# CS & IT ENGINEERING





LOGIC

Lecture No: 09



TOPICS TO BE COVERED Inference Rule

Quantifier

**Negation of Quantifier** 

**Nested Quantifier** 

$$p \rightarrow q$$



$$p \leftarrow c$$

$$r \rightarrow \neg q$$

$$p \rightarrow q$$

2: 
$$P \rightarrow a$$
  $P \rightarrow a$   $P \rightarrow \gamma R = R \rightarrow \gamma P$ 
 $R \rightarrow \gamma a = a \rightarrow \gamma R$ 
 $R \rightarrow \gamma R$ 



$$p \rightarrow (q \rightarrow r)$$
  
 $\neg q \rightarrow \neg p$ 

$$Q \rightarrow R(2A)$$



Q.3

$$p \rightarrow (q \rightarrow r)$$

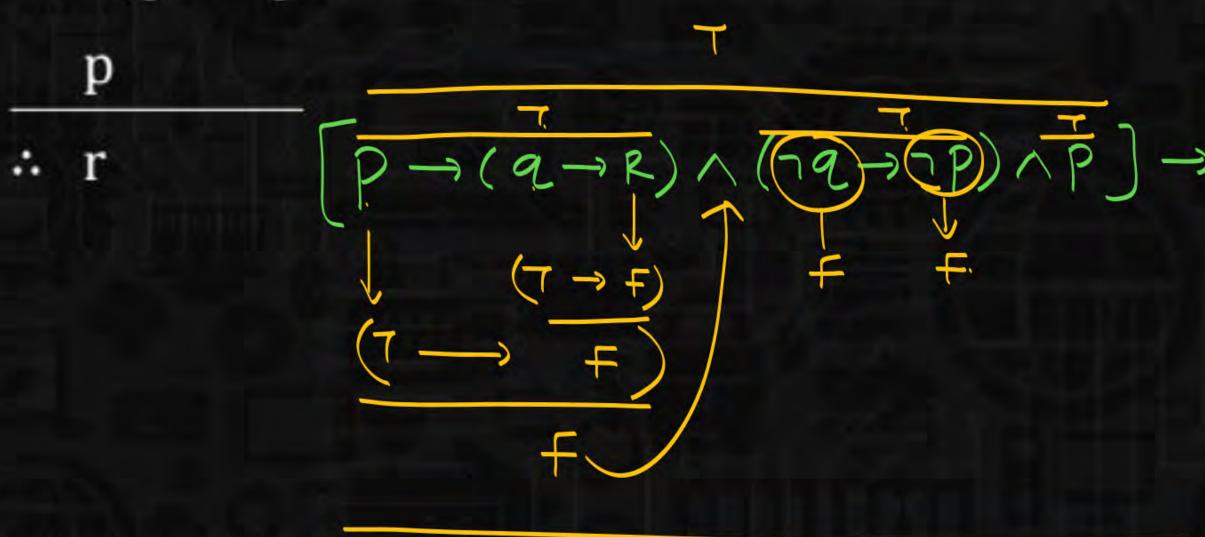


P=T

R=F

9=7

$$\neg q \rightarrow \neg p$$



Q.4

$$p \wedge q$$

$$p \rightarrow (r \land q)$$

$$r \rightarrow (s \lor t)$$

$$\neg s$$

$$\dot{}$$

$$p \rightarrow (q \rightarrow r)$$



$$t \rightarrow q$$

$$\neg S$$

$$\therefore \neg r \rightarrow \neg t$$



$$(\neg p \lor \neg q) \rightarrow (r \land s)$$



$$r \rightarrow t$$

$$\frac{\neg t}{\Rightarrow p}$$

$$r \rightarrow t$$

$$p$$

$$r \rightarrow t$$

(Addition) J 7RV75.

$$\begin{array}{c} (\gamma p \vee \gamma q) \rightarrow (R \wedge S) \\ \neg (R \wedge S) \rightarrow \gamma (\gamma p \vee \gamma q) \\ (\gamma R \vee \gamma S) \rightarrow p \wedge q \\ \hline \gamma R \vee \gamma S \\ \hline p \wedge q \longrightarrow p \end{array}$$
modus ponens.

