Logical Agents

Bùi Tiến Lên

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KHOA CÔNG NGHỆ THÔNG TIN
TRƯỜNG ĐẠI HỌC KHOA HỌC TỰ NHIÊN

Contents

Knowledge-based Agents

Problem-solving agents

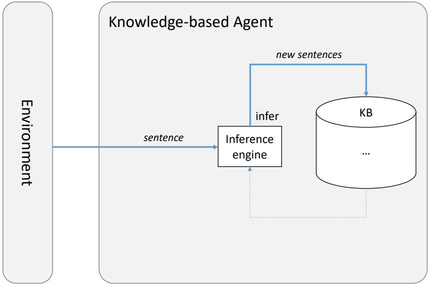
- The problem-solving agents know things, but only in a very limited, inflexible sense.
  - E.g., the 8-puzzle agent cannot deduce that with odd parity cannot be reached from states with even parity
- CSP enables some parts of the agent to work domain-independently
  - Represent states as assignments of values to variables
  - Allow for more efficient algorithms

Knowledge-based agents

- Supported by **logic** – a general class of representation
- Knowledge-based agents can combine and recombine information to suit myriad purposes.
  - Accept new tasks in the form of explicitly described goals
  - Achieve competence by learning new knowledge of the environment
  - Adapt to changes by updating the relevant knowledge

Knowledge-based agents (cont.)

- **Knowledge base** (KB): A set of sentences or facts in a *formal* language
  - Each sentence represents some assertion about the world.
  - Axiom = the sentence that is not derived from other sentences
- **Inference**: Using **inference engine** to derive (infer) new sentences from old ones
  - Add new sentences to the knowledge base and query what is known



A generic knowledge-based agent

```
function KB-AGENT(percept) returns an action
persistent: KB, a knowledge base
            t, a counter, initially 0, indicating time
  TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))
  action ← ASK(KB, MAKE-ACTION-QUERY(t))
  TELL(KB, MAKE-ACTION-SENTENCE(action, t))
  t ← t + 1
  return action
```

- Given a percept, the agent adds the percept to its knowledge base, asks the knowledge base for the best action, and tells the knowledge base that it has in fact taken that action.

Building an Agent

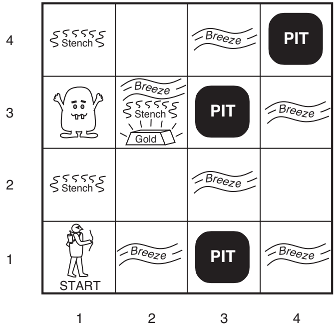
- **Declarative** approach to building an agent
  - **Tell** it what it needs to know, then it can **Ask** itself what to do – answers should follow from the KB
- Procedural approach
  - Encode desired behaviors directly as program code.
- Combined approach → Partially autonomous
- Learning approach → Fully autonomous
  - Provide a knowledge-based agent with mechanisms that allow it to learn for itself

The Wumpus World

Wumpus World PEAS description

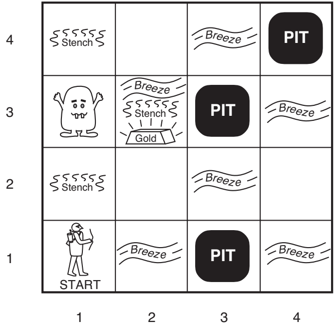
The **wumpus world** is a cave consisting of rooms connected by passageways

- **Performance measure**
  - $+1000$ for climbing out of the cave with gold
  - -1000 for falling into a pit or being eaten by the wumpus
  - -1 each action taken
  - -10 for using the arrow
  - The game ends when agent dies or climbs out of the cave



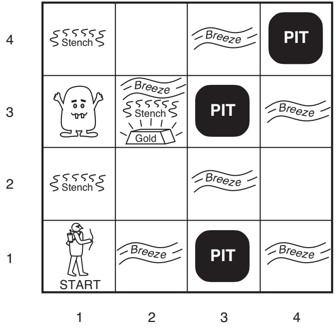
Wumpus World PEAS description (cont.)

- **Environment**
  - A 4×4 grid of rooms
  - Agent starts in the square $[1, 1]$, facing to the right
  - The locations of **gold** and **wumpus** are random
  - Each square can be a **pit**, with probability 0.2



Wumpus World PEAS description (cont.)

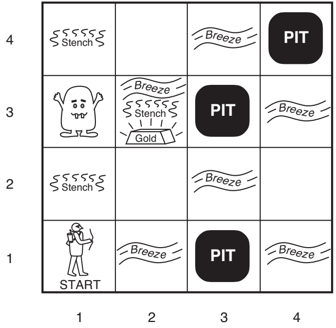
- **Actuators**: The agent can
  - *Forward*
  - *Left turn* by 90°
  - *Right turn* by 90°
  - *Shooting* kills wumpus if you are facing it (the agent has only one arrow)
  - *Grabbing* picks up gold if in same square
  - *Releasing* drops the gold in same square



Wumpus World PEAS description (cont.)

