Specification of Source §2 Stepper—2022 edition

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Reduction

The *reducer* $\Rightarrow$ is a partial function from programs/statements/expressions to programs/statements/expressions (with slight abuse of notation via overloading) and $\Rightarrow^*$ is its reflexive transitive closure. A *reduction* is a sequence of programs $p_1 \Rightarrow \cdots \Rightarrow p_n$, where p_n is not reducible, i.e. there is no program q such that $p_n \Rightarrow q$. Here, the program p_n is called the *result of reducing* p_1.

If p_n, the result of reducing a program, is of the form $v;$, we call v the result of the program evaluation. If p_n is the empty sequence of statements, we declare the result of the program evaluation to be the value undefined. Reduction can get "stuck": the result can be a program that has neither of these two forms, which corresponds to a runtime error.

A *value* is a primitive number expression, primitive boolean expression, a primitive string expression, a function definition expression or a function declaration expression.

The *substitution* function $p[n := v]$ on programs/statements/expressions replaces every free occurrence of the name n in statement p by value v. Care must be taken to introduce and preserve co-references in this process; substitution can introduce cyclic references in the result of the substitution. For example, n may occur free in v, in which case every occurrence of n in p will be replaced by v such that n in v refers cyclically to the node at which the replacement happens.

Programs

Program-intro: In a sequence of statements, we can always reduce the first one that isn't a value statement.

$$\frac{statement \Rightarrow statement'}{[value;] \ statement\ldots \Rightarrow [value;] \ statement'\ldots}$$

Program-reduce: When the first two statements in a program are value statements, the first one can be removed.

$$\frac{v1, v2 \text{ are values}}{v1; \ v2; \ statement\ldots \Rightarrow v2; \ statement\ldots}$$

Eliminate-function-declaration: Function declarations as the first statement optionally after one value statement are substituted in the remaining statements.

$$\frac{f = \texttt{function} \ name \ (\ parameters\ ) \ block}{[value;] \ f \ statement\ldots \ \Rightarrow \ [value;] \ statement\ldots[name := f]}$$

Eliminate-constant-declaration: Constant declarations as the first statement optionally after one value statement are substituted in the remaining statements.

$$\frac{c = \texttt{const} \ name \ \texttt{=} \ v}{[value;] \ c \ statement\ldots \ \Rightarrow \ [value;] \ statement\ldots[name := v]}$$

Statements: Expression statements

Expression-statement-reduce: An expression statement is reducible if its expression is reducible.

$$\frac{e \;\Rightarrow\; e'}{e; \;\Rightarrow\; e';}$$

Statements: Constant declarations

Evaluate-constant-declaration: The right-hand expressions in constant declarations are evaluated.

$$\frac{expression \;\Rightarrow\; expression'}{\texttt{const } name \texttt{ = } expression \;\Rightarrow\; \texttt{const } name \texttt{ = } expression'}$$

Statements: Conditionals

Conditional-statement-predicate: A conditional statement is reducible if its predicate is reducible.

$$\frac{e \;\Rightarrow\; e'}{\texttt{if (} e \texttt{) \{ } \cdots \texttt{\} else \{ } \cdots \texttt{\}} \;\Rightarrow\; \texttt{if (} e' \texttt{) \{ } \cdots \texttt{\} else \{ } \cdots \texttt{\}}}$$

SICP-JS Exercise 4.8 specifies that a conditional statement where the taken branch is non-value-producing is value-producing, with its value being **undefined**.

Conditional-statement-consequent: Immediately within a program or block statement, a conditional statement whose predicate is true reduces to the consequent block.

$$\overline{\texttt{if (true) \{ } program_1 \texttt{\} else \{ } program_2 \texttt{\}} \;\Rightarrow\; \texttt{\{ undefined; } \;\; program_1 \texttt{\}}}$$

Conditional-statement-alternative: Immediately within a program or block statement, a conditional statement whose predicate is false reduces to the alternative block.

$$\overline{\texttt{if (false) \{ } program_1 \texttt{\} else \{ } program_2 \texttt{\}} \;\Rightarrow\; \texttt{\{ undefined; } \;\; program_2 \texttt{\}}}$$

We can remove the **undefined;** if the conditional statement is in a block expression without affecting the return value of the block expression. This is performed to reduce the surprise factor.

