CS & IT ENGINEERING

Types of Graphs Part 2

Lecture No. 4

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TOPICS TO BE COVERED

01 Bipartite graph

02 Star Graph

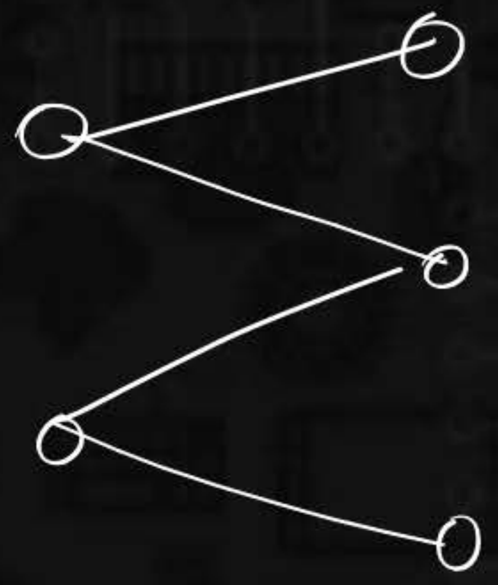
03 Line graph

04 Complement Graph

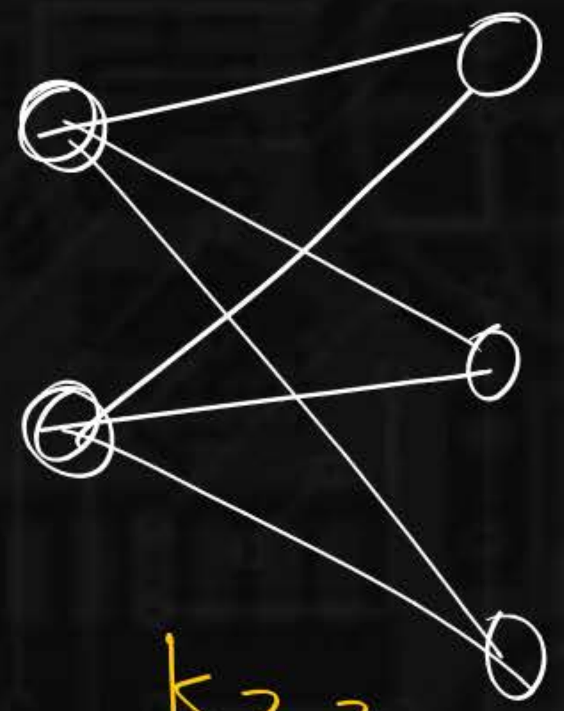
05 Isomorphic Graph

Types of graph

Bipartite Graph:



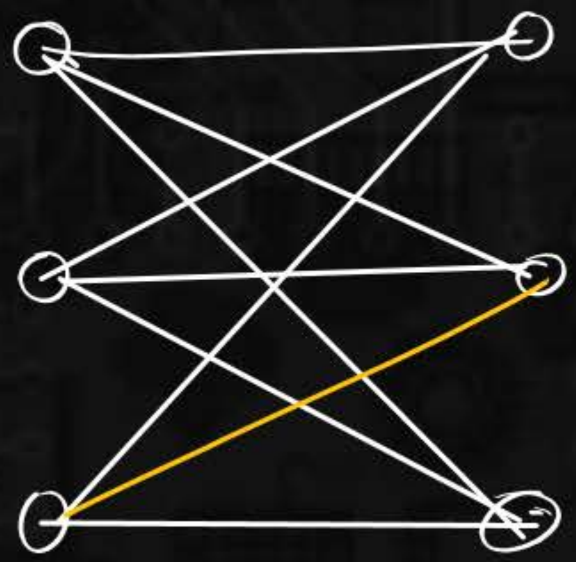
Complete bipartite Graph. $(K_{m,n})$



$K_{2,3}$.

