

# CS & IT ENGINEERING

## Operating Systems

### Memory Management



**Lecture No. 8**



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TOPICS TO BE  
COVERED

**Multi-Level Paging**

**Segmentation**

**Problem Solving**



$1K + 3K = \underline{4K}$       Multi-Level Paging / Hierarchical Paging / Tree-level

$L.A = 32 \text{ bits}; P.S = 4 \text{ KB}; N = 1M$

Im Pager VAS = 4GB

P.A.S

## 2-level Paging

2<sup>nd</sup> level

$$P = 4B$$

PS

P.T.



1K

24

13

Po

91

$P_2$

$P_3$

$P_x$

Py

Pr

۲۰

۱۱

 $f_2$ 

१५

fy



151



$P_x$

Py

P1

Po

Pr

Pick

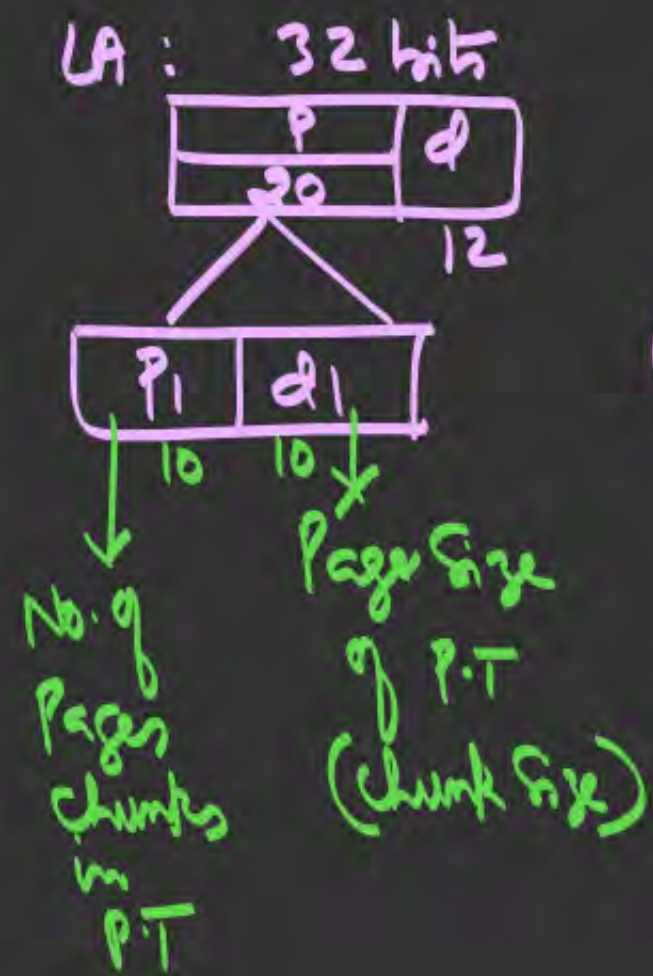
②

$P_3$

$$\begin{array}{r} 4KB \\ \underline{4B} \\ = 1K \end{array}$$

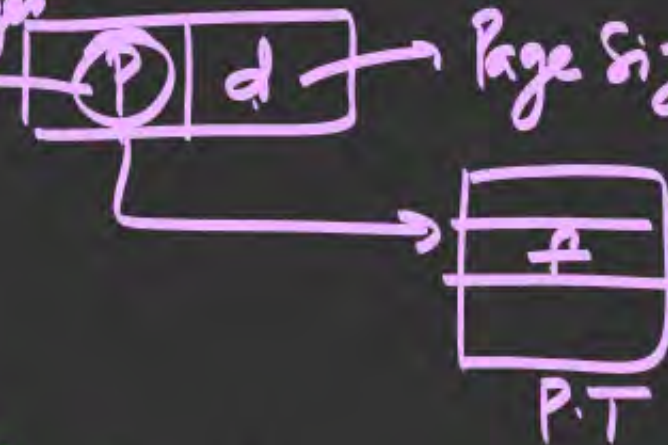
10





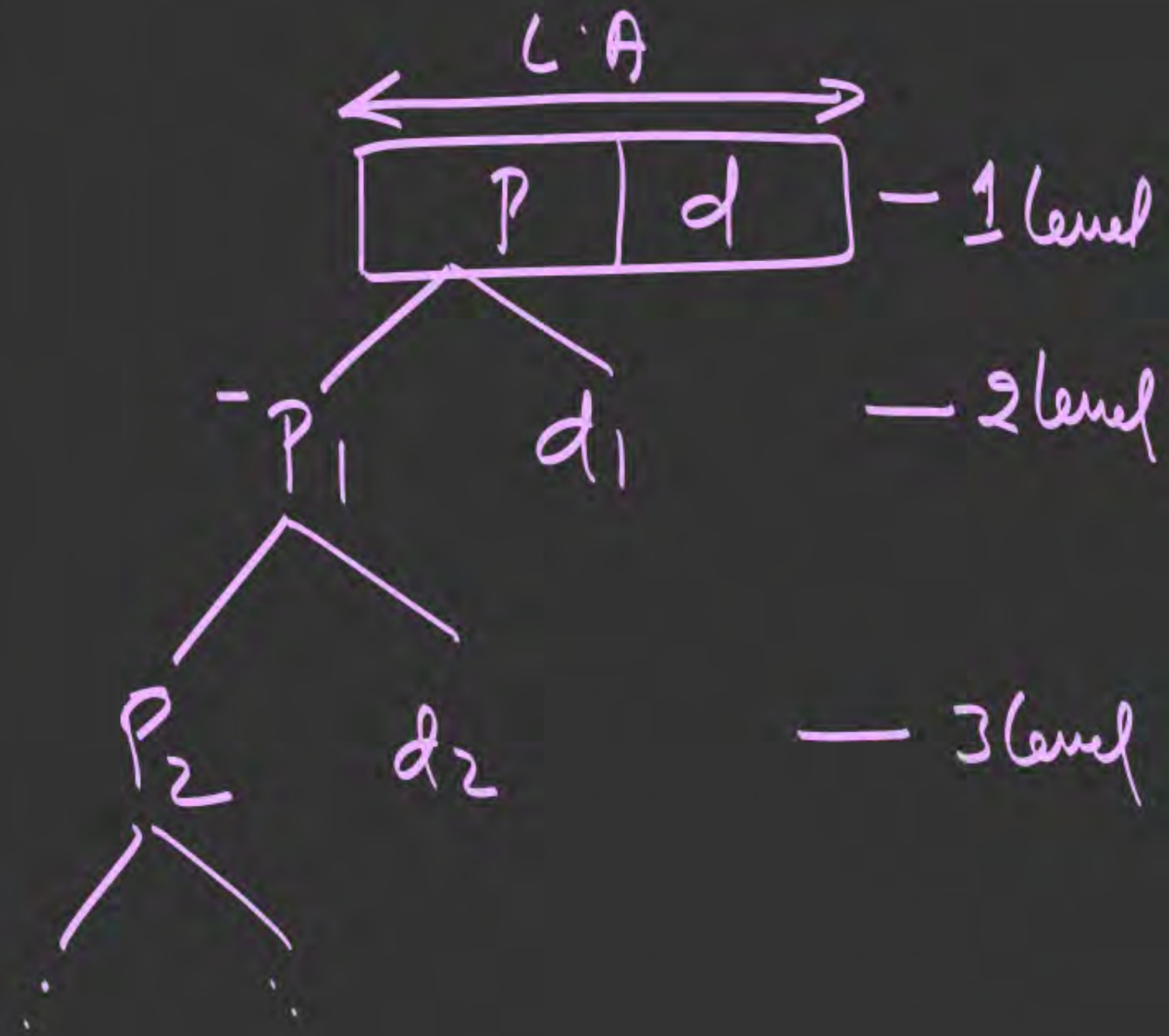
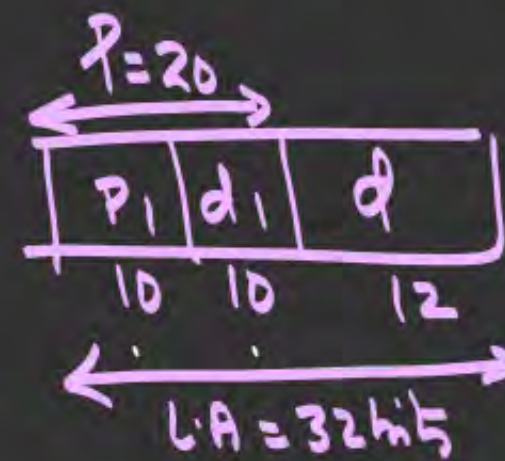
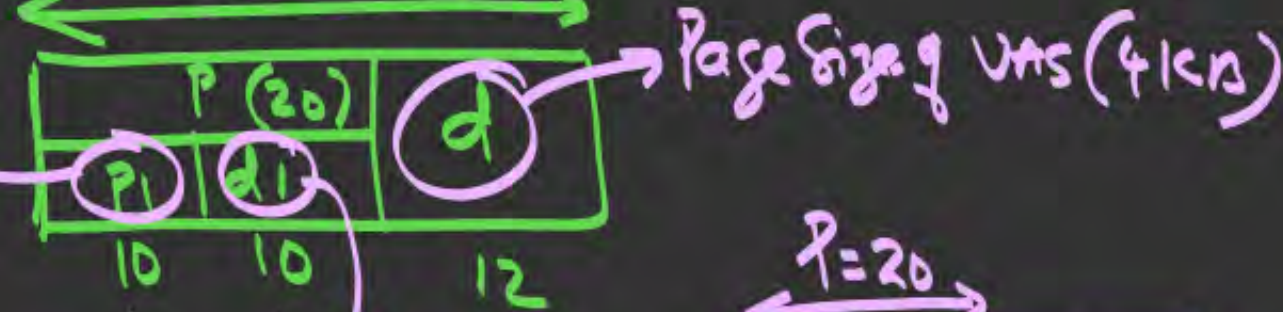
P.S = 4KB

