\begin{array}{ccc} x & & y \\ 1 & \longrightarrow & 9 \\ 2 & \longrightarrow & 8 \\ 3 & \longrightarrow & 7 \\ 0 & \longrightarrow & 0 \\ 0 & \longrightarrow & 0 \end{array}$$

$$\begin{array}{ccc} x & & y \\ 0 & \longrightarrow & 0 \\ 0 & \longrightarrow & 0 \\ 0 & \longrightarrow & 0 \\ 0 & \longrightarrow & 0 \end{array}$$

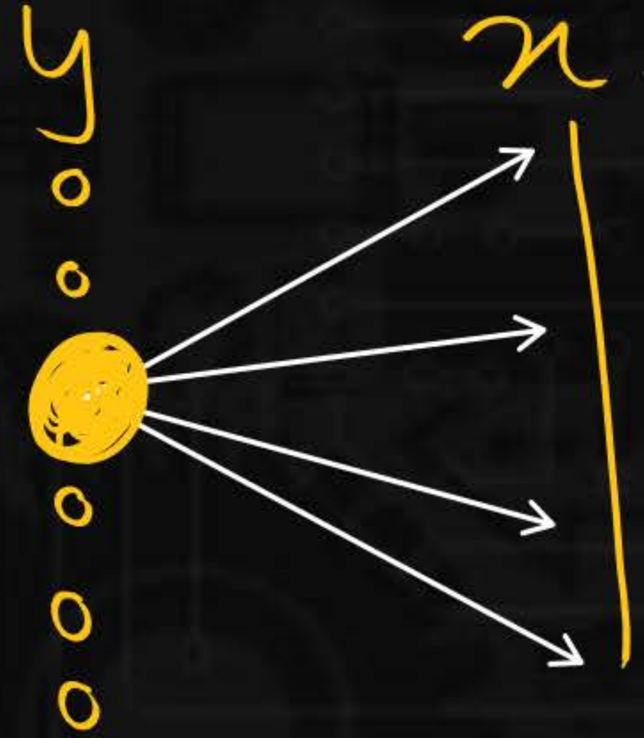
$\forall x \exists y.$

$D : \mathbb{Z}$

$P(x, y) : x + y = 10$

$\exists y \forall x (x + y = 10) \to$ false

there exist y for all of x.



it is forcing y to be constant but it is not possible.

eg :

$x + y = 10$

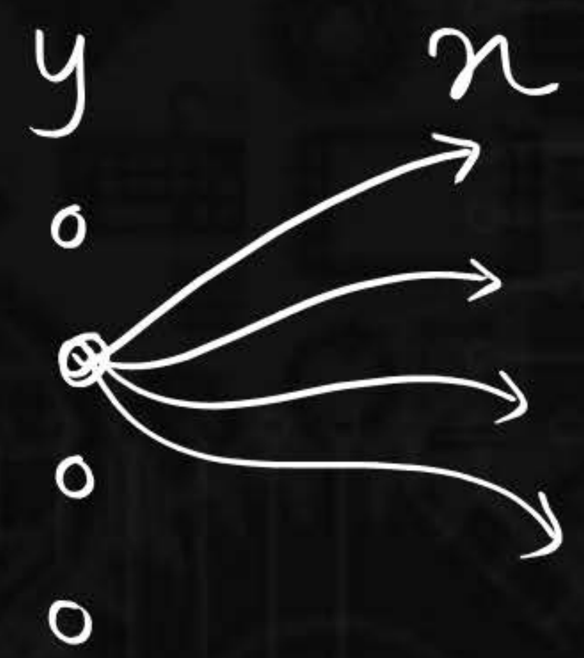
$y = 1 \quad x = 10 - 1 = 9.$

$y = 2 \quad x = 10 - 2 = 8$

$x + y = 10$

$\exists y \forall x (x + y = 10) \rightarrow$ false

there exist y for all of x



Demand.