Types of graph

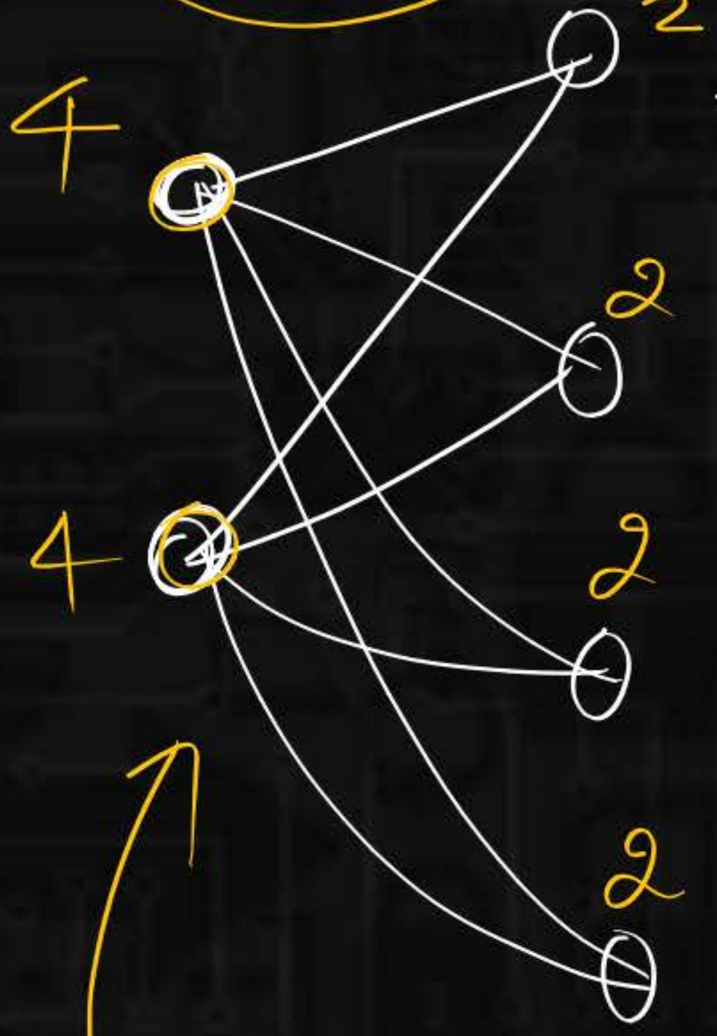
$K_{3,3}$



$e = 3 \times 3$

$V = 3 + 3$

$K_{2,4}$



Total vertices $= 2 + 4$

Total edges $= 2 \times 4$

$\Delta(K_{2,4}) = 4$

$\delta(K_{2,4}) = 2$

$K_{m,n}$

Total vertices $= m + n$

Total edges $= m \times n$

$\Delta(K_{m,n}) = \max(m,n)$

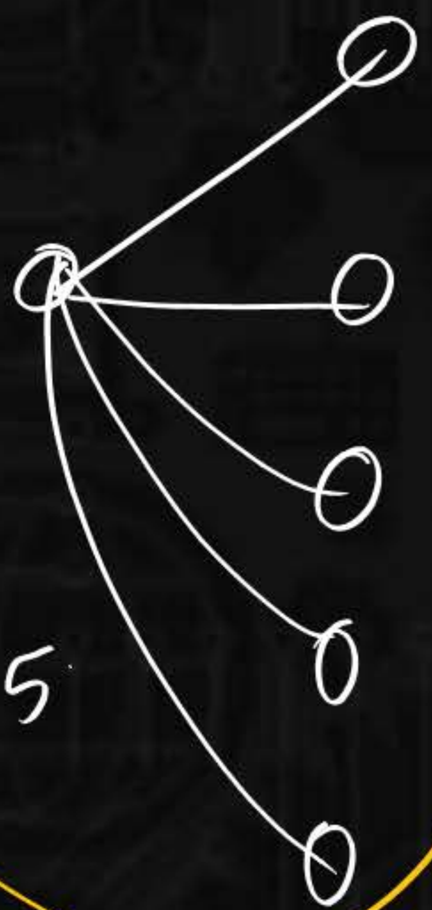
$\delta(K_{m,n}) = \min(m,n)$

Types of graph

Total vertices = 6

Case 1: $e = 5$

1 5

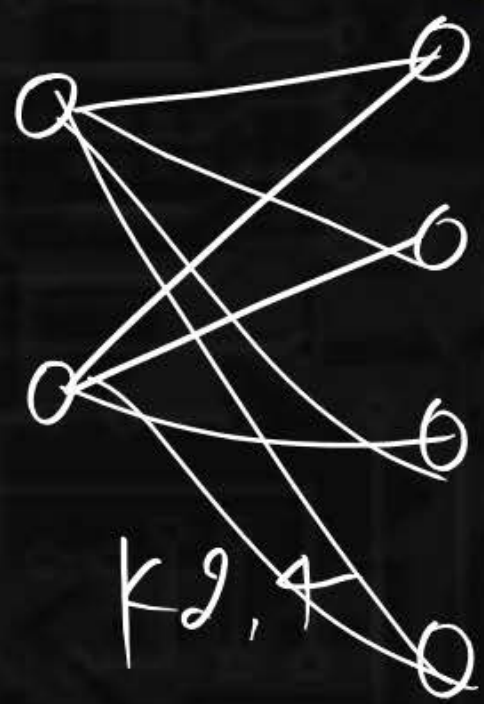


$K_{1,5}$

Case 2:

2 4

$e = 8$

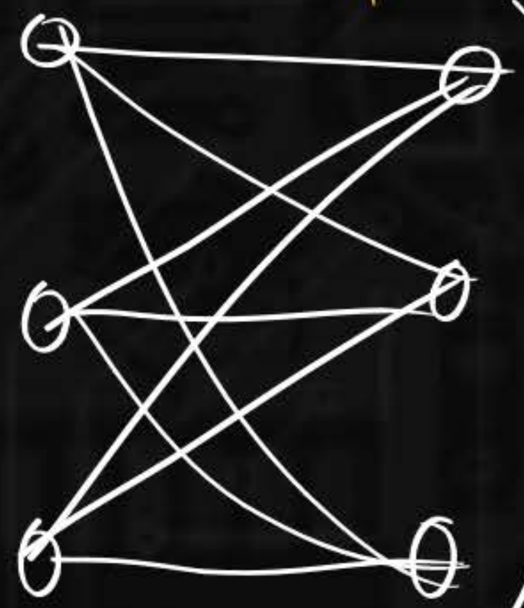


$K_{2,4}$

Case 3:

3 3

$e = 9$



$K_{3,3}$

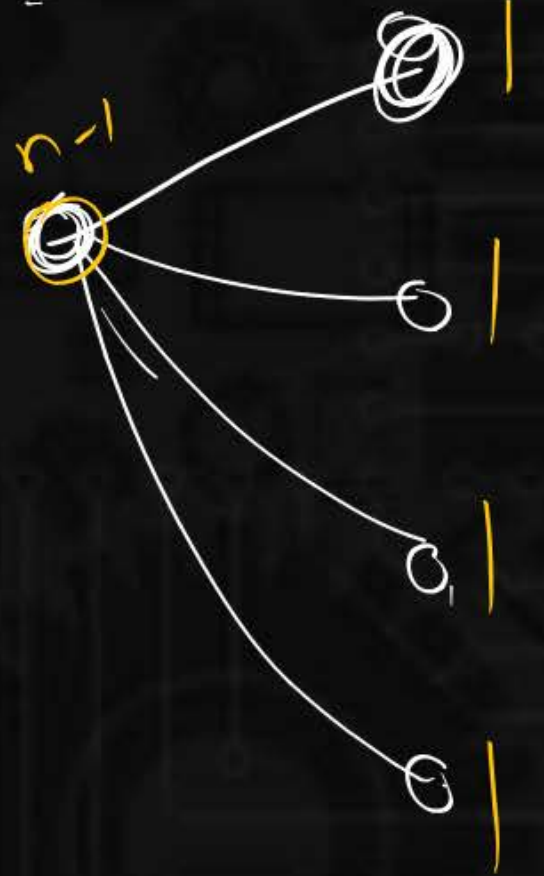
Types of graph

Star Graph : $(K_{1,n-1})$

$n = 5$

Draw star Graph.

