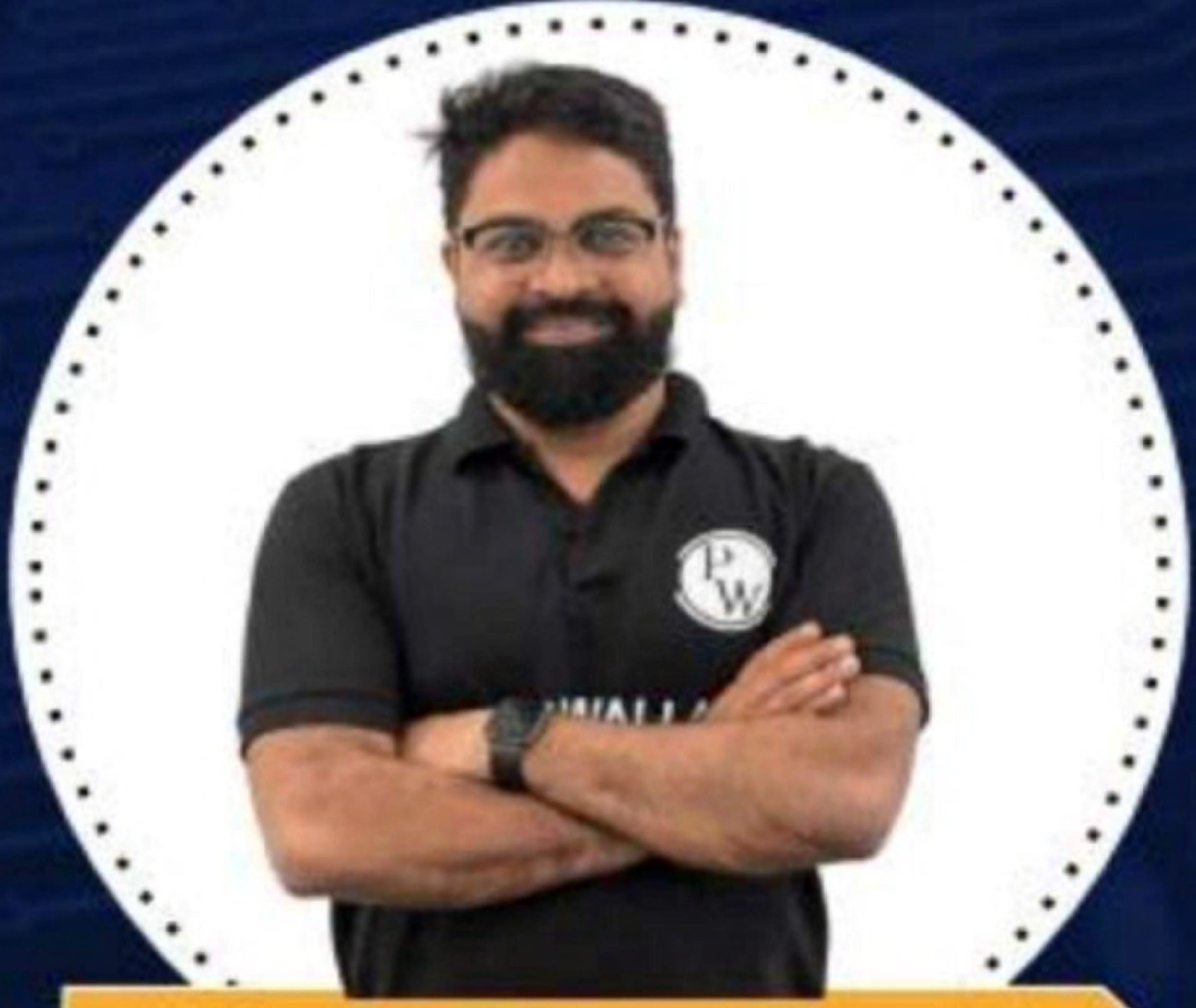
CS & IT ENGINEERING

LOGIC

Lecture No: 10



Satish Sir

TOPICS TO BE COVERED

- Propositional Logic
- First Order Logic

Q.1 Indicate which of the following well-formed formulae are valid: (GATE - 90)

c) $P \wedge (\neg P \vee \neg Q) \to Q$

$\to \neg P \vee \neg Q$

$\to P$

$\neg Q$

$((P \to Q) \wedge (Q \to R)) \to (P \to R)$

(valid)