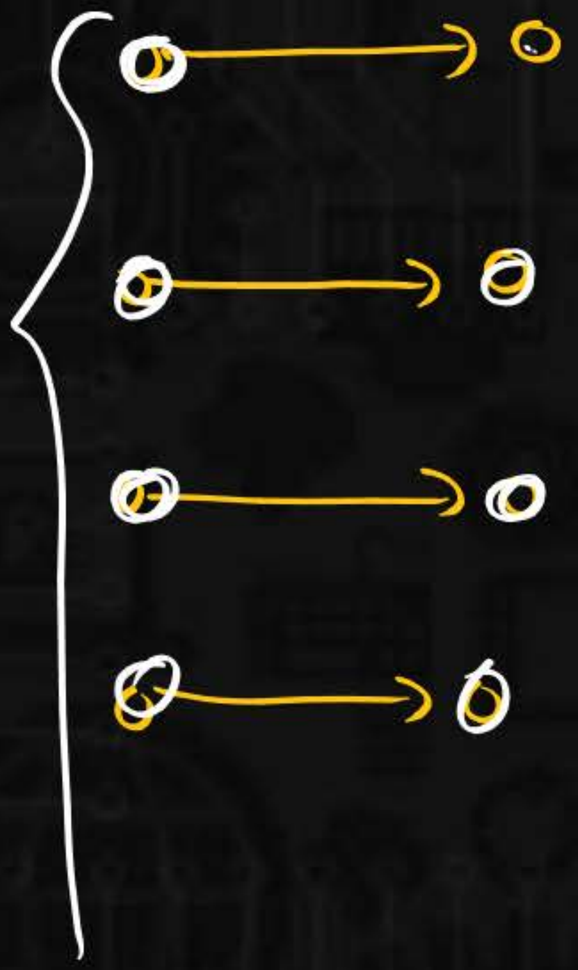
y to be constant.

Reality:

$x + y = 10$

$y = 10 - \boxed{\vdots}$

$$\overset{T}{\forall x\, \exists y\, (x+y=10)} \not\longrightarrow \overset{F}{\exists y\, \forall x\, (x+y=10)}$$