$K_{1,4}$



Total vertices
$= 1 + n - 1 = n$.

Total edges $= (n-1)$

$\Delta(K_{1,n-1}) = n-1$.

$\delta(K_{1,n-1}) = 1$.

Bipartite Graph $n = 6$

what will be minimum no. of edges it will have

bipartite + min no. of edges $e = 5$

Star Graph $K_{1,5}$

Types of graph

bipartite Graph having $n = 6$, what will be maximum no. of edges?

Types of graph

$\underline{\underline{n=6}}$. B.P + maximum edges.

1 5

$K_{1,5}$

$e = 5$

2 4

$K_{2,4}$

$e = 8$

$e = 9$

3 3

$K_{3,3}$

$e = 9$

Types of graph

$$\boxed{K_{m,n} \quad e = m \times n}$$

B.P + maximum
(n vertices) edges

$K_{n/2,\, n/2}$

Types of graph

$n = 6$

$K_{3,3}$

$e = 9$

$n = 7$

$K_{1,6}$

$K_{2,5}$

$K_{3,4}$

$K_{4,3}$

$e = 12$

n

$\frac{n}{2} \times \frac{n}{2}$

$\left\lfloor \frac{n^2}{4} \right\rfloor$

Types of graph

* no. of edges in bipartite Graph $\leq \left\lfloor \frac{n^2}{4} \right\rfloor$

* $\boxed{\text{Bipartite graph}}$ + maximum no. of edges

  Total vertices = n $e = \left\lfloor \frac{n^2}{4} \right\rfloor$

* $K_{m,n}$ $e = m \times n$.

Types of graph

if graph contain more than $\frac{n^2}{4}$ edges

it contains triangle.

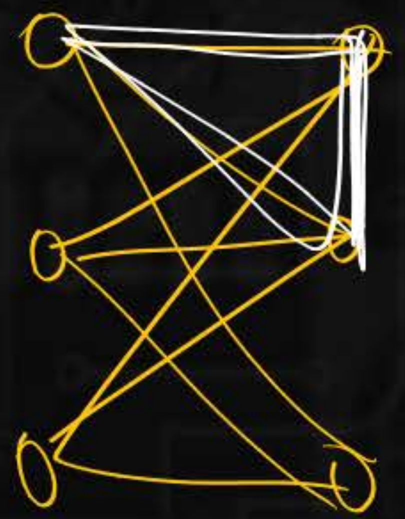
$n = 6 \quad K_{3,3}$

$e = 9$

B P $n = 6 \quad e = 9 + 1$

$K_{3,3}$

$K_{3,3}$



it is not B.P Graph

Types of graph

if Graph contains 2n vertices, it contains more than n^2 edges then it contains Triangle.

Types of graph

1.) B.P does not contain odd length cycle.

2.) In bipartite graph no. of edges $\leq \left\lfloor \frac{n^2}{4} \right\rfloor$ *

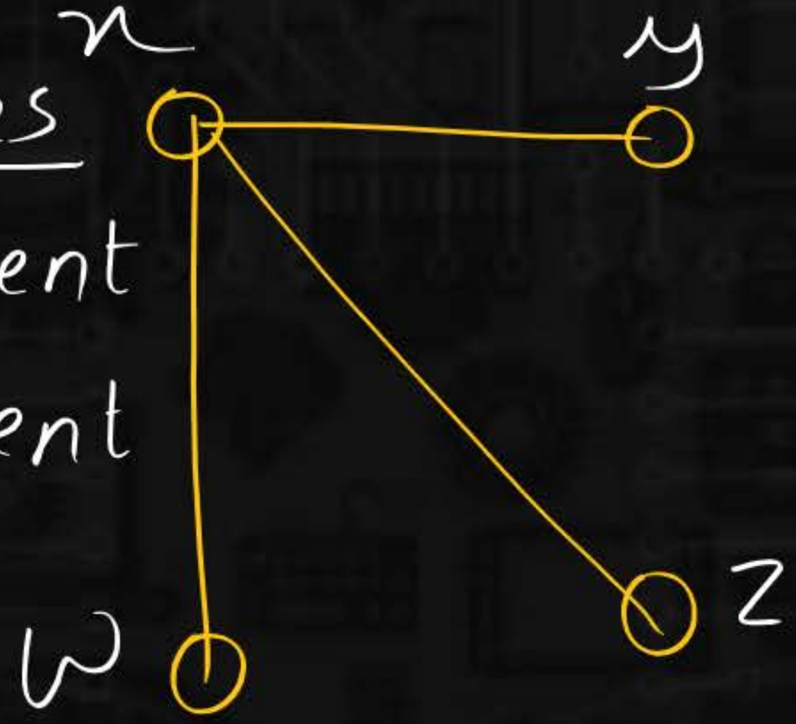
3) In Graph having n vertices & more than $n^2/4$ edges it contains triangle.