Conditional-statement-blockexpr-consequent: Immediately within a block expression, a conditional statement whose predicate is true reduces to the consequent block.

$$\overline{\texttt{\{ } [value;] \texttt{ if (true) \{ } program_1 \texttt{\} else \{ } program_2 \texttt{\} \}} \;\Rightarrow\; \texttt{\{}[value;] \texttt{ \{ } program_1 \texttt{\}\}}}$$

Conditional-statement-blockexpr-alternative: Immediately within a block expression, a conditional statement whose predicate is false reduces to the alternative block.

$$\overline{\texttt{\{ } [value;] \texttt{ if (false) \{ } program_1 \texttt{\} else \{ } program_2 \texttt{\} \}} \;\Rightarrow\; \texttt{\{}[value;] \texttt{ \{ } program_2 \texttt{\}\}}}$$

Statements: Blocks

Block-statement-intro: A block statement is reducible if its body is reducible.

$$\frac{program \;\Rightarrow\; program'}{\texttt{\{ } program \texttt{ \}} \;\Rightarrow\; \texttt{\{ } program' \texttt{ \}}}$$

Block-statement-single-reduce: A block statement consisting of a single value statement reduces to that value statement.

$$\overline{\{value;\} \Rightarrow value;}$$

Block-statement-empty-reduce: An empty block statement (or a non-value- producing one, which will reduce to the empty block statement) is non-value-producing, i.e. reduces to ε.

$$\overline{\{\} \Rightarrow \varepsilon}$$

Expresssions: Blocks

Block-expression-intro: A block expression is reducible if its body is reducible.

$$\frac{program \;\Rightarrow\; program'}{\{\; program \;\} \;\Rightarrow\; \{\; program' \;\}}$$

Block-expression-single-reduce: A block expression consisting of a single value statement reduces to **undefined**.

$$\overline{\{value;\} \Rightarrow \texttt{undefined}}$$

Block-expression-empty-reduce: An empty block expression reduces to **undefined**.

$$\overline{\{\} \Rightarrow \texttt{undefined}}$$

Block-expression-return-reduce-1: In a block expression starting with a value statement and then a return statement, the value statement can be removed.

$$\overline{\{value;\; \texttt{return}\; return\text{-}expression;\; statement\ldots\} \;\Rightarrow\; \{\texttt{return}\; return\text{-}expression;\; statement\ldots\}}$$

Block-expression-return-reduce-2: A block expression starting with a return statement reduces to the expression of the return statement.

$$\overline{\{\texttt{return}\; return\text{-}expression;\; statement\ldots\} \Rightarrow return\text{-}expression}$$

Block expressions are currently used only as expanded forms of functions.

Expressions: Binary operators

Left-binary-reduce: An expression with binary operator can be reduced if its left sub-expression can be reduced.

$$\frac{e_1 \;\Rightarrow\; e_1'}{e_1\; binary\text{-}operator\; e_2 \;\Rightarrow\; e_1'\; binary\text{-}operator\; e_2}$$

And-shortcut-false: An expression with binary operator **&&** whose left sub-expression is **false** can be reduced to **false**.

$$\overline{\texttt{false \&\&}\; e \;\Rightarrow\; \texttt{false}}$$

And-shortcut-true: An expression with binary operator **&&** whose left sub-expression is **true** can be reduced to the right sub-expression.

$$\overline{\texttt{true \&\&}\; e \;\Rightarrow\; e}$$

Or-shortcut-true: An expression with binary operator `||` whose left sub-expression is `true` can be reduced to `true`.

