Disc / Cont Distn.s

$X \sim U[a, b]$

1 **Uniform** :- $f_X(x) = \frac{1}{b-a}, \quad a \le x \le b$

→ $E(X) = \frac{b+a}{2}$, $V(X) = \frac{(b-a)^2}{12}$

$$M_X(t) = \begin{cases} \frac{e^{bt} - e^{at}}{t(b-a)}, & t \ne 0 \\ 1, & t = 0. \end{cases}$$

$$F(x) = \begin{cases} \frac{x-a}{b-a}, & a \le x \le b \\ 0, & x < a \\ 1, & x > b. \end{cases}$$

$\mu_n'(a) = \frac{(b-a)^n}{n+1} \Rightarrow \mu_3 = 0,$

$\mu_4 = \frac{(b-a)^4}{80}$

$\therefore \gamma_1 = 0, \quad \gamma_2 = -1.2$

So, symm & platykurtic.

$\Rightarrow \gamma_1 = \frac{\mu_3}{\mu_2^{3/2}}$; $\gamma_2 = \frac{\mu_4}{\mu_2^2} - 3$

Skewness ; kurtosis

$\gamma(n,\theta)$

② **Gamma Dist^n** : [$n=1 \to$ exponential dist^n]

$$f_X(x) = \frac{1}{\Gamma n} e^{-\frac{x}{\theta}} \left(\frac{x}{\theta}\right)^{n-1} \cdot \frac{1}{\theta}, \quad x>0$$

$X \sim \text{Gamma}(\theta, n)$ $[\theta > 0, \; n > 0.]$

$\to E(X) = n\theta, \quad V(X) = n\theta^2$

$\mu_r' = \theta^r \dfrac{\Gamma r + n}{\Gamma n}, \quad r > -n$ [all non -ve moments exist]

VVR

$HM = [\mu_{-1}']^{-1} = \dfrac{1}{E\left(\frac{1}{X}\right)} = \theta\cdot(n-1) \quad (n>1)$

$\to M_X(t) = (1-\theta t)^{-n}, \quad (\theta t < 1)$

$\to K_r = n\theta^r \underline{|r-1}$

Thus, $\gamma_1 = \dfrac{2}{\sqrt{n}}, \quad \gamma_2 = \dfrac{6}{n}$.

Thus, +vely skew & leptokurtic

③ Beta Dist^n :→ MGF exists, but no compact form

$$f_X(x) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \quad 0<x<1$$

$\alpha, \beta > 0$

$$\mu_r' = \frac{B(\alpha+r, \beta)}{B(\alpha,\beta)}, \quad \longrightarrow \; r > -\alpha$$

$\alpha > 1$

$$E(X) = \frac{\alpha}{\alpha+\beta}, \quad HM = \frac{\alpha-1}{\alpha+\beta-1}$$

So, $E(X) > V(X)$

$$V(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

GM be G, $\ln G = \frac{\partial}{\partial\alpha}\left[\ln B(\alpha,\beta)\right]$

④ 2nd kind Beta :-

$$f_X(x) = \frac{1}{B(\alpha,\beta)} \frac{x^{\alpha-1}}{(1+x)^{\alpha+\beta}}, \quad x>0$$

$\alpha, \beta > 0$

$$\mu_r' = \frac{B(\alpha+r, \beta-r)}{B(\alpha,\beta)}, \quad E(X) = \frac{\alpha}{\beta-1}$$

$-\alpha < r < \beta$

$$V(X) = \frac{\alpha(\alpha+\beta-1)}{(\beta-1)^2(\beta-2)}, \quad HM = \frac{\alpha-1}{\beta}.$$

If $X \sim$ 2nd kind $B(\alpha, \beta)$

Then, $Y = \frac{X}{1+X} \sim$ 1st kind $B(\alpha, \beta)$.

$B(m,n) = \int_0^{\pi/2} 2 \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$.