Types of graph

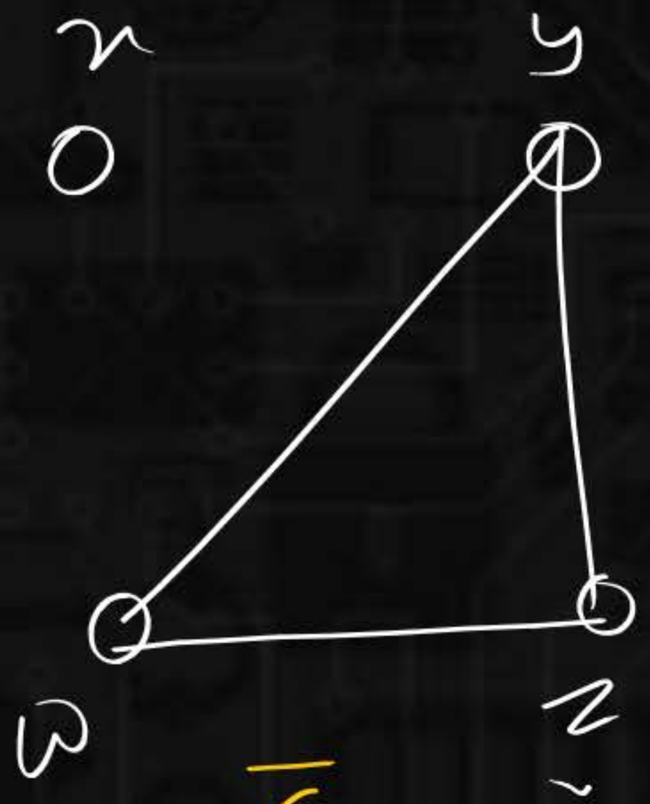
$G + \bar{G} = K_4$

Complement Graph : ($\bar{G}$)

edges
present
absent



∴ G



$\bar{G}$

edges
absent
present

Types of graph

1. $G + \bar{G} = K_n$

2. $e(G) + e(\bar{G}) = \frac{n(n-1)}{2}$

Consider a B.P of having 6 vertices & 9 edges what will be total edges in the complement of this graph?

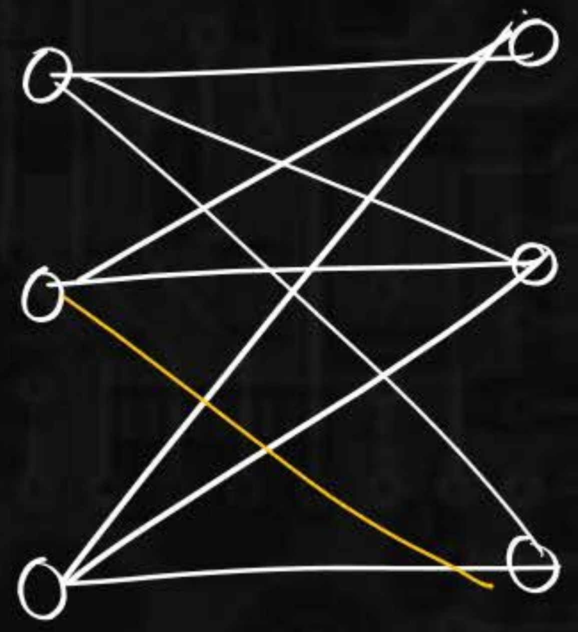
G $n = 6$, $e = 9$

$e(G) + e(\bar{G}) = \frac{n(n-1)}{2}$

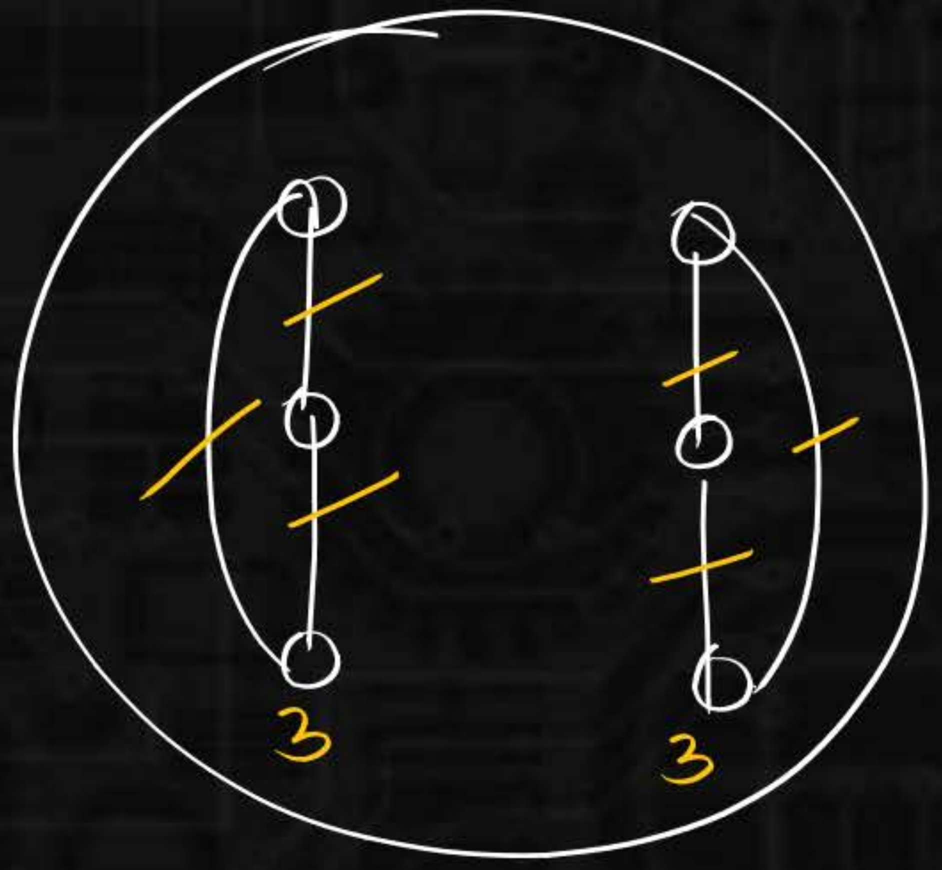
$9 + x = \frac{\cancel{6}^{3} \times 5}{\cancel{2}}$

