Specification of Source §2 Stepper—2021 edition

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Reduction

The *reducer* $\Rightarrow$ is a partial function from programs/statements/expressions to programs/statements/expressions (with slight abuse of notation via overloading) and $\Rightarrow^*$ is its reflexive transitive closure. A *reduction* is a sequence of programs $p_1 \Rightarrow \cdots \Rightarrow p_n$, where p_n is not reducible, i.e. there is no program q such that $p_n \Rightarrow q$. Here, the program p_n is called the *result of reducing* p_1.
A *value* is a primitive number expression, primitive boolean expression, a primitive string expression, a function definition expression or a function declaration statement.
The *substitution* function $p[n := v]$ on programs/statements/expressions replaces every free occurrence of the name n in statement p by value v. Care must be taken to introduce and preserve co-references in this process; substitution can introduce cyclic references in the result of the substitution. For example, n may occur free in v, in which case every occurrence of n in p will be replaced by v such that n in v refers cyclically to the node at which the replacement happens.

Programs

First-statement: In a sequence of statements, we can always reduce the first one.

$$\frac{\textit{statement} \;\Rightarrow\; \textit{statement}'}{\textit{statement} \ldots \Rightarrow \textit{statement}' \ldots}$$

Eliminate-function-declaration: Function declarations as first statements are substituted in the remaining statements.

$$\frac{f = \texttt{function}\ \textit{name}\ \texttt{(}\ \textit{parameters}\ \texttt{)}\ \textit{block}}{f\ \textit{statement} \ldots \;\Rightarrow\; \textit{statement} \ldots [\textit{name} := f]}$$

Eliminate-constant-declaration: Constant declarations as first statements are substituted in the remaining statements.

$$\frac{c = \texttt{const}\ \textit{name}\ \texttt{=}\ v}{c\ \textit{statement} \ldots \;\Rightarrow\; \textit{statement} \ldots [\textit{name} := v]}$$

Eliminate-Values: Values as first statemments are discarded, if they are preceding one or more statements in a statement sequence.

$$\frac{v \text{ is a value}}{v\texttt{;}\ \textit{statement}+ \;\Rightarrow\; \textit{statement}+}$$

Statements: Constant declarations

Evaluate-constant-declaration: The right-hand expressions in constant declarations are evaluated.

$$\frac{\textit{expression} \;\Rightarrow\; \textit{expression}'}{\texttt{const}\ \textit{name}\ \texttt{=}\ \textit{expression} \;\Rightarrow\; \texttt{const}\ \textit{name}\ \texttt{=}\ \textit{expression}'}$$

Statements: Conditionals

Conditional-statement-predicate: A conditional statement is reducible if its predicate is reducible.

$$\frac{e \;\Rightarrow\; e'}{\texttt{if (} e \texttt{) \{ } \cdots \texttt{\} else \{ } \cdots \texttt{\}} \;\Rightarrow\; \texttt{if (} e' \texttt{) \{ } \cdots \texttt{\} else \{ } \cdots \texttt{\}}}$$

Conditional-statement-consequent: A conditional statement whose predicate is true reduces to the consequent block.

$$\frac{}{\texttt{if (true) \{ } \textit{statement}_1 \texttt{\} else \{ } \textit{statement}_2 \texttt{\}} \;\Rightarrow\; \texttt{\{ } \textit{statement}_1 \texttt{\}}}$$

Conditional-statement-alternative: A conditional statement whose predicate is false reduces to the alternative block.

$$\frac{}{\texttt{if (false) \{ } \textit{statement}_1 \texttt{\} else \{ } \textit{statement}_2 \texttt{\}} \;\Rightarrow\; \texttt{\{ } \textit{statement}_2 \texttt{\}}}$$

Statements: Blocks

Block-statement-reduce: A block statement is reducible if its program is reducible.

$$\frac{\textit{program} \;\Rightarrow\; \textit{program}'}{\texttt{\{ } \textit{program} \texttt{ \}} \;\Rightarrow\; \texttt{\{ } \textit{program}' \texttt{ \}}}$$

Block-statement-undefined: A block statement whose body only contains a single value statement reduces to the value `undefined`.

$$\frac{}{\texttt{\{ } v \texttt{ ; \}} \;\Rightarrow\; \texttt{undefined}}$$

Block-statement-reduce-return: A block statement whose body only contains a single return statement can be reduced by reducing the return expression.

