CS & IT ENGINEERING

GRAPH THEORY

Lecture No. 2

Degree Sequence In Graphs

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Degree Sequence in Graphs

01 Degree Sequence

02 Graphical sequence

03 Havell-Hakimi Thm

04 Inequalities Thm

05 Theorem no . 7

Degree Sequence

Thm1: $\sum d(v_i) = 2e$

Thm2: no of odd degree vertices will be even.

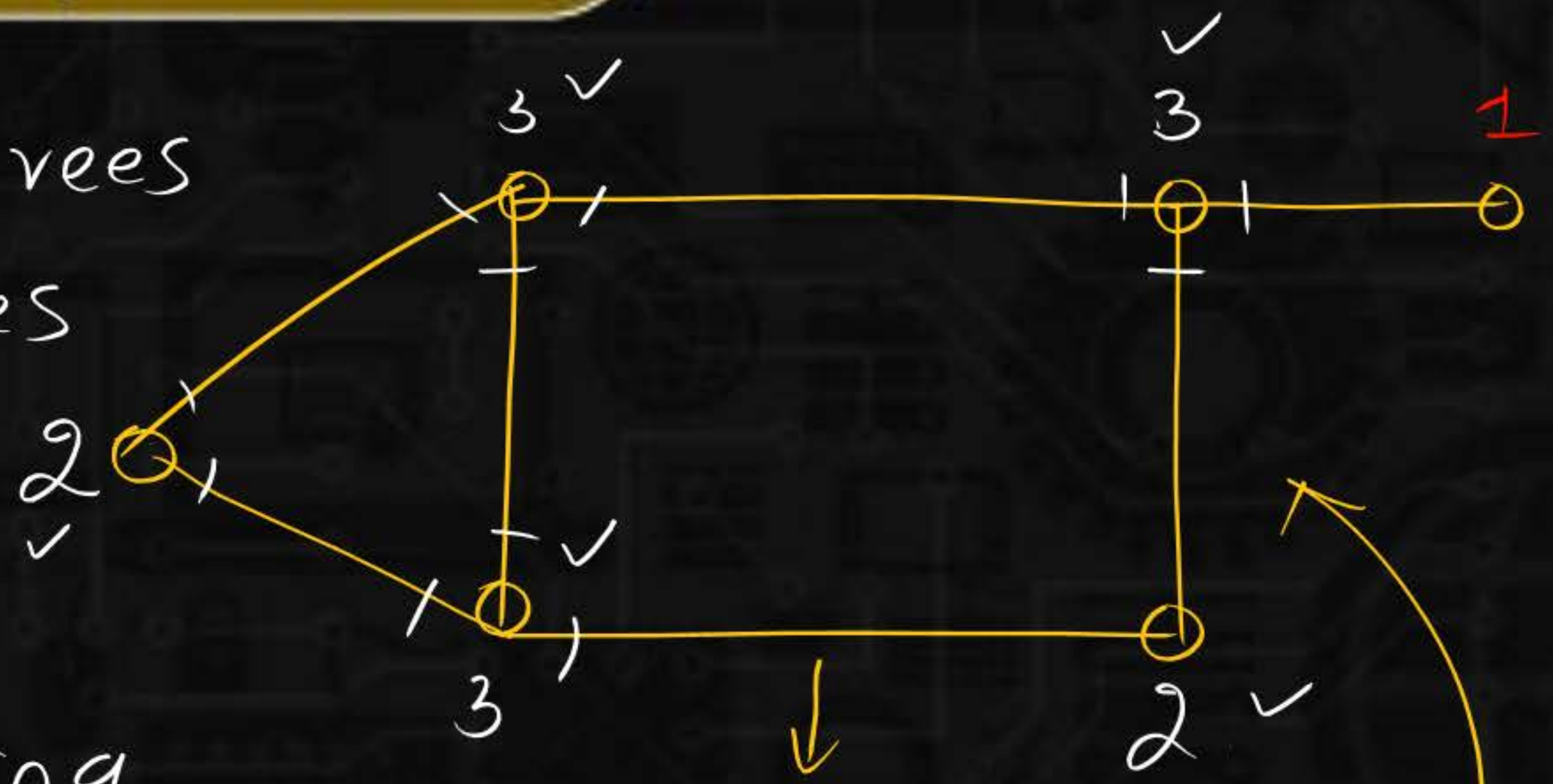
Thm3: $\leq n-1$.

Thm5: $2^{n(n-1)/2}$

Thm4: $\leq \frac{n(n-1)}{2}$

Degree Sequence

writing degrees of all vertices either in increasing or decreasing order



3 3 3 2 2 1

OR

1 2 2 3 3 3.

Degree Sequence

Q: What will be total no. of edges

5 2 2 2 2 1.

Thm 1: $\sum d(v_i) = 2e$.

$\underbrace{5+2+2+2+2+1}$ $= 2e$.