- **Sensors**: The agent has five sensors
  - In the square containing the wumpus and in the directly (not diagonally) adjacent squares, the agent will perceive a *Stench*.
  - In the squares directly adjacent to a pit, the agent will perceive a *Breeze*.
  - In the square where the gold is, the agent will perceive a *Glitter*.
  - When an agent walks into a wall, it will perceive a *Bump*.
  - When the wumpus is killed, it emits a woeful *Scream* that can be perceived anywhere in the cave

[*Stench*, *Breeze*, *None*, *None*, *None*]



Characterize the Wumpus World

- Fully Observable
  - No – only local perception
- Deterministic
  - Yes – outcomes exactly specified
- Episodic
  - No – sequential at the level of actions
- Static
  - Yes – Wumpus and Pits do not move
- Discrete
  - Yes
- Single-agent
  - Yes – Wumpus is essentially a natural feature

Exploring a wumpus world with inference

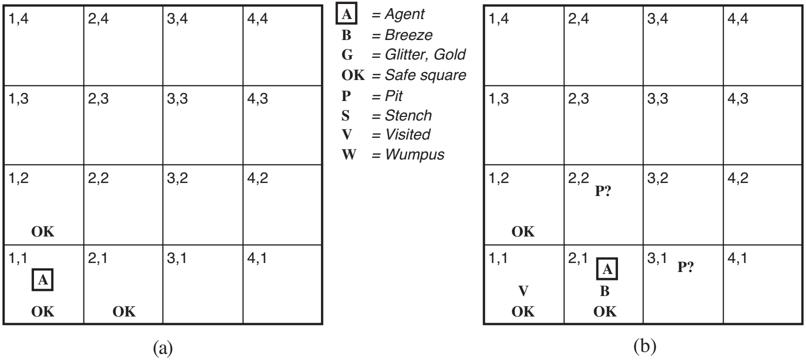


Figure 1: The first step taken by the agent in the wumpus world. (a) The initial situation, after percept [*None*, *None*, *None*, *None*, *None*]. (b) After one move, with percept [*None*, *Breeze*, *None*, *None*, *None*].

Exploring a wumpus world with inference (cont.)

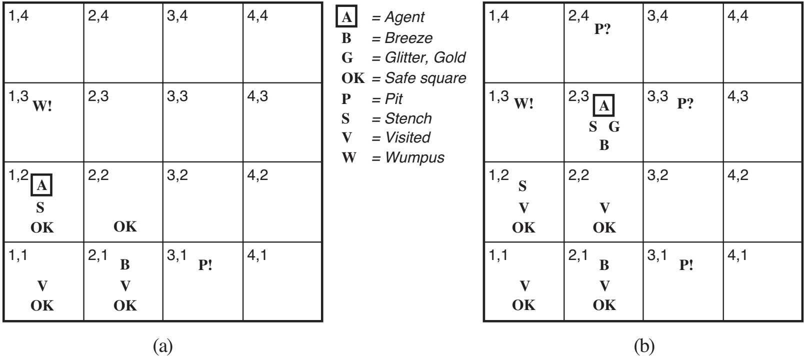
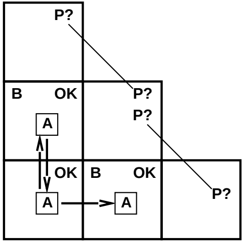
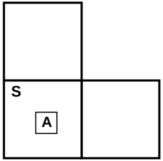


Figure 2: Two later stages in the progress of the agent. (a) After the third move, with percept [*Stench, None, None, None, None*]. (b) After the fifth move, with percept [*Stench, Breeze, Glitter, None, None*].

More



- Breeze in (1,2) and (2,1) $\implies$ no safe actions
- Assuming pits uniformly distributed, (2,2) has pit with probability 0.86 vs. 0.31



- Smell in (1,1) $\implies$ cannot move
- Can use a strategy of coercion:
  - shoot straight ahead
  - wumpus was there $\implies$ dead $\implies$ safe
  - wumpus wasn't there $\implies$ safe

Logic

Logic in general

Concept 1

- **Logics** are formal languages for representing information such that conclusions can be drawn
- **Syntax** defines the **sentences** in the language
- **Semantics** define the "meaning" of sentences; i.e., define **truth** of a sentence in a world

Logic in general (cont.)

Example 1

The language of arithmetic

- $x + 2 \geq y$ is a sentence
- $x2 + y >$ is not a sentence
- $x + 2 \geq y$ is true iff the number $x + 2$ is no less than the number y
- $x + 2 \geq y$ is true in a world where $x = 7, y = 1$
- $x + 2 \geq y$ is false in a world where $x = 0, y = 6$

Worlds, models, and events

Concept 2

- A **world** is a particular state of affairs in which the value of each variable is known
- **Models** are formally structured worlds with respect to which truth can be evaluated
  - If a sentence α is true in model m, we say that m **satisfies** α or sometimes m **is a model of** α
  - $M(\alpha)$ is the set of all models of α
  - $M(\alpha) = \{\omega : \omega \models \alpha\}$
- $M(\alpha)$ is called the **event** denoted by α

Worlds, models, and events (cont.)

Figure 3: A set of worlds, also known as truth assignments, variable assignments, or variable instantiations

world/model	Earthquake	Burglary	Alarm
ω_1	true	true	true
ω_2	true	true	false
ω_3	true	false	true
ω_4	true	false	false
ω_5	false	true	true
ω_6	false	true	false
ω_7	false	false	true
ω_8	false	false	false

Entailment

Concept 3

- **Entailment** means that one thing *follows from* another
- "α entails β" or "α follows from β"

$$\alpha \models \beta \tag{1}$$

iff in every world where α is true, β is also true **or**

$$M(\alpha) \subseteq M(\beta) \tag{2}$$

Entailment (cont.)