$$\frac{e \;\Rightarrow\; e'}{\texttt{\{ return } e \texttt{ ; \}} \;\Rightarrow\; \texttt{\{ return } e' \texttt{ ; \}}}$$

Block-statement-eliminate-return: A block statement whose body only contains a single return value reduces to that value.

$$\frac{}{\texttt{\{ return } v \texttt{ ; \}} \;\Rightarrow\; v}$$

Statements: Expression statements

Expression-statement-reduce: An expression statement is reducible if its expression is reducible.

$$\frac{e \;\Rightarrow\; e'}{e\texttt{;} \;\Rightarrow\; e'\texttt{;}}$$

Expressions: Binary operators

Left-binary-reduce: An expression with binary operator can be reduced if its left sub-expression can be reduced.

$$\frac{e_1 \;\Rightarrow\; e_1'}{e_1 \textit{ binary-operator } e_2 \;\Rightarrow\; e_1' \textit{ binary-operator } e_2}$$

And-shortcut-false: An expression with binary operator `&&` whose left sub-expression is `false` can be reduced to `false`.

$$\frac{}{\texttt{false \&\& } e \;\Rightarrow\; \texttt{false}}$$

And-shortcut-true: An expression with binary operator `&&` whose left sub-expression is `true` can be reduced to the right sub-expression.

$$\frac{}{\texttt{true \&\& } e \;\Rightarrow\; e}$$

Or-shortcut-true: An expression with binary operator `||` whose left sub-expression is `true` can be reduced to `true`.

$$\frac{}{\texttt{true || } e \;\Rightarrow\; \texttt{true}}$$

Or-shortcut-false: An expression with binary operator `||` whose left sub-expression is `false` can be reduced to the right sub-expression.

$$\frac{}{\texttt{false || } e \;\Rightarrow\; e}$$

Right-binary-reduce: An expression with binary operator can be reduced if its left sub-expression is a value and its right sub-expression can be reduced.

$$\frac{e_2 \;\Rightarrow\; e_2', \text{and } \textit{binary-operator}\text{is not } \texttt{\&\&} \text{ or } \texttt{||}}{v \textit{ binary-operator } e_2 \;\Rightarrow\; v \textit{ binary-operator } e_2'}$$

Prim-binary-reduce: An expression with binary operator can be reduced if its left and right sub-expressions are values and the corresponding function is defined for those values.

$$\frac{v \text{ is result of } v_1 \textit{ binary-operator } v_2}{v_1 \textit{ binary-operator } v_2 \;\Rightarrow\; v}$$

Expressions: Unary operators

Unary-reduce: An expression with unary operator can be reduced if its sub-expression can be reduced.

$$\frac{e \;\Rightarrow\; e'}{\textit{unary-operator } e \;\Rightarrow\; \textit{unary-operator } e'}$$

Prim-unary-reduce: An expression with unary operator can be reduced if its sub-expression is a value and the corresponding function is defined for that value.

$$\frac{v' \text{ is result of } \textit{unary-operator } v}{\textit{unary-operator } v \;\Rightarrow\; v'}$$

Expressions: conditionals

Conditional-predicate-reduce: A conditional expression can be reduced, if its predicate can be reduced.

$$\frac{e_1 \ \Rightarrow \ e_1'}{e_1 \ \texttt{?} \ e_2 \ \texttt{:} \ e_3 \ \Rightarrow \ e_1' \ \texttt{?} \ e_2 \ \texttt{:} \ e_3}$$

Conditional-true-reduce: A conditional expression whose predicate is the value **`true`** can be reduced to its consequent expression.

$$\frac{}{\texttt{true ?} \ e_1 \ \texttt{:} \ e_2 \ \Rightarrow \ e_1}$$

Conditional-false-reduce: A conditional expression whose predicate is the value **`false`** can be reduced to its alternative expression.

$$\frac{}{\texttt{false ?} \ e_1 \ \texttt{:} \ e_2 \ \Rightarrow \ e_2}$$

Expressions: function application

Application-functor-reduce: A function application can be reduced if its functor expression can be reduced.

$$\frac{e \ \Rightarrow \ e'}{e \ \texttt{(} \ \textit{expressions} \ \texttt{)} \ \Rightarrow \ e' \ \texttt{(} \ \textit{expressions} \ \texttt{)}}$$

Application-argument-reduce: A function application can be reduced if one of its argument expressions can be reduced and all preceding arguments are values.