No. of Pages



No. of Pages (chunks)  
in  
I.P.T  
= No. of entries in  
O.P.T

LA: 32 bits

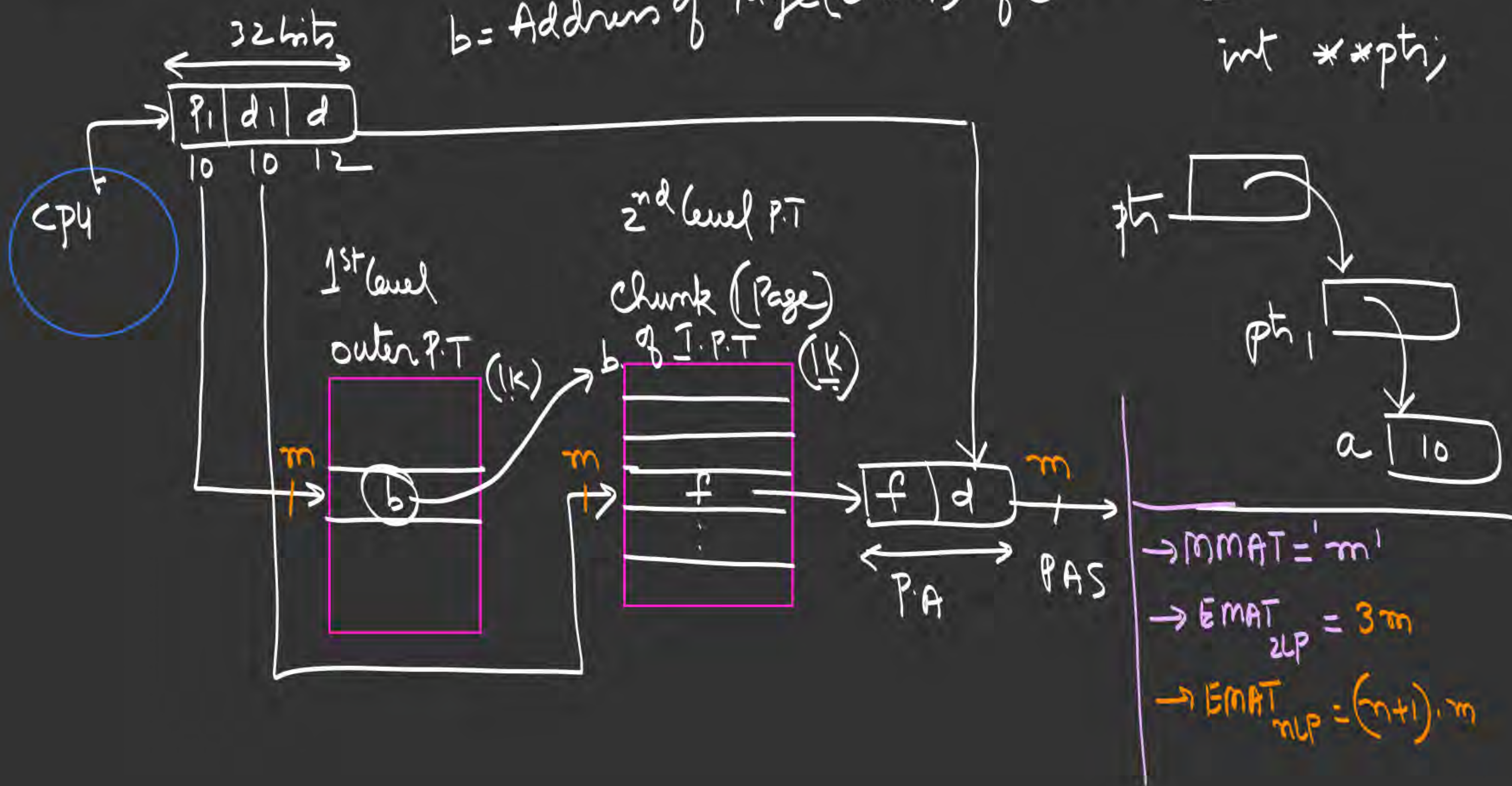




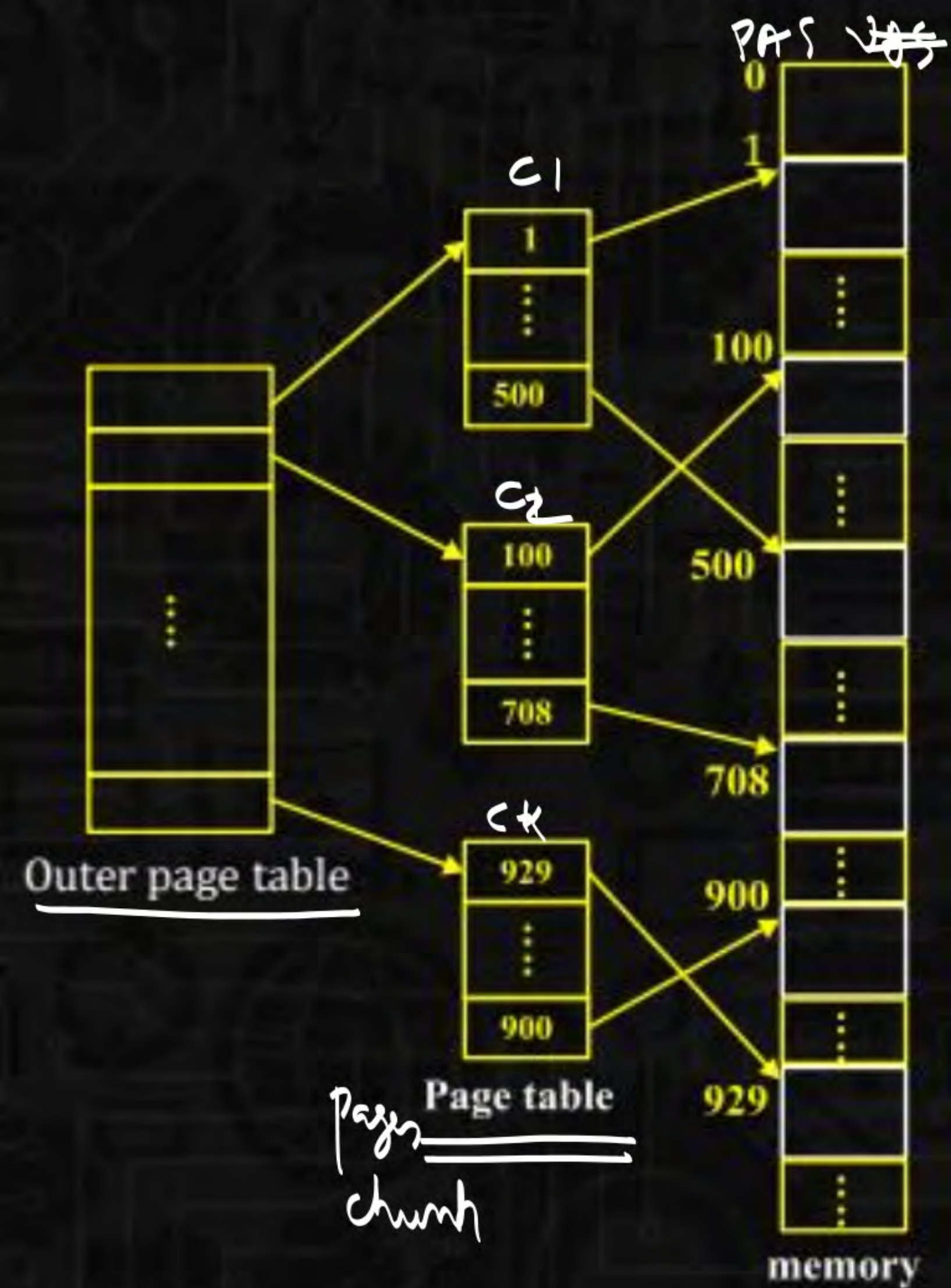
LA: 32 bits; P.S = 4KB;

$b = \text{Address of Page (chunk) of I.P.T}$  Perf.