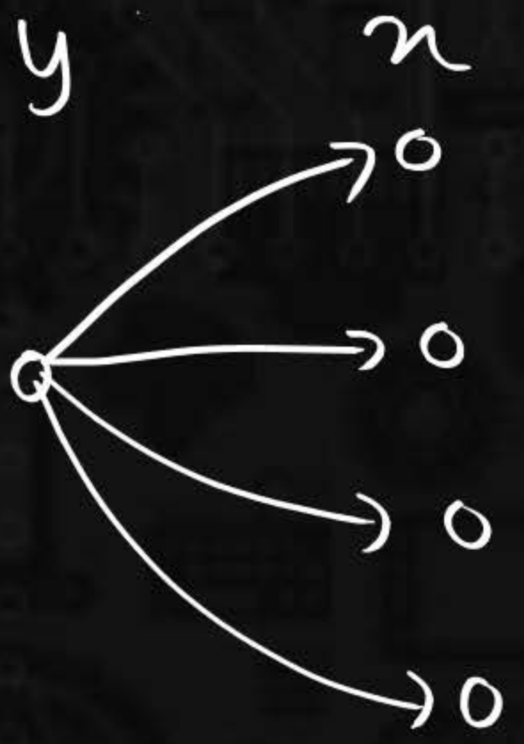
$2 \not\rightarrow 3$

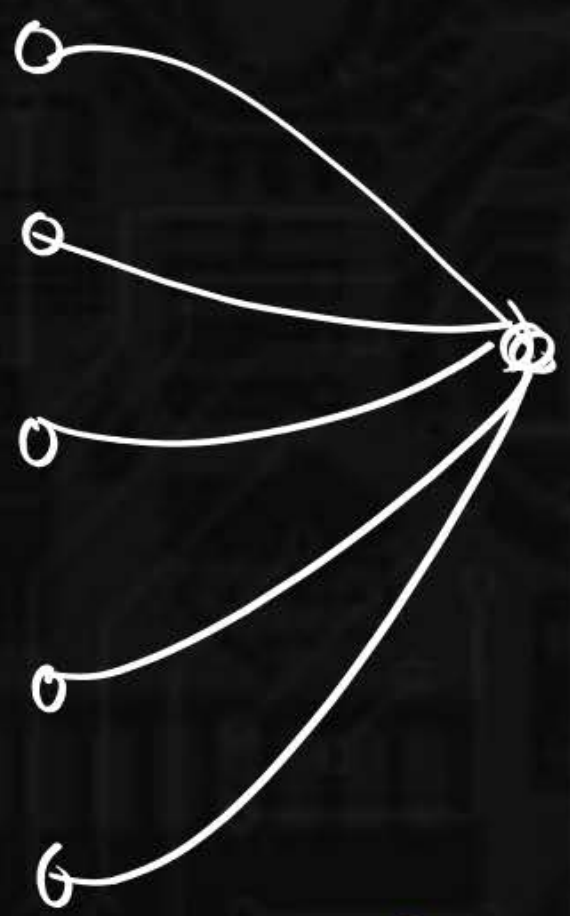
$$\boxed{\forall x \exists y \not\rightarrow \exists y \forall x}$$