(5) Normal Dist^n :- $X \sim N(\mu, \sigma^2)$

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < \infty$$

$\mu_{2r+1} = 0$; $\mu_{2r} = \sigma^{2r}(2r-1)(2r-3)\cdots 3\cdot 1$

$\therefore \gamma_1 = 0, \ \gamma_2 = 0$

$\mathrm{Var}(S^2) = \frac{2\sigma^4}{n-1}$

$MGF = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$

$MD_\mu = \sigma\sqrt{\frac{2}{\pi}}$; $V(|X-\mu|) = \sigma^2\left(1 - \frac{2}{\pi}\right)$

$$1 - \frac{1}{x^2} \le \frac{x\{1-\Phi(x)\}}{\phi(x)} \le 1 .$$

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}, \quad a > 0$$

Dist^n of $|X|$ $(=Y)$:- $f_Y(y) = \begin{cases} 0, & y \le 0 \\ 2 f_X(y), & y > 0 \end{cases}$

$X \sim N(0, \sigma^2)$

↳ folded normal dist^n.

Scanned by CamScanner

⑥ Log-Normal distⁿ :- ⊙

$X \sim \Lambda(\mu, \sigma^2)$ iff $\log_e X \sim N(\mu, \sigma^2)$ ✓

$$f_X(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2}, \quad x>0$$

$\mu_r' = e^{\mu r + \frac{1}{2}\sigma^2 r^2}$; $HM = e^{\mu - \frac{1}{2}\sigma^2}$

$\therefore E(X) = e^{\mu + \frac{1}{2}\sigma^2}$, $GM = e^{\mu}$.

$V(X) = e^{2\mu + \sigma^2}(e^{\sigma^2} - 1)$.

$M_e = e^{\mu}$; M_0 (mode) $= e^{\mu - \sigma^2}$.

pth quantile, $\xi_p = e^{\mu + \sigma Z_p}$ (✓)

[Z_p : pth quantile of $N(0,1)$ distⁿ].

+vely skew.
[∵ QD > 0]

AM > GM > HM

$e^{\mu + \frac{1}{2}\sigma^2} > e^{\mu} > e^{\mu - \frac{1}{2}\sigma^2}$

$\mu > M_e > M_0$

$M_0 = e^{\mu - \sigma^2}$

$\boxed{\text{Discrete}} \Rightarrow$

$$\mu_{[r]} = \frac{d^r}{dt^r} P_x(t) \Big|_{t=1}.$$

$$\mu_r' = \frac{d^r}{dt^r} M_x(t) \Big|_{t=0}.$$

$\hookrightarrow$ coeff of $\frac{t^r}{\lfloor r}$ in $M_x(t) = E(e^{tx})$

$p(x) = $ Coeff of t^x in $P_x(t) = E(t^x)$

$K_r = $ Coeff of $\frac{t^r}{\lfloor r}$ in $K_x(t) = \ln M_x(t)$

$\mu_1' = K_1$; $\mu_2 = K_2$; $\mu_3 = K_3$; $\mu_4 = K_4 + 3K_2^2$

(7) Degenerate distn :

$p(x) = \begin{cases} 1, & x = c \\ 0, & \text{o.w.} \end{cases}$

$K_x(t) = ct$

$K_r = 0 \;\; \forall r > 1.$

$\therefore K_1 = ct.$

$\mu_r' = c^r$

$E(X) = c$

$V(X) = 0$

mgf $= e^{ct}.$

⑧ **Uniform distⁿ:**

$$p(x) = \begin{cases} \frac{1}{n}, & x = a + ih, \; i = 1(1)n \\ 0, & \text{o.w.} \end{cases}$$

$$E(X) = a + \frac{h(n+1)}{2} \; ; \; V(X) = \frac{h^2(n^2-1)}{12}$$

$$\mu_r = \frac{h^r}{n} \sum_{1}^{n} \left(i - \frac{n+1}{2}\right)^r$$

$$\mu_3 = 0$$

$$\mu_4 = \frac{h^4 (n^2-1)(3n^2-7)}{240}.$$

Thus, $\gamma_1 = 0$, $\gamma_2 = \beta_2 - 3 < 0$

where $\beta_2 = 1.8 \times \frac{n^2 - \frac{7}{3}}{n^2-1}$

$$MD_\mu = \begin{cases} |h| \times \frac{n^2-1}{4n}, & n = \text{odd} \\ |h| \times \frac{n}{4}, & n = \text{even}. \end{cases}$$