$x = 15 - 9$
$= 6$

Types of graph



$K_{3,3}$

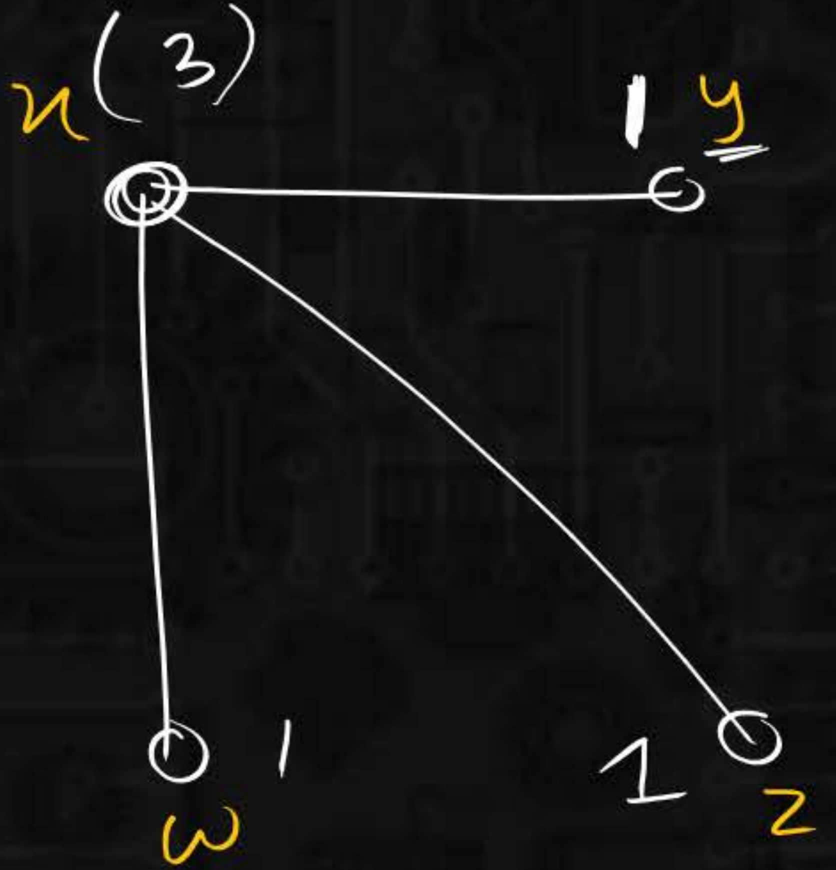


$e = 6$

$\overline{K_{3,3}}$

$$e(G) + e(\overline{G}) = \frac{n(n-1)}{2}$$

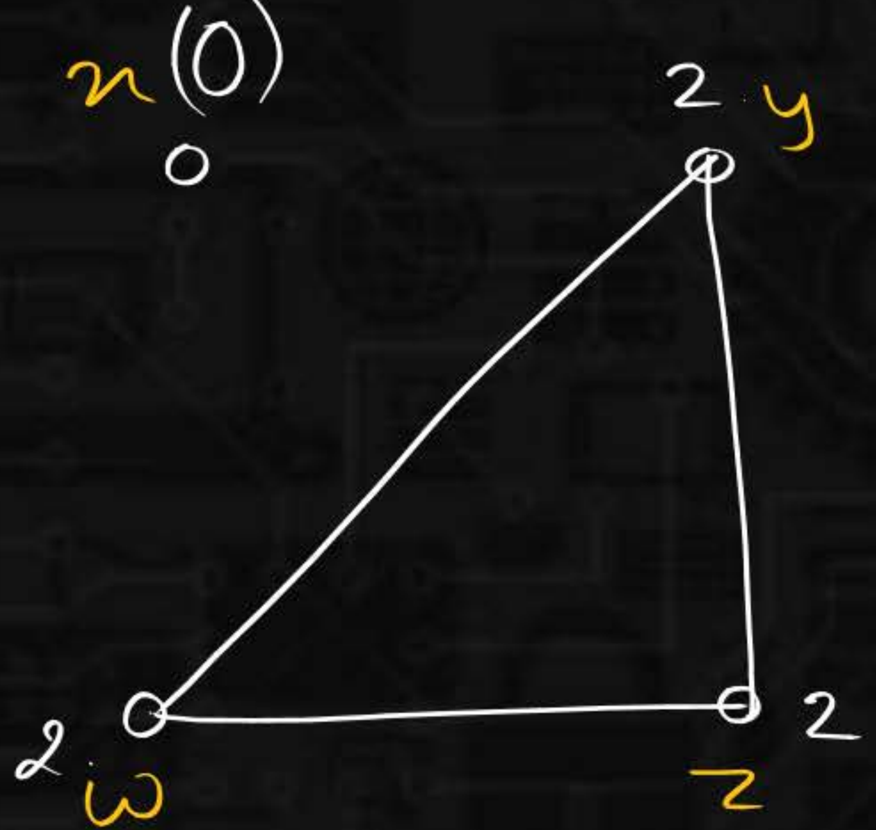
Types of graph



K_4 3 3 3 3

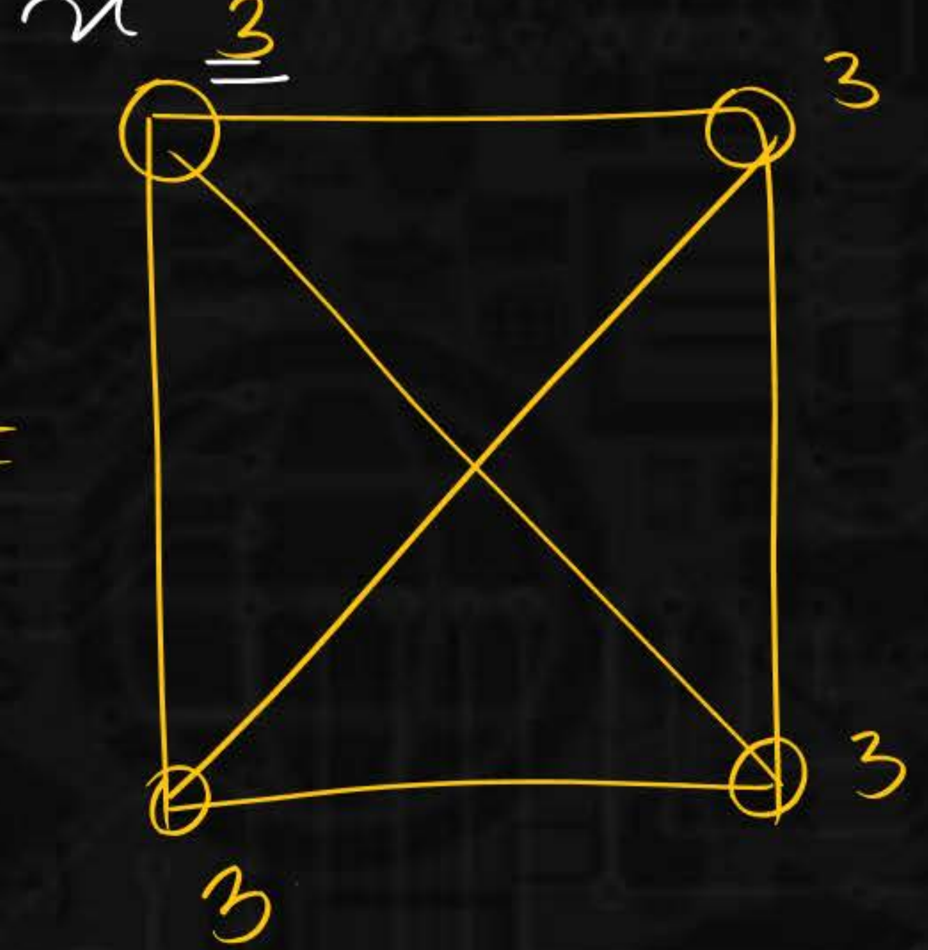
G 3 1 1 1

$\overline{G}$ 0 2 2 2



0 2 2 2

=



Types of graph

Consider a Graph having degrees 5 2 2 2 2 1.

What will be degree sequence of $\overline{G}$, what will be edges in $\overline{G}$?

K_6	5	5	5	5	5	5
G	5	2	2	2	2	1
$\rightarrow \overline{G}$	0	3	3	3	3	4

$\sum d(v_i) = 2e$

$16 = 2e$

$e = 8$

Types of graph

K_n $n-1, n-1, n-1, \ldots n-1$

G $d_1, d_2, d_3 \ldots d_n$

$\overline{G}$ $\underline{n-1-d_1}, \underline{n-1-d_2}, n-1-d_3 \ldots n-1-d_n$

Types of graph

Consider star Graph of 6 vertices, what will be total edges in the complement of this graph?

