CS & IT ENGINEERING

Coloring in Graphs

Lecture No.9

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TOPICS TO BE COVERED

01 Properly coloring

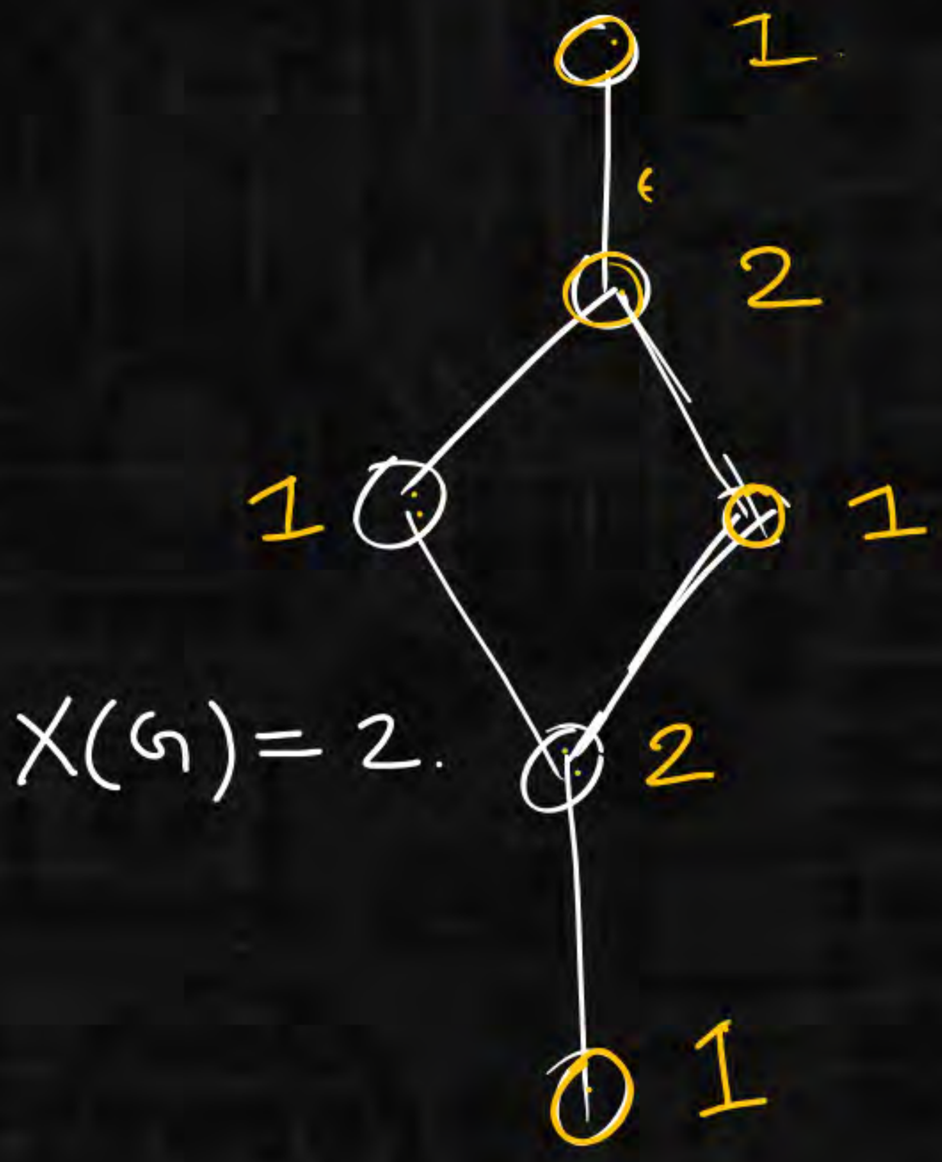
02 Chromatic number

03 Chromatic Number in Graphs

04 Subgraphs

05 Graph operations

$X(G) = K.$ K-colorable Graph.



Chromatic no:

$X(G)$: { min no of colors + $\boxed{\text{such that adjacent should not have same color}}$ → properly coloring