$$\overline{\texttt{true} \mathbin{\texttt{||}} e \;\Rightarrow\; \texttt{true}}$$

Or-shortcut-false: An expression with binary operator `||` whose left sub-expression is `false` can be reduced to the right sub-expression.

$$\overline{\texttt{false} \mathbin{\texttt{||}} e \;\Rightarrow\; e}$$

Right-binary-reduce: An expression with binary operator can be reduced if its left sub-expression is a value and its right sub-expression can be reduced.

$$\frac{e_2 \;\Rightarrow\; e_2', \text{and } \textit{binary-operator} \text{ is not } \texttt{\&\&} \text{ or } \texttt{||}}{v \; \textit{binary-operator} \; e_2 \;\Rightarrow\; v \; \textit{binary-operator} \; e_2'}$$

Prim-binary-reduce: An expression with binary operator can be reduced if its left and right sub-expressions are values and the corresponding function is defined for those values.

$$\frac{v \text{ is result of } v_1 \; \textit{binary-operator} \; v_2}{v_1 \; \textit{binary-operator} \; v_2 \;\Rightarrow\; v}$$

Expressions: Unary operators

Unary-reduce: An expression with unary operator can be reduced if its sub-expression can be reduced.

$$\frac{e \;\Rightarrow\; e'}{\textit{unary-operator} \; e \;\Rightarrow\; \textit{unary-operator} \; e'}$$

Prim-unary-reduce: An expression with unary operator can be reduced if its sub-expression is a value and the corresponding function is defined for that value.

$$\frac{v' \text{ is result of } \textit{unary-operator} \; v}{\textit{unary-operator} \; v \;\Rightarrow\; v'}$$

Expressions: conditionals

Conditional-predicate-reduce: A conditional expression can be reduced, if its predicate can be reduced.

$$\frac{e_1 \;\Rightarrow\; e_1'}{e_1 \mathbin{\texttt{?}} e_2 \mathbin{\texttt{:}} e_3 \;\Rightarrow\; e_1' \mathbin{\texttt{?}} e_2 \mathbin{\texttt{:}} e_3}$$

Conditional-true-reduce: A conditional expression whose predicate is the value `true` can be reduced to its consequent expression.

$$\overline{\texttt{true} \mathbin{\texttt{?}} e_1 \mathbin{\texttt{:}} e_2 \;\Rightarrow\; e_1}$$

Conditional-false-reduce: A conditional expression whose predicate is the value `false` can be reduced to its alternative expression.

$$\overline{\texttt{false} \mathbin{\texttt{?}} e_1 \mathbin{\texttt{:}} e_2 \;\Rightarrow\; e_2}$$

Expressions: function application

Application-functor-reduce: A function application can be reduced if its functor expression can be reduced.

$$\frac{e \;\Rightarrow\; e'}{e \;(\; \textit{expressions} \;) \;\Rightarrow\; e' \;(\; \textit{expressions} \;)}$$

Application-argument-reduce: A function application can be reduced if one of its argument expressions can be reduced and all preceding arguments are values.

$$\frac{e \;\Rightarrow\; e'}{v \;(\; v_1 \ldots v_i \; e \ldots \;) \;\Rightarrow\; v \;(\; v_1 \ldots v_i \; e' \ldots \;)}$$

Function-declaration-application-reduce: The application of a function declaration can be reduced, if all arguments are values.

$$\frac{f = \texttt{function } n \;(\; x_1 \ldots x_n \;) \; \textit{block}}{f \;(\; v_1 \ldots v_n \;) \;\Rightarrow\; \textit{block}[x_1 := v_1] \ldots [x_n := v_n][n := f]}$$

Function-definition-application-reduce: The application of a function definition can be reduced, if all arguments are values.