Q.8



$$u \rightarrow r$$

$$(r \land s) \rightarrow (p \lor t)$$

$$q \rightarrow (u \land s)$$

$$\neg t$$



# Negate and simplify each of the following.



1) 
$$\exists n [p(n) \vee q(n)]$$
 $aegate$ 
 $\exists n [p(n) \vee q(n)]$ 

Hn[7p(n)ハフの(m)

A. 
$$\exists x[p(x) \lor q(x)]$$

B. 
$$\forall x[p(x) \land \neg q(x)]$$

$$\forall x [p(x) \to q(x)]$$

$$\exists x [(p(x) \lor q(x)) \to r(x)]$$





$$p(x): x^2 - 7x + 10 = 0 (n = 2, 5)$$

$$q(x): x^2 - 2x - 3 = 0 \ (n = -1, 3)$$

$$r(x): x < 0$$
 (negative)

Determine the truth or falsity of the following statements, where the universe is all integers.

a) 
$$\forall n [p(n) \rightarrow \tau R(n)]$$

7 R(n): +ve

A. 
$$\forall x[p(x) \rightarrow \neg r(x)]$$

$$\forall x[p(x) \rightarrow \neg r(x)] \qquad \qquad \gamma \leftarrow 2 \qquad \gamma(2) \qquad \rightarrow \neg R(2)$$

B. 
$$\forall x[q(x)\rightarrow r(x)]$$

$$\exists x[q(x)\rightarrow r(x)]$$

$$\exists x[p(x) \rightarrow r(x)]$$

$$N=5$$
.  $T \rightarrow T$ .





$$p(x): x^2 - 7x + 10 = 0 (n = 2, 5)$$

$$q(x): x^2 - 2x - 3 = 0 \ (\lambda = 4,3)$$

$$q(x): x^2 - 2x - 3 = 0$$

$$r(x): x < 0$$
 (negative)

Determine the truth or falsity of the following statements, where the universe is all integers.

b)  $\forall n (2(n) \rightarrow R(n))$ 

A. 
$$\forall x[p(x) \rightarrow \neg r(x)] \quad \pi = 3$$

$$\forall x[q(x)\rightarrow r(x)](false)$$

$$\exists x[q(x)\rightarrow r(x)]$$

$$\exists x[p(x) \rightarrow r(x)]$$





$$p(x): x^2 - 7x + 10 = 0 (n = 2, 5)$$

$$q(x): x^2 - 2x - 3 = 0$$
 (  $\chi = 4,3$ )

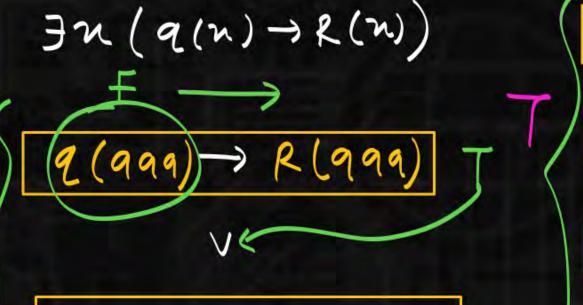
$$r(x): x < 0$$
 (negative)

7 R(n): +ve

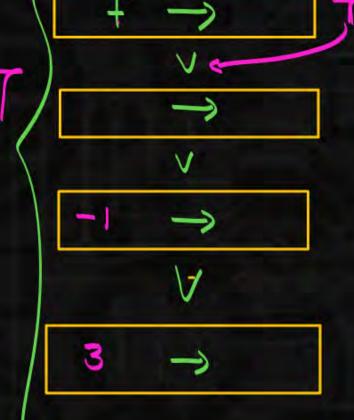
Determine the truth or falsity of the following statements, where the universe is all integers.

A. 
$$\forall x[p(x) \rightarrow \neg r(x)] \gamma = 999$$

- $\forall x[q(x)\rightarrow r(x)]$
- $\exists x[q(x)\rightarrow r(x)](\neg vw)$
- $\exists x[p(x)\rightarrow r(x)](\neg v)$











$$p(x): x^2 - 7x + 10 = 0 (n = 2, 5)$$

$$q(x): x^2 - 2x - 3 = 0 \ (\lambda = 4,3)$$

$$r(x): x < 0$$
 (negative)

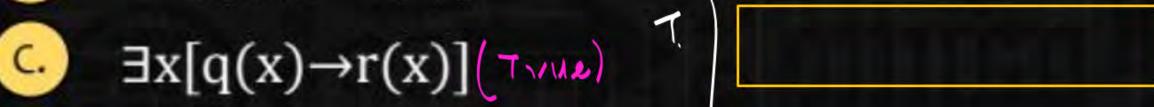
Determine the truth or falsity of the following statements, where the universe is all integers.