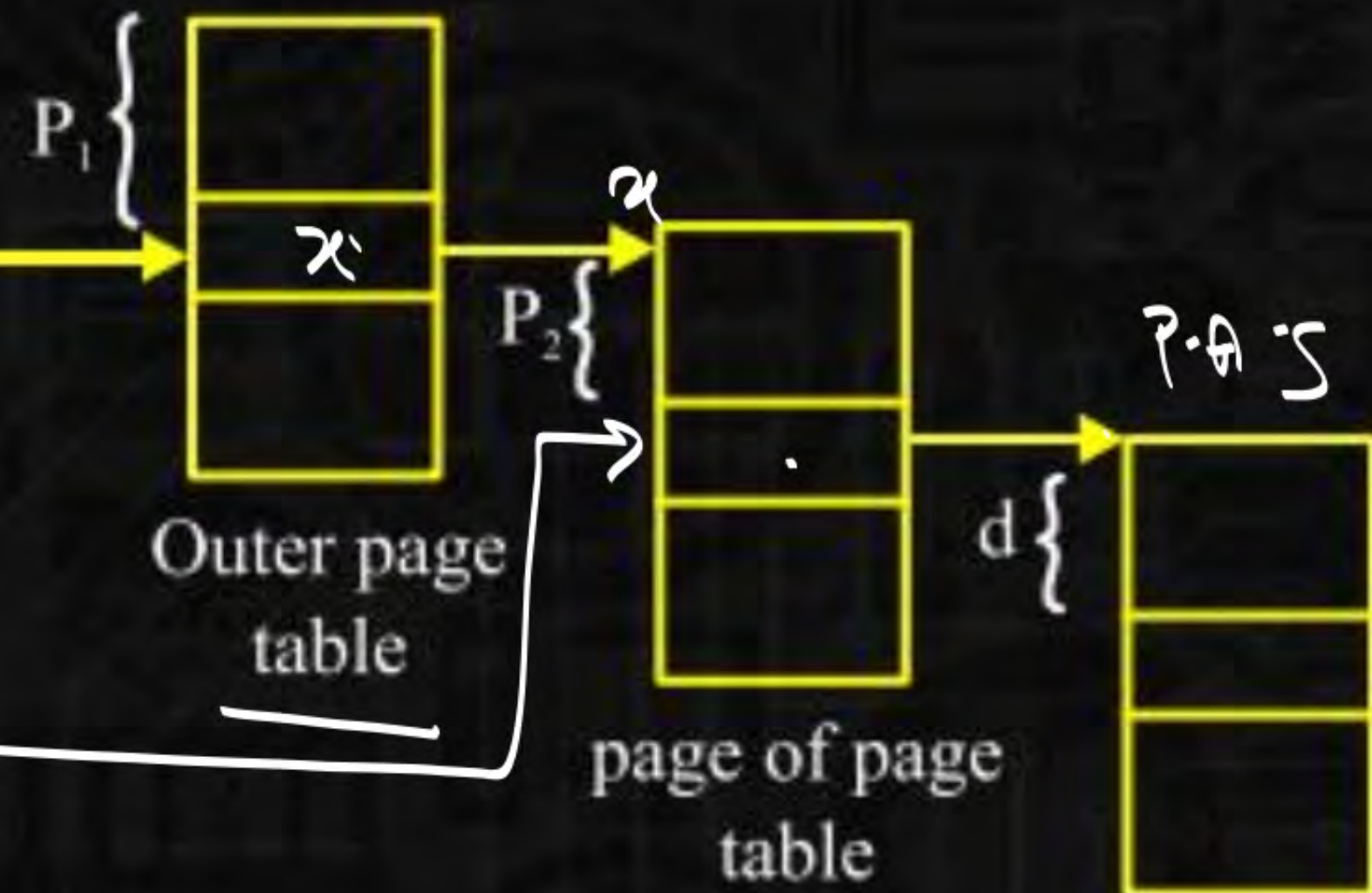
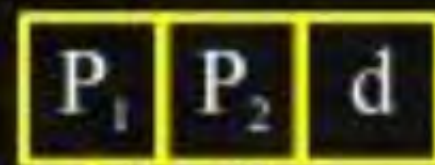
$\text{int}^{**} \text{pth};$