$$\frac{f = \;(\; x_1 \ldots x_n \;) \;\texttt{=>}\; b, \text{ where } b \text{ is an expression or block}}{f \;(\; v_1 \ldots v_n \;) \;\Rightarrow\; b[x_1 := v_1] \ldots [x_n := v_n]}$$

Substitution

Identifier: An identifier with the same name as x is substituted with e_x.

$$\overline{x[x := e_x] \; = \; e_x}$$

$$\frac{name \; \neq \; x}{name[x := e_x] \; = \; name}$$

Expression statement: All occurrences of x in e are substituted with e_x.

$$\overline{e\texttt{;}\,[x := e_x] \; = \; e\,[x := e_x]\texttt{;}}$$

Binary expression: All occurrences of x in the operands are substituted with e_x.

$$\overline{(e_1 \; \textit{binary-operator} \; e_2)[x := e_x] \; = \; e_1[x := e_x] \; \textit{binary-operator} \; e_2[x := e_x]}$$

Unary expression: All occurrences of x in the operand are substituted with e_x.

$$\overline{(\textit{unary-operator} \; e)[x := e_x] \; = \; \textit{unary-operator} \; e[x := e_x]}$$

Conditional expression: All occurrences of x in the operands are substituted with e_x.

$$\overline{(e_1 \; \texttt{?} \; e_2 \; \texttt{:} \; e_3)[x := e_x] \; = \; e_1[x := e_x] \; \texttt{?} \; e_2[x := e_x] \; \texttt{:} \; e_3[x := e_x]}$$

Logical expression: All occurrences of x in the operands are substituted with e_x.

$$\overline{(e_1 \; \texttt{||} \; e_2)[x := e_x] \; = \; e_1[x := e_x] \; \texttt{||} \; e_2[x := e_x]}$$

$$\overline{(e_1 \; \texttt{\&\&} \; e_2)[x := e_x] \; = \; e_1[x := e_x] \; \texttt{\&\&} \; e_2[x := e_x]}$$

Call expression: All occurrences of x in the arguments and the function expression of the application e are substituted with e_x.

$$\overline{(e \; \texttt{(} \; x_1 \ldots x_n \; \texttt{)}\,)[x := e_x] \; = \; e[x := e_x] \; \texttt{(} \; x_1[x := e_x] \ldots x_n[x := e_x] \; \texttt{)}}$$

Function declaration: All occurrences of x in the body of a function are substituted with e_x under given circumstances.

1. Function declaration where x has the same name as a parameter.

$$\frac{\exists i \in \{1, \cdots, n\} \; \text{s.t.} \; x \; = \; x_i}{(\texttt{function} \; name \; \texttt{(} \; x_1 \ldots x_n \; \texttt{)} \; block)[x := e_x] \; = \; \texttt{function} \; name \; \texttt{(} \; x_1 \ldots x_n \; \texttt{)} \; block}$$

2. Function declaration where x does not have the same name as a parameter.

(a) No parameter of the function occurs free in e_x.

$$\frac{\forall i \in \{1, \cdots, n\} \text{ s.t. } x \neq x_i, \ \forall j \in \{1, \cdots, n\} \text{ s.t. } x_j \text{ does not occur free in } e_x}{(\textbf{function } \textit{name} \ (\ x_1 \ldots x_n \ ) \ \textit{block})[x := e_x]}$$
$$=$$
$$\textbf{function } \textit{name} \ (\ x_1 \ldots x_n \ ) \ \textit{block}[x := e_x]$$

(b) A parameter of the function occurs free in e_x.

$$\frac{\forall i \in \{1, \cdots, n\} \text{ s.t. } x \neq x_i, \ \exists j \in \{1, \cdots, n\} \text{ s.t. } x_j \text{ occurs free in } e_x}{(\textbf{function } \textit{name} \ (\ x_1 \ldots x_j \ldots x_n \ ) \ \textit{block})[x := e_x]}$$
$$=$$
$$(\textbf{function } \textit{name} \ (\ x_1 \ldots y \ldots x_n \ ) \ \textit{block}[x_j := y])[x := e_x]$$

Substitution is applied to the whole expression again as to recursively detect and re-name all parameters of the function declaration that clash with variables that occur free in e_x, at which point (i) takes place. Note that the name y is not declared in, nor occurs in *block* and e_x.