$T \longrightarrow F$

True

$\exists y \forall x \xrightarrow{\text{valid}} \forall x \exists y.$





$2 \not\to 3$

$\forall x \exists y \not\Rightarrow \exists y \forall x$

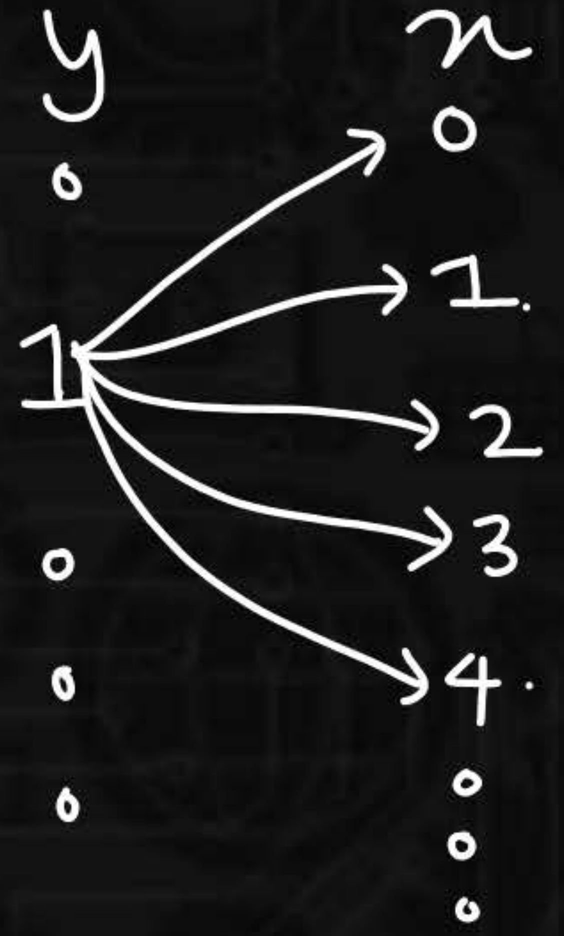
$3 \to 2$

$\exists y \forall x \to \forall x \exists y.$

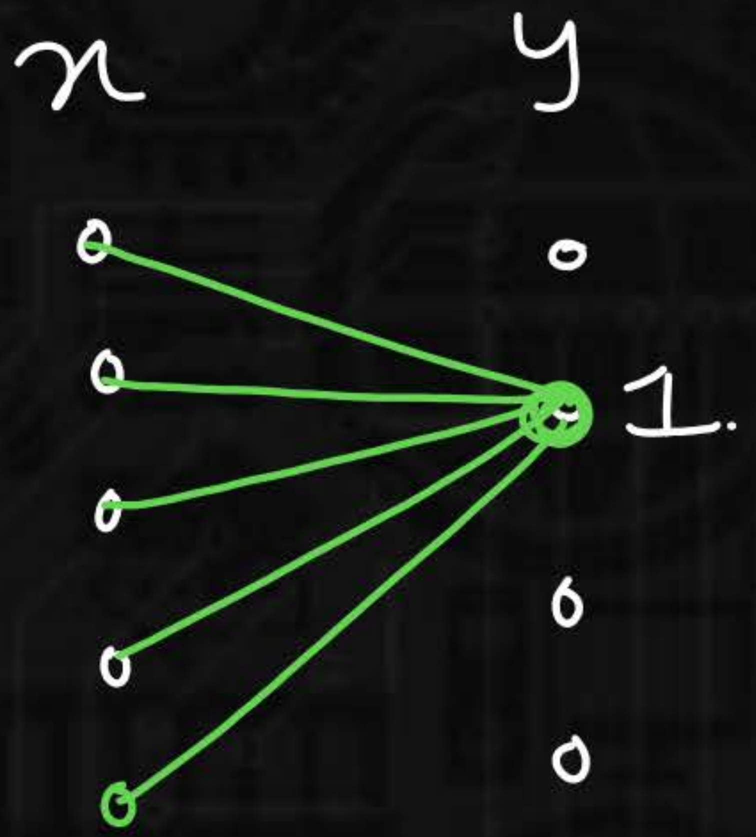
3:

$$\exists y \forall x \left(\frac{x}{y} \in z\right) \longrightarrow \forall x \exists y \left(\frac{x}{y} \in z\right)$$

2:



$$\frac{0 \quad 0 \quad 0 \quad 0 \cdots 0}{y=1}$$



Let $p(x, y)$ denote the open statement " x divides y," where the universe for each of the variables x, y comprises all integers

Determine the truth value of each of the following statements;

D : Z.

i) $p(3,7)$ – F.
ii) $p(3,27) \rightarrow$ T
iii) $\forall y\, p(1, y)$ – T.
iv) $\forall x\, p(x, 0)$ – F
v) $\forall x\, p(x, x)$ – F
vi) $\forall y \exists x\, p(x, y)$ – T
vii) $\exists y \forall x\, p(x, y)$ – T $\longrightarrow$
viii) $\forall x \forall y [(p(x, y) \land p(y, x)) \rightarrow (x = y)]$

$\frac{y}{x} \land \frac{x}{y} \rightarrow x = y.$

$\left(\frac{2}{-2} \land \frac{-2}{2} \rightarrow 2 \neq -2\right)$

(false)

iv) $\forall x\, P(x, 0)$

$x = 0$

v) $\forall x\, P(x, x)$ (F)

$\frac{x}{x}$ $x=0$

valid all Z.
except 0.

i) False

ii $\rightarrow$ T.

iii $\rightarrow$ T.

iv) F.

v) F

vi) $P(x, y) : \frac{y}{x}$

$\underline{\underline{\forall y}} \; \exists x \left(\frac{y}{x} \right)$ — True.

$$\left(\frac{0 \quad 0 \quad 0 \quad 0 \quad 0 \cdots\cdots 0}{x = 1} \right)$$

vii) $\underline{\underline{\exists y}} \; \underline{\underline{\forall x}} \left(\frac{y}{x} \right)$ $x = 0$

false

$y \qquad x$

$$\left(\frac{y}{0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \cdots} \right)$$

$x = 0$

0

$x = 0$

$$\forall x \forall y \left[\frac{y}{x} \wedge \frac{x}{y} \rightarrow x = y \right]$$

$x = -2$
$y = 2$

$$\underbrace{\frac{2}{-2} \wedge \frac{-2}{2}}_{T} \longrightarrow \underbrace{2 = -2}_{F}$$

False

x	y
1	1 (T)
2	4 (T)
2	7 (T)
2	−2 (F)

3	5 (T)
2	7 (T)
−2	2 (F)

$$\underbrace{\left(\frac{x}{y}\right) \wedge \frac{y}{x}}_{F} \rightarrow x = y$$