A. $((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$

B. $(P \Rightarrow Q) \Rightarrow (\sim P \Rightarrow Q)$ (Invalid) (1)

C. $(P \wedge (\sim P \vee \sim Q)) \Rightarrow Q$ (Invalid) (3)

D. $((P \Rightarrow R) \vee (Q \Rightarrow R)) \Rightarrow ((P \vee Q) \Rightarrow R)$ (Invalid) (2)

b) $\underbrace{(P \to Q)}_{(F \to F)} \to \underbrace{(\neg P \to Q)}_{T \quad F}$

$\overline{T} \quad \overline{F}$

$T \longrightarrow F$

false

$P = F$

$Q = F$

P	Q	
F	F	F

Q.1 Indicate which of the following well-formed formulae are valid: (GATE - 90)

c) $P \wedge (\neg P \vee \neg Q) \rightarrow Q$

$\rightarrow \neg P \vee \neg Q$

$\rightarrow P$

$\overline{\qquad\qquad}$

$\neg Q$

(valid)

$$\overline{(P \rightarrow R) \vee (Q \rightarrow R)}^{\,T} \equiv ((P \wedge Q) \rightarrow R)$$

A. $((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$

B. $(P \Rightarrow Q) \Rightarrow (\sim P \Rightarrow Q)$ (Invalid)

C. $(P \wedge (\sim P \vee \sim Q)) \Rightarrow Q$ (Invalid

D. $((P \Rightarrow R) \vee (Q \Rightarrow R)) \Rightarrow ((P \vee Q) \Rightarrow R)$ (Invalid) (2)

F

Q.2 Which of the following is/are tautology?

(GATE-92)

$\overline{(a \vee b)}^{T} \to \overline{(b \wedge c)}^{F}$

$\left\{\begin{array}{l} b = T \\ c = F \end{array}\right.$ $(\underline{\ \vee T}) \to (T \wedge F)$

(Invalid) $T \to F \equiv F$. (Type-1)

F

A. $(a \vee b) \to (b \wedge c)$

God's Rule

b) $\overline{(a \wedge b)}^{T} \to \overline{(b \vee c)}^{F}$ $b = F$, $c = F$

$(a \wedge b) \to (b \vee c)$

B. $(a \wedge b) \to (b \vee c)$ (Valid)

$\wedge F$, F F

$\neg(a \wedge b) \vee (b \vee c)$

$\overline{F \to}$ True

$\neg a \vee \neg b \vee b \vee c.$

C. $(a \vee b) \to (b \to c)$

$a \wedge b \to b \to b \vee c.$

$\vee T \vee$

$\overline{}$ T.

D. $(a \to b) \to (b \to c)$

Q.2 Which of the following is/are tautology?

(GATE-92)

A. $(a \vee b) \rightarrow (b \wedge c)$ (Invalid)

B. $(a \wedge b) \rightarrow (b \vee c)$ (valid)

C. $(a \vee b) \rightarrow (b \rightarrow c)$ (Invalid)

D. $(a \rightarrow b) \rightarrow (b \rightarrow c)$ (Invalid)

c) $\overline{(a \vee b)}^{T} \rightarrow \overline{(b \rightarrow c)}^{F}$

$(a \vee b) \downarrow (\vee T)$, $b \downarrow T$, $c \downarrow F$

$b = T$, $c = F$.

$T \rightarrow F$

F.

$b = T$, $c = F$

$b = T$ $c = F$.

d) $\overline{(a \rightarrow b)}^{T} \rightarrow \overline{(b \rightarrow c)}^{F}$

$(a \rightarrow b) \downarrow (\rightarrow T)$, $b \downarrow T$, $c \downarrow F$

$T \rightarrow F$

F

Q.3 The proposition $p\wedge(\sim p\vee q)$ is

(GATE - 93)

Type $(1, 3) \rightarrow$

Type $(2) \leftrightarrow$

Type-2.

$p\wedge(\neg p\vee q)$

$\underline{(p\wedge\neg p)}\vee(p\wedge q)$

$F\vee(p\wedge q)$

$= p\wedge q.$

$p\vee F\equiv p.$

$p = T$	$p = F$
$T\vee F\equiv T$	$F\vee F\equiv F.$

A. a tautology

B. $\Leftrightarrow(p\wedge q)$ ✓

C. $\Leftrightarrow(p\vee q)$

D. a contradiction

$$\boxed{\begin{array}{l} p\wedge(\neg p\vee q)\equiv(p\wedge q) \\ p\wedge(\neg p\vee q) \\ (p\wedge(p\rightarrow q))\longrightarrow q. \end{array}}$$

$\longleftrightarrow \;|\; \equiv$

$A \longrightarrow B \qquad B \not\rightarrow A$

viceversa is not true.