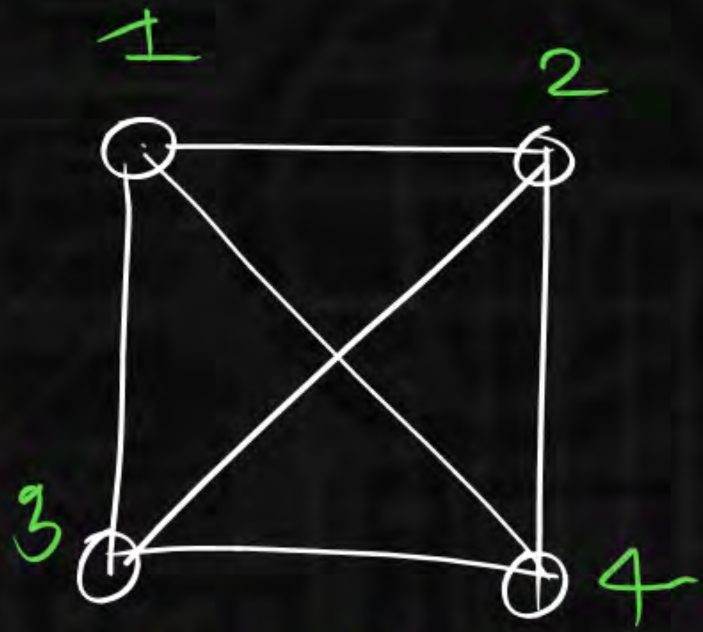
1.  $\chi(G) = 1$

2. Complete Graph (K_n) $(n \geq 1)$

$\chi(K_1) = 1$

$\chi(K_2) = 2$

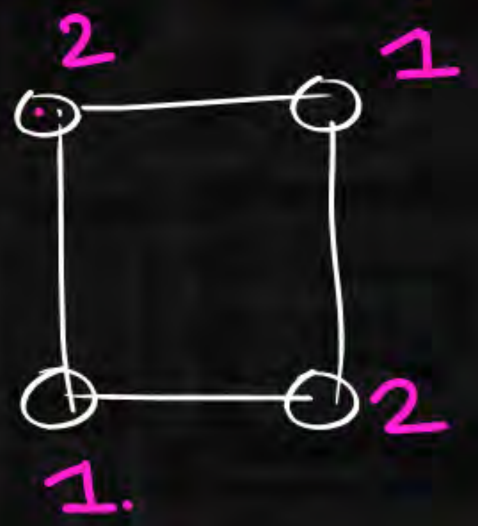
$\chi(K_3) = 3$

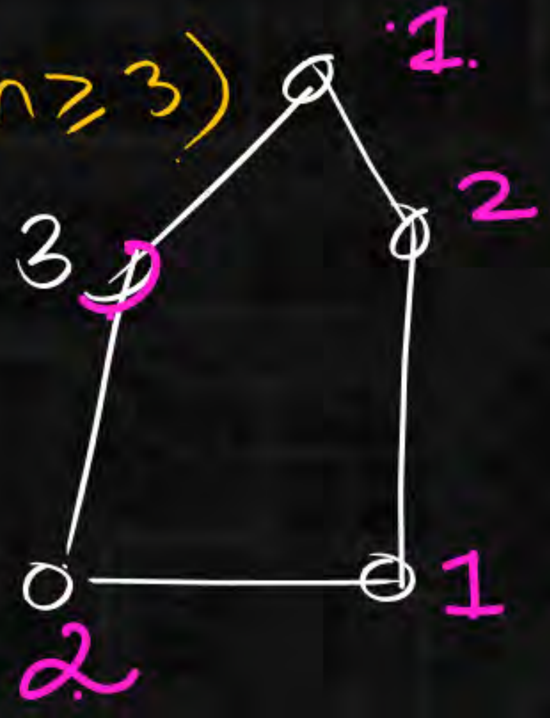


$\chi(K_4) = 4$

$\chi(K_n) = n$

Cycle Graph (C_n) ($n \geq 3$)





C_3

$\chi(C_n) = 2$ n is even.

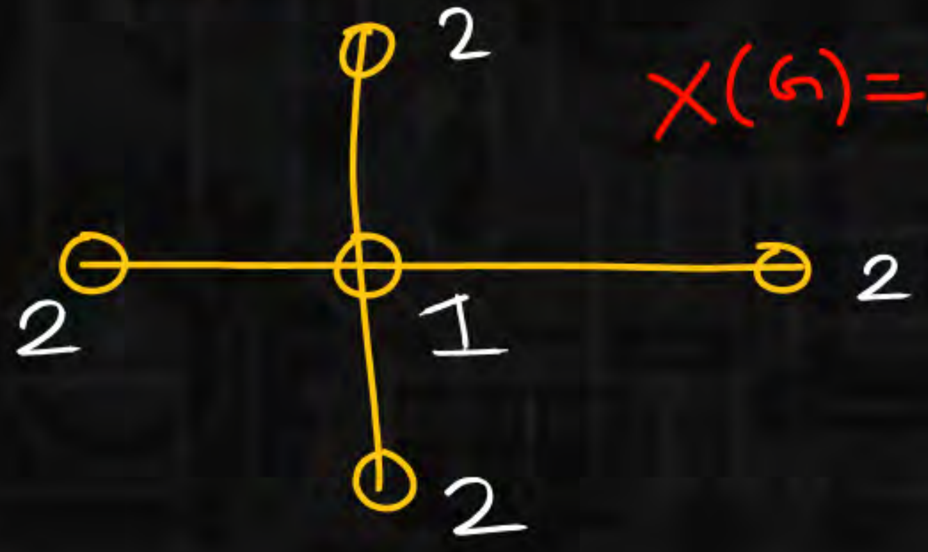
$\chi(C_n) = 3$ n is odd

$\chi(C_3) = 3$ $\quad \chi(C_4) = 2$ $\quad \chi(C_5) = 3$

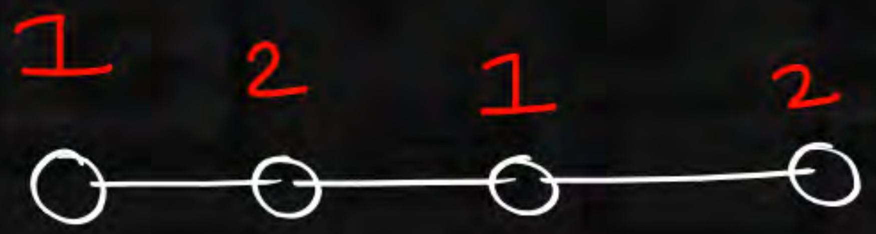
* Every even length cycle is 2-colorable (True)

* Every 2-colarable graph is even length cycle (false)

Tree :



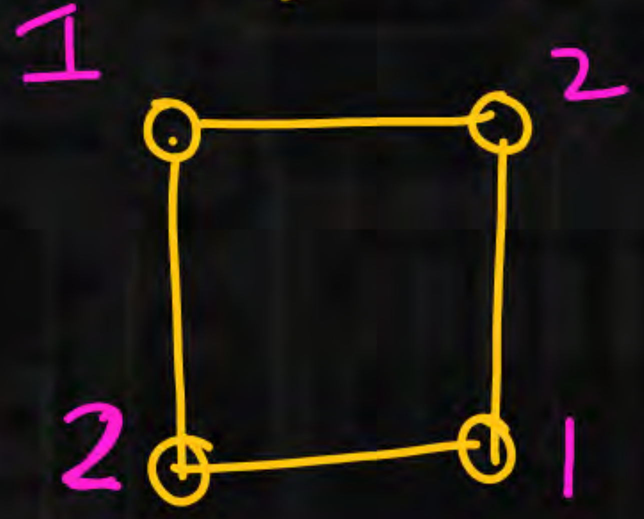
$\chi(G) = 2$



$\chi(G) = 2$

Every Tree is 2-colorable (True)

Every 2-colorable graph is Tree.
(false)



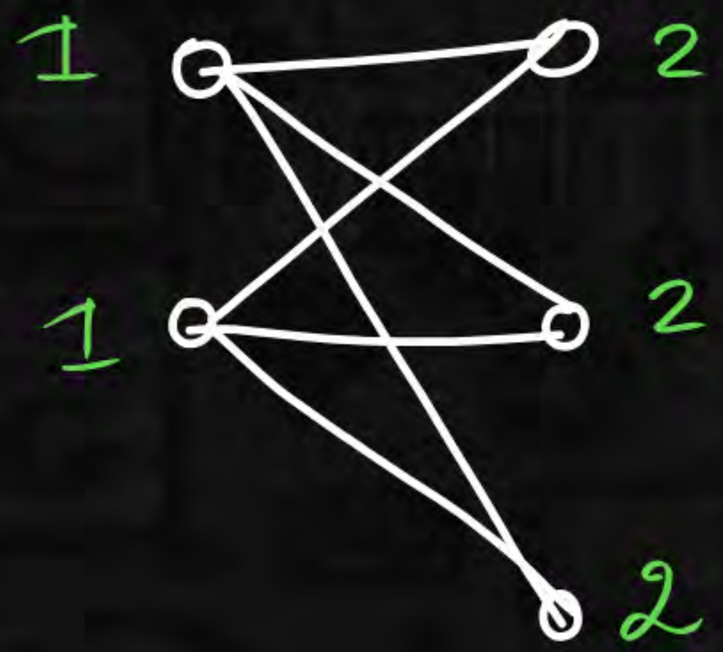
Every even length cycle is 2-colorable. (True)

