Uninformed Search

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Contents

Uninformed search strategies

- No additional information about states beyond that provided in the problem definition
- All they can do is generate successors and distinguish a goal state from a non-goal state.
- Also called **blind search**
- Each strategy is an modified instance of the general **tree/graph search** algorithm

Breadth-first search

Breadth-first search

- **Breadth-first search** (BFS) is a simple strategy in which the root node is expanded first, then all the successors of the root node are expanded next, then *their* successors, and so on
- In general, all the nodes are expanded at a given depth in the search tree before any nodes at the next level are expanded

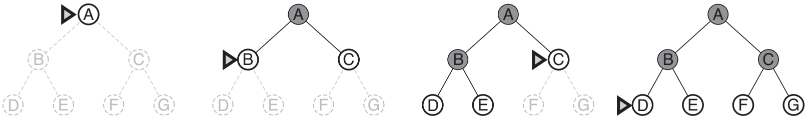


Figure 1: Breadth-first search on a simple binary tree. At each stage, the node to be expanded next is indicated by a marker.

Algorithm

- Frontier is a FIFO queue

```
function BREADTH-FIRST-SEARCH(problem) returns a solution, or failure
  node ← a node with STATE = problem.INITIAL-STATE, PATH-COST = 0
  if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
  frontier ← node
  explored ← ∅
  loop do
    if frontier = ∅ then return failure
    node ← POP(frontier)
    add node.STATE to explored
    for each action in problem.ACTIONS(node.STATE) do
      child ← CHILD-NODE(problem, node, action)
      if child.STATE is not in explored or frontier then
        if problem.GOAL-TEST(child.STATE) then return SOLUTION(child)
        frontier ← INSERT(child, frontier)
```

Properties of breadth-first search

- **Time**: $1 + b + b^2 + b^3 + \ldots + b^d = O(b^d)$
- **Space**: $O(b^d)$ ($O(b^{d-1})$ for explored and $O(b^d)$ for frontier)
- **Complete**: Yes (if b is finite)
- **Optimal**: not optimal in general

Depth	Nodes	Time	Memory
2	110	.11 milliseconds	107 kilobytes
4	11,110	11 milliseconds	10.6 megabytes
6	10^6	1.1 seconds	1 gigabyte
8	10^8	2 minutes	103 gigabytes
10	10^{10}	3 hours	10 terabytes
12	10^{12}	13 days	1 petabyte
14	10^{14}	3.5 years	99 petabytes
16	10^{16}	350 years	10 exabytes

Time and memory requirements for breadth-first search. The numbers shown assume branching factor $b = 10$; 1 million nodes/second; 1000 bytes/node.

Uniform-cost search

Uniform-cost search

- **Uniform-cost search** (UCS) expands the node n with the **lowest path cost** $g(n)$
- Implementation: frontier is a priority queue ordered by g
  - Equivalent to Dijkstra's algorithm
- The goal test is applied to a node when it is selected for expansion
- A test is added in case a better path is found to a node currently on the frontier.

Algorithm

- Frontier is a priority queue

```
function UNIFORM-COST-SEARCH(problem) returns a solution, or failure
  node ← a node with STATE = problem.INITIAL-STATE, PATH-COST = 0
  frontier ← node # a priority queue ordered by PATH-COST
  explored ← ∅
  loop do
    if frontier = ∅ then return failure
    node ← POP(frontier) # chooses the lowest-cost node in frontier
    if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
    add node.STATE to explored
    for each action in problem.ACTIONS(node.STATE) do
      child ← CHILD-NODE(problem, node, action)
      if child.STATE is not in explored or frontier then
        frontier ← INSERT(child, frontier)
      else if child.STATE is in explored with higher PATH-COST then
        replace that frontier node with child
```

Properties of uniform-cost search

- **Time**: $O(b^{1+\lfloor \frac{C^*}{\epsilon} \rfloor})$ where C^* is the cost of the optimal solution
- **Space**: $O(b^{1+\lfloor \frac{C^*}{\epsilon} \rfloor})$
- **Complete** Yes, if step cost $\geq \epsilon$ (small positive constant)
- **Optimal** Yes, nodes expanded in increasing order of $g(n)$

Proof

Using contradiction method

- Suppose UCS terminates at a goal state n with a path cost $g(n) = C$.
- If C is not the optimal value then there exists another unexplored goal state n' with $g(n') < C$
- Therefore, there must exist a node n'' on the frontier that is on the optimal path to n' (graph separation property)
- But $g(n'') < g(n') < g(n) \rightarrow n''$ must expand before n, a contradiction.