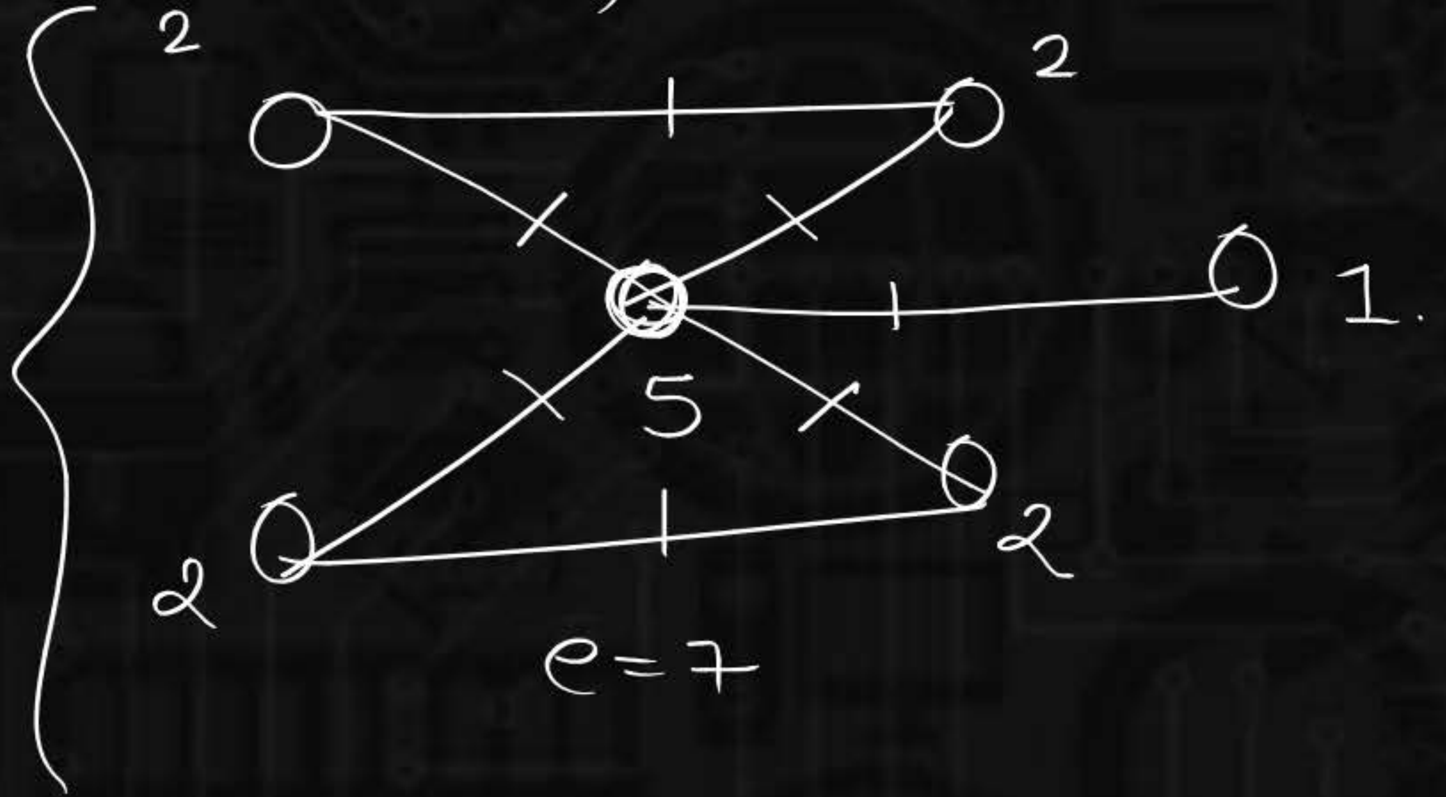
$14 = 2e$

$e = 7$

Drawing:

