Instance Based Learning

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Contents

Notation

symbol	meaning
$a, b, c, N \ldots$	scalar number
$\boldsymbol{w}, \boldsymbol{v}, \boldsymbol{x}, \boldsymbol{y} \ldots$	column vector
$\boldsymbol{X}, \boldsymbol{Y} \ldots$	matrix
$\mathbb{R}$	set of real numbers
$\mathbb{Z}$	set of integer numbers
$\mathbb{N}$	set of natural numbers
$\mathbb{R}^D$	set of vectors
$\mathcal{X}, \mathcal{Y}, \ldots$	set
$\mathcal{A}$	algorithm

operator	meaning
$\boldsymbol{w}^\top$	transpose
$\boldsymbol{X}\boldsymbol{Y}$	matrix multiplication
$\boldsymbol{X}^{-1}$	inverse

Parametric vs Non-parametric Models

Parametric Models

- In the models that we have seen, we select a hypothesis space $\mathcal{H}$ and adjust a *fixed set of parameters* $\boldsymbol{w}$ with the training data $\mathcal{D}$
- We assume that the parameters $\boldsymbol{w}$ summarize the training data $\mathcal{D}$ and we can forget about it

Non-parametric Models

- A non parametric model is one that can not be characterized by a fixed set of parameters
- A family of non parametric models is **Instance Based Learning**

k-Nearest Neighbor

k-Nearest Neighbor

Key idea: just store all training examples $\mathcal{D} = \{(\boldsymbol{x}_i, y_i)\}$

Nearest neighbor

- Given query instance $\boldsymbol{x}_q$, first locate *the nearest neighbhor* $\boldsymbol{x}^{(1)}$, then estimate

$$h(\boldsymbol{x}_q) = y^{(1)} \tag{1}$$

k-**Nearest neighbor** (k is a *hyper-parameter*)

- Given $\boldsymbol{x}_q$, take vote among its k *nearest neighbors* $\{\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}, \ldots, \boldsymbol{x}^{(k)}\}$ (if discrete-valued target function)

$$h(\boldsymbol{x}_q) = vote\{\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}, \ldots, \boldsymbol{x}^{(k)}\} \tag{2}$$

- Take mean of the values of k *nearest neighbors* (if real-valued target function)

$$h(\boldsymbol{x}_q) = \frac{\sum_{i=1}^{k} y^{(i)}}{k} \tag{3}$$

When To Consider Nearest Neighbor

- Instances map to points in $\mathbb{R}^D$
- Less than 20 attributes per instance
- Lots of training data $\mathcal{D}$

Advantages

- No training, just store data
- Learn complex target functions
- Don't lose information

Disadvantages

- Slow at query time
- Easily fooled by irrelevant attributes

Distance

Some common distances in space $\mathbb{R}^D$

- The Minkowski distance of order $p > 0$

$$d(\boldsymbol{x}, \boldsymbol{y}) = L_p(\boldsymbol{x}, \boldsymbol{y}) = \left(\sum_{i=1}^{D} |x_i - y_i|^p \right)^{1/p} \tag{4}$$

- Euclidean distance

$$d(\boldsymbol{x}, \boldsymbol{y}) = L_2(\boldsymbol{x}, \boldsymbol{y}) = \sqrt{\sum_{i=1}^{D} (x_i - y_i)^2} \tag{5}$$

Distance (cont.)

- Manhattan distance

$$d(\boldsymbol{x}, \boldsymbol{y}) = L_1(\boldsymbol{x}, \boldsymbol{y}) = \sum_{i=1}^{D} |x_i - y_i| \tag{6}$$

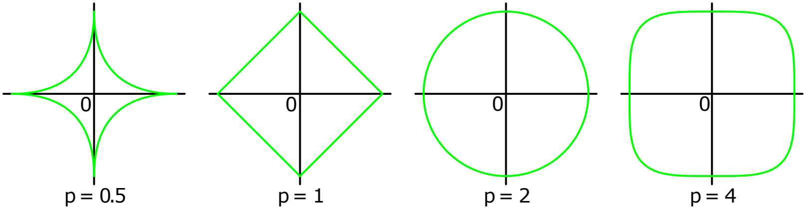
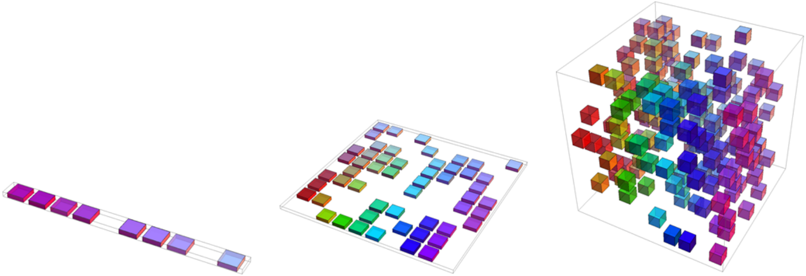


Figure 1: Contours of the distance from the origin O for various values of the parameter p

The Curse of dimensionality

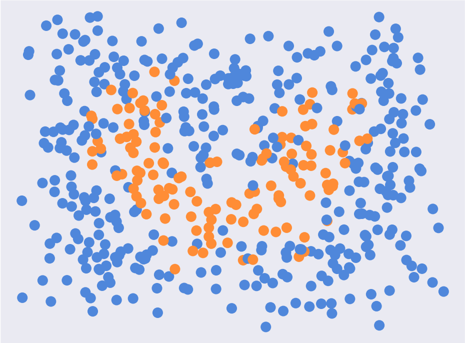
- The more dimensions we have, the more examples we need
- The number of examples that we have in a volume of space *decreases exponentially* with the number of dimensions
  - If the number of dimensions is very high the nearest neighbours can be very far away



Effects of Hyper-parameters

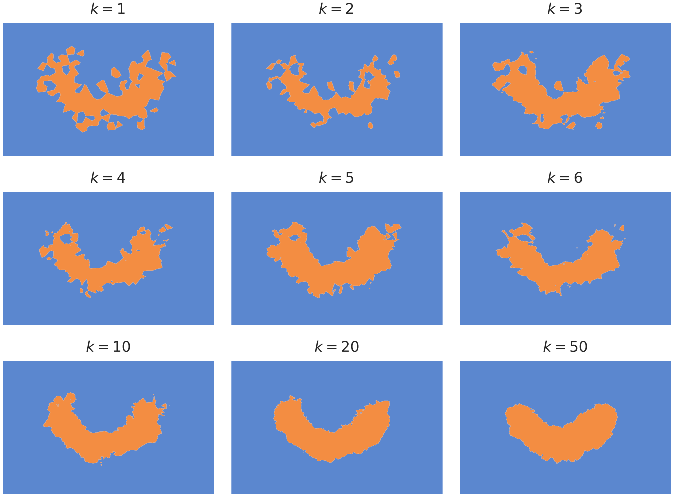
k Parameter

- Data set $\mathcal{D}$ with 500 samples belonging to two classes



k Parameter (cont.)

- Decision regions for various values of k



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