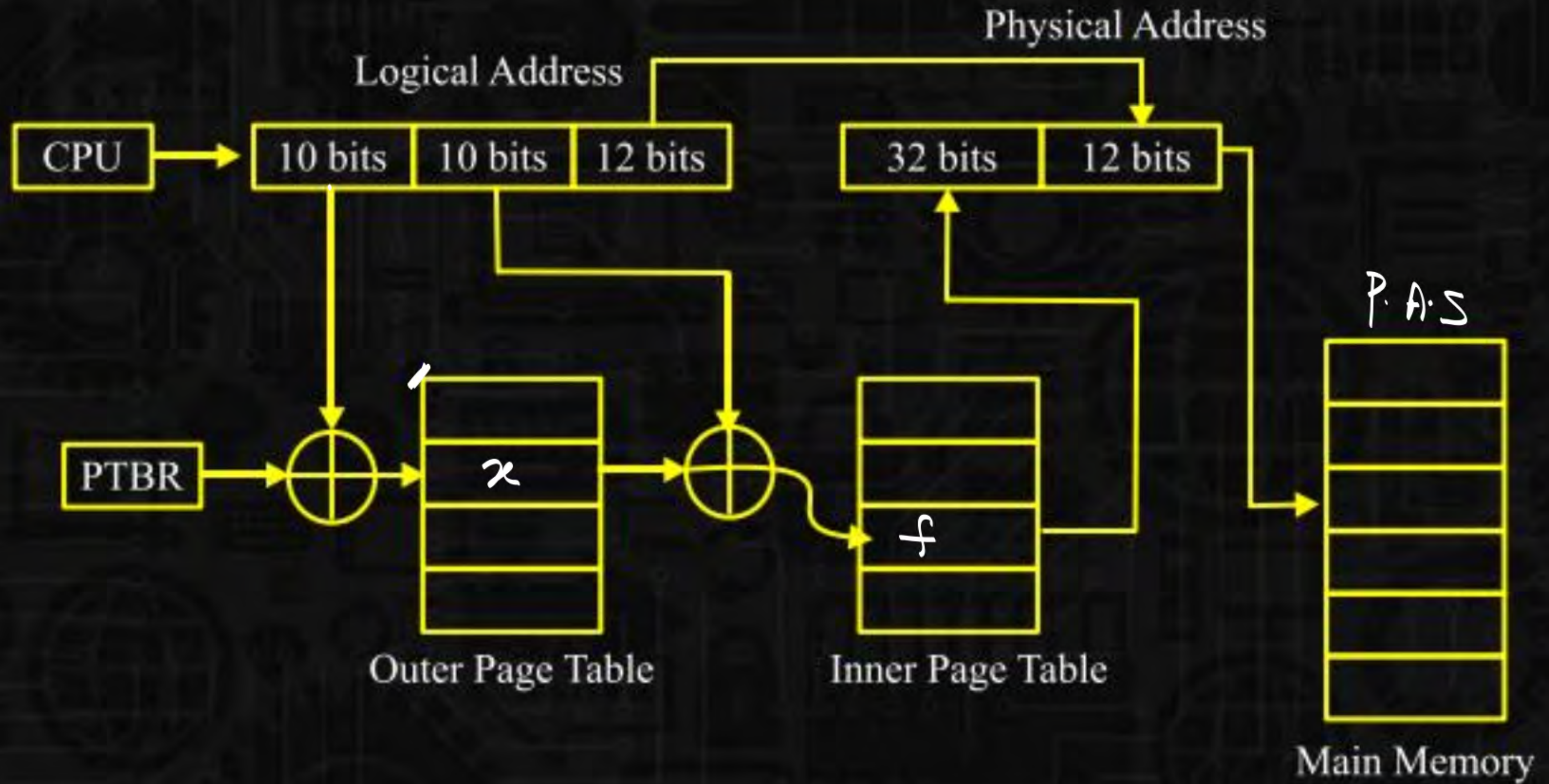




Logical address





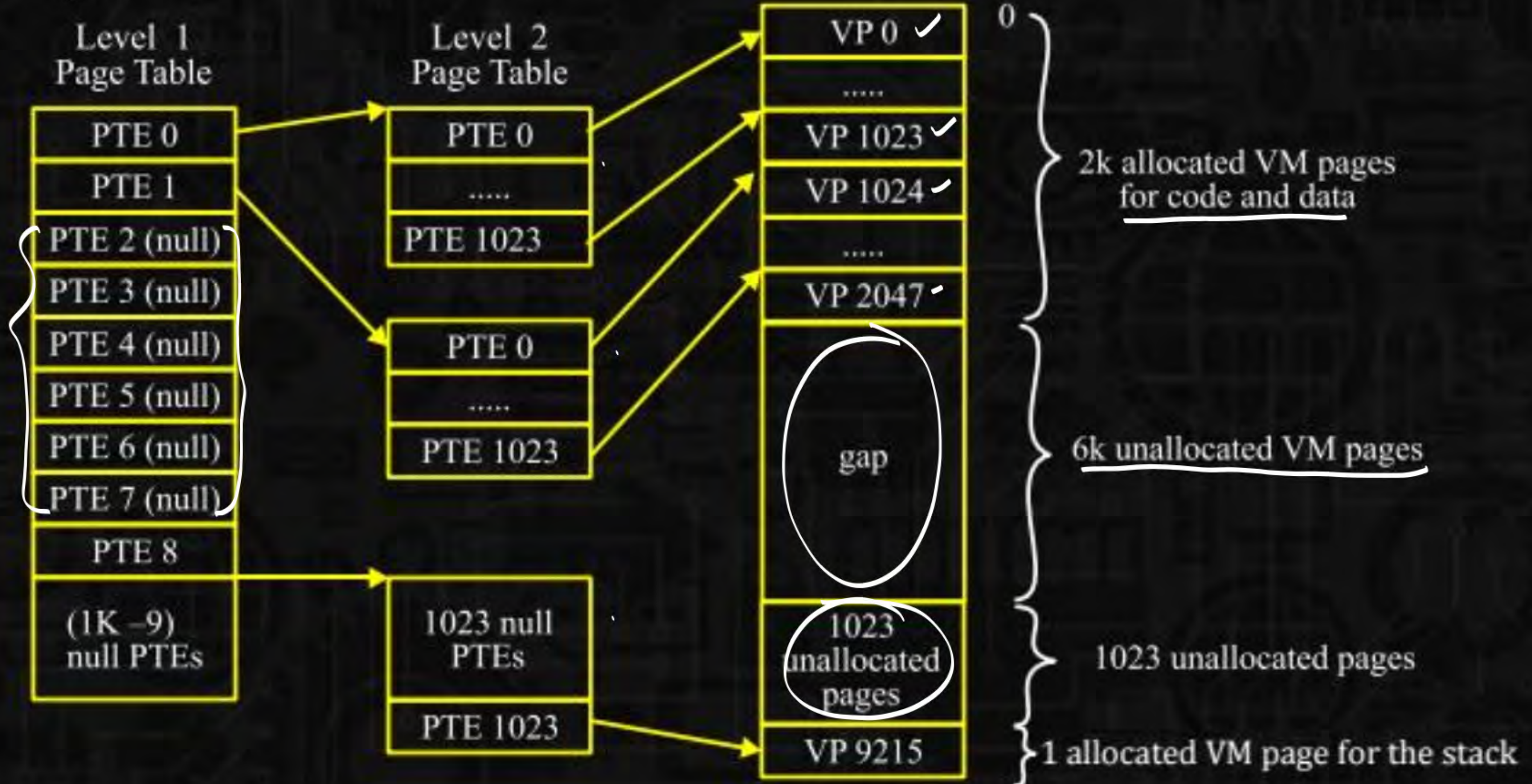




# Multi-Level Page Table

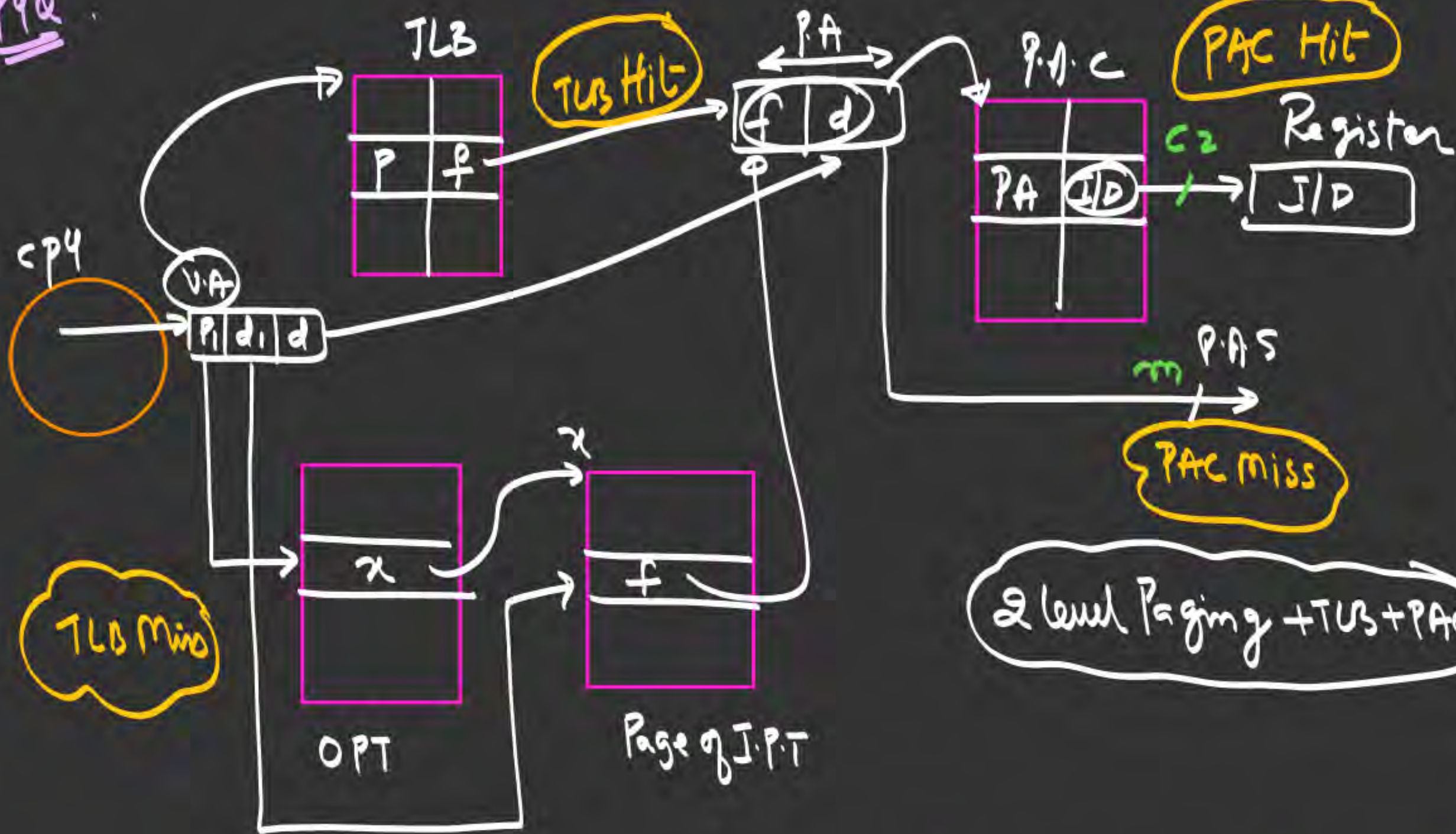
Virtual  
Memory

outer P.T





Q4Q



- $MMAT = m$
- $TLBAT = c_1$
- $TLB Hit ratio = x_1$
- $PAC AT = c_2$
- $PAC Hit ratio = x_2$

2 level Paging + TLB + PAC

$$EMAT_{ZLP} = x_1 \left[ c_1 + x_2 c_2 + (1 - x_2) (c_2 + m) \right] + (1 - x_1) \left[ c_1 + \underline{m} + \underline{m} + x_2 c_2 + (1 - x_2) (c_2 + m) \right]$$





In the context of operating systems, which of the following statements is/are correct with respect to paging?

MSQ

- A. ✗ Page size has no impact on internal fragmentation.
- B. ✓ Paging helps solve the issue of external fragmentation.
- C. ✓ Paging incurs memory overheads <sup>Space (PT's)</sup>
- D. ✗ Multi-level paging is necessary to support pages of different sizes