→ 5 2 2 2 2 1

Total no. of vertices = 6



e = 7

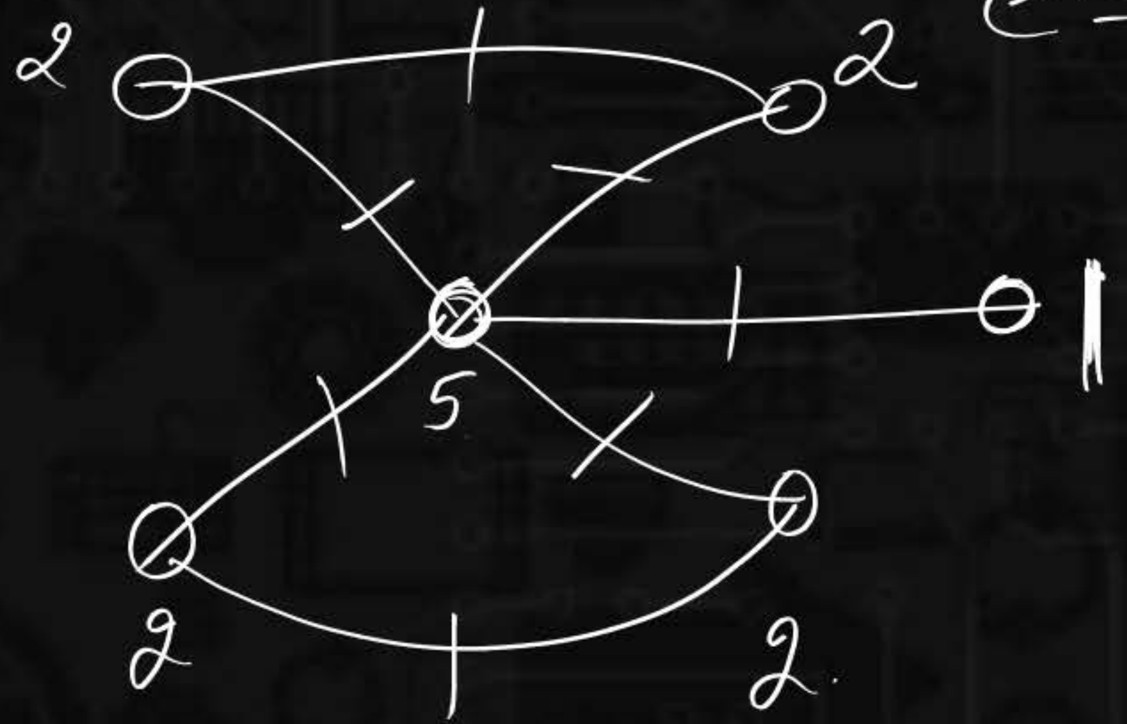
Degree Sequence

$\{5, 2, 2, 2, 2, 1\}$

Total vertices = 6

$\sum d(v_i) = 2e$

$e = 7$



Degree Sequence

What will be edge in 3, 3, 3, 1 ?

3 3 3 1

$\sum d(v_i) = 2e$

$3+3+3+1 = 2e$

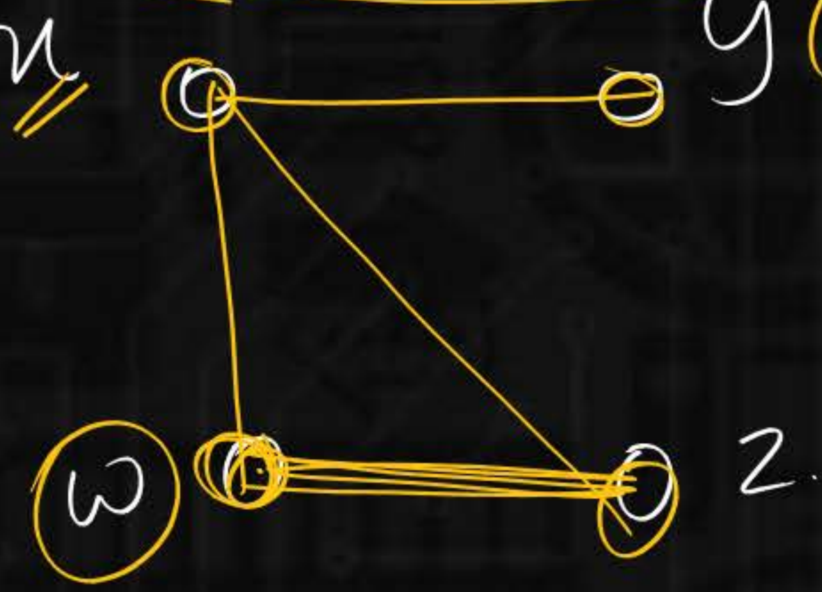
$10 = 2e$

$\boxed{e=5}$

w →

Total no. of vertices = 4

3 3 3 1



w having 1.

Dema
2

Degree Sequence

DS { 5 2 2 2 2 1 ⟶ simple Graph.

DS (3 3 3 1 ⟶ no simple Graph.

Graphical sequence:

D.S ⟶ simple graph.

5 2 2 2 2 1.
Graphical sequence

3 3 3 1.
it is not graphical sequence

Degree Sequence

Which one of the following is graphical sequence?

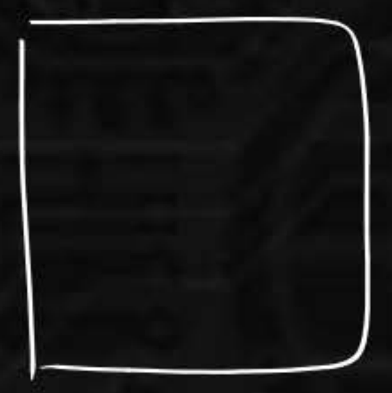
1) 5 4 3 2 1 (✗)

2) 4 4 3 2 1 (✗)

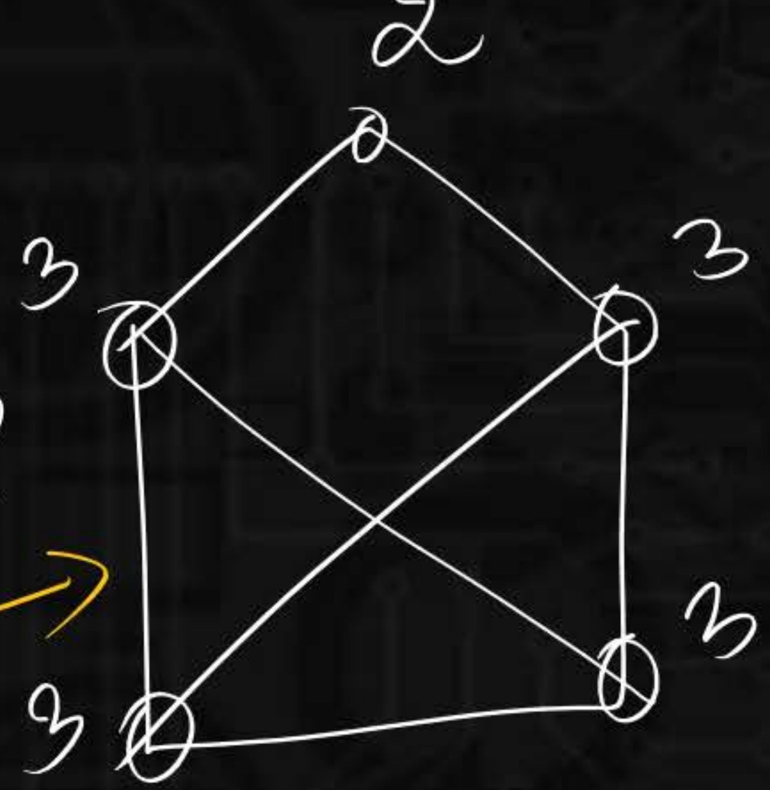
3) 2 2 2 2 (✓)