Every 2-colorable is even length cycle. (false)

→ Every Tree is 2-colorable (True)

→ Every 2-colorable is Tree (false)

Bipartite Graph :



Every B.P Graph is 2-colorable.

* Every 2-colorable is B.P Graph.

  - → Tree
  - OR.
  - → Even length cycle.

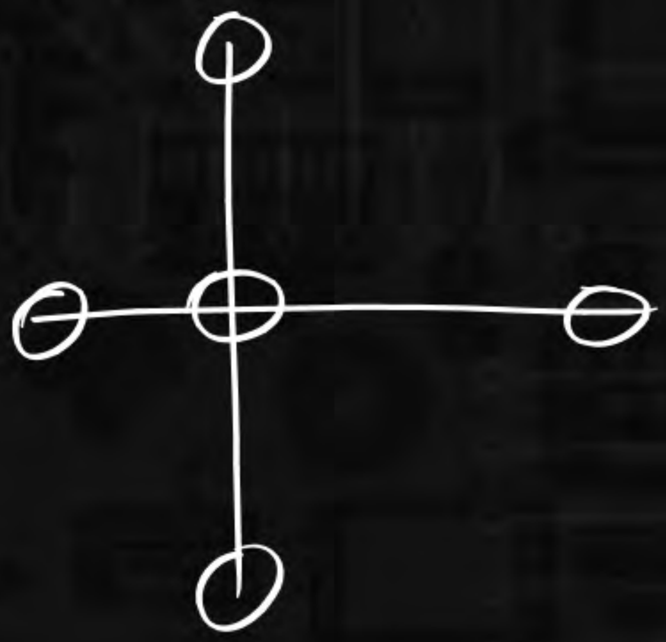
(True)

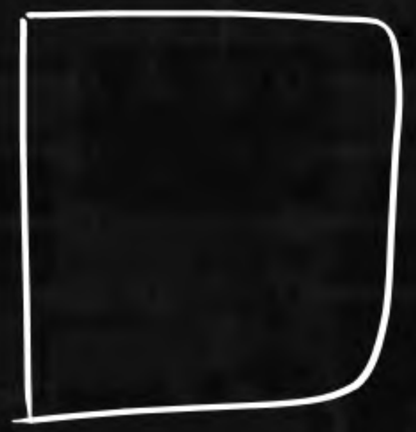
→ B.P does not contains odd length cycle

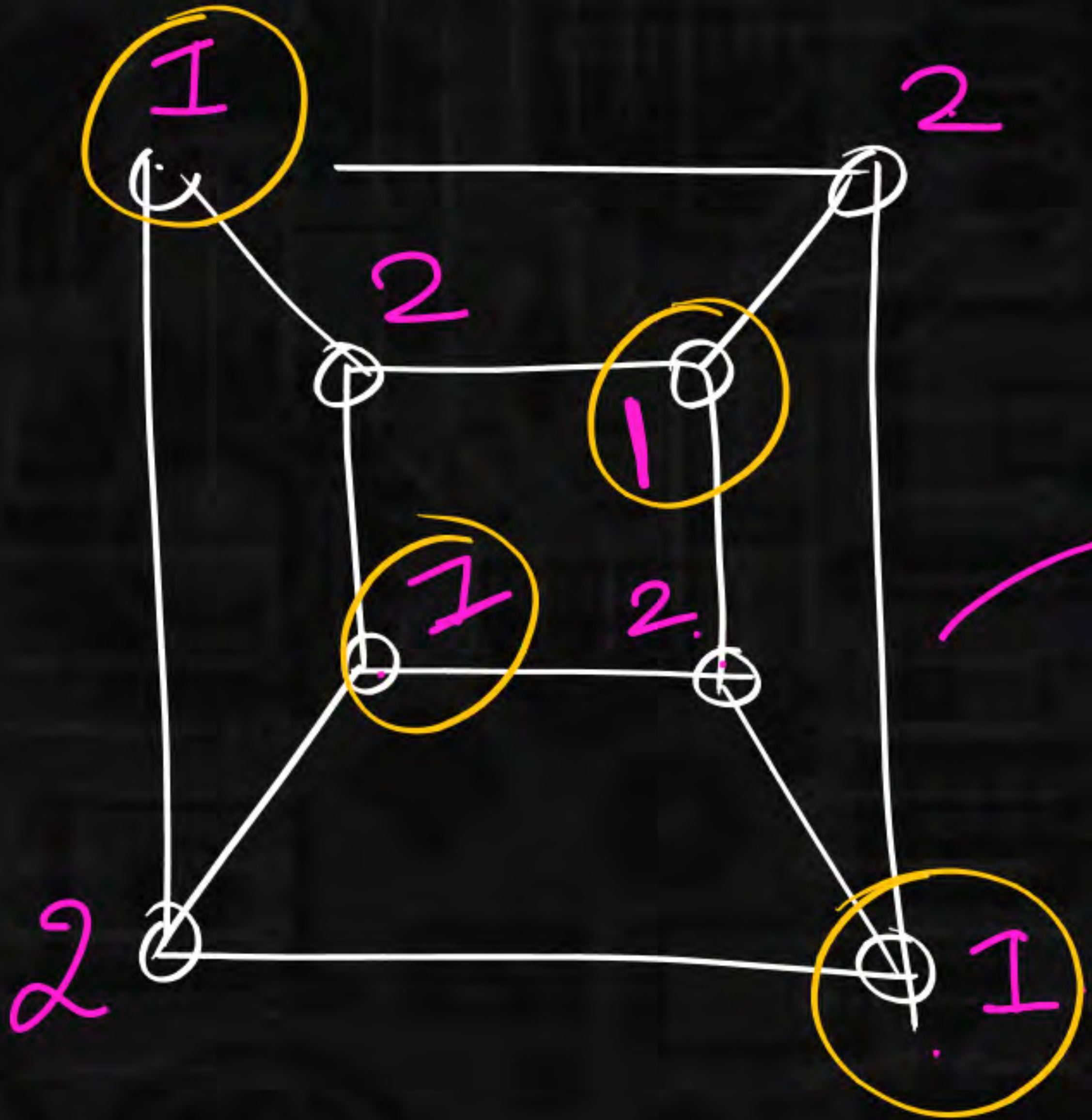
- → Tree.
- → Even length cycle.