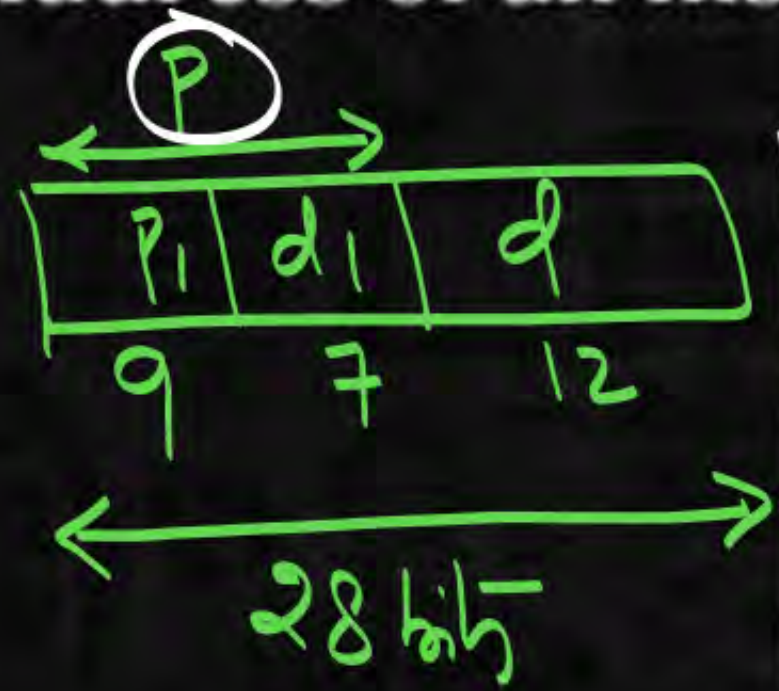
(B & C) ✓



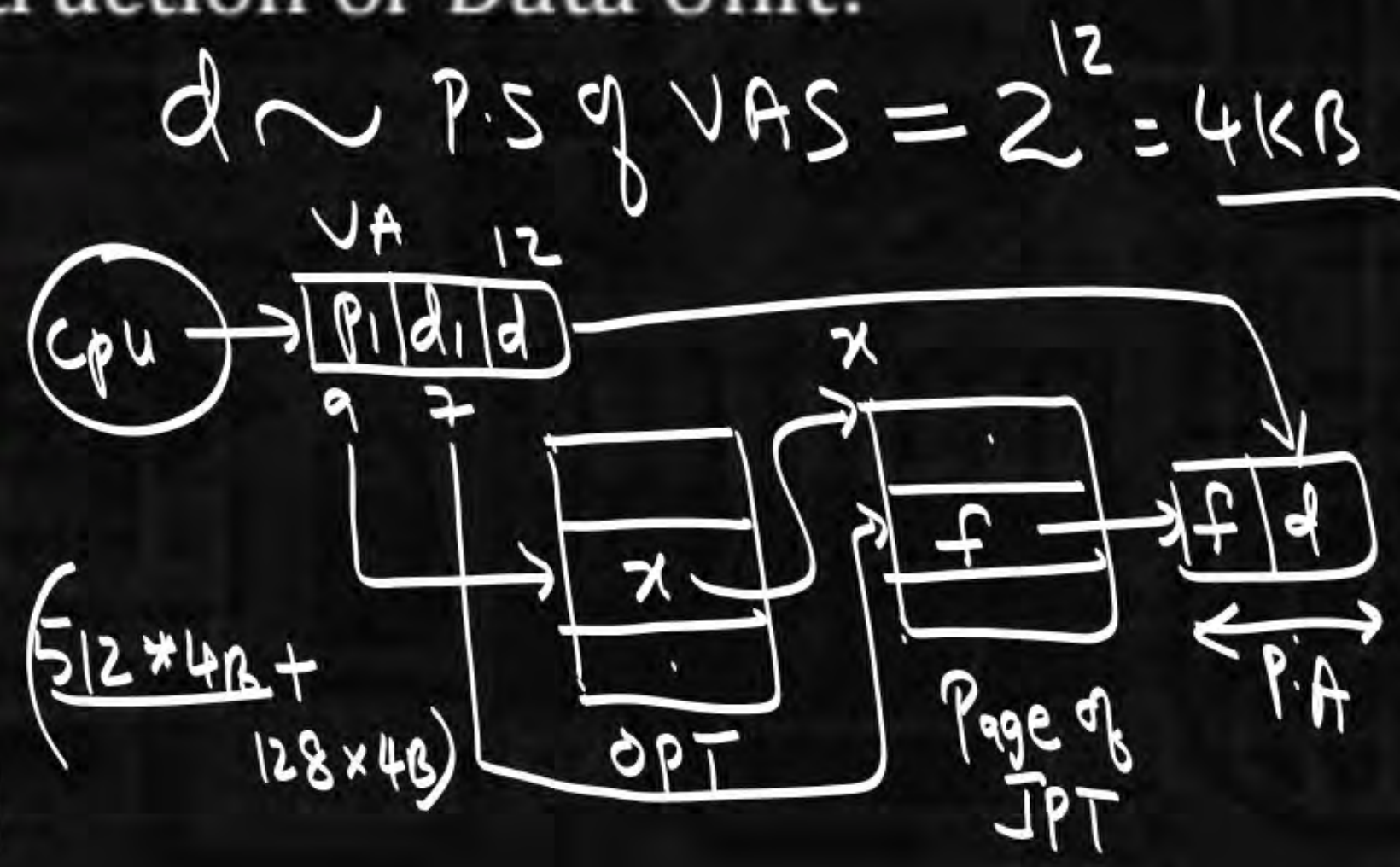


Consider a System using 2 Level Paging Architecture. The Top level 9 bits of the Virtual Address are used to index into the outer Page Table. The next 7 bits of the Address are used to index into next level Page Table. If the size of Virtual Address is 28 bits. Then

- (i) How large are the Pages and How many are there in Virtual Address Space?
- (ii) If P.T.E at both levels is 32 bits in size then what is the Space Overhead needed to translate Virtual Address to Physical Address of an Instruction or Data Unit?



$(2KB + 512B)$   
 $(2.5KB)$



$(512 * 4B + 128 * 4B)$

$P \sim \text{No. of pages in VAS}$   
 $N = 2^{16} = 64K$





Consider a Computer System using 3 Level Paging Architecture with a uniform Page Size at all levels of Paging. The size of Virtual Address is 46 bits. Page Table Entries at all levels of Paging is 32 bits. What must be the Page Size in Bytes such that the Outer Page Table exactly fits in one frame of Memory. Assume Page Size is power of 2 in Bytes. Show the Virtual Address format indicating the number of bits required to access all the three levels of Page Tables and the Page offset of Virtual Address Space.