4) 3 3 3 3 2 (✓)

3) 2 2 2 2



4) 3 3 3 3 2



Degree Sequence

1) 5 4 3 2 1

it is not graphical.

Reason 1: Total vertices = 5

max degree $\leq n-1$

≤ 4

Reason 2 5 4 3 2 1

no. of odd vertices should be even

Degree Sequence

4 4 3 2 1

1. $n = 5$
   max. degree $\le n-1$ ✓
   ≤ 4.

2. no. of odd vertices should be even ✓

3

4 4 3 2 1.

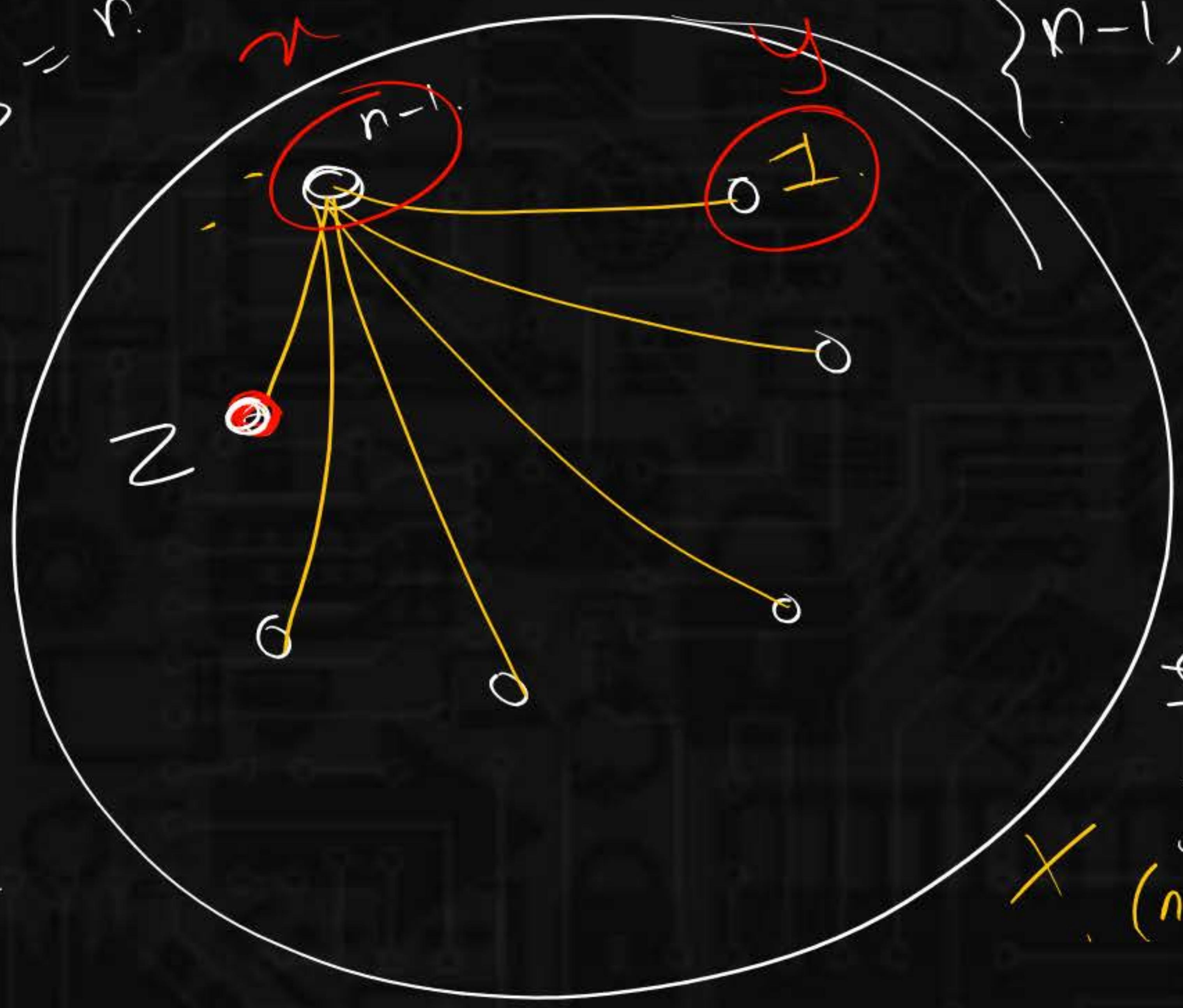
it is not simple graph.