$A \Longleftrightarrow B$

$A \longrightarrow B \wedge B \longrightarrow A$

$$\begin{array}{l} p \rightarrow q \\ \underline{p \qquad} \\ \therefore q \end{array} \qquad \begin{array}{l} p \rightarrow q \\ \underline{q \qquad} \\ p \end{array}$$

(fallacy)
(invalid)

$$\begin{array}{l} p \\ \underline{p \rightarrow q} \\ \therefore q \end{array}$$

$(p \wedge (p \rightarrow q)) \rightarrow q$.

Implication not equivalent

$p \wedge (p \rightarrow q) \equiv p \wedge q$.

Q.4 What is the converse of the following assertion?

$\underbrace{\boxed{\text{I stay}}}_{p}$ only if $\underbrace{\boxed{\text{you go}}}_{q}$ (GATE - 98)

stay → go

converse

go → stay

if p, q ≡ p → q

q if p

q when p

q whenever p

p only if q

A. $\overbrace{\underbrace{\text{I stay}}_{q}}^{q}$ if $\overbrace{\underbrace{\text{you go}}_{p}}^{p}$ ≡ go → stay

B. If I stay then you go ≡ stay → go

C. If you do not go then I do not stay ≡ ¬go → ¬stay

D. If I do not stay then you go ¬stay → go

Q.6 "If X then Y unless Z " is represented by which of the following formulas in propositional logic? (" ~ " is negation, " ∧" is conjunction, and " → " is implication)

(GATE - 02)

A. $(X \wedge \sim Z) \rightarrow Y$

B. $(X \wedge Y) \rightarrow \sim Z$

C. $X \rightarrow (Y \wedge \sim Z)$

D. $(X \rightarrow Y) \wedge \sim Z$

$P \rightarrow q \equiv q\ \boxed{\text{unless}}\ \neg p$

-ve

if x then y unless z. P q.

$(x \rightarrow y)$ unless z

-ve

$\neg z \rightarrow (x \rightarrow y)$

$z \vee (\neg x \vee y) \equiv z \vee \neg x \vee y$

a) $(x \wedge \neg z) \rightarrow y$

$\neg(x \wedge \neg z) \vee y$

$\neg x \vee z \vee y$

b) $(x \wedge y) \rightarrow \neg z$

$\neg(x \wedge y) \vee \neg z$

$\neg x \vee \neg y \vee \neg z$

c) $x \rightarrow (y \wedge \neg z)$

$\neg x \vee (y \wedge \neg z)$ not same

d) $(x \rightarrow y) \wedge \neg z$

$$p \to q \equiv \neg p \vee q$$

Q.7 The following propositional statement is

$(P \rightarrow (Q \vee R)) \rightarrow ((P \wedge Q) \rightarrow R)$

(GATE - 04)

$\underbrace{(\overset{T}{P} \rightarrow \overset{F}{(Q \vee R)})}_{F} \longrightarrow (\quad) \longrightarrow T$

$P = T$, $Q = F$, $R = F$

$\overset{T}{\overline{(P \rightarrow (Q \vee R))}} \rightarrow \overset{F}{\overline{((\underset{T}{P \wedge Q}) \rightarrow \underset{F}{R})}}$

$P = T$, $Q = T$, $R = F$

$T \rightarrow (T \vee F)$

$\underbrace{(T \rightarrow T)}_{T} \longrightarrow F$

A. Satisfiable but not valid → contingency

B. Valid

C. A contradiction

D. None of the above

P	Q	R	
T	T	F	F
T	F	F	T

Q.8 Let p,q,r and s be four primitive statements. Consider the following arguments: (GATE - 04)

$s \vee q$

P: $[(\sim p \vee q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (\sim s \rightarrow q)$ (valid)

Q: $[(\sim p \wedge q) \wedge [q \rightarrow (p \rightarrow r)]] \rightarrow \sim r$ (Invalid)

R: $[[(q \wedge r) \rightarrow p] \wedge (\sim q \vee p)] \rightarrow r$

S: $[p \wedge (p \rightarrow r) \wedge (q \vee \sim r)] \rightarrow q$ (valid)

Which of the above arguments are valid?