$$\frac{e \ \Rightarrow \ e'}{v \ \texttt{(} \ v_1 \ldots v_i \ e \ldots \ \texttt{)} \ \Rightarrow \ v \ \texttt{(} \ v_1 \ldots v_i \ e' \ldots \ \texttt{)}}$$

Function-declaration-application-reduce: The application of a function declaration can be reduced, if all arguments are values.

$$\frac{f = \texttt{function} \ n \ \texttt{(} \ x_1 \ldots x_n \ \texttt{)} \ \textit{block}}{f \ \texttt{(} \ v_1 \ldots v_n \ \texttt{)} \ \Rightarrow \ \textit{block}[x_1 := v_1] \ldots [x_n := v_n][n := f]}$$

Function-definition-application-reduce: The application of a function definition can be reduced, if all arguments are values.

$$\frac{f = \texttt{(} \ x_1 \ldots x_n \ \texttt{)} \ \texttt{=>} \ b, \text{ where } b \text{ is an expression or block}}{f \ \texttt{(} \ v_1 \ldots v_n \ \texttt{)} \ \Rightarrow \ b[x_1 := v_1] \ldots [x_n := v_n]}$$

Substitution

Identifier: An identifier with the same name as x is substituted with e_x.

$$\frac{}{x[x := e_x] \ = \ e_x}$$

$$\frac{\textit{name} \ \neq \ x}{\textit{name}[x := e_x] \ = \ \textit{name}}$$

Expression statement: All occurrences of x in e are substituted with e_x.

$$\frac{}{e\texttt{;}\,[x := e_x] \ = \ e\,[x := e_x]\texttt{;}}$$

Binary expression: All occurrences of x in the operands are substituted with e_x.

$$\frac{}{(e_1 \ \textit{binary-operator}\ e_2)[x := e_x] \ = \ e_1[x := e_x] \ \textit{binary-operator}\ e_2[x := e_x]}$$

Unary expression: All occurrences of x in the operand are substituted with e_x.

$$\frac{}{(\textit{unary-operator}\ e)[x := e_x] \ = \ \textit{unary-operator}\ e[x := e_x]}$$

Conditional expression: All occurrences of x in the operands are substituted with e_x.

$$\frac{}{(e_1 \ \texttt{?}\ e_2 \ \texttt{:}\ e_3)[x := e_x] \ = \ e_1[x := e_x] \ \texttt{?}\ e_2[x := e_x] \ \texttt{:}\ e_3[x := e_x]}$$

Logical expression: All occurrences of x in the operands are substituted with e_x.

$$\frac{}{(e_1 \ \texttt{||}\ e_2)[x := e_x] \ = \ e_1[x := e_x] \ \texttt{||}\ e_2[x := e_x]}$$

$$\frac{}{(e_1 \ \texttt{\&\&}\ e_2)[x := e_x] \ = \ e_1[x := e_x] \ \texttt{\&\&}\ e_2[x := e_x]}$$

Call expression: All occurrences of x in the arguments and the function expression of the application e are substituted with e_x.

$$\frac{}{(e \ \texttt{(}\ x_1 \ldots x_n \ \texttt{)})[x := e_x] \ = \ e[x := e_x] \ \texttt{(}\ x_1[x := e_x] \ldots x_n[x := e_x] \ \texttt{)}}$$

Function declaration: All occurrences of x in the body of a function are substituted with e_x under given circumstances.

① Function declaration where x has the same name as a parameter.

$$\frac{\exists\, i \in \{1, \cdots, n\} \text{ s.t. } x \;=\; x_i}{(\texttt{function}\ \textit{name}\ \texttt{(}\ x_1 \ldots x_n\ \texttt{)}\ \textit{block})[x := e_x] \;=\; \texttt{function}\ \textit{name}\ \texttt{(}\ x_1 \ldots x_n\ \texttt{)}\ \textit{block}}$$

② Function declaration where x does not have the same name as a parameter.

(i) No parameter of the function occurs free in e_x.

$$\frac{\forall\, i \in \{1, \cdots, n\} \text{ s.t. } x \;\neq\; x_i,\;\; \forall\, j \in \{1, \cdots, n\} \text{ s.t. } x_j \text{ does not occur free in } e_x}{\begin{array}{c}(\texttt{function}\ \textit{name}\ \texttt{(}\ x_1 \ldots x_n\ \texttt{)}\ \textit{block})[x := e_x] \\ = \\ \texttt{function}\ \textit{name}\ \texttt{(}\ x_1 \ldots x_n\ \texttt{)}\ \textit{block}[x := e_x]\end{array}}$$