Reason:

$n-1, n-1 \ldots\ldots 1$

Degree Sequence

Total vertices = n.



z O ✗
x ✗
y ✗

$\{n-1, n-1 \ldots\ldots\ldots 1.$

$n-1 \; (n-1) \ldots\ldots 1$

Simple graph is not possible

eg:

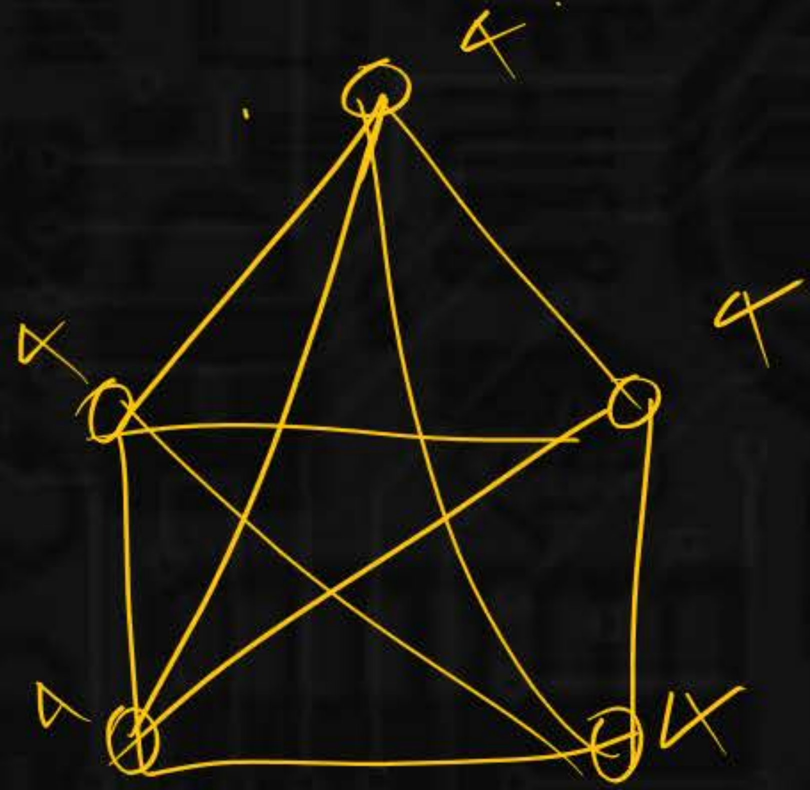
✗ 3 3 3 1
(n−1) (n−1) (pend)

Degree Sequence

$\{n-1, n-1 \ldots \ldots 1$ (no simple graph)

$n = 5$

4 4 4 4 4



no simple graph.

4 4 ……… 1

Degree Sequence

<u>Graphical sequence.</u>

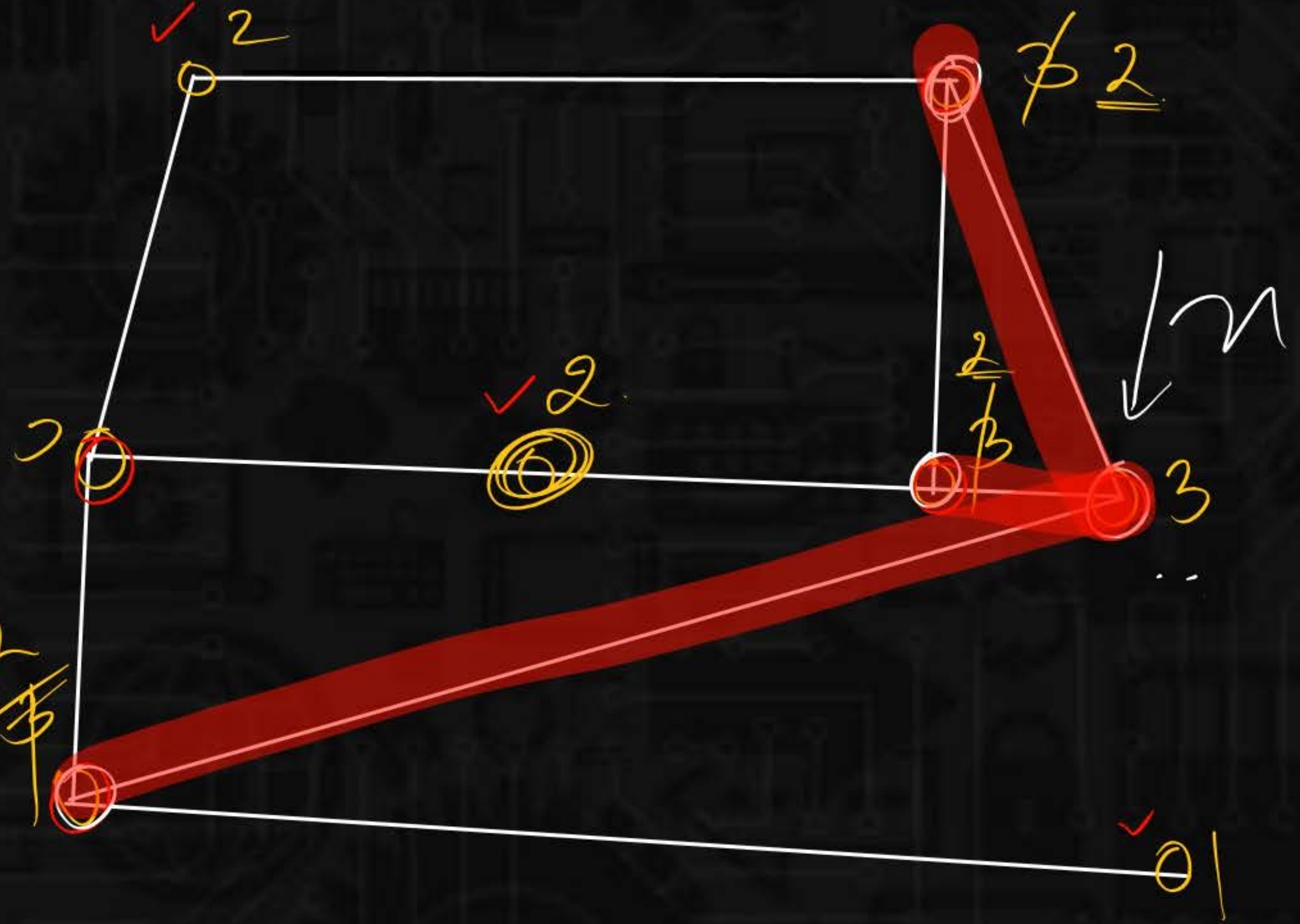
- max. degree $\leq n-1$.
- no. of odd vertices should be even
- $n-1, n-1 \cdots\cdots 1($