2-colorable

↓







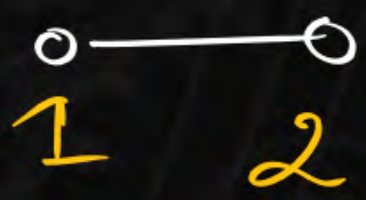
$G \longrightarrow B.P$

$\chi(G) = 2 \longrightarrow B.P$

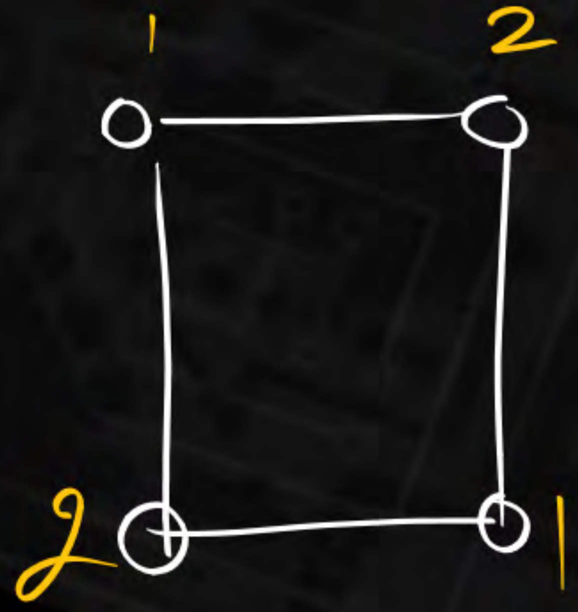
Hypecube:

$\chi(G) = 2.$

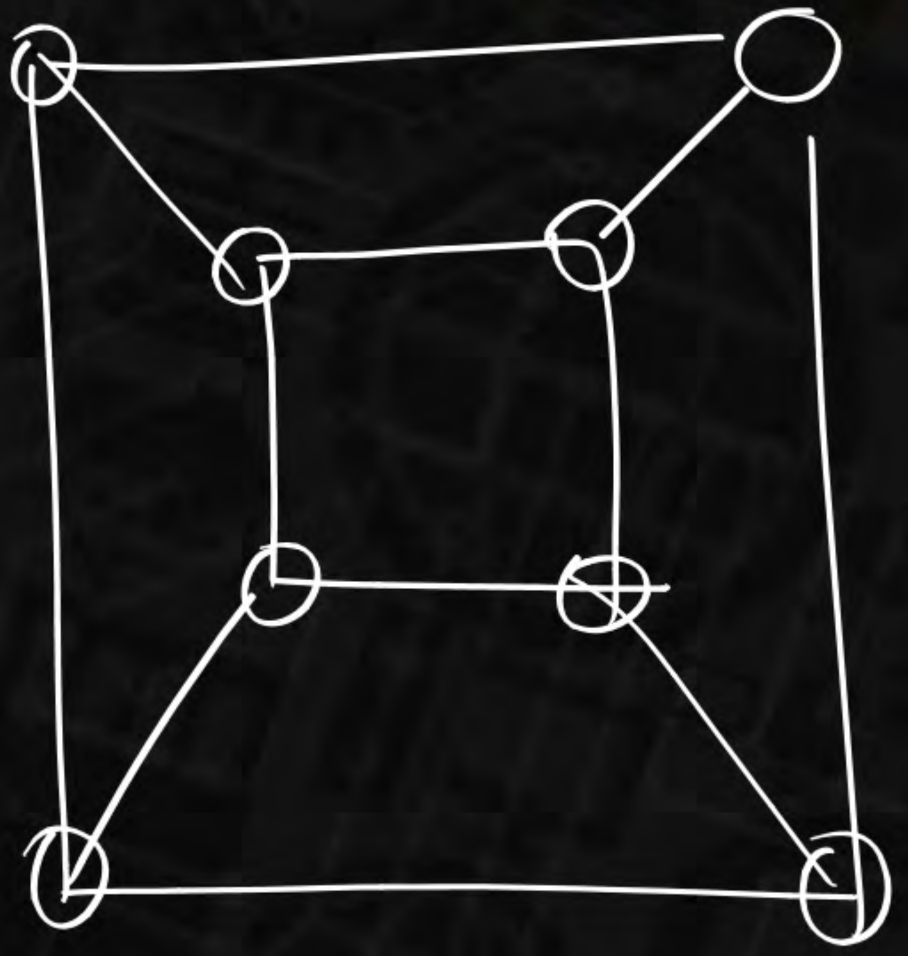
Q1



Q2



Q3



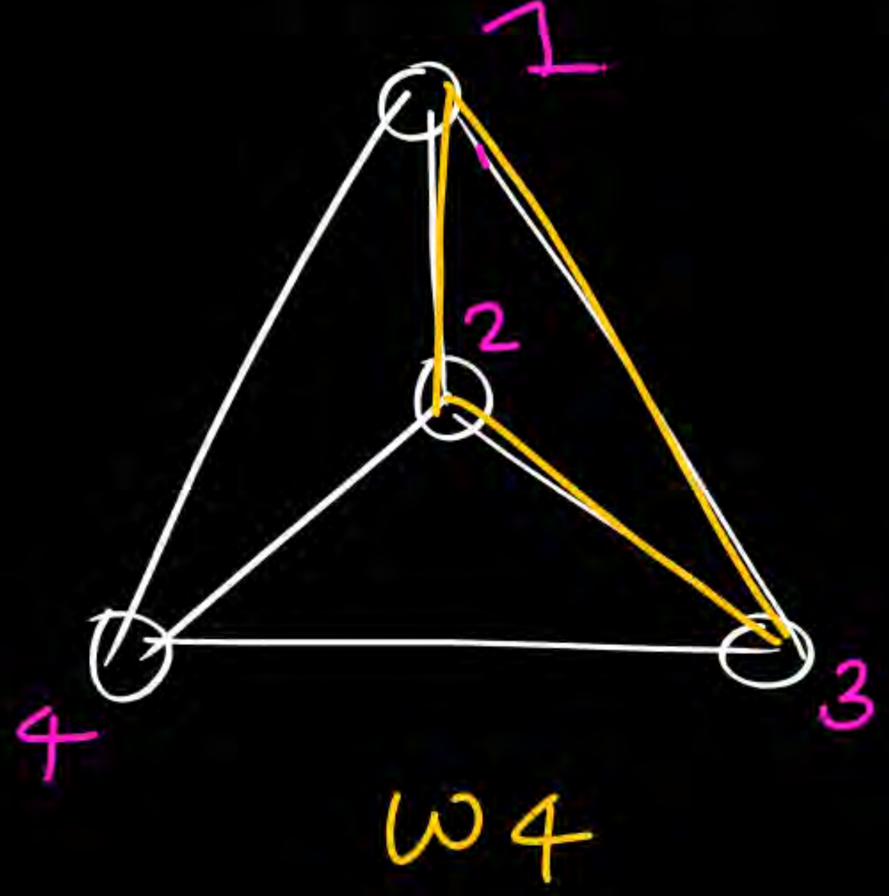
eg: Tree.

$$\begin{tikzcd} \text{Every Hypercube} \arrow[r] \arrow[rr, bend right] & \text{2 colorable} \arrow[r] \arrow[l, bend left, "/"] & \text{B.P Graph} \arrow[ll, bend right, "/"'] \arrow[l, bend right] \end{tikzcd}$$

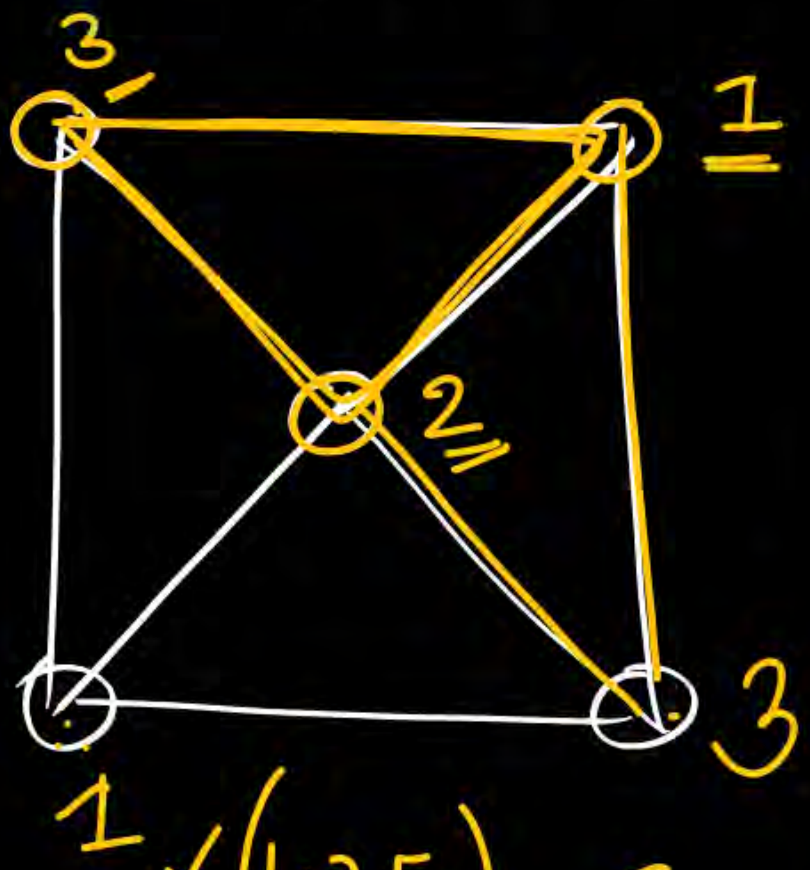
G

$\rightarrow$ B.P

Wheel Graph (W_n) ($n \geq 4$)