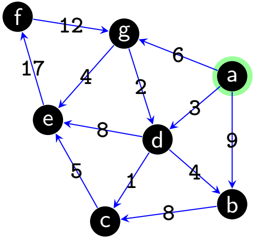
■

Ilustration

Table 1: Each node has PATH-COST and PARENT

node	frontier
	a(0;null)

Figure 2: Find a shortest path from a to f



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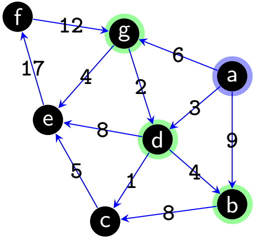

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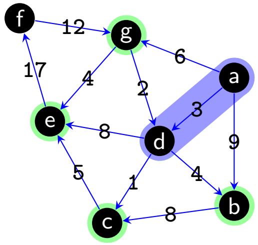

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c(4;d)	b(7;d) g(6;a) e(9;c)

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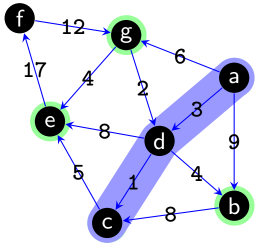

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c(4;d)	b(7;d) g(6;a) e(9;c)
g(6;a)	b(7;d) e(9;c)

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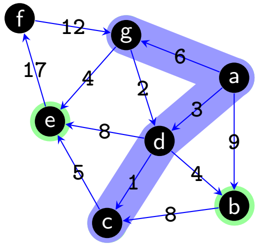

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c(4;d)	b(7;d) g(6;a) e(9;c)
g(6;a)	b(7;d) e(9;c)
b(7;d)	e(9;c)

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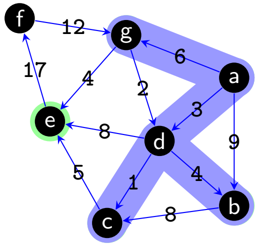

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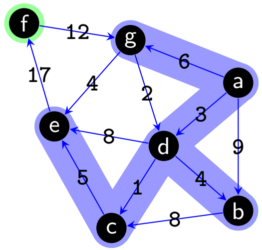
c(4;d)	b(7;d) g(6;a) e(9;c)
g(6;a)	b(7;d) e(9;c)
b(7;d)	e(9;c)
e(9;c)	f(26;e)

Figure 2: Find a shortest path from a to f



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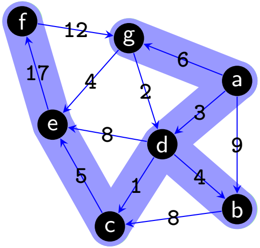
g(6;a)	b(7;d) e(9;c)
b(7;d)	e(9;c)
e(9;c)	f(26;e)
f(26;e)	∅

Figure 2: Find a shortest path from a to f



Depth-first search

Depth-first search

- **Depth-first search** (DFS) expands deepest unexpanded node
- Implementation: frontier is a LIFO Stack

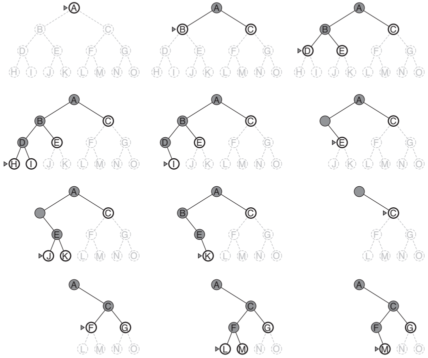


Figure 3: Depth-first search on a binary tree. The unexplored region is shown in light gray. Explored nodes with no descendants in the frontier are removed from memory. Nodes at depth 3 have no successors and M is the only goal node.

Properties of DFS

- **Time**: $O(b^m)$
  - Terrible if m is much larger than d, but if solutions are dense, may be much faster than breadth-first
- **Space**: $O(bm)$, i.e., linear space!
- **Complete**:
  - No in infinite-depth spaces
  - Yes in finite spaces
- **Optimal**: No, the "leftmost" solution, regardless of depth or cost

Depth-limited search

Depth-limited Search

- **Depth-limited Search** (DLS) is a standard DFS with a predetermined depth limit l, i.e., nodes at depth l are treated as if they have no successors
  - infinite problems solved
- Depth limits can be based on knowledge of the problem