⑨ **Bernoulli → Bin(1, p).**

→ $\sum_{i=1}^{n} i^4 = \left(\sum i^2\right) \times \left(\frac{3n^2+3n-1}{5}\right)$

(9) Binomial Distn :-

$$p(x) = \begin{cases} \binom{n}{x} p^x q^{n-x}, & x = 0(1)n \\ 0, \text{ o.w.} & \begin{pmatrix} 0<p<1 \\ q=1-p \end{pmatrix} \end{cases}$$

$\mu'_{[r]} = (n)_r p^r$

$\mu'_r \Rightarrow \mu'_{[r]}$ [Coeff. : 1 ($\mu'_1 = \mu'_{[1]}$)

11, 131, 1671]

$\mu'_4 = \mu'_{[4]} + 6\mu'_{[3]} + 7\mu'_{[2]} + \mu'_{[1]}$

$\mu_{r+1} = pq\left[nr\mu_{r-1} + \frac{d\mu_r}{dp}\right], r = 1, 2 \ldots$

$\therefore \mu'_1 = np, \quad \mu_2 = npq \quad [\mu_0 = 1, \mu_1 = 0]$

$\mu_3 = npq(q-p)$, &

$\mu_4 = 3n^2p^2q^2 + npq(1-6pq)$

$\therefore \gamma_1 = \frac{q-p}{\sqrt{npq}}, \quad \gamma_2 = \frac{1-6pq}{npq}$

$\therefore$ +vely skew / symm / -vely skew as $p \lesseqgtr \frac{1}{2}$

L/m/p as $pq \lesseqgtr \frac{1}{6}$; As $n \to \infty$,

$(\gamma_1, \gamma_2) \to (0, 0)$.

For $p \neq \frac{1}{2}$, $\beta_2 = c\beta_1 + 3$,

$$c = 1 - \frac{2pq}{(q-p)^2} < 1.$$

Mode $= \begin{cases} k \text{ \& } k-1 & \text{, if } k = \text{integer} \\ [k] & \text{, if } k \text{ } \cancel{\text{fraction}} = \text{frac}^n \end{cases}$

$k = (n+1)p$

$M_x(t) = (q+pe^t)^n$; $P_x(t) = (q+pt)^n$

$MD_\mu = 2npq \times \binom{n-1}{k} p^k q^{n-1-k}$

$\rightarrow \sqrt{\frac{2npq}{\pi}}$, as $n \to \infty$ $\left(k = [np] \right)$

$\rightarrow$ $P(X \leq k) = P(Y \leq q)$, $[q = 1-p]$

where $X \sim Bin(n,p)$; $Y \sim \beta(n-k, k+1)$

↳ use $\binom{n}{x} = \binom{k}{x} \times \frac{B(n-k, k-x+1)}{B(n-k, k+1)}$

$X \sim Bin(n,p) \Rightarrow n-X \sim Bin(n,q)$

$K_{r+1} = pq \frac{d}{dp}(K_r)$ with $K_1 = np$.

$MD_\mu = 2p(n-k) f(k)$ → pmf of $Bin(n,p)$.

Fitting :- $\frac{p(x)}{p(x-1)} = \frac{n-x+1}{x} \times \frac{p}{q}$, $p(0) = q^n$

(10) Poisson Distⁿ:

$$p(x) = \begin{cases} e^{-\lambda} \dfrac{\lambda^x}{\lfloor x}, & \lambda > 0,\ x = 0, 1, 2 \ldots \\ 0, & \text{o.w.} \end{cases}$$

We can take $\lambda = vt$ ($v \to$ mean rate of occurrence):

$\lambda \to$ mean no of occ.