(ii) A parameter of the function occurs free in e_x.

$$\frac{\forall\, i \in \{1, \cdots, n\} \text{ s.t. } x \;\neq\; x_i,\;\; \exists\, j \in \{1, \cdots, n\} \text{ s.t. } x_j \text{ occurs free in } e_x}{\begin{array}{c}(\texttt{function}\ \textit{name}\ \texttt{(}\ x_1 \ldots x_j \ldots x_n\ \texttt{)}\ \textit{block})[x := e_x] \\ = \\ (\texttt{function}\ \textit{name}\ \texttt{(}\ x_1 \ldots y \ldots x_n\ \texttt{)}\ \textit{block}[x_j := y])[x := e_x]\end{array}}$$

Substitution is applied to the whole expression again as to recursively detect and rename all parameters of the function declaration that clash with variables that occur free in e_x, at which point (i) takes place. Note that the name y is not declared in, nor occurs in *block* and e_x.

Lambda expression: All occurrences of x in the body of a lambda expression are substituted with e_x under given circumstances.

① Lambda expression where x has the same name as a parameter.

$$\frac{\exists\, i \in \{1, \cdots, n\} \text{ s.t. } x \;=\; x_i}{(\texttt{(}\ x_1 \ldots x_n\ \texttt{)}\ \texttt{=>}\ \textit{block})[x := e_x] \;=\; \texttt{(}\ x_1 \ldots x_n\ \texttt{)}\ \texttt{=>}\ \textit{block}}$$

② Lambda expression where x does not have the same name as a parameter.

(i) No parameter of the lambda expression occurs free in e_x.

$$\frac{\forall\, i \in \{1, \cdots, n\} \text{ s.t. } x \;\neq\; x_i,\;\; \forall\, j \in \{1, \cdots, n\} \text{ s.t. } x_j \text{ does not occur free in } e_x}{(\texttt{(}\ x_1 \ldots x_n\ \texttt{)}\ \texttt{=>}\ \textit{block})[x := e_x] \;=\; \texttt{(}\ x_1 \ldots x_n\ \texttt{)}\ \texttt{=>}\ \textit{block}[x := e_x]}$$

(ii) A parameter of the lambda expression occurs free in e_x.

$$\frac{\forall\, i \in \{1, \cdots, n\} \text{ s.t. } x \;\neq\; x_i,\;\; \exists\, j \in \{1, \cdots, n\} \text{ s.t. } x_j \text{ occurs free in } e_x}{(\texttt{(}\ x_1 \ldots x_j \ldots x_n\ \texttt{)}\ \texttt{=>}\ \textit{block})[x := e_x] \;=\; (\texttt{(}\ x_1 \ldots y \ldots x_n\ \texttt{)}\ \texttt{=>}\ \textit{block}[x_j := y])[x := e_x]}$$

Substitution is applied to the whole expression again as to recursively detect and rename all parameters of the lambda expression that clash with variables that occur free in e_x, at which point (i) takes place. Note that the name y is not declared in, nor occurs in *block* and e_x.

Block expression: All occurrences of x in the statements of a block expression are substituted with e_x under given circumstances.

① Block expression in which x is declared.

$$\frac{x \text{ is declared in } \textit{block}}{\textit{block}[x := e_x] \;=\; \textit{block}}$$

② Block expression in which x is not declared.

(i) No names declared in the block occurs free in e_x.

$$\frac{x \text{ is not declared in } \textit{block},\;\; \textit{name} \text{ declared in } \textit{block} \text{ does not occur free in } e_x}{\textit{block}[x := e_x] \;=\; [\textit{block}[0][x := e_x],\; \ldots,\; \textit{block}[n][x := e_x]]}$$

(ii) A name declared in the block occurs free in e_x.

$$\frac{x \text{ is not declared in } \textit{block},\;\; \textit{name} \text{ declared in } \textit{block} \text{ occurs free in } e_x}{\textit{block}[x := e_x] \;=\; [\textit{block}[0][\textit{name} := y],\; \ldots,\; \textit{block}[n][\textit{name} := y]][x := e_x]}$$

Substitution is applied to the whole expression again as to recursively detect and rename all declared names of the block expression that clash with variables that occur free in e_x, at which point (i) takes place. Note that the name y is not declared in, nor occurs in *block* and e_x.