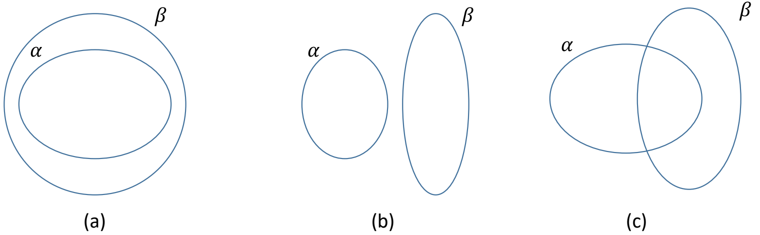
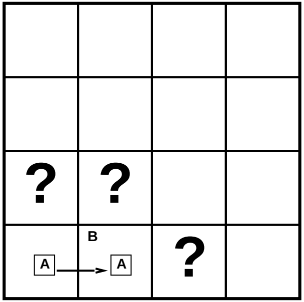
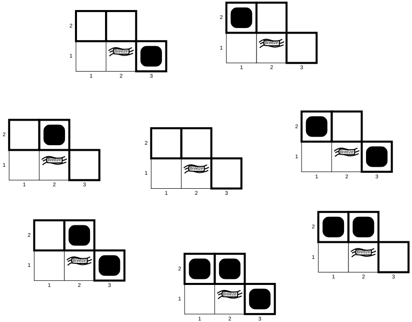


Figure 4: Possible relationships between α and β. (a) $\alpha \models \beta$ (b) $\alpha \models \neg\beta$ (c) $\alpha \not\models \beta$ and $\beta \not\models \alpha$

Wumpus models

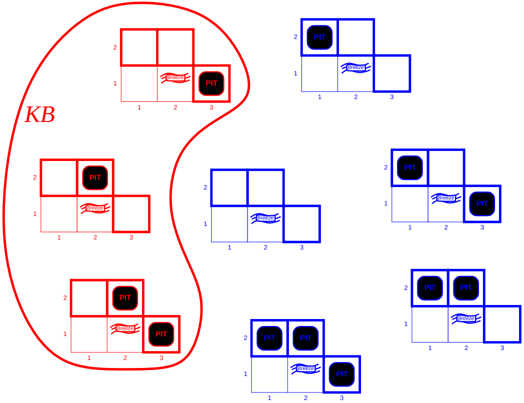
- Situation after the agent detecting nothing in [1,1], moving right and feel breeze in [2,1]
- Consider possible models for x? (assuming only pits)
  - 3 Boolean choices $\implies$ 8 possible models





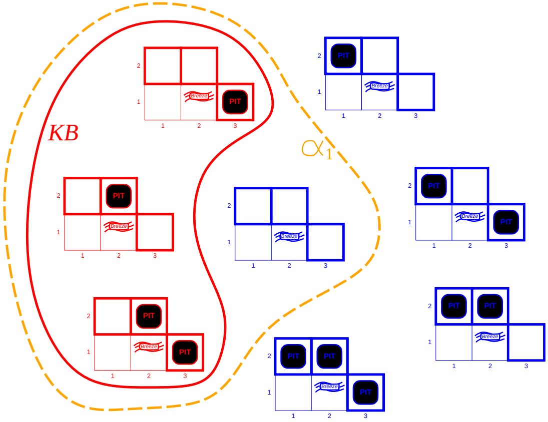
Knowledge base

- The agent *building* **knowledge base** *KB* from wumpus-world rules + observations



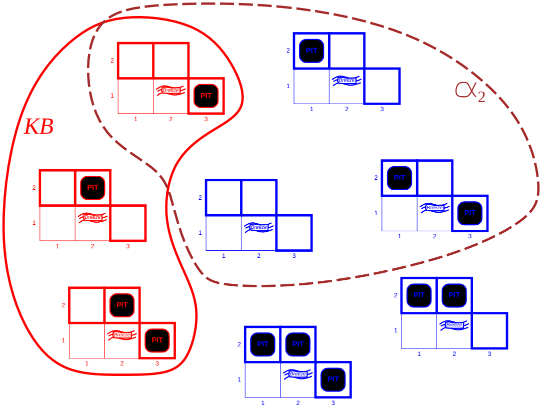
Entailment

- α_1 = "[1,2] is safe", $KB \models \alpha_1$, proved by **model checking**



Entailment

- α_2 = "[2,2] is safe", $KB \not\models \alpha_2$



Inference

Concept 4

Sentence α can be derived from KB by **procedure** i; denoted by

$$KB \vdash_i \alpha$$

- consequences of KB are a haystack
- α is a needle.
- entailment $=$ needle in haystack
- inference $=$ finding it

Soundness: i is sound if whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$
Completeness: i is complete if whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$

World and representation

- if KB is true in the real world, then any sentence α derived from KB by a sound inference procedure is also true in the real world

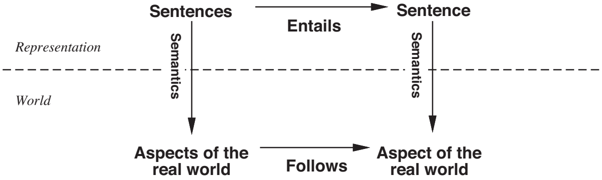


Figure 5: Sentences are physical configurations of the agent, and reasoning is a process of constructing new physical configurations from old ones. Logical reasoning should ensure that the new configurations represent aspects of the world that actually follow from the aspects that the old configurations represent.

Propositional Logic: A Very Simple Logic

Syntax

Propositional logic (**a formal language**) is the simplest logic – illustrates basic ideas.