A. 
$$\forall x[p(x) \rightarrow \neg r(x)]$$

d) 
$$\exists n (p(n) \rightarrow r(n))$$



B.  $\forall x[q(x)\rightarrow r(x)]$ 



 $\exists x[p(x)\rightarrow r(x)](1\rightarrow v)$ 







$$p(x,y): y - x = y + x^2$$

where the universe for each of the variables x,y comprises all integers. Determine the truth value for

each of the following statements. P(n,y) n=1 4=1

a) 
$$p(0,0)(\tau)$$
 e)  $\exists yp(1,y)$ 

b) 
$$p(1,1)(f)$$
 f)  $\forall x \exists y p(x,y)$ 

$$p(n,y): y-n=y+n^2$$

$$|-|=|+|^2$$

0 = 2

c) 
$$p(0,1)$$

c) 
$$p(0,1)$$
 g)  $\exists y \forall x p(x,y)$ 

d) 
$$\forall yp(0,y)$$
 h)  $\forall y\exists xp(x,y)$ 

d) 
$$\forall yp(0,y)$$
 h)  $\forall y\exists xp(x,y)$ 

Pw

$$p(x,y): y-x=y+x^2$$
 d)  $\forall y(0,y) \quad y-x-y+x^2$ 

where the universe for each of the variables x,y comprises all integers. Determine the truth value for each of the following statements.

a) 
$$p(0,0)(\tau)$$
 e)  $\exists yp(1,y)$ 

b) 
$$p(1,1)(f)$$
 f)  $\forall x \exists y p(x,y)$ 

c) 
$$p(0,1)(T)$$
 g)  $\exists y \forall x p(x,y)$ 

d) 
$$\forall yp(0,y) \rightarrow h) \forall y\exists xp(x,y)$$

c) 
$$p(0,1)$$
  $n=0$   $y=1$   
 $y-n=y+n^2$   
 $|-0=|+(0)^2$ 



$$p(x,y): y-x=y+x^2$$
   
  $y-x=y+x^2$    
  $y-1=y+x^2$    
  $y-1=y+x^2$ 

where the universe for each of the variables x,y comprises all integers. Determine the truth value for each of the following statements.

9-2-1+4

a) 
$$p(0,0)(\tau)$$
 e)  $\exists yp(1,y)(false)$   $\forall n \exists y(y-n=y+n^2)$ 

c) 
$$p(0,1)(\tau)$$
 g)  $\exists y \forall x p(x,y)$ 

d) 
$$\forall yp(0,y) \rightarrow h) \forall y\exists xp(x,y)$$



$$p(x,y): y - x = y + x^2$$

where the universe for each of the variables x,y comprises all integers. Determine the truth value for each of the following statements.

a) 
$$p(0,0)(\tau, e) \exists yp(1,y)$$

b) 
$$p(1,1)(f)$$
 f)  $\forall x \exists y p(x,y)$ 

c) 
$$p(0,1)(7)$$
 g)  $\exists y \forall x p(x,y)(f_{obs})$ 

d) 
$$\forall yp(0,y) \rightarrow h) \forall y\exists xp(x,y)$$





$$p(x,y): y - x = y + x^2$$

where the universe for each of the variables x,y comprises all integers. Determine the truth value for each of the following statements.

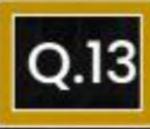
a) 
$$p(0,0)(\tau, e) \exists yp(1,y)$$

b) 
$$p(1,1)(f)$$
 f)  $\forall x \exists y p(x,y)$ 

c) 
$$p(0,1)(T)$$
 g)  $\exists y \forall x p(x,y)$ 

d) 
$$\forall yp(0,y) \rightarrow h) \forall y\exists xp(x,y)$$





# Determine whether each of the following statements is true or false.



The universe comprises all integers.

a) 
$$\forall x \exists y \exists z (x = 7y + 5z) (7)$$

b) 
$$\forall x \exists y \exists z (x = 4y + 6z)(f)$$

2. 
$$r \rightarrow s$$

$$p \rightarrow q$$

$$r \lor p$$

$$\therefore s \lor q$$

4. 
$$p \rightarrow (r \rightarrow s)$$
  
 $\sim r \rightarrow \sim p$   
 $p$   
 $\therefore s$ 

5. 
$$(p \land q) \rightarrow \sim t$$
  
 $w \lor r$   
 $w \rightarrow p$   
 $r \rightarrow q$   
 $\therefore (w \lor r) \rightarrow \sim t$ 

$$6. \sim t \rightarrow \sim r$$

$$\sim s$$

$$t \rightarrow w$$

$$r \lor s$$

$$\therefore w$$

7. 
$$(p \land q) \rightarrow \sim t$$
  
 $w \lor r$   
 $w \rightarrow p$   
 $r \rightarrow q$   
 $\therefore (w \lor r) \rightarrow \sim t$ 

8. 
$$p$$

$$p \rightarrow q$$

$$q \rightarrow r$$

$$\therefore r$$

$$11. r$$

$$p \rightarrow q$$

$$q \rightarrow r$$

$$\vdots p$$