2 2 2 2

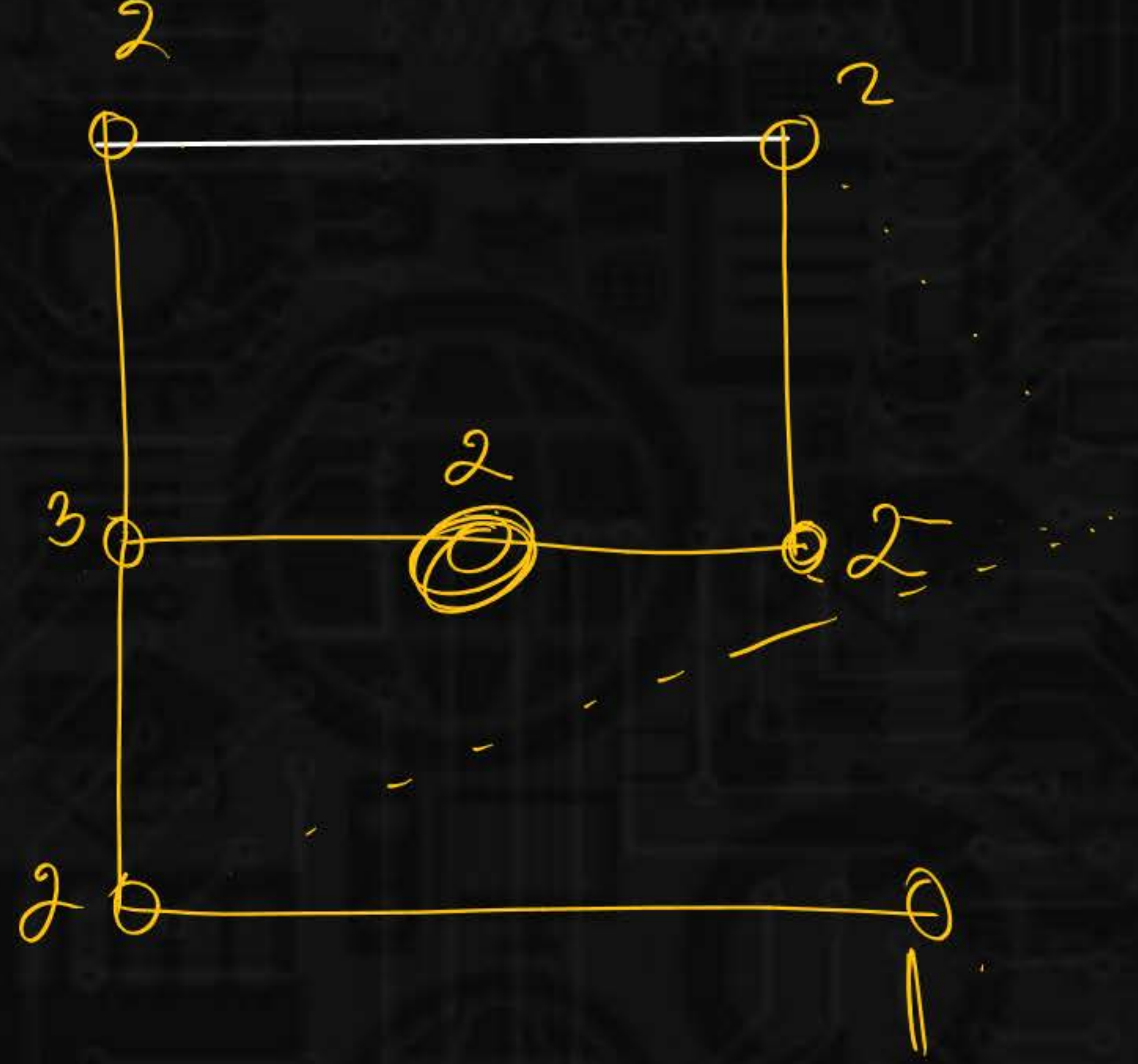
3 3 3 3 2

Degree Sequence

3 3 3 3 3 2 2 1



3 2 2 2 2 2 1

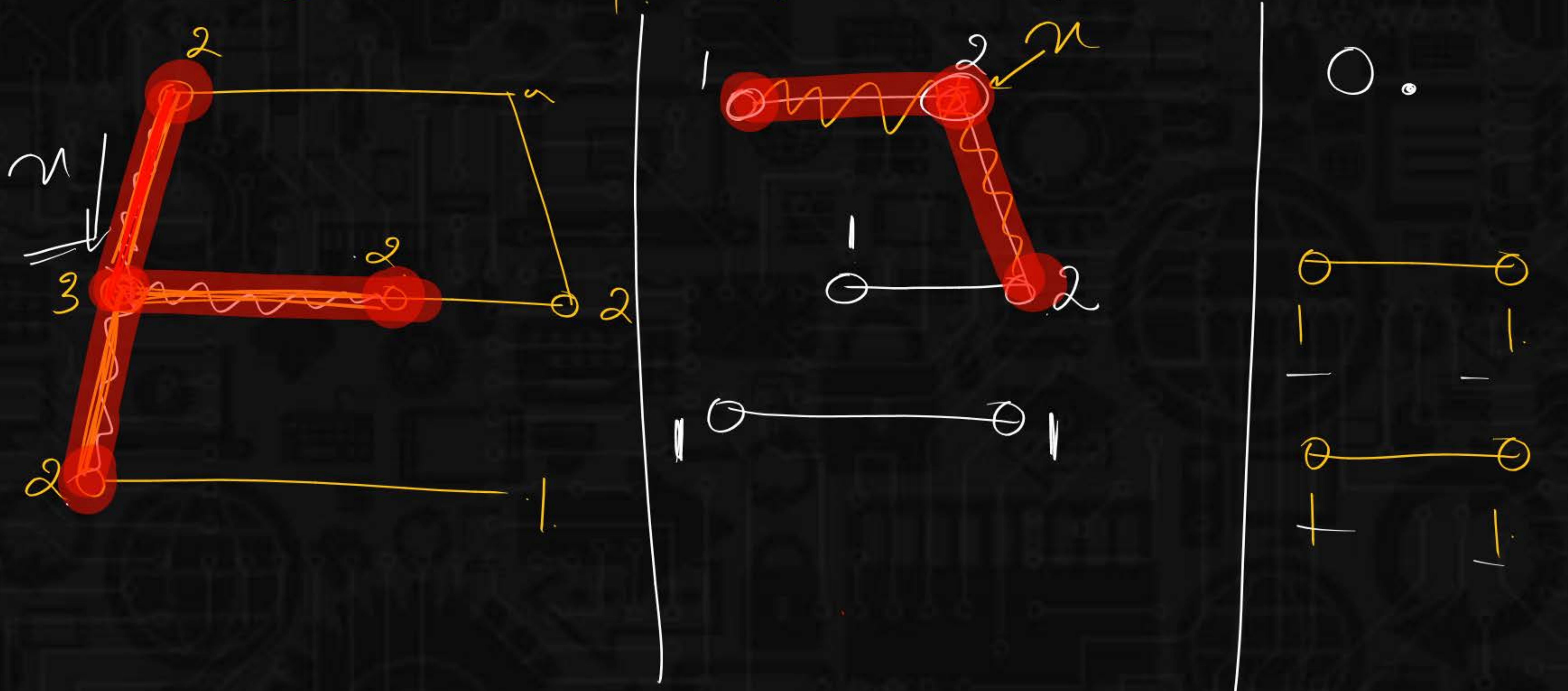


Degree Sequence

3 2 2 2 2 2 1

2 2 1 1 1 1

1 1 1 1 0



Degree Sequence

Havell-Hakimi:

check (D.S → G.S)

→ cut
→ mark
→ D1t(1)
→ orderin

1100

$\cancel{3}$ 3 3 3 3 2 2 1
2 2 2 3 2 2 1
$\cancel{3}$ 2 2 2 2 2 1
1 1 1 2 2 1
$\cancel{2}$ 2 1 1 1 1 (ordering)
1 0 1 1 1
$\cancel{1}$ 1 1 1 0 (ordering)
0 1 1 0

Degree Sequence

cut
mark.
dlt
ordering

$\cancel{3}\ 3\ 3\ 3\ 3\ 2\ 2\ 1$

$2\ 2\ 2\ 3\ 2\ 2\ 1$

$\cancel{3}\ 2\ 2\ 2\ 2\ 2\ 1$ (ordering)

$1\ 1\ 1\ 2\ 2\ 1$

$\cancel{2}\ 2\ 1\ 1\ 1\ 1$ (ordering)

$1\ 0\ 1\ 1\ 1$

$\cancel{1}\ 1\ 1\ 1\ 0$

$0\ 1\ 1\ 0$

$1\ 1\ 0\ 0$

Degree Sequence

$\cancel{3}\ 3\ 3\ 1$ (use Havel Hakimi)

$\cancel{2}\ 2\ 0$

$1\ (-1)$

it is not possible.