The **syntax** of propositional logic defines

- **Constants**:

$$True, \; False$$

- **Symbols**: stand for propositions

$$A, \; B, \; B_{1,1}, \; P_{2,1}$$

- **Logical connectives** (**operator**)

connectives	**meaning**	**example**
$\neg$	negation (NOT)	$\neg S$
$\wedge$	conjunction (AND)	$S_1 \wedge S_2$
$\vee$	disjunction (OR)	$S_1 \vee S_2$
$\Longrightarrow$	implication	$S_1 \Longrightarrow S_2$
$\Longleftrightarrow$	equivalence, biconditional	$S_1 \Longleftrightarrow S_2$

Syntax (cont.)

- A BNF (Backus–Naur Form) grammar of sentences in propositional logic, along with operator precedences, from highest to lowest.

$$
\begin{array}{rcl}
\mathit{Sentence} & \rightarrow & \mathit{AtomicSentence} \mid \mathit{ComplexSentence} \\
\mathit{AtomicSentence} & \rightarrow & \mathit{True} \mid \mathit{False} \mid P \mid Q \mid R \mid \ldots \\
\mathit{ComplexSentence} & \rightarrow & (\mathit{Sentence}) \mid [\mathit{Sentence}] \\
& \mid & \neg\mathit{Sentence} \\
& \mid & \mathit{Sentence} \land \mathit{Sentence} \\
& \mid & \mathit{Sentence} \lor \mathit{Sentence} \\
& \mid & \mathit{Sentence} \implies \mathit{Sentence} \\
& \mid & \mathit{Sentence} \iff \mathit{Sentence} \\
\\
\textsc{Operator Precedence} & : & \neg, \land, \lor, \implies, \iff
\end{array}
$$

Semantic

- The semantics defines the rules for determining the truth of a sentence with respect to a particular model m
  - Each model m specifies *true*/*false* for each proposition symbol
  - Arbitrary sentence can be evaluateed by **recursive process** PL_TRUE and **truth tables**

Figure 6: Truth tables for the five logical connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \implies Q$	$P \iff Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

Semantic (cont.)

```
function PL-TRUE?(α,model) returns true or false
  if α is a symbol then return LOOKUP(α, model)
  if OP(α) = ¬ then return NOT(PL-TRUE?(ARG1(α),model))
  if OP(α) = ∧ then return AND(PL-TRUE?(ARG1(α),model),
                               PL-TRUE?(ARG2(α),model))
  if OP(α) = ∨ then return OR(PL_TRUE?(ARG1(α),model),
                              PL-TRUE?(ARG2(α),model))
  if OP(α) = ⟹ then return ...
  if OP(α) = ⟺ then return ...
```

Inference

Problem

Given a set of sentences KB and α. Prove that

$$KB \models \alpha$$

- **Method 1: model-checking** (enumeration)
  - Time complexity: $O(2^n)$ (if KB and α contain n symbols $\rightarrow$ there are 2^n models)
  - Space complexity: $O(n)$ (depth-first)
  - OK for propositional logic (finitely many worlds); not easy for first-order logic
- **Method 2: theorem-proving**
  - Search for a sequence of proof steps (applications of inference rules) leading from to

Model checking

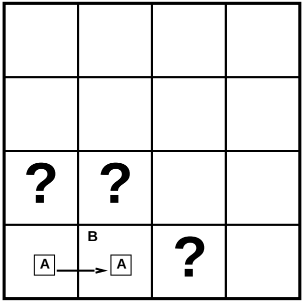
```
function TT-ENTAILS?(KB, α) returns true or false
  inputs: KB, the knowledge base, a sentence in propositional logic
          α, the query, a sentence in propositional logic
  symbols ← a list of the propositional symbols in KB and α
  return TT-CHECK-ALL(KB, α, symbols, ∅)

function TT-CHECK-ALL(KB, α, symbols, model)
returns true or false
  if EMPTY?(symbols) then
    if PL-TRUE?(KB, model) then return PL-TRUE?(α, model)
    else return true
  else
    P ← First(symbols)
    rest ← Rest(symbols)
    return (TT-CHECK-ALL(KB, α, rest, model ∪ {P = true})
            and
            TT-CHECK-ALL(KB, α, rest, model ∪ {P = false}))
```

A simple knowledge base in Wumpus world

Symbols for each $[x, y]$ location:

- $P_{x,y}$ is true if there is a pit in $[x, y]$.
- $W_{x,y}$ is true if there is a wumpus in $[x, y]$, dead or alive.
- $B_{x,y}$ is true if the agent perceives a breeze in $[x, y]$.
- $S_{x,y}$ is true if the agent perceives a stench in $[x, y]$.



Sentences in Wumpus world

$$\begin{aligned}
R_1 &: \neg P_{1,1} \\
R_2 &: B_{1,1} \iff (P_{1,2} \lor P_{2,1}) \\
R_3 &: B_{2,1} \iff (P_{1,1} \lor P_{2,2} \lor P_{3,1}) \\
R_4 &: \neg B_{1,1} \\
R_5 &: B_{2,1}
\end{aligned}$$

Inference in Wumpus world

- A truth table constructed for the knowledge base given in the text. *KB* is true if R_1 through R_5 are true, which occurs in just 3 of the 128 rows
- The agent makes some conclusion
  - $KB \models \neg P_{1,2}$ means there is no pit in [1,2]
  - $KB \not\models \neg P_{2,2}$ means there might (or might not) be a pit in [2,2]

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

Propositional Theorem Proving

Inference rules approach

- **Theorem proving**: Apply rules of inference directly to the sentences in *KB* to construct a proof of the desired sentence without consulting models.
  - More efficient than model checking when the number of models is large but the length of the proof is short
- **Note**: Logical systems is **monotonicity**, which says that the set of entailed sentences can only *increase* as information is added to the knowledge base. For any sentences α and β,

$$\textbf{\textit{if}}\ KB \models \alpha\ \textbf{\textit{then}}\ KB \wedge \beta \models \alpha$$

Logical equivalence

Concept 5

Two sentences α and β are logically **equivalent** if they are true in the same set of models. We denotes as $\alpha \equiv \beta$

$$\alpha \equiv \beta \text{ if and only if } \alpha \models \beta \text{ and } \beta \models \alpha$$

Logical equivalence (cont.)