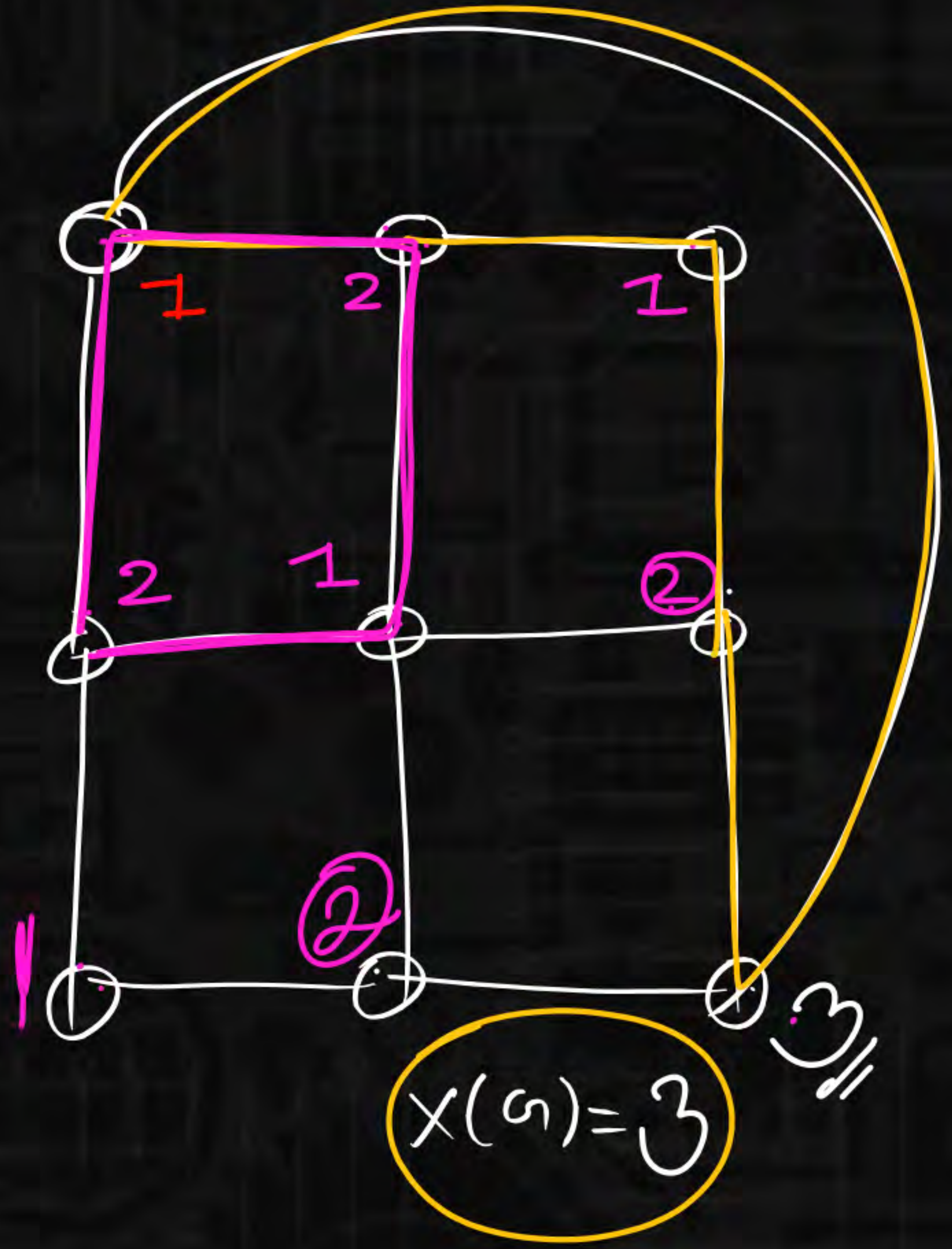
$X(W4) = 4$



$X(W5) = 3$

$X(W_n) = 3$ n is odd

$X(W_n) = 4$ n is even



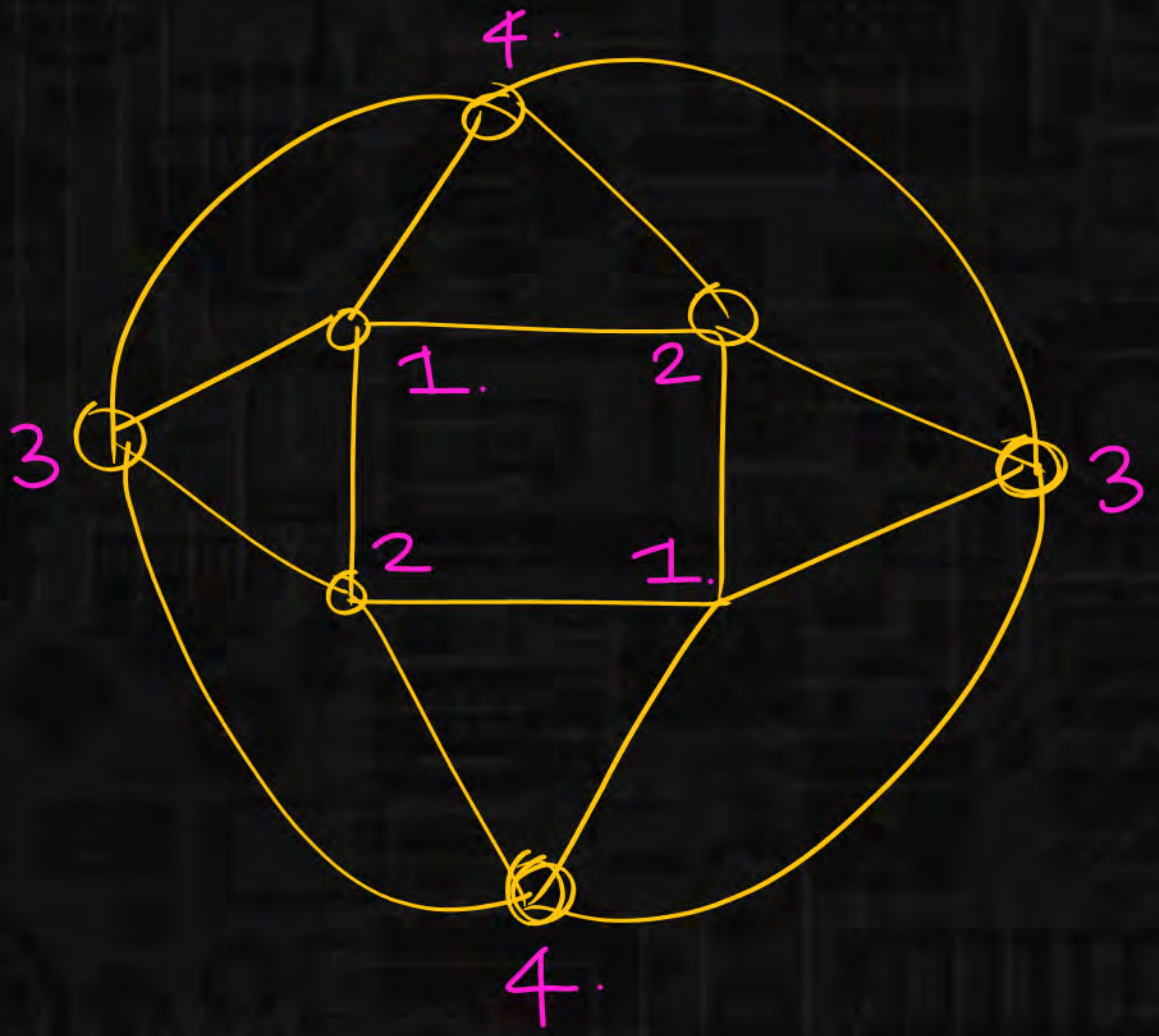
GATE.