A. P and Q only

B. P and R only

C. P and S only

D. P, Q, R and S

$\neg p \vee q \rightarrow \neg p \vee q$

$\neg R \vee s$

$p \vee r \vdash p \vee s \vdash q \vee s$

Q. $\neg p \wedge q$

$$\frac{q \rightarrow (p \rightarrow R)}{\neg R}$$

$\neg p \wedge q$

$\downarrow$ simp

q

$$\frac{q \rightarrow (p \rightarrow R)}{p \rightarrow R}$$

simp

$\neg p$

$$\frac{p \rightarrow R \equiv \neg R \rightarrow \neg p}{\neg R}$$

fallacy

Q.8 Let p,q,r and s be four primitive statements. Consider the following arguments: (GATE - 04)

P: $[(\sim p \vee q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (\sim s \rightarrow q)$

Q: $[(\sim p \wedge q) \wedge [q \rightarrow (p \rightarrow r)]] \rightarrow \sim r$

R: $[[(q \wedge r) \rightarrow p] \wedge (\sim q \vee p)] \rightarrow r$

S: $[p \wedge (p \rightarrow r) \wedge (q \vee \sim r)] \rightarrow q$

Which of the above arguments are valid?

A. P and Q only

B. P and R only

C. P and S only

D. P, Q, R and S

$[\overline{(q \wedge R) \rightarrow p}^{T} \wedge \overline{(\neg q \vee p)}^{T}] \rightarrow \overline{R}^{F}$ R=F.

F

$(F \rightarrow)$

$(T) \wedge (\vee T)$

R) $(q \wedge R) \rightarrow p \equiv \neg(q \wedge R) \vee p$

$\dfrac{\neg q \vee p}{R}$ $\equiv \neg q \vee \neg R \vee p$

$\neg q \vee p$

Q.9 A logical binary relation ⊙, is defined as follows: Let ~ be the unary negation (NOT) operator, with higher precedence then ⊙. Which one of the following is equivalent to A∧B ?

(GATE - 06)

A	B	A ⊙ B	A ∧ B	¬A ⊙ B	¬(A ⊙ ¬B)
True	True	True	T		T
True	False	True	F		
False	True	False	F		
False	False	True	F		

a) A = T B = T

¬A ⊙ B

F ⊙ T ≡ F

b) A = T B = T

¬(A ⊙ ¬B)

¬(T ⊙ F)

¬(T) ≡ F

c) A = T B = T

¬(¬A ⊙ ¬B)

¬(F ⊙ F)

¬(T)

≡ F

d) ¬(¬A ⊙ B)

¬(F ⊙ T)

¬(F) ≡ T

A. (~A⊙B)

B. ~(A⊙~B)

C. ~(~A⊙~B)

D. ~(~A⊙B)

Q.9 A logical binary relation $\odot$, is defined as follows:
Let $\sim$ be the unary negation (NOT) operator, with higher precedence then $\odot$. Which one of the following is equivalent to $A \wedge B$?
(GATE - 06)

$A = T \quad B = T$

$A \wedge B = T$

A	B	A $\odot$ B	$A \wedge B$
True	True	True	T
True	False	True	F
False	True	False	F
False	False	True	F

A B C D

A. $(\sim A \odot B) \equiv F$

B. $\sim(A \odot \sim B)$

C. $\sim(\sim A \odot \sim B)$

D. $\sim(\sim A \odot B)$

$\neg($

$= f$

Q.9 A logical binary relation $\odot$, is defined as follows: Let $\sim$ be the unary negation (NOT) operator, with higher precedence then $\odot$. Which one of the following is equivalent to $A \wedge B$?

(GATE - 06)

A	B	$A \odot B$
True	True	✓True
True	False	✓True
False	True	0 False
False	False	✓True

A. $(\sim A \odot B)$

B. $\sim(A \odot \sim B)$

C. $\sim(\sim A \odot \sim B)$

D. $\sim(\sim A \odot B)$

$A \odot B \equiv B \rightarrow A$ $\quad A \wedge B.$

$\equiv \neg B \vee A$

Q.10 P and Q are two propositions. Which of the following logical expressions are equivalent? **(GATE - 08)**

I. $P \vee \sim Q$

II. $\sim(\sim P \wedge Q)$ (Type-2)

III. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

IV. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

A. Only I and II

B. Only I, II and III

C. Only I, II and IV

D. All of I, II, III and IV

Q.15 Consider the following two statements.