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) \text{ commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) \text{ commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) \text{ associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) \text{ associativity of } \vee \\
\neg(\neg\alpha) &\equiv \alpha \text{ double-negation elimination} \\
(\alpha \implies \beta) &\equiv (\neg\beta \implies \neg\alpha) \text{ contraposition} \\
(\alpha \implies \beta) &\equiv (\neg\alpha \vee \beta) \text{ implication elimination} \\
(\alpha \iff \beta) &\equiv ((\alpha \implies \beta) \wedge (\beta \implies \alpha)) \text{ biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg\alpha \vee \neg\beta) \text{ De Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg\alpha \wedge \neg\beta) \text{ De Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \text{ distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \text{ distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

Validity

Concept 6

A sentence is **valid** if it is true in *all* models

- Valid sentences are also known as **tautologies**

Theorem 1 (Deduction theorem)

For any sentences α and β, $\alpha \models \beta$ if and only if the sentence $(\alpha \implies \beta)$ is valid.

Satisfiability

Concept 7

A sentence is **satisfiable** if it is true in, or satisfied by, *some* model

Some useful results

- α is valid iff $\neg\alpha$ is unsatisfiable
- α is satisfiable iff $\neg\alpha$ is not valid
- $\alpha \models \beta$ iff the sentence $(\alpha \wedge \neg\beta)$ is unsatisfiable (**refutation** or **contradiction)**

The SAT problem

The problem of determining the satisfiability of sentences in propositional logic was the first problem proved to be NP-complete

Inference and Proofs

- **Application of inference rules**
  - Legitimate (sound) generation of new sentences from old
  - **Proof** = a sequence of inference rule applications to the desired goal
  - Can use inference rules as operators in a **standard search algorithm**. Typically require translation of sentences into a **normal form**
- Some important inference rules

Modus ponens $$\frac{\alpha \implies \beta, \quad \alpha}{\beta}$$

Modus tollens $$\frac{\alpha \implies \beta, \quad \neg\beta}{\neg\alpha}$$

And-introduction $$\frac{\alpha, \quad \beta}{\alpha \wedge \beta}$$

And-elimination $$\frac{\alpha \wedge \beta}{\alpha}$$

Inference rules: An example

Problem

Given $KB = \{P \wedge Q, P \implies R, Q \wedge R \implies S\}$, prove that $KB \models S$

Solution

#	Sentence	Explanation
1	$P \wedge Q$	from KB
2	$P \implies R$	from KB
3	$Q \wedge R \implies S$	from KB
4	P	(1) and-elimination
5	R	(4,2) modus ponens
6	Q	(1) and-elimination
7	$Q \wedge R$	(5,6) and-introduction
8	S	(3,7) modus ponens

■

Inference rules: An example in Wumpus world

Problem

In Wumpus wolrd, given $KB = \{R_1, R_2, R_3, R_4, R_5\}$, prove that $KB \models \neg P_{1,2}$

Solution

#	Sentence	Explanation
R_1	$\neg P_{1,1}$	from KB
R_2	$B_{1,1} \iff (P_{1,2} \lor P_{2,1})$	from KB
R_3	$B_{2,1} \iff (P_{1,1} \lor P_{2,2} \lor P_{3,1})$	from KB
R_4	$\neg B_{1,1}$	from KB
R_5	$B_{2,1}$	from KB
R_6	$(B_{1,1} \implies (P_{1,2} \lor P_{2,1})) \land ((P_{1,2} \lor P_{2,1}) \implies B_{1,1})$	Bi-conditional elimination to R_2
R_7	$(P_{1,2} \lor P_{2,1}) \implies B_{1,1}$	And-elimination to R_6
R_8	$\neg B_{1,1} \implies \neg(P_{1,2} \lor P_{2,1})$	Contrapositives to R_7
R_9	$\neg(P_{1,2} \lor P_{2,1})$	Modus ponens to R_4, R_8
R_{10}	$\neg P_{1,2} \land \neg P_{2,1}$	De Morgan's rule to R_9
R_{11}	$\neg P_{1,2}$	And-elimination to R_{10}

■

Proving by search

Any search algorithms can be applied to find a sequence of steps that constitutes a proof:

- **INITIAL STATE**: the initial knowledge base KB.
- **ACTIONS**: the set of actions consists of all the inference rules applied to all the sentences that match the top half of the inference rule.
- **RESULT**: the result of an action is to add the sentence in the bottom half of the inference rule.
- **GOAL**: the goal is a state that contains the sentence we are trying to prove.

Resolution

Concept 8

Conjunctive Normal Form (CNF—universal)

$$\textit{conjunction of } \underbrace{\textit{disjunctions of literals}}_{\textit{clauses}}$$

A BNF (Backus–Naur Form) grammar for conjunctive normal form

$$\begin{aligned} \mathit{CNFSentence} &\rightarrow \mathit{Clause}_1 \wedge \ldots \wedge \mathit{Clause}_n \\ \mathit{Clause} &\rightarrow \mathit{Literal}_1 \vee \ldots \vee \mathit{Literal}_m \\ \mathit{Literal} &\rightarrow \mathit{Symbol} \mid \neg \mathit{Symbol} \\ \mathit{Symbol} &\rightarrow P \mid Q \mid R \ldots \end{aligned}$$

Resolution (cont.)