2

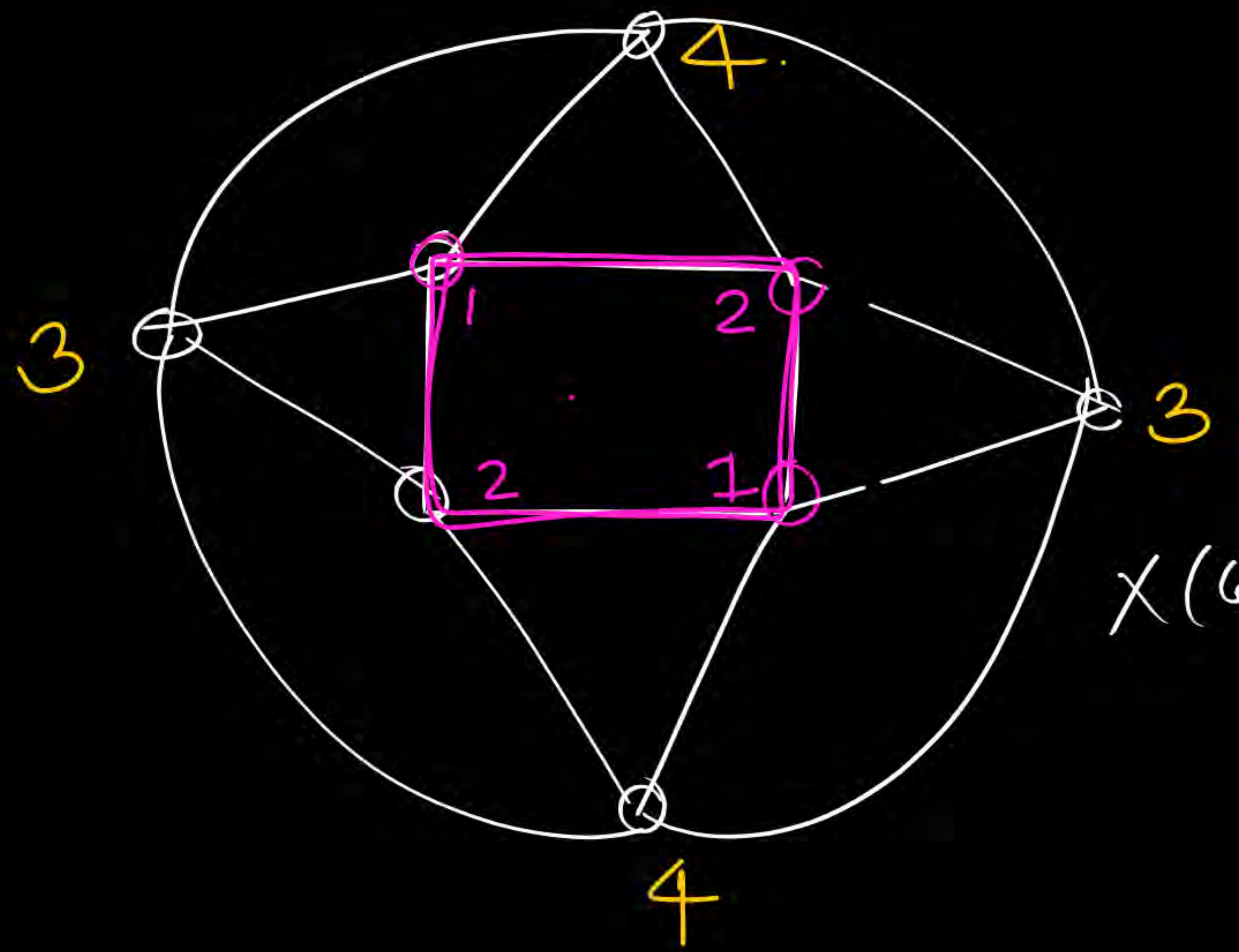
3

4

none of these.



$\chi(G) = 4$



$\chi(G) = 4$

Disconnected graph with 10 vertices & maximum edges what will be Chromatic no?