Lambda expression: All occurrences of x in the body of a lambda expression are substituted with e_x under given circumstances.

1. Lambda expression where x has the same name as a parameter.

$$\frac{\exists i \in \{1, \cdots, n\} \text{ s.t. } x = x_i}{((\ x_1 \ldots x_n \ ) \ \texttt{=>} \ \textit{block})[x := e_x] \ = \ (\ x_1 \ldots x_n \ ) \ \texttt{=>} \ \textit{block}}$$

2. Lambda expression where x does not have the same name as a parameter.

   (a) No parameter of the lambda expression occurs free in e_x.

$$\frac{\forall i \in \{1, \cdots, n\} \text{ s.t. } x \neq x_i, \ \forall j \in \{1, \cdots, n\} \text{ s.t. } x_j \text{ does not occur free in } e_x}{((\ x_1 \ldots x_n \ ) \ \texttt{=>} \ \textit{block})[x := e_x] \ = \ (\ x_1 \ldots x_n \ ) \ \texttt{=>} \ \textit{block}[x := e_x]}$$

   (b) A parameter of the lambda expression occurs free in e_x.

$$\frac{\forall i \in \{1, \cdots, n\} \text{ s.t. } x \neq x_i, \ \exists j \in \{1, \cdots, n\} \text{ s.t. } x_j \text{ occurs free in } e_x}{((\ x_1 \ldots x_j \ldots x_n \ ) \ \texttt{=>} \ \textit{block})[x := e_x] \ = \ ((\ x_1 \ldots y \ldots x_n \ ) \ \texttt{=>} \ \textit{block}[x_j := y])[x := e_x]}$$

   Substitution is applied to the whole expression again as to recursively detect and re-name all parameters of the lambda expression that clash with variables that occur free in e_x, at which point (i) takes place. Note that the name y is not declared in, nor occurs in *block* and e_x.

Block expression: All occurrences of x in the statements of a block expression are substituted with e_x under given circumstances.

1. Block expression in which x is declared.

$$\frac{x \text{ is declared in } \textit{block}}{\textit{block}[x := e_x] \;=\; \textit{block}}$$

2. Block expression in which x is not declared.

   (a) No names declared in the block occurs free in e_x.

   $$\frac{x \text{ is not declared in } \textit{block}, \quad \textit{name} \text{ declared in } \textit{block} \text{ does not occur free in } e_x}{\textit{block}[x := e_x] \;=\; [\textit{block}[0][x := e_x], \; \dots, \; \textit{block}[n][x := e_x]]}$$

   (b) A name declared in the block occurs free in e_x.

   $$\frac{x \text{ is not declared in } \textit{block}, \quad \textit{name} \text{ declared in } \textit{block} \text{ occurs free in } e_x}{\textit{block}[x := e_x] \;=\; [\textit{block}[0][\textit{name} := y], \; \dots, \; \textit{block}[n][\textit{name} := y]][x := e_x]}$$

   Substitution is applied to the whole expression again as to recursively detect and rename all declared names of the block expression that clash with variables that occur free in e_x, at which point (i) takes place. Note that the name y is not declared in, nor occurs in *block* and e_x.