$\mu'_{[r]} = \lambda^r$, $K_r = \lambda$; $\gamma_1 = \frac{1}{\sqrt{\lambda}}$, $\gamma_2 = \frac{1}{\lambda}$

$\mu'_1 = \lambda$, $\mu_2 = \lambda$, $\mu_3 = \lambda$, $\mu_4 = 3\lambda^2 + \lambda$

$M_x(t) = e^{-\lambda} \cdot e^{\lambda e^t}$; $P_x(t) = e^{-\lambda} \cdot e^{\lambda t}$

$MD\mu = 2\lambda e^{-\lambda} \frac{\lambda^k}{\lfloor k}$, $k = [\lambda]$.

$= 2\lambda f(k)$

$$P(X \le k) = P(Y > \lambda) \left[= 1 - \overbrace{I\left(\frac{\lambda}{\sqrt{k+1}}, k\right)}^{P(Y \le \lambda)} \right]$$

$X \sim$ Poisson(λ) ; $Y \sim$ Gamma$(k+1)$.

where $I(u, k) = \dfrac{\int_0^\lambda e^{-t} t^k \, dt}{\Gamma k+1} = P(Y \le \lambda)$

$u = \dfrac{\lambda}{\sqrt{k+1}}$

Scanned by CamScanner

If $X_1 \sim \text{Poisson}(\lambda_1)$
$X_2 \sim \text{Poisson}(\lambda_2)$ } indep.

Then, (i) $X_1 + X_2 \sim \text{Poisson}(\lambda_1 + \lambda_2)$

(ii) $X_1 \mid (X_1 + X_2) \sim \text{Bin}\left(x, \frac{\lambda_1}{\lambda_1 + \lambda_2}\right)$

$$\mu'_{r+1} = \lambda \sum_{j=0}^{r} \binom{r}{j} \mu'_j .$$

$$\text{Mode} = \begin{cases} \lambda, \lambda - 1 & \text{if } \lambda \in I^+ \\ [\lambda] & , \text{ o.w.} \end{cases}$$

$$\mu_{r+1} = \lambda \left[r\mu_{r-1} + \frac{d\mu_r}{d\lambda} \right]$$

$p(x-1) \lesseqgtr p(x)$, ...

Fitting:- $\frac{p(x)}{p(x-1)} = \frac{\lambda}{x}$, $p(0) = e^{-\lambda}$

$\to H(N, Np, n)$

(11) Hypergeometric Distr^n :

$$\text{pmf} = \frac{\binom{Np}{x}\binom{Nq}{n-x}}{\binom{N}{n}}, \quad x = 0(1)\,n$$

$Hyp(N, n, p)$

$E(X) = np$; $\to V(X) = npq \cdot \frac{N-n}{N-1}$

$$\mu'_{[r]} = \frac{(n)_r \times (Np)_r}{(N)_r}$$

Mode :- $\boxed{k = \frac{(n+1)(Np+1)}{N+2}}$

$\hookrightarrow \begin{cases} k,\ k-1 & \text{when } k \in I^+ \\ [k] & \text{, otherwise} \end{cases}$

$$MD_\mu = \frac{2(Np-k)(n-k)}{N}\, \underset{\downarrow}{f(k)}$$

$= 2\left(p - \frac{k}{N}\right)(n-k)\, f(k)$ Hyp with $x=k$

~~If X ⊥ Y are~~

If $X_1 \sim Bin(n_1, p)$; $X_2 \sim Bin(n_2, p)$ (indep.), Then $(X_1 \mid X_1 + X_2 = x)$

$x_1 = 0(1)\,x \mid \sim Hyp\left(n_1 + n_2, x, \frac{n_1}{n_1+n_2}\right)$

(12) Geometric :- ($r=1$ in -ve Bin) $\rightarrow$ Geometric (p)

$$f(x) = pq^x, \quad x = 0, 1, 2, \ldots$$

$$F(x) = 1 - q^{x+1}, \quad \sum_{x=0}^{\infty} q^x = (1-q)^{-1}$$

$$E(x) = \frac{q}{p}, \quad \therefore \sum_{1}^{\infty} x q^{x-1} = (1-q)^{-2}$$

$$V(x) = \frac{q}{p^2}, \quad \therefore \sum_{2}^{\infty} x(x-1) q^{x-2} = 2(1-q)^{-3}.$$