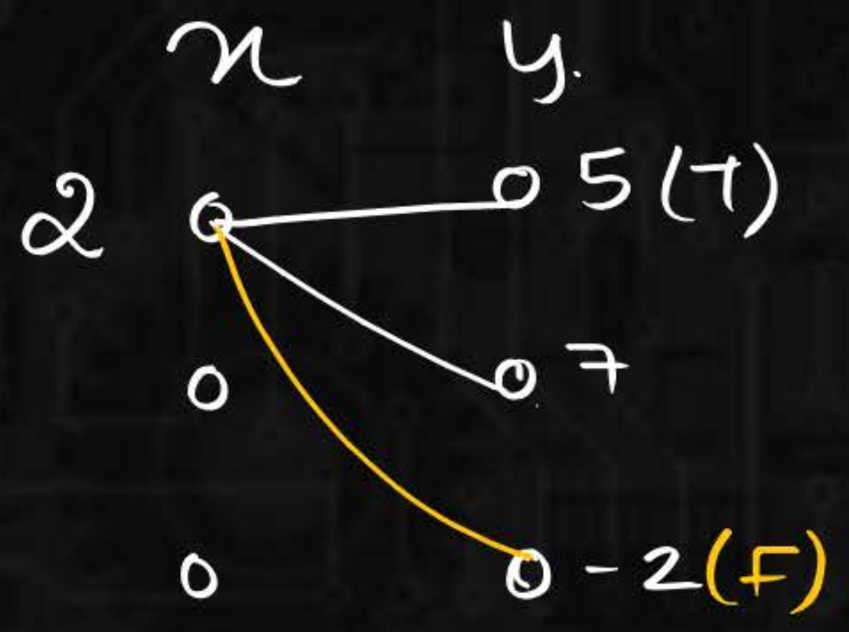
$F \longrightarrow$

$$\left(\frac{3}{5}\right) \wedge \frac{5}{3} \longrightarrow 3 = 5$$

$F \wedge \quad \rightarrow$

$F \rightarrow$

T



$\forall x \forall y$ → false

$\frac{2}{5} \wedge \frac{5}{2} \rightarrow 2=5$

$[F \wedge \ ] \rightarrow$

$\frac{F \rightarrow}{T}$

$\frac{2}{7} \wedge \frac{7}{2} \rightarrow 2=7$

$\frac{[F \wedge \ ] \rightarrow}{T}$

$\frac{2}{-2} \wedge \frac{-2}{2} \rightarrow \underline{2=-2}$

$T \wedge T \rightarrow F$

$T \rightarrow F \equiv F.$

$P(x, y) : \frac{y}{x} \quad D : Z.$

False.

1) $\forall x \exists y \left(\frac{y}{x}\right)$

$x = 0$

3) $\exists y \forall x. \left(\frac{y}{x}\right)$ — false $\left(\frac{y.}{\circ \circ \circ \circ\, x = 0 \ldots}\right)$

$x = 0$

2) $\forall y \exists x \left(\frac{y}{x}\right)$ — True

$\frac{\circ \circ \circ \circ \circ \circ}{x = 1}$

4) $\exists x \forall y \left(\frac{y}{x}\right)$ — True.

$\left(\frac{\circ \circ \circ \circ \circ}{x = 1}\right)$

$D : \mathbb{Z}.$

$\exists m \forall n\, (m \cdot n = 1)$

(false.

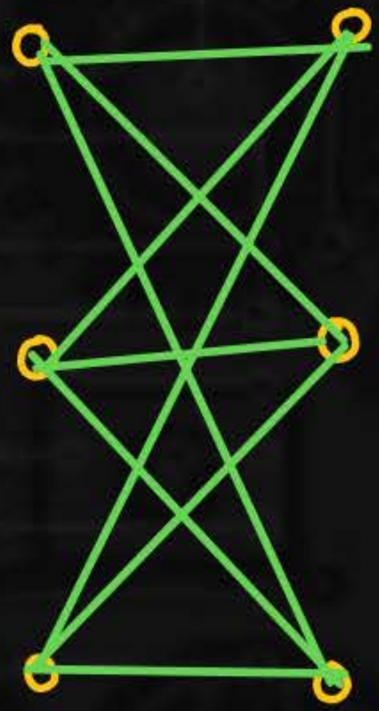
$\times \begin{array}{|c|} \hline \\ 2 \\ 3 \\ 4 \\ \\ \\ \hline \end{array} = 1.$