$\cancel{4}\ 4\ 3\ 2\ 1$

$\cancel{3}\ 2\ 1\ 0$

$(1\ 0\ -1)$

Degree Sequence

GATE Graphical?

A) 7 6 5 4 4 3 2 1 (graphical)

B) 6 6 6 6 3 3 2 2 (×)

C) 7 6 6 4 4 3 2 2 (✓)

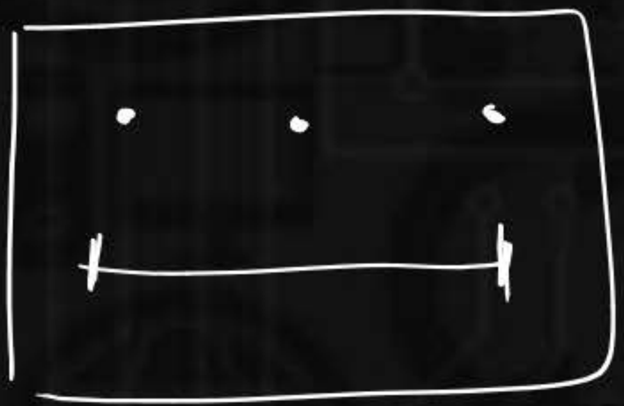
d) 8 7 7 6 4 4 2 1 (not graphical)

~~7~~ 6 5 4 4 3 2 1

~~5~~ 4 3 3 2 1 0 (ord)

~~3~~ 2 2 1 0 0 (ord)

1 1 0 0 0



Degree Sequence

b) $\cancel{6}$ 6 6 6 3 3 2 2

5 5 5 2 2 1 2

$\cancel{5}$ 5 5 2 2 2 1 (ord)

$\cancel{4}$ 4 1 1 1 1 (ord)

3 0 0 0 1

3 1 0 0 0

not graphical

c) $\cancel{7}$ 6 6 4 4 3 2 2

$\cancel{5}$ 5 3 3 2 1 1

4 2 2 1 0 1

$\cancel{4}$ 2 2 1 1 0 (ord)

1 1 0 0 0 (ord)

Graphical

Degree Sequence

Graphical sequence (follow steps)

eg

5 4 3 2 1

1. max degree $\leq n-1$
2. no. of odd vertices should be even.
3. $n-1, n-1 \cdots\cdots 1$ (sequence is not possible)
4. → if all degrees are distinct simple graph is not possible
5. kuch nahi aya → use Havell Hakimi (Directly)

Degree Sequence

Thm7: In Simple Graph atleast 2 vertices will have same degree

let's take all vertices as distinct degree.

Total vertices = n.

Distinc degrees: $\underbrace{\{1, 2, 3 \ldots n-1\}}$ → Total distinct degrees $= n-1$.

Degree Sequence

Total vertices	Total distinct degrees
V_1	1
V_2	2
V_3	3
V_4	0
$\vdots$	$\vdots$
V_n	$n-1$

$n = 5$

Distinct:

V_1	1
V_2	2
V_3	3
V_4	4
V_5	

Degree Sequence

$\boxed{\text{repeat} \rightarrow \text{Degrees}}$

$\uparrow$ → Assump:

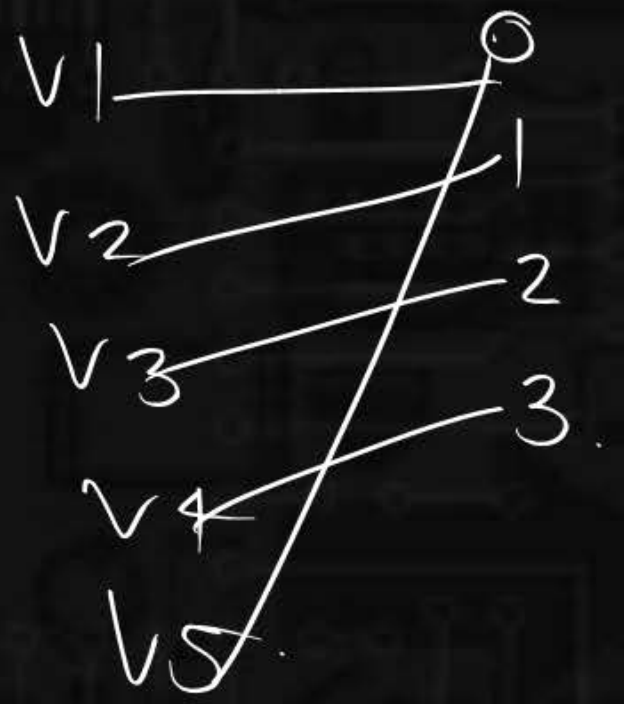
Degrees Distinct

V1	1
V2	2
V3	3
V4	4
V5	

Degree Sequence

Case 2 : $\{0, \ldots \ldots n-2\}$.

$n = 5$.



Degree Sequence

Case 2 : $\{0, \ldots, n-2\}$ Total distinct degrees $= n-1$,

Total vertices $= n$.

n-1 vertices

v1

$\downarrow$

0.

$\le n-2$.


THANK YOU GW
SOLDIERS !