Mode, $\mu_0 = 0$

Memory Loss Prop :-

$$P(X \leq t+k \mid X \geq t) = P(X \leq k)$$

$$MGF = \frac{p}{1-qe^t}; \quad PGF = \frac{p}{1-qt}.$$

⊙ X_1, X_2 are iid geometric Γ pmf, $f(y) = \frac{1}{1+x}$, $y = 0(1)x$.

$\Rightarrow (X_1 \mid X_1 + X_2 = x)$ is uniform

(13) Negative Binomial distⁿ ⇒

$$f(x) = \binom{r+x-1}{x} p^r q^x , \quad x = 0, 1, 2, \ldots$$

$$= \binom{-r}{x} p^r (-q)^x \rightarrow -ve\,Bin(r, p)$$

from $(Q-P)^{-r} \longrightarrow$ $$= \binom{r+x-1}{x} P^x Q^{-(r+x)} \quad \left[p = \frac{1}{Q}, \; q = \frac{P}{Q} \right]$$

$$\text{Mode} = \begin{cases} k \;\&\; k-1 & \text{if } k \in I^+ \\ [k], & \text{o.w.} \end{cases} \text{, where}$$

$$k = (r-1)\frac{q}{p}.$$

$$MD_\mu = 2\frac{(k+1)}{p} f(k+1) = 2(r+k)\frac{q}{p} f(k)$$

$$\mu'_{[k]} = (r+k-1)_{k} \times \left(\frac{q}{p}\right)^k$$

$$\mu = \frac{rq}{p} , \quad Var = \frac{rq}{p^2}$$

R

$$\mu_{k+1} = q\left[\frac{rk}{p^2}\mu_{k-1} - \frac{d\mu_k}{dp}\right]$$

$$\rightarrow P(X \le k) = P(Y \le p) = I_p(r, k+1)$$

$$M_X(t) = (Q - Pe^t)^{-r} = p^r(1 - qe^t)^{-r}$$

$$P_X(t) = p^r(1-qt)^{-r}$$

Scanned by CamScanner

$$\mu_3 = \frac{rq(1+q)}{p^3},$$

$$\mu_4 = \frac{rq}{p^4}\left(p^2 + 3rq + 6q\right)$$

Fitting :-

$$\frac{p(x)}{p(x-1)} = \frac{r+x-1}{x} q \; ; \; p(0) = p^r$$

∴ +vely skew & leptokurtic

$k, k-1$ or $k+1$ }
$[k]$, o.w. }

Mode

$\ni k = [\mu]$

MDu

Bin $\Rightarrow$ $k = (n+1)p$, $2p(n-k)f(k)$

Poisson $\Rightarrow$ $k = \lambda$, $2\lambda f(k)$

Hyp $\Rightarrow$ $k = (n+1)\left(\frac{Np+1}{N+2}\right)$, $2\left(p - \frac{k}{N}\right)(n-k)\,f(k)$

-ve Bin $\Rightarrow$ $k = (r-1)\frac{q}{p}$, $2\frac{(k+1)}{p}f(k+1)$

⊛ $E(X^n) = \int_0^\infty n \cdot x^{n-1} P[X > x]\, dx$

(14) Power Series Dist^n:-

$p(x) = a_x \dfrac{\theta^x}{g(\theta)}$, $a_x, \theta, g(\theta) > 0$, $\boxed{x = 0, 1, 2, \ldots}$

Cases:- (i) **Bin:-** $\theta = \dfrac{p}{q}$, $g(\theta) = (1+\theta)^n$

(ii) **Poisson:-** $\theta = \lambda$, $g(\theta) = e^{+\theta}$.