**Handwritten Solution:**

**Let**  $P.S = 2^x$  B

**Let**  $P.S = 2^{13} = 8KB$  ✓

**Virtual Address Space (VAS) =  $2^{46}$**

**Outer Page Table (OPT) (3LP):**

$\left[ \frac{2^{46-3x+2+2+2}}{2^B} \right]$

**Second Level Page Table (2-LP):**

$\left[ \frac{2^{46-2x+2+2}}{2^B} \right]$

**Third Level Page Table (3-LP):**

$\left[ \frac{2^{46-x+2}}{2^B} \right]$

**Final Equation:**

$\left[ \frac{2^{46-3x+2+2+2}}{2^B} \right] = 1$

**Solving for x:**

$46 - 3x + 2 + 2 + 2 = 0$

$46 - 3x = -6$

$46 + 6 = 3x$

$52 = 3x$

$x = 13$



Let  $VAS = 2^S$  Bytes  $VA = S$  bits

$P.S = 2^x$  Bytes

$PTE = 2^c$  Bytes

No. of levels of Paging = 'l'

Size of outer P.T =  $2^{S-l \cdot x + l \cdot c}$  Bytes

$$= \left( \frac{2^S}{2^{lx}} \right) \cdot 2^c = 2^{S-lx+c}$$

$VA = 46$  bits

$PTE = 4$  Bytes

$P.S = 2^x$  By

$l = 3 \rightarrow 2^{13} = 8 \text{ KB}$

$LA = 46$  bits

$P_1$	$P_2$	$P_3$	$d$
-------	-------	-------	-----

$y$   $y$   $y$   $x$

$\left( \frac{y}{2} \right) 4B$

$$3y + x = 46 - \textcircled{1}$$

$OPT = \left( \frac{y}{2} \right) 4B$

$$x = y + 2 - \textcircled{2}$$

$x = 13$

$$= 2^{y+2} \text{ By} = 2^x \text{ By}$$

$$3y + y + 2 = 46$$

$$4y = 44$$

$$\therefore y = 11$$



Q

Consider a Computer System with 57 bit Virtual Addressing, Using Multi-Level Tree Structured Page Tables with L Levels for Virtual to Physical Address Translation. Page size is 4KB and Page Table Entry is of 8 Bytes at all levels.



The value of L is \_\_\_\_\_.

$$VA = 57 \text{ bits}$$

$$\text{No. of levels} = 'L'$$

$$P.S = 4KB$$

$$PTE = 8B$$



$$OPT = \left[ \frac{57 - L \cdot 12 + L \cdot 3}{2} \right] \text{ By}$$

$$= \left[ \frac{57 - 12L + 3L}{2} \right] = 2 \text{ B}$$

$$57 - 9L = 12$$

$$L = 5$$

$$\begin{aligned} \text{No. of entries that} \\ \text{Can be stored in one page} &= \frac{4KB}{8B} \\ &= \frac{2^{12}}{2^3} = 2^9 = 512 \end{aligned}$$

$$OPT = \frac{1 \text{ Page}}{\text{Frame}}$$

$$OPT = \frac{4KB}{2^{12}}$$

$$VA: 57 \text{ bits}$$

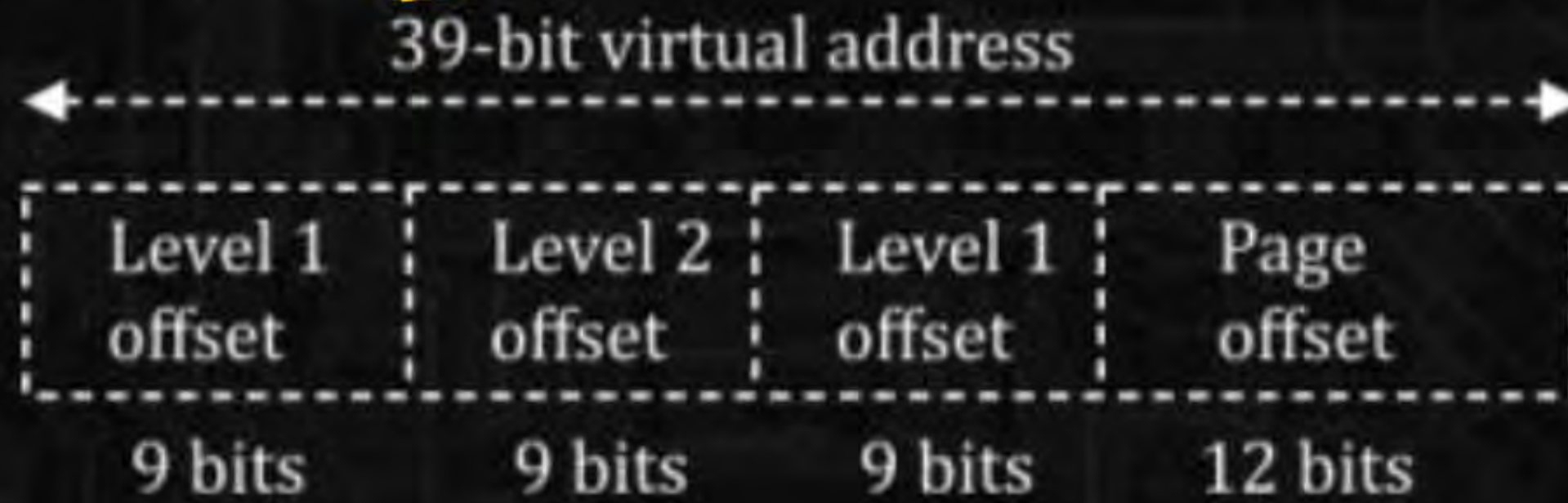


$$\begin{aligned} 9 \cdot L &= 45 \\ L &= \frac{45}{9} = 5 \end{aligned}$$





Consider a three-level page table to translate a 39-bit virtual address to a physical address as shown below.



$$2^{31} = \text{(31 bit)}$$

The page size is 4KB ( $1\text{KB} = 2^{10}$  bytes) and page table entry size at every level is 8 bytes. A process P is currently using 2GB ( $1\text{GB} = 2^{30}$  bytes) virtual memory which is mapped to (2 GB) of physical memory. The minimum amount of memory required for the page table of P across all levels is \_\_\_\_ KB.

$$(OPT + 2^{nd} \text{ level PT} + 3^{rd} \text{ level P.T})$$

$$4 = 100$$

(00100) (0100)



$$V.A.S = 64KB$$

$$Text = S_1 B$$

$$Data = S_2 B$$

$$Stack = S_3 B$$

$$P.S = ? 2^x$$

$$VAS = (2^{16-x})$$

$$Text =$$

$$Data =$$

$$Stack =$$

$$\underline{\underline{Ans: 512 B}}$$

< Internal Freq. >  
Hint Freq.