Conversion to CNF: a sentence $B_{1,1} \iff (P_{1,2} \lor P_{2,1})$

1. Eliminate $\iff$, replacing $\alpha \iff \beta$ with $(\alpha \implies \beta) \land (\beta \implies \alpha)$.

$$(B_{1,1} \implies (P_{1,2} \lor P_{2,1})) \land ((P_{1,2} \lor P_{2,1}) \implies B_{1,1})$$

2. Eliminate $\implies$, replacing $\alpha \implies \beta$ with $\neg\alpha \lor \beta$.

$$(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg(P_{1,2} \lor P_{2,1}) \lor B_{1,1})$$

3. Move $\neg$ inwards using de Morgan's rules and double-negation:

$$(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land ((\neg P_{1,2} \land \neg P_{2,1}) \lor B_{1,1})$$

4. Apply distributivity law ($\lor$ over $\land$) and flatten:

$$(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg P_{1,2} \lor B_{1,1}) \land (\neg P_{2,1} \lor B_{1,1})$$

Resolution (cont.)

Resolution inference rule (for CNF):

$$\frac{\ell_1 \vee \cdots \vee \ell_k, \qquad m_1 \vee \cdots \vee m_n}{\ell_1 \vee \cdots \vee \ell_{i-1} \vee \ell_{i+1} \vee \cdots \vee \ell_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n}$$

where ℓ_i and m_j are complementary literals

The resolution algorithm

- Proof by contradiction: To show that $KB \models \alpha$, prove that $KB \wedge \neg\alpha$ is unsatisfiable

```
function PL-RESOLUTION(KB, α) returns true or false
inputs: KB, the knowledge base, a sentence in propositional logic
        α, the query, a sentence in propositional logic
  clauses ← the set of CNF clauses of KB ∧ ¬α
  new ← ∅
  loop do
    for each pair of clauses Ci, Cj in clauses do
      resolvents ← PL-RESOLVE(Ci, Cj)
      if resolvents contains the empty clause then return true
      new ← new ∪ resolvents
    if new ⊆ clauses then return false
    clauses ← clauses ∪ new
```

- The function PL-RESOLVE returns the set of all possible clauses obtained by resolving its two inputs. The function PL-RESOLUTION is **complete**.

Inference in Wumpus world

- $KB = \{(B_{1,1} \iff (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1}\}$ and $\alpha = \neg P_{1,2}$
- **Note**: many resolution steps are pointless.

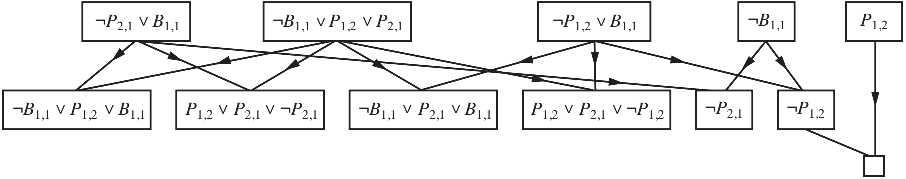


Figure 7: Partial application of PL-RESOLUTION to a simple inference in the wumpus world. $\neg P_{1,2}$ is shown to follow from the first four clauses in the top row

Forward and backward chaining

- In many practical situations, the full power of resolution is not needed. Some real-world knowledge bases satisfy certain restrictions (Horn form) on the form of sentences

Concept 9

Horn Form (restricted)

conjunction of *Horn clauses*

A BNF (Backus–Naur Form) grammar for Horn form

$$\begin{aligned} HornClauseForm &\rightarrow DefiniteClauseForm \mid Symbol \\ DefiniteClauseForm &\rightarrow (Symbol_1 \wedge \ldots \wedge Symbol_n) \implies Symbol \\ Symbol &\rightarrow P \mid Q \mid R \ldots \end{aligned}$$

Forward and backward chaining (cont.)

- **Modus ponens inference rule** (for Horn Form): complete for Horn *KB*s

$$\frac{\alpha_1, \ldots, \alpha_n, \alpha_1 \wedge \cdots \wedge \alpha_n \implies \beta}{\beta}$$

- Can be used with **forward chaining** or **backward chaining**.
- These algorithms are very natural and run in *linear* time

Forward chaining (FC)

- **Idea**: fire any rule whose *premises* are satisfied in the KB, add its *conclusion* to the KB, until query is found
- **Example**: Given the following KB, prove that $KB \models Q$

$$
\begin{aligned}
P &\implies Q\\
L \land M &\implies P\\
B \land L &\implies M\\
A \land P &\implies L\\
A \land B &\implies L\\
A&\\
B&
\end{aligned}
$$

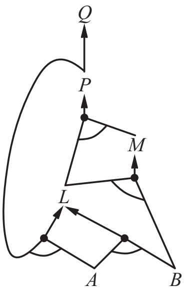


Figure 8: The corresponding AND–OR graph.