(iii) **-ve Bin:-** $\theta = q$, $g(\theta) = (1-\theta)^{-r}$

Here, $E(x) = \theta \dfrac{d}{d\theta}[\ln g(\theta)]$,

$$V(x) = E(x) + \theta^2 \frac{d^2}{d\theta^2}[\ln g(\theta)]$$

$MGF = \dfrac{g(\theta e^t)}{g(\theta)}$; $PGF = \dfrac{g(\theta t)}{g(\theta)}$

Recursion Rela^n:- $\mu_{k+1} = \theta \dfrac{d\mu_k}{d\theta} + k \mu_2 \mu_{k-1}$.

Cont. Dist^n s ⇒

⑮ **Cauchy Dist^n:** $X \sim C(\mu, \sigma)$

$$f_X(x) = \frac{\sigma}{\pi[\sigma^2 + (x-\mu)^2]}, \quad x \in \mathbb{R}$$

$(\mu \in \mathbb{R}, \sigma > 0)$

$$F_X(x) = \frac{1}{\pi}\left[\tan^{-1}\left(\frac{x-\mu}{\sigma}\right) + \frac{\pi}{2}\right]$$

Median $= \mu$ $=$ mode | Heavy tailed dist^n.

⊙ Only fractional order moments exist: $E(X^r) = \frac{1}{\pi} B\left(\frac{1-r}{2}, \frac{r+1}{2}\right), \; \underline{-1 < r < 1}$

↳ If $X \sim C(0,1)$ ⇒

Standard Cauchy dist^n, $\rightarrow F_X(x) = \frac{1}{\pi}\left[\tan^{-1}x + \frac{\pi}{2}\right]$

(platykurtic dist^n) $f_X(x) = \frac{1}{\pi(1+x^2)}, \; x \in \mathbb{R}$

MGF does not exist $[\text{CF} = e^{-|t|}]$

Mean & Var doesn't exist

① (16) Shifted Exponential distⁿ ⇒

$f_X(x) = \frac{1}{\theta} e^{-\frac{x-a}{\theta}}, \; x \geq a \; (\theta > 0).$

$F_X(x) = \begin{cases} 1 - e^{-\frac{x-a}{\theta}}, & x \geq a \\ 0, & x \leq a \end{cases}$ // $\boxed{\mu = a + \theta}$

$\xi_p = a - \ln(1-p)^{\theta}$; $MD_\mu = \theta \ln 2$

$QD = \frac{\theta}{2} \ln 3$; $\mu_r'(a) = \theta^r \cdot \lfloor r$ $(r > -1)$

$\mu = a + \theta$; $Var = \theta^2$; $\underline{Me = a + \theta \ln 2}$,

$\mu_3 = 2\theta^3$, $\mu_4 = 9\theta^4$

$\gamma_1 = 2$, $\gamma_2 = 6$

~~[illegible]~~ → $X \sim exp(\theta)$

④ **Exp Distⁿ**:- $\frac{1}{\theta} e^{-\frac{x}{\theta}}$, $x > 0$ ← also

$M_X(t) = (1-\theta t)^{-1}$, $\left(t < \frac{1}{\theta}\right)$ ⇒ $\underline{K_r = \theta^r \lfloor r-1}$.

Loss of memory ~~[illegible]~~ ⇒ $P[X > a+b \mid X > a] = P[X > b]$

$\mu_r' = \theta^r \lfloor r$ $(r > -1)$. [↳ Gamma$(\theta, 1)$]

(17) Laplace / Double-Exp distⁿ (μ, λ)

$$f_X(x) = \frac{1}{2\lambda} e^{-\left|\frac{x-\mu}{\lambda}\right|}, \; x \in \mathbb{R}$$

leptokurtic

$$(\mu \in \mathbb{R}, \lambda > 0)$$

$$= \begin{cases} \frac{1}{2\lambda} e^{-\left(\frac{x-\mu}{\lambda}\right)}, & x \geq \mu \\ \frac{1}{2\lambda} e^{\left(\frac{x-\mu}{\lambda}\right)}, & x < \mu \end{cases}$$

$$\mu_r'(\mu) = \begin{cases} 0, & r = \text{odd} \\ \lambda^r \, \underline{|r}, & r = \text{even} \end{cases}$$