S_1: If a candidate is known to be corrupt, then he will not be elected $C \rightarrow \neg E \equiv E \rightarrow \neg C$ $E \rightarrow \neg C$

S_2: If a candidate is kind, he will be elected $K \rightarrow E$ $K \rightarrow \neg C$

Which one of the following statements follows from S1 and S2 as per sound inference rules of logic? $K \rightarrow \neg C$ **(GATE - 15 - set 2)**

pdf ⇒ friday.

OR

$C \rightarrow \neg K$

$C \rightarrow K$ ×

A. If a person is known to be corrupt, he is kind

$\neg C \rightarrow \neg K$

B. If a person is not known to be corrupt, he is not kind

$K \rightarrow \neg C$

C. If a person is kind, he is not known to be corrupt

$\neg K \rightarrow \neg C$

D. If a person is not kind, he is not known to be corrupt

Q.16 Let p,q,r,s represent the following propositions.

p: $x \in \{8,9,10,11,12\}$

q: x is a composite number

r. x is a perfect square

s: x is a prime number

(GATE - 16 - set 1)

The integer $x \geq 2$ which satisfies $\sim((p \Rightarrow q) \wedge (\sim r \vee \sim s))$ is

Q.17 Let p,q and r be propositions and the expression $(p \rightarrow q) \rightarrow r$ be a contradiction.
Then, the expression $(r \rightarrow p) \rightarrow q$ is

(GATE-17-Set 1)

A. a tautology

B. a contradiction

C. always TRUE when p is FALSE

D. always TRUE when q is TRUE

Q.18 Let p,q,r denote the statements "It is raining", "It is cold" and "It is pleasant", respectively. Then the statement "It is not raining and it is pleasant, and it is not pleasant only if it is raining and it is cold" is represented by

(GATE-17-Set2)

A. $(\sim p \wedge r) \wedge (\sim r \rightarrow (p \wedge q))$

B. $(\sim p \wedge r) \wedge ((p \wedge q) \rightarrow \sim r)$

C. $(\sim p \wedge r) \vee ((p \wedge q) \rightarrow \sim 1)$

D. $(\sim p \wedge r) \vee (r \rightarrow (p \wedge q))$

Determine whether each of the following statements is true or false. If false, provide a counterexample.

The universe comprises all integers.

a) $\forall x \exists y \exists z (x = 7y + 5z)$

b) $\forall x \exists y \exists z (x = 4y + 6z)$

Q.19 Which of the following predicate calculus statements is/are valid?

(GATE - 92)

A. $((\forall x)P(x) \vee (\forall x)Q(x)) \rightarrow (\forall x)\{P(x) \vee Q(x)\}$

B. $\{(\exists x)P(x) \wedge (\exists x)Q(x)\} \rightarrow (\exists x)\{P(x) \wedge Q(x)\}$

C. $(\forall x)\{P(x) \vee Q(x)\} \rightarrow \{(\forall x)P(x) \vee (\forall x)Q(x)\}$

D. $(\exists x)\{P(x) \vee Q(x)\} \rightarrow \sim\{(\forall x)P(x) \vee (\exists x)Q(x)\}$

Q.20 Identify the correct translation into logical notation of the following assertion.

Some boys in the class are taller than all the girls

Note: taller (x, y) is true if x is taller than y.

(GATE - 04)

A. $(\exists x)(boy(x) \rightarrow (\forall y)(girl(y) \land taller(x, y)))$

B. $(\exists x)(boy(x) \land (\forall y)(girl(y) \land taller(x, y)))$

C. $(\exists x)(boy(x) \rightarrow (\forall y)(girl(y) \rightarrow taller(x, y)))$

D. $(\exists x)(boy(x) \land (\forall y)(girl(y) \rightarrow taller(x, y)))$

Q.21 Let $a(x,y)$, $b(x,y)$ and $c(x,y)$ be three statements with variables x and y chosen from some universe.

Consider the following statement

$(\exists x)(\forall y)[(a(x,y) \wedge b(x,y)) \wedge \sim c(x,y)]$

Which one of the following is its equivalent?