$$e(K_{1,5}) + e(\overline{G}) = \frac{n(n-1)}{2}$$

$$e(\overline{G}) = 15 - 5$$

$$= 10.$$

Types of graph

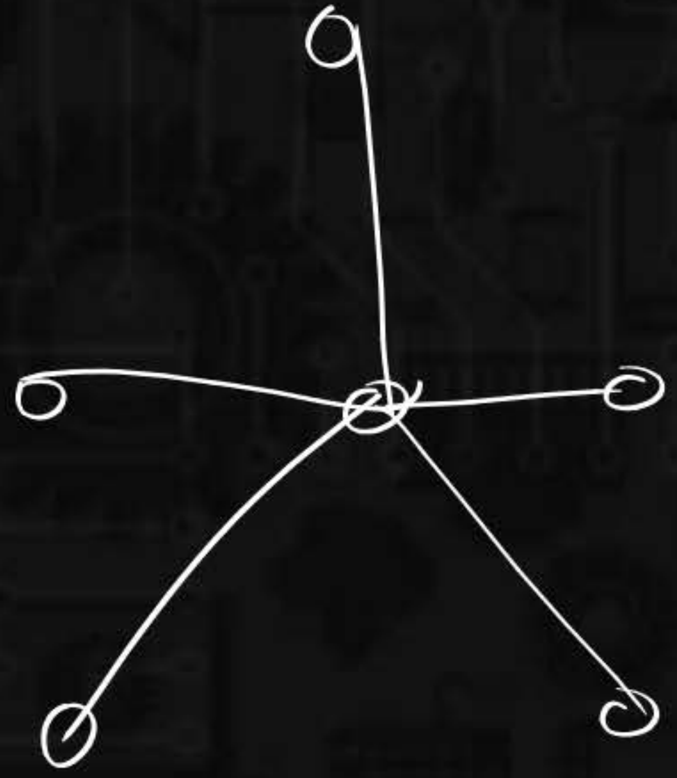
$e(K_{1,5}) = 5 \quad n = 6$

$e(G) + e(\bar{G}) = \frac{n(n-1)}{2}$

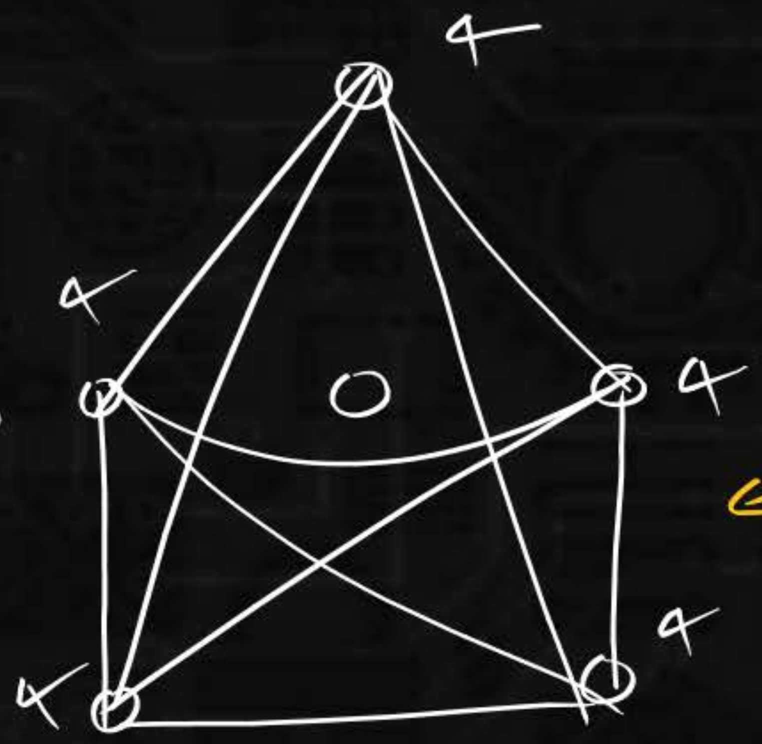
$5 + x = \frac{6 \times 5}{2}$

$x = 15 - 5$

$= 10$



$K_{1,5}$



$e = 10$

K_1, K_5

Types of graph

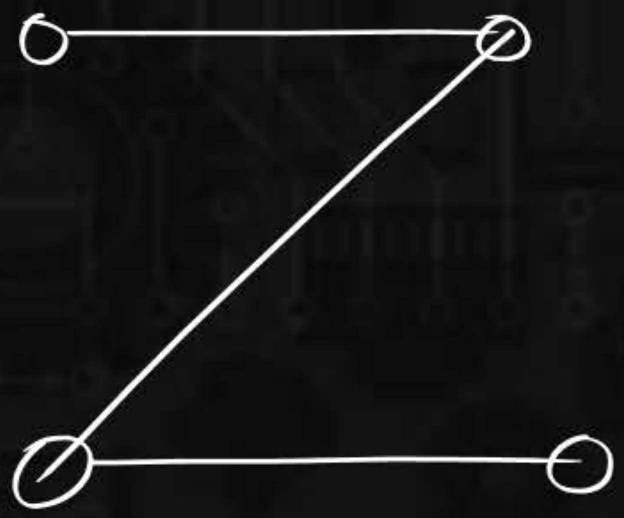
<u>note</u> : if we take complement of $K_{1,n-1}$.

then we will get

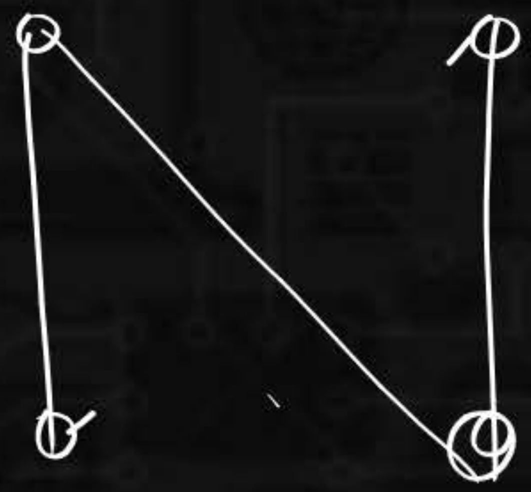
K_1 and complete Graph of $(n-1)$ vertices.

Types of graph

G



$\overline{G}$



$G \equiv \overline{G}$ (Self-complement Graph)