Variable declaration: All occurrences of x in the declarators of a variable declaration are substituted with e_x.

$$\overline{\textit{declarations}[x := e_x] \;=\; [\textit{declarations}[0][x := e_x] \dots \textit{declarations}[n][x := e_x]]}$$

Return statement: All occurrences of x in the expression that is to be returned are substituted with e_x.

$$\overline{(\texttt{return } e\texttt{; })[x := e_x] \;=\; \texttt{return } e[x := e_x]\texttt{;}}$$

Conditional statement: All occurrences of x in the condition, consequent, and alternative expressions of a conditional statement are substituted with e_x.

$$\overline{(\texttt{if (} e \texttt{) } \textit{block} \texttt{ else } \textit{block})[x := e_x] \;=\; \texttt{if (} e[x := e_x] \texttt{) } \textit{block}[x := e_x] \texttt{ else } \textit{block}[x := e_x]}$$

Array expression: All occurrences of x in the elements of an array are substituted with e_x.

$$\overline{[x_1, \; \dots, \; x_n][x := e_x] \;=\; [x_1[x := e_x], \; \dots, \; x_n[x := e_x]]}$$

Free names

Let $\rhd$ be the relation that defines the set of free names of a given Source expression; the symbols p_1 and p_2 shall henceforth refer to unary and binary operations, respectively. That is, p_1 ranges over $\{\,!\,\}$ and p_2 ranges over $\{\,|\,|\,,\, \&\&,\, +,\, -,\, *,\, /,\, ===,\, >,\, <\,\}$.

Identifier:

$$\frac{}{x \,\rhd\, \{x\}}$$

$$\frac{}{\mathit{name} \,\rhd\, \varnothing}$$

Boolean:

$$\frac{}{\mathtt{true} \,\rhd\, \varnothing}$$

$$\frac{}{\mathtt{false} \,\rhd\, \varnothing}$$

Expression statement:

$$\frac{e \,\rhd\, S}{e\texttt{;} \,\rhd\, S}$$

Unary expression:

$$\frac{e \,\rhd\, S}{p_1(e) \,\rhd\, S}$$

Binary expression:

$$\frac{e_1 \,\rhd\, S_1,\ \ e_2 \,\rhd\, S_2}{p_1(e_1, e_2) \,\rhd\, S_1 \cup S_2}$$

Conditional expression:

$$\frac{e_1 \,\rhd\, S_1,\ \ e_2 \,\rhd\, S_2,\ \ e_3 \,\rhd\, S_3}{e_1 \,\texttt{?}\, e_2 \,\texttt{:}\, e_3 \,\rhd\, S_1 \cup S_2 \cup S_3}$$

Call expression:

$$\frac{e \,\rhd\, S,\ \ e_k \,\rhd\, T_k}{e(e_1,\, \ldots,\, e_n) \,\rhd\, S \cup T_1 \cup \ldots \cup T_n}$$

Function declaration:

$$\frac{\mathit{block} \,\rhd\, S}{\texttt{function}\ \mathit{name}\ \texttt{(}\ x_1 \ldots x_n\ \texttt{)}\ \mathit{block} \,\rhd\, S - \{x_1,\, \ldots,\, x_n\}}$$

Lambda expression:

$$\frac{\mathit{block} \,\rhd\, S}{\texttt{(}\ x_1 \ldots x_n\ \texttt{)}\ \texttt{=>}\ \mathit{block} \,\rhd\, S - \{x_1,\, \ldots,\, x_n\}}$$

Block expression:

$$\frac{block[k] \;\triangleright\; S_k, \;\; T \text{ contains all names declared in } block}{block \;\triangleright\; (S_1 \cup \ldots \cup S_n) - T}$$

Constant declaration:

$$\frac{e \;\triangleright\; S}{\texttt{const } name \texttt{ = } e\texttt{;} \;\triangleright\; S}$$

Return statement:

$$\frac{e \;\triangleright\; S}{\texttt{return } e\texttt{;} \;\triangleright\; S}$$

Conditional statement:

$$\frac{e \;\triangleright\; S, \;\; block_1 \;\triangleright\; T_1, \;\; block_2 \;\triangleright\; T_2}{\texttt{if (} e \texttt{)} \; block_1 \; \texttt{else} \; block_2 \;\triangleright\; S \cup T_1 \cup T_2}$$