Variable declaration: All occurrences of x in the declarators of a variable declaration are substituted with e_x.

$$\frac{}{\textit{declarations}[x := e_x] \;=\; [\textit{declarations}[0][x := e_x] \ldots \textit{declarations}[n][x := e_x]]}$$

Return statement: All occurrences of x in the expression that is to be returned are substituted with e_x.

$$\frac{}{(\texttt{return}\ e\texttt{;})[x := e_x] \;=\; \texttt{return}\ e[x := e_x]\texttt{;}}$$

Conditional statement: All occurrences of x in the condition, consequent, and alternative expressions of a conditional statement are substituted with e_x.

$$\frac{}{(\texttt{if (}\ e\ \texttt{)}\ \textit{block}\ \texttt{else}\ \textit{block})[x := e_x] \;=\; \texttt{if (}\ e[x := e_x]\ \texttt{)}\ \textit{block}[x := e_x]\ \texttt{else}\ \textit{block}[x := e_x]}$$

Array expression: All occurrences of x in the elements of an array are substituted with e_x.

$$\frac{}{[x_1,\; \ldots,\; x_n][x := e_x] \;=\; [x_1[x := e_x],\; \ldots,\; x_n[x := e_x]]}$$

Free names

Let $\triangleright$ be the relation that defines the set of free names of a given Source expression; the symbols p_1 and p_2 shall henceforth refer to unary and binary operations, respectively. That is, p_1 ranges over $\{\texttt{!}\}$ and p_2 ranges over $\{\texttt{||}, \texttt{\&\&}, \texttt{+}, \texttt{-}, \texttt{*}, \texttt{/}, \texttt{===}, \texttt{>}, \texttt{<}\}$.

Identifier:

$$\frac{}{x \;\triangleright\; \{x\}}$$

$$\frac{}{\textit{name} \;\triangleright\; \varnothing}$$

Boolean:

$$\frac{}{\texttt{true} \;\triangleright\; \varnothing}$$

$$\frac{}{\texttt{false} \;\triangleright\; \varnothing}$$

Expression statement:

$$\frac{e \;\triangleright\; S}{e\texttt{;} \;\triangleright\; S}$$

Unary expression:

$$\frac{e \;\triangleright\; S}{p_1(e) \;\triangleright\; S}$$

Binary expression:

$$\frac{e_1 \;\triangleright\; S_1,\;\; e_2 \;\triangleright\; S_2}{p_1(e_1, e_2) \;\triangleright\; S_1 \cup S_2}$$

Conditional expression:

$$\frac{e_1 \;\triangleright\; S_1,\;\; e_2 \;\triangleright\; S_2,\;\; e_3 \;\triangleright\; S_3}{e_1 \;\texttt{?}\; e_2 \;\texttt{:}\; e_3 \;\triangleright\; S_1 \cup S_2 \cup S_3}$$

Call expression:

$$\frac{e \;\triangleright\; S,\;\; e_k \;\triangleright\; T_k}{e(e_1, \ldots, e_n) \;\triangleright\; S \cup T_1 \cup \ldots \cup T_n}$$

Function declaration:

$$\frac{\textit{block} \;\triangleright\; S}{\texttt{function}\; \textit{name} \;\texttt{(}\; x_1 \ldots x_n \;\texttt{)}\; \textit{block} \;\triangleright\; S - \{x_1, \ldots, x_n\}}$$

Lambda expression:

$$\frac{\textit{block} \;\triangleright\; S}{\texttt{(}\; x_1 \ldots x_n \;\texttt{)}\; \texttt{=>}\; \textit{block} \;\triangleright\; S - \{x_1, \ldots, x_n\}}$$

Block expression:

$$\frac{\mathit{block}[k] \;\triangleright\; S_k,\;\; T \text{ contains all names declared in } \mathit{block}}{\mathit{block} \;\triangleright\; (S_1 \cup \ldots \cup S_n) - T}$$

Constant declaration:

$$\frac{e \;\triangleright\; S}{\texttt{const}\ \mathit{name}\ \texttt{=}\ e\texttt{;} \;\triangleright\; S}$$

Return statement:

$$\frac{e \;\triangleright\; S}{\texttt{return}\ e\texttt{;} \;\triangleright\; S}$$

Conditional statement:

$$\frac{e \;\triangleright\; S,\;\; \mathit{block}_1 \;\triangleright\; T_1,\;\; \mathit{block}_2 \;\triangleright\; T_2}{\texttt{if (}\ e\ \texttt{)}\ \mathit{block}_1\ \texttt{else}\ \mathit{block}_2 \;\triangleright\; S \cup T_1 \cup T_2}$$