$[\mu_r]$

Mode = median = mean = μ, Var = $2\lambda^2$

$$F_X(x) = \begin{cases} \frac{1}{2} e^{\frac{x-\mu}{\lambda}}, & x < \mu \\ 1 - \frac{1}{2} e^{-\frac{x-\mu}{\lambda}}, & x \geq \mu \end{cases}$$

$$M_X(t) = \frac{e^{\mu t}}{1 - \lambda^2 t^2}$$

$|t| < \frac{1}{\lambda}$

$$M_{X-\mu}(t) = \frac{1}{1 - t^2\lambda^2}$$

$f_X(x) = \frac{1}{2} e^{-|x|}$, $-\infty < x < \infty$

$\mu = 0, \lambda = 1 \Rightarrow$ Std. DE/Laplace distⁿ

$X \sim$ std Laplace $\Rightarrow |X| \sim$ std exp.

$\hookrightarrow M_X(t) = (1 - t^2)^{-1}$

(18) **Pareto Distⁿ** :- P(x_0, α)

$$f_X(x) = \frac{\alpha}{x_0}\times\left(\frac{x_0}{x}\right)^{\alpha+1}, \quad \forall\ x \geq x_0$$

$$P(x_0, \alpha) = \frac{\alpha\, x_0^{\alpha}}{x^{\alpha+1}} \quad \begin{pmatrix} x_0 > 0 \\ \alpha > 0 \end{pmatrix}$$

$$F_X(x) = \begin{cases} 0, & x < x_0 \\ 1 - \left(\frac{x_0}{x}\right)^{\alpha}, & x \geq x_0 \end{cases}$$

$(\alpha > 1)$

$$E(X) = \frac{\alpha x_0}{\alpha - 1}\ ; \quad V(X) = \frac{\alpha x_0^2}{(\alpha-1)^2(\alpha-2)}, \ (\alpha > 2)$$

$$HM = x_0\left(1 + \frac{1}{\alpha}\right)\ ; \quad GM = x_0 \times e^{1/\alpha}$$

$$M_e = x_0 \times 2^{1/\alpha}, \ \alpha > 0 \quad \Big| \quad M_0 = x_0$$

MGF does not exist | +vely skew

$$E|X|^r = \frac{\alpha x_0^r}{\alpha - r}, \quad r < \alpha$$

μ_r'

$$\underset{AM}{x_0 \times \frac{\alpha}{\alpha - 1}} > \underset{GM}{x_0 \times e^{1/\alpha}} > \underset{HM}{x_0 \times \left(\frac{\alpha + 1}{\alpha}\right)}$$

$$\mu \ '' > \underset{M_e}{x_0 \times 2^{1/\alpha}} > x_0 \ (M_0)$$

(19) Logistic $dist^n$; $L(\alpha, \beta)$

$$F_X(x) = \left[1 + e^{-\frac{x-\alpha}{\beta}}\right]^{-1}, \quad x \in \mathbb{R}, \quad (\alpha \in \mathbb{R}, \beta > 0)$$

* **Std. Logistic** :- Symm $dist^n$.

$\alpha = 0, \beta = 1 \rightarrow$ $F_Y(y) = \dfrac{1}{1+e^{-y}}$ ✓

$\therefore f_Y(y) = \dfrac{e^{-y}}{(1+e^{-y})^2}, \; y \in \mathbb{R}$.

$MD_\mu = 2 \ln 2$. $[\mu = 0]$ ✓

MGF, $M_Y(t) = \dfrac{\pi t}{\sin \pi t}$.

μ_2'

$Var = \dfrac{\pi^2}{3}$; $\mu_{2r+1} = 0 \; \forall r$.

Leptokurtic $\left[\because \mu_4 = \mu_4' = \dfrac{7\pi^4}{15}\right]$

Symmetrical > 3.

— ☺ —