$\exists m \exists n\, (m \cdot n = 1) \rightarrow$ True

$m = 1 \quad n = 1$

1

$\forall x \forall y$

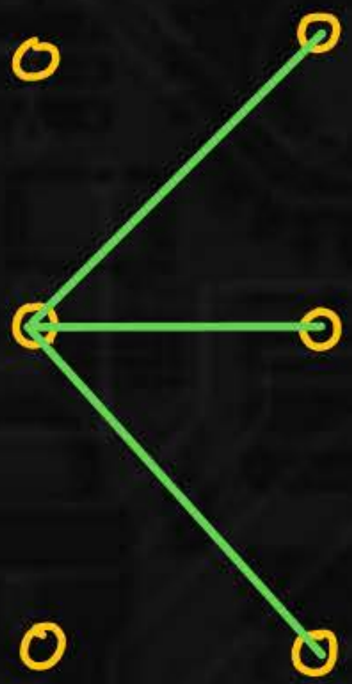


9 edges

↳ True

2 **3**

$\forall x \exists y \underset{\longleftarrow}{\not\longrightarrow} \exists y \forall x$

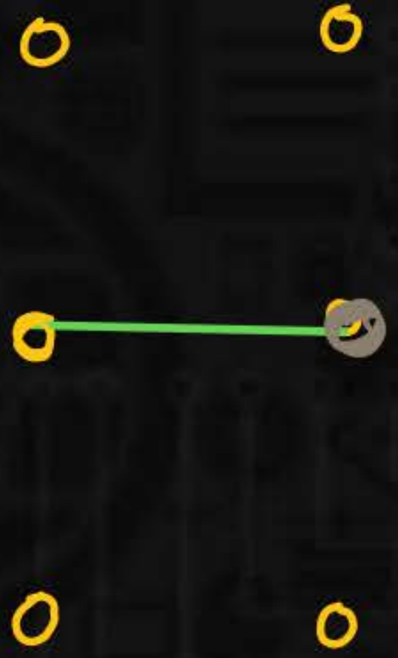


Domain is fixed.

open stmt is also fixed.

4.

$\exists x \exists y$.



at least 1 edge

Type-5.

9 edges.

1. $\forall x \forall y \equiv \forall y \forall x$

$\downarrow$ $\qquad$ $\downarrow$

$\left(\frac{x}{y}\right)$ $\exists y \forall x$ $\qquad$ $\exists x \forall y$ $\rightarrow$ no Relation.

$\downarrow$ $\qquad$ $\downarrow$

$\left(\frac{x}{y}\right)$ $\forall x \exists y$ $\qquad$ $\forall y \exists x$

$\downarrow$ $\qquad$ $\downarrow$

T. $\exists x \exists y \equiv \exists y \exists x$.

$\forall x \forall y \rightarrow$ all.

$\forall y \forall x \rightarrow$ all.

all $\rightarrow \exists x \exists y$

all $\rightarrow \exists y \exists x$.

Let the universe for the variables in the following statements consist of all real numbers. In each case negate and simplify the given statement.

a) $\forall x \forall y[(x > y) \rightarrow (x - y > 0)]$

b) $\forall x \forall y\{(x < y) \rightarrow \exists z(x < z < y)]$

c) $\forall x \forall y\{(|x| = |y|) \rightarrow (y = \pm x)\}$

$$\forall x \forall y\,[(x>y) \rightarrow (x-y>0)]$$

$$\neg \forall x \forall y\,[\neg(x>y) \vee (x-y>0)]$$

$$\exists x \exists y\,[(x>y) \wedge \neg(x-y>0)]$$

$\neg(x-y>0)$

$(x-y \leq 0)$

p	$\neg p$
$>$	$\leq$
$<$	$\geq$


THANK YOU GW
SOLDIERS !