The forward-chaining algorithm

```
function PL-FC-ENTAILS?(KB, q) returns true or false
inputs: KB, the knowledge base, a set of propositional definite clauses
        q, the query, a proposition symbol
  count ← a table, where count[c] is the number of symbols in c's premise
  inferred ← a table, where inferred[s] is initially false for all symbols
  agenda ← a queue of symbols, initially symbols known to be true in KB
  while agenda ≠ ∅ do
    p ← POP(agenda)
    if p = q then return true
    if inferred[p] = false then
      inferred[p] ← true
      for each clause c in KB where p is in c.PREMISE do
        decrement count[c]
        if count[c] = 0 then add c.CONCLUSION to agenda
  return false
```

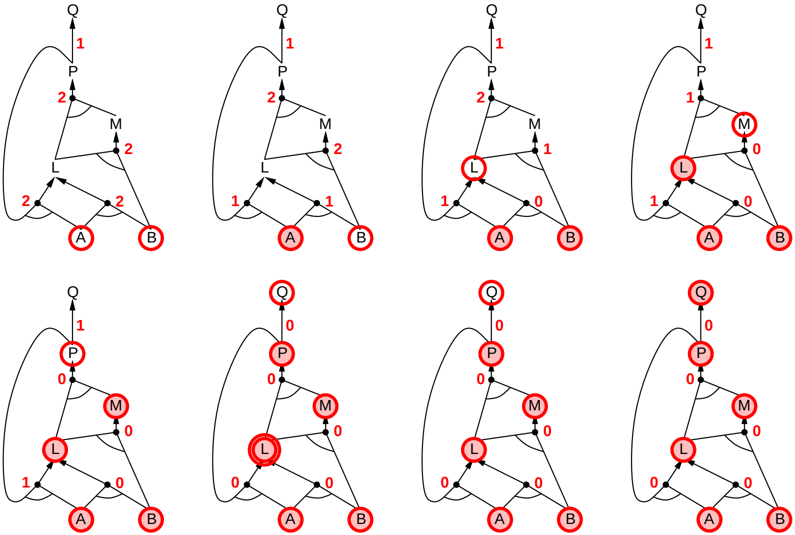
Proof of completeness

FC derives every atomic sentence that is entailed by KB

1. FC reaches a **fixed point** where no new atomic sentences are derived
2. Consider the final state as a model m, assigning true/false to symbols
3. Every clause in the original KB is true in m
   Proof: Suppose a clause $a_1 \wedge \ldots \wedge a_k \Rightarrow b$ is false in m
    - Then $a_1 \wedge \ldots \wedge a_k$ is true in m and b is false in m
    - Therefore the algorithm has not reached a fixed point!
4. 4. Hence m is a model of KB
5. 5. If $KB \models q$, q is true in *every* model of KB, including m

- **General idea**: construct any model of KB by sound inference, check α

Forward chaining example



Backward chaining (BC)

Idea: work backwards from the query q:

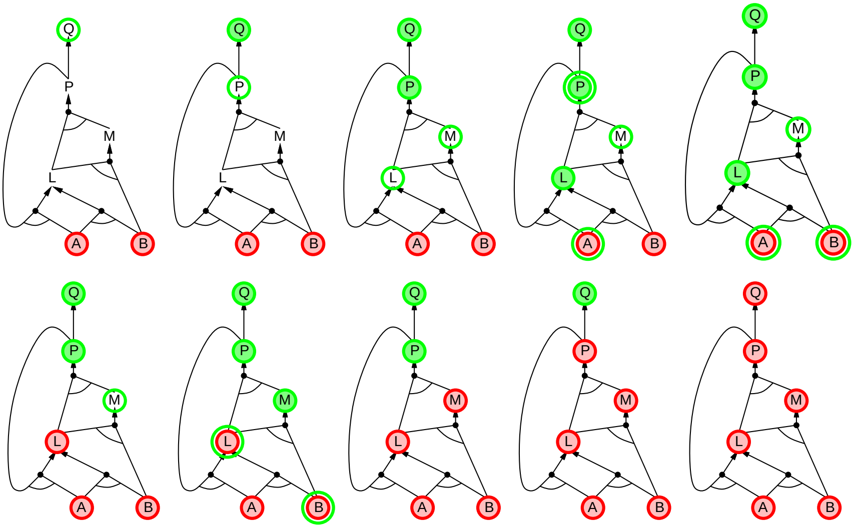
- to prove q by BC,
  - check if q is known already, or
  - prove by BC all premises of some rule concluding q

Avoid loops: check if new subgoal is already on the goal stack

Avoid repeated work: check if new subgoal

1. has already been proved true, or
2. has already failed

Backward chaining example



Forward vs. backward chaining

- FC is **data-driven**, cf. automatic, unconscious processing,
  - e.g., object recognition, routine decisions
  - May do lots of work that is irrelevant to the goal
- BC is **goal-driven**, appropriate for problem-solving,
  - e.g., Where are my keys? How do I get into a PhD program?
  - Complexity of BC can be *much less* than linear in size of *KB*

Effective Propositional Model Checking

Efficient propositional inference

Problem

The SAT problem is checking satisfiability of sentence α

- Application of SAT: testing entailment, $\alpha \models \beta$, can be done by testing **unsatisfiability** of $\alpha \wedge \neg\beta$.

Two families of efficient algorithms for general propositional inference based on **model checking**

1. Complete backtracking search algorithms
   - DPLL algorithm (proposed by Davis, Putnam, Logemann and Loveland)
2. Incomplete local search algorithms (hill-climbing)
   - WalkSAT algorithm

The DPLL algorithm

- Determine whether an input propositional logic sentence (**in CNF**) is satisfiable
- A recursive, depth-first enumeration of possible models.
- Improvements over truth table enumeration
  1. Early termination
  2. Pure symbol heuristic
  3. Unit clause heuristic

The DPLL algorithm (cont.)

- **Early termination**
  - A clause is true if any literal is true.
  - A sentence is false if any clause is false.
  - Avoid examination of entire subtrees in the search space
  - E.g., $(B \vee C) \wedge (B \vee D)$ is true if B is true, regardless C and D

The DPLL algorithm (cont.)