$n = 10$ D.C $(K \geq 2)$

$e = (n-K)(n-K+1)/2$

$K = 2$ $\quad K = 3$

$\downarrow$

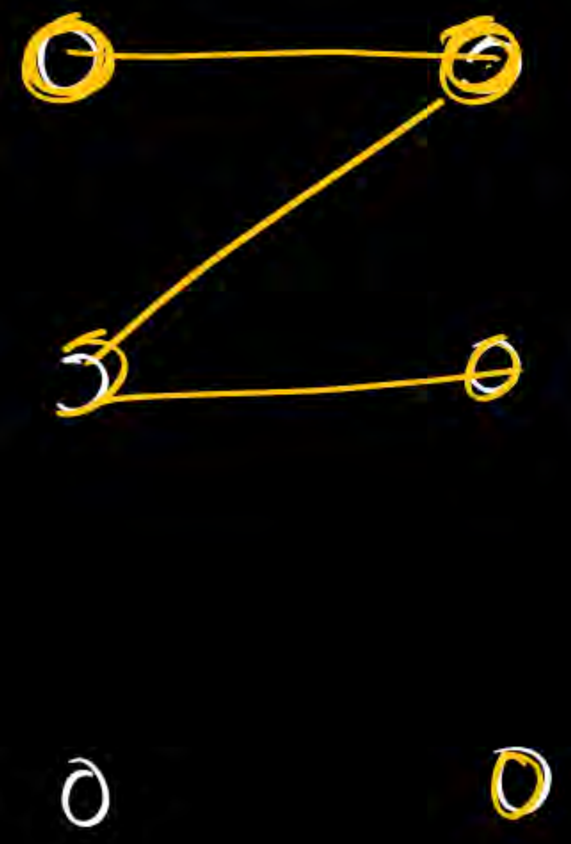
maximum edges

K_1 $\quad K_9$

$\chi(G) = 9$

$K_{n,n}$

y



x y

2 ways × 1 ways

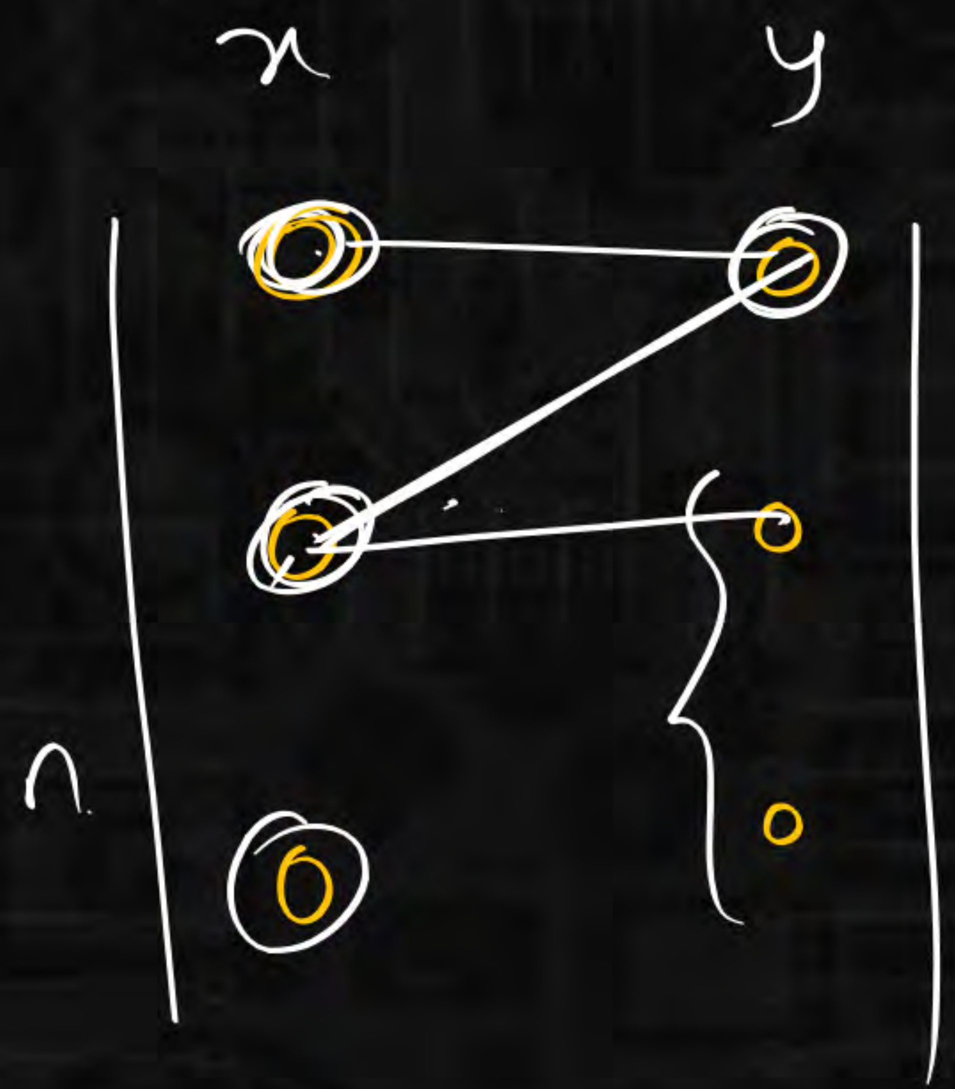
3 ways × 2 ways

$$\frac{(n-1)(n-2)\cdots}{(n-1)!}$$

$$\frac{n \times (n-1) \cdots \_\_}{n!}$$

$$\frac{n!\,(n-1)!}{2}$$

x y



$n = 3$

$$\frac{n!\,(n-1)!}{2}$$

x

2 ways ×

× 1 ways

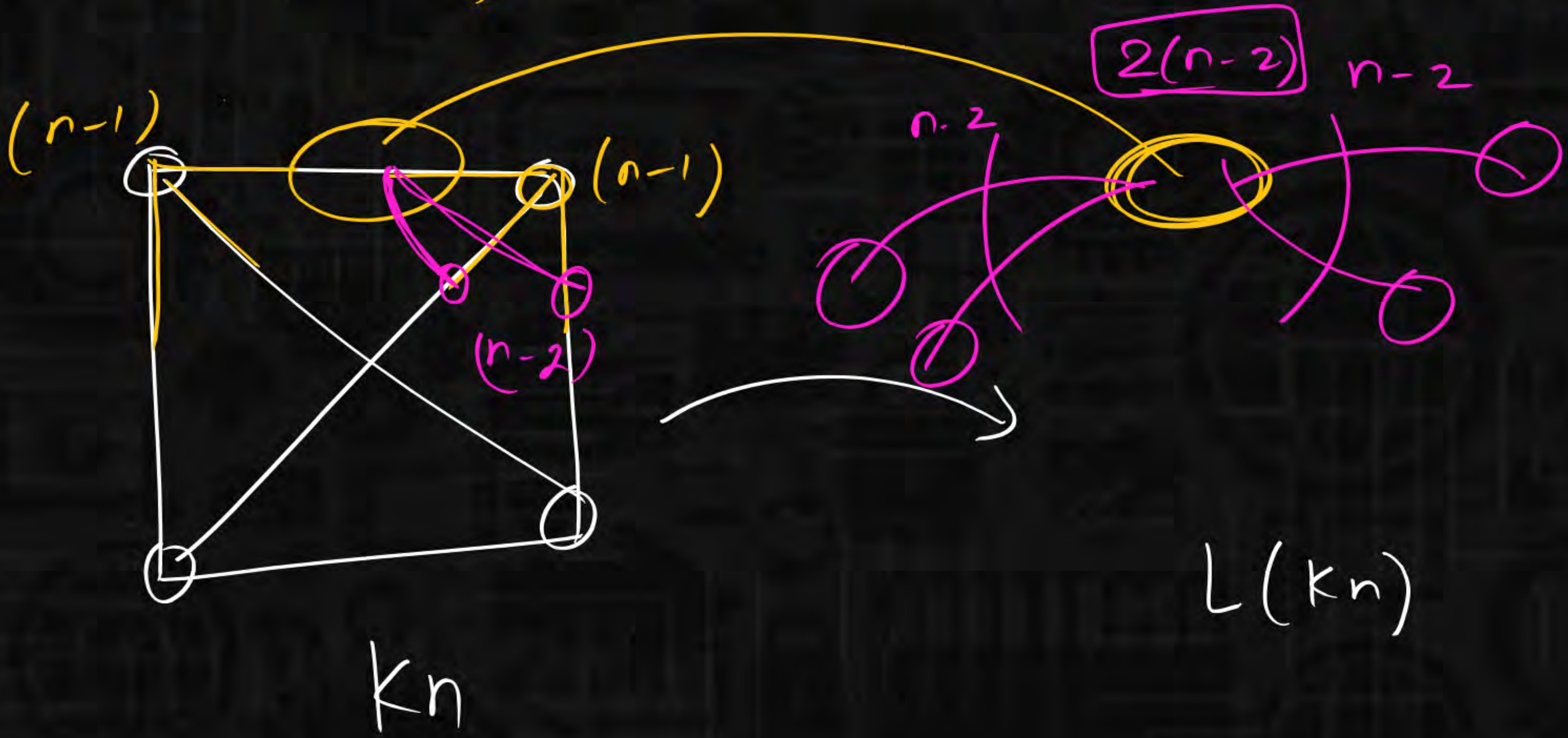
$$\frac{(n-1)(n \cdots 2}{(n-1)!}$$

y

3 ways × 2 ways

$$\frac{n \times (n-1) \ldots}{n!}$$

Degree of each vertex in L(kn) is 2(n-2)




THANK YOU GW
SOLDIERS !