$$e(G) + e(\overline{G}) = \frac{n(n-1)}{2}$$

$$e + e = \frac{n(n-1)}{2}$$

Types of graph

$$e + e = \frac{n(n-1)}{2}$$

$$2e = \frac{n(n-1)}{2}$$

$$e = \frac{n(n-1)}{4}$$

$n = 4 \quad e = \frac{4 \cdot 3}{4} = 3$

$n = 5 \quad e = \frac{5 \cdot 4}{4} = 5$

$n = 6 \quad e = \frac{6 \cdot 5}{4} = \frac{15}{2} \quad (\times)$

$n = 7 \quad e = \frac{7 \cdot 6}{4} = \text{—} \quad (\times)$

Types of graph

* Self-complement Graph.
  it is possible when

$\frac{n}{4}$ OR $\frac{n-1}{4}$

$\frac{n-0}{4}$ OR $\frac{n-1}{4}$

$n \equiv 0 \pmod 4$ OR $n \equiv 1 \pmod 4$

***

$$\boxed{n \equiv 0 \text{ OR } 1 \pmod 4}$$

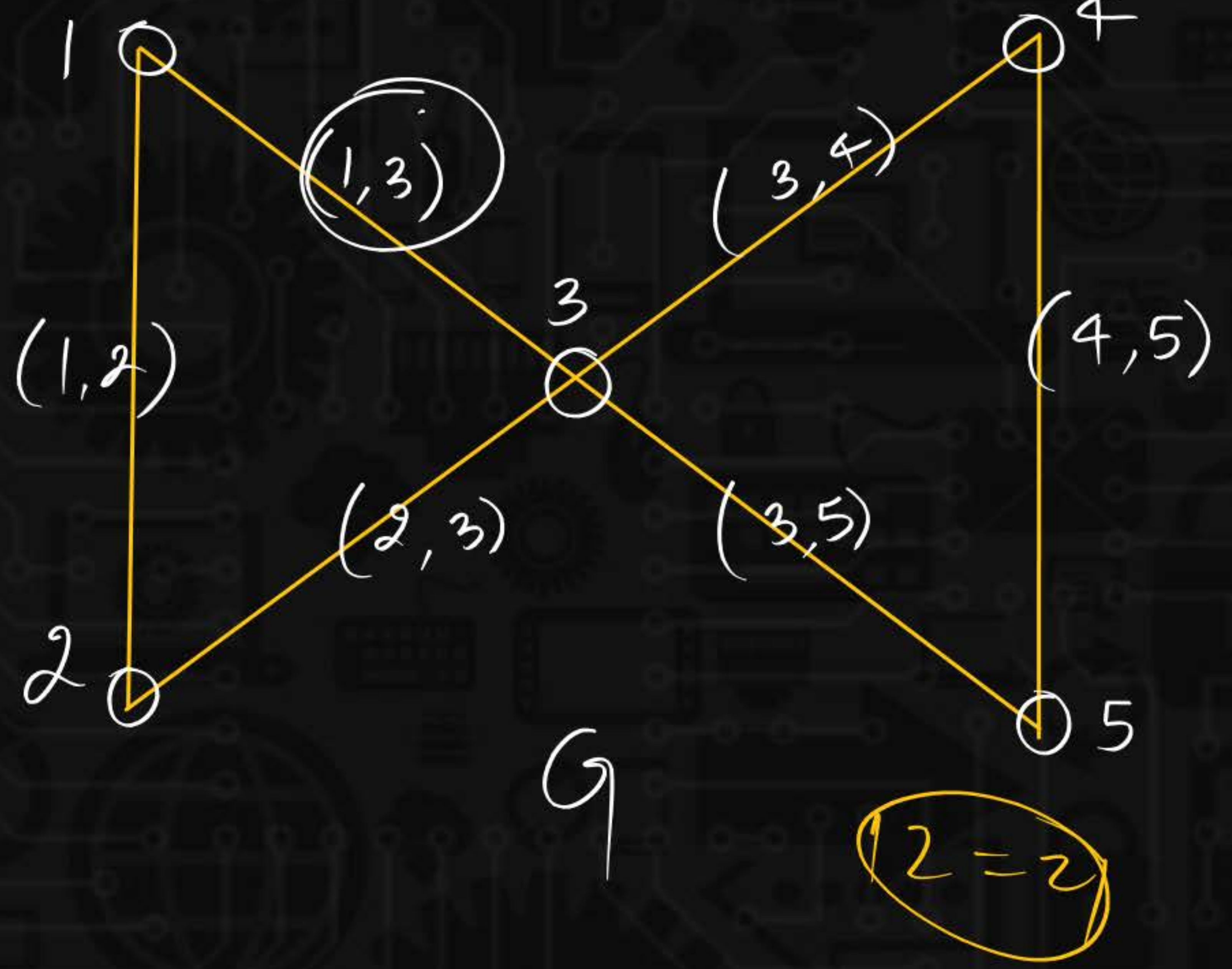
$a \quad b$

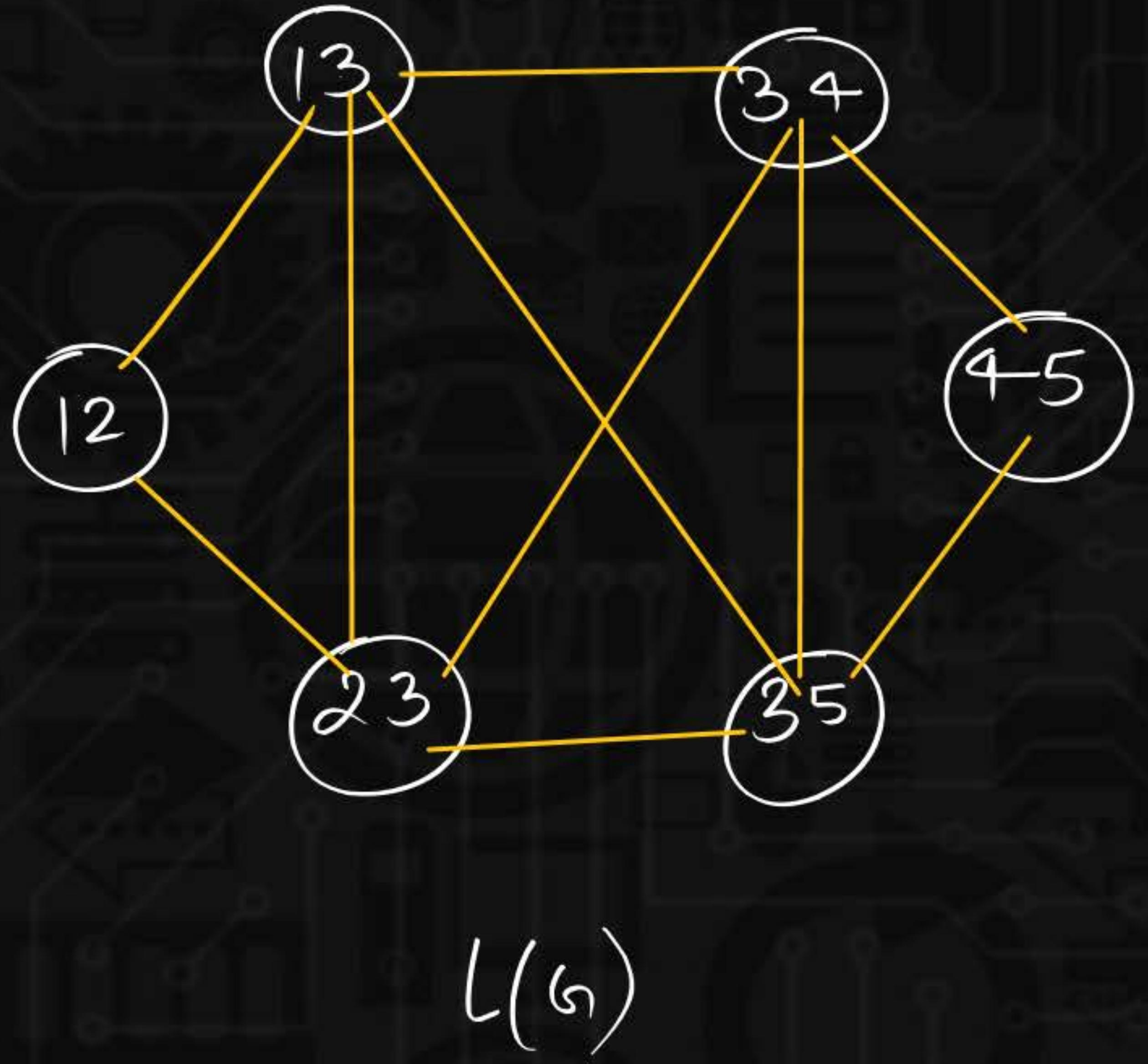
1. $a \equiv b \pmod 4$ $\quad 1 \equiv 5 \pmod 4$

2. a, b are having same remainder w.r.t 4

3. $\frac{a-b}{n} \in Z = a \equiv b \pmod n$

Line Graph (L(G))






THANK YOU GW
SOLDIERS !