(GATE - 04)

A. $(\forall x)(\exists y)[(a(x,y) \vee b(x,y)) \rightarrow c(x,y)]$

B. $(\exists x)(\forall y)[(a(x,y) \vee b(x,y)) \wedge \sim c(x,y)]$

C. $\sim[(\forall x)(\exists y)[(a(x,y) \wedge b(x,y)) \rightarrow c(x,y)]$

D. $\sim[(\forall x)(\exists y)[(a(x,y) \vee b(x,y)) \rightarrow c(x,y)]$

Q.22 Which one of the following is the most appropriate logical formula to represent the statement:

"Gold and silver ornaments are precious"

The following notations are used:

$G(x)$:x is a gold ornament.

$S(x)$:x is a silver ornament.

$P(x)$:x is precious.

(GATE - 09)

A. (a) $\forall x(P(x) \rightarrow (G(x) \wedge S(x)))$

B. (b) $\forall x((G(x) \wedge S(x)) \rightarrow P(x))$

C. (c) $\exists x((G(x) \wedge S(x)) \rightarrow P(x))$

D. (d) $\forall x((G(x) \vee S(x)) \rightarrow P(x))$

Q.23 Consider the following well-formed formulae:

I. $\sim\forall x(P(x))$

II. $\sim\exists x(P(x))$

III. $\sim\exists x(\sim P(x))$

IV. $\exists x(\sim P(x))$

Which of the above are equivalent? **(GATE - 09)**

A. I and II

B. II and III

C. I and IV

D. II and IV

Q.24 Which one of the following options is correct given three positive integers x, y and z, and a predicate

$P(x): \sim(x=1) \wedge \forall y\{(\exists z(x=y * z) \Rightarrow (y=x) \vee (y=1)\}$

(GATE - 11)

A. P(x) being true means that x is a prime number

B. P(x) being true means that x is a number other than 1

C. P(x) is always true irrespective of the value of x

D. P(x) being true means that x has exactly two factors other than 1 and x

Q.25 Consider the statement:
"Not all that glitters is gold"
Predicate glitters (x) is true if x glitters and predicate gold(x) is true if x is gold.
Which one of the following logical formulae represents the above statement? (GATE-14-Set1)

A. $\forall x$: glitters $(x) \Rightarrow \sim$gold(x)

B. $\forall x$: gold $(x) \Rightarrow$ glitters(x)

C. $\exists x$: gold$(x) \wedge \sim$glitters(x)

D. $\exists x$: glitters $(x) \wedge \sim$gold(x)

Q.26 Which one of the following well-formed formulae in predicate calculus is NOT valid?

(GATE-16-Set 2)

A. $((\forall x\, p(x) \Rightarrow \forall x\, q(x)) \Rightarrow (\exists x \sim p(x) \vee \forall x\, q(x))$

B. $(\exists x\, p(x) \vee \exists x\, q(x)) \Rightarrow \exists x(p(x) \vee q(x))$

C. $(\exists x(p(x) \wedge q(x)) \Rightarrow (\exists x\, p(x) \wedge \exists x\, q(x))$

D. $\forall x(p(x) \vee q(x)) \Rightarrow (\forall x\, p(x) \vee \forall x\, q(x))$

Q.27 Consider the first-order logic sentence $F: \forall x(\exists y R(x, y))$. Assuming non-empty logical domain, which of the sentences below are implied by F ?

I. $\exists y(\exists x R(x,y))$

II. $\exists y(\forall x R(x,y))$

III. $\forall y(\exists x R(x,y))$

IV. $\sim x(\forall y \sim R(x,y))$

(GATE-17-Set1)

A. IV only

B. I and IV only

C. II only

D. II and III only

Q.29 Consider the first order predicate formula φ :

$$(\forall x[(\forall z\ z|x \Rightarrow ((z=x) \vee (z=1)))$$
$$\Rightarrow \exists w(w>x) \wedge (\forall z\ z|w \Rightarrow ((w=z) \vee (z=1)))])$$

Here ' a|b ' denotes that ' a divides b ', where a and b are integers.

Consider the following sets:

S1. {1,2,3……100}

S2. Set of all positive integers

S3. Set of all integers

(GATE-19)

Which of the above sets satisfy φ ?

A. (a) S2 and S3

B. (b) S1, S2 and S3

C. (c) S1 and S2

D. (d) S1 and S3


THANK YOU GW
SOLDIERS !