Algorithm

- A recursive implementation of depth-limited tree search.

```
function DEPTH-LIMITED-SEARCH(problem, limit)
returns a solution, or failure/cutoff
  return RECURSIVE-DLS(MAKE-NODE(problem.INITIAL-STATE), problem, limit)
function RECURSIVE-DLS(node, problem, limit)
returns a solution, or failure/cutoff
  if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
  else if limit = 0 then return cutoff
  else
    cutoff_occurred? ← false
    for each action in problem.ACTIONS(node.STATE) do
      child ← CHILD-NODE(problem, node, action)
      result ← RECURSIVE-DLS(child, problem, limit − 1)
      if result = cutoff then cutoff_occurred? ← true
      else if result ≠ failure then return result
    if cutoff_occurred? then return cutoff
    else return failure
```

Properties

- **Time**
  - $O(b^l)$
- **Space**
  - $O(bl)$
- **Completeness**
  - Maybe no if $l < d$
- **Optimality**
  - No if $l > d$

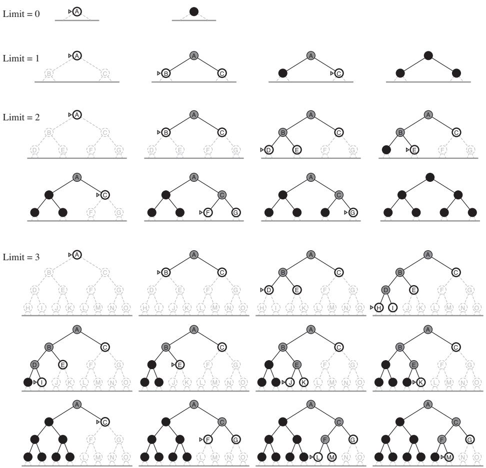
Iterative deepening depth-first search

Iterative deepening depth-first search

- General strategy, often used in combination with depth-first tree search to find the best depth limit
- Gradually increasing the limit until a goal is found
  - The depth limit reaches the depth d of the shallowest goal node.

```
function ITERATIVE-DEEPENING-SEARCH(problem)
returns a solution, or failure
  for depth = 0 to ∞ do
    result ← DEPTH-LIMITED-SEARCH(problem, depth)
    if result ≠ cutoff then return result
```

Illustration



Four iterations of iterative deepening search on a binary tree.

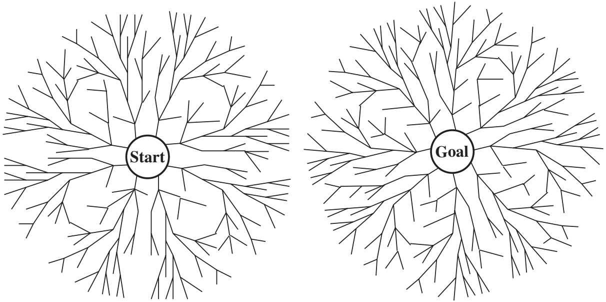
Properties

- **Time complexity**
  - $db^1 + (d-1)b^2 + 1b^d = O(b^d)$
- **Space complexity**
  - $O(bd)$, similar to DFS
- **Completeness**
  - Yes when the branching factor b is finite
- **Optimality**
  - No in general

Bidirectional search

Bidirectional search

- Two simultaneous searches
  - From the **initial state** towards
  - From the **goal state** backwards
- Hoping that two searches meet in the middle



Properties

- **Time and Space complexity**: $O(b^{d/2})$
- **Goal test**: Whether the frontiers of two searches intersect
- **Optimality**: Maybe no
- It sounds attractive, but what is the **tradeoff**?
- Space requirement for the frontiers of at least one search
- Not easy to search backwards (predecessors required)
    - In case there are more than 1 goals
    - Especially if the goal is an abstract description (no queen attacks another queen)

Comparing uninformed search strategies

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Itervative Deepen-ing	Bidirec-tional
Complete	Yes[a]	Yes[a,b]	No	No	Yes[a]	Yes[a,d]
Time	$O(b^d)$	$O(b^{1+\lfloor \frac{C^*}{\epsilon} \rfloor})$	$O(b^m)$	$O(b^l)$	$O(b^d)$	$O(b^{\frac{d}{2}})$
Space	$O(b^d)$	$O(b^{1+\lfloor \frac{C^*}{\epsilon} \rfloor})$	$O(bm)$	$O(bl)$	$O(bd)$	$O(b^{\frac{d}{2}})$
Optimal	Yes[c]	Yes	No	No	Yes[c]	Yes[c,d]

Table 2: Evaluation of tree-search strategies. b is the branching factor; d is the depth of the shallowest solution; m is the maximum depth of the search tree; l is the depth limit. Superscript caveats are as follows: [a] complete if b is finite; [b] complete if step costs $\geq \epsilon$ for positive ϵ; [c] optimal if step costs are all identical; [d] if both directions use breadth-first search.

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