- **Pure symbol heuristic**
  - **Pure symbol**: always appears with the same "sign" in all clauses.
  - Make a pure symbol literal true → can never make a clause false
  - For example, given a sentence $A \vee \neg B, \neg B \vee \neg C, A \vee C \rightarrow B$ and C are pure, D is impure.
- **Unit clause heuristic**
  - **Unit clause**: only one literal in the clause → the only literal in a unit clause must be **true** → cause "cascade" of forced assignments (**unit propagation**)
  - For example, given a sentence $B, \neg B \vee \neg C$, if the model contains $B = \mathit{true}$ then $C = \mathit{false}$

Algorithm

```
function DPLL-SATISFIABLE?(s) returns true or false
inputs: s, a sentence in propositional logic.
  clauses ← the set of clauses in the CNF representation of s
  symbols ← a list of the proposition symbols in s
  return DPLL(clauses, symbols, ∅)

function DPLL(clauses, symbols, model) returns true or false
  if every clause in clauses is true in model then return true
  if some clause in clauses is false in model then return false
  P, value ← FIND-PURE-SYMBOL(symbols, clauses, model)
  if P ≠ ∅ then
    return DPLL(clauses, symbols − P, model ∪ {P = value})
  P, value ← FIND-UNIT-CLAUSE(clauses, model)
  if P ≠ ∅ then
    return DPLL(clauses, symbols − P, model ∪ {P = value})
  P ← FIRST(symbols); rest ← REST(symbols)
  return DPLL(clauses, rest, model ∪ {P = true}) or
         DPLL(clauses, rest, model ∪ {P = false})
```

Success of DPLL

- 1962 – DPLL invented
- 1992 – 300 propositions
- 1997 – 600 propositions (satz)
- Additional techniques:
  - Learning conflict clauses at backtrack points
  - Randomized restarts
  - 2002 (zChaff) 1,000,000 propositions – encodings of hardware verification problems

The WalkSAT algorithm

Incomplete, local search algorithm

- **Evaluation function**: The min-conflict heuristic of minimizing the number of unsatisfied clauses
- Balance between greediness and randomness
- When the algorithm returns a model
  - The input sentence is indeed satisfiable
- When it returns failure
  - The sentence is unsatisfiable OR we need to give it more time
- WALKSAT cannot always detect unsatisfiability
- It is most useful when a solution is expected to exist
- For example,
  - An agent cannot reliably use WALKSAT to prove that a square is safe in the Wumpus world.
  - Instead, it can say, "I thought about it for an hour and couldn't come up with a possible world in which the square isn't safe."

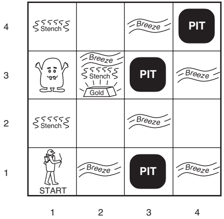
Algorithm

```
function WALKSAT(clauses, p, max_flips)
returns a satisfying model or failure
inputs: clauses, a set of clauses in propositional logic
        p, the probability of choosing to do a "random walk" move,
           typically around 0.5
        max_flips, number of flips allowed before giving up
  model ← a random assignment of true/false to the symbols
              in clauses
  for i = 1 to max_flips do
    if model satisfies clauses then return model
    clause ← a randomly selected clause from clauses that is false
                in model
    with probability p
      flip the value in model of a randomly selected symbol
        from clause
    else
      flip whichever symbol in clause maximizes
        the number of satisfied clauses
  return failure
```

Agents Based on Propositional Logic

Propositional Logic Based Agent

- Agent has to act given only local perception
- Agent is installed with two kinds of knowledge base
  - "Hardcode" knowledge base
    IF *glitter* ***THEN*** *grab gold*
    IF *wumpus or pit around* ***THEN*** *avoid it*
  - "Softcode" knowledge base *KB*



$$\neg P_{1,1}, \neg W_{1,1}$$
$$B_{x,y} \iff (P_{x,y+1} \vee P_{x,y-1} \vee P_{x+1,y} \vee P_{x-1,y})$$
$$S_{x,y} \iff (W_{x,y+1} \vee W_{x,y-1} \vee W_{x+1,y} \vee W_{x-1,y})$$
$$W_{1,1} \vee W_{1,2} \vee \ldots \vee W_{4,3} \vee W_{4,4}$$
$$\neg W_{1,1} \vee \neg W_{1,2}, \ldots$$

For a 4×4 wumpus world, the *KB* begin with a total of 155 sentences containing 64 distinct symbols

Algorithm

```
function PL-WUMPUS-AGENT(percept) returns an action
inputs: percept, a list, [stench,breeze,glitter]
static: KB, a knowledge base
        x, y, , the agent's position (initially 1,1)
        orientation, orientation (initially right)
        visited, an array indicating which squares have been visited,
                initially false
        action, the agent's most recent action, initially null
        plan, an action sequence, initially empty
  update x,y,orientation, visited based on action
  if stench then TELL(KB,S_{x,y}) else TELL(KB,¬S_{x,y})
  if breeze then TELL(KB,B_{x,y}) else TELL(KB,¬B_{x,y})
  if glitter then action ← grab
  else if plan ≠ ∅ then action ← POP(plan)
  else if for some fringe square [i,j], ASK(KB,(¬P_{i,j} ∧ ¬W_{i,j})) is true or
          for some fringe square [i,j], ASK(KB,(P_{i,j} ∨ W_{i,j})) is false then
    plan ← A*-GRAPH-SEARCH(ROUTE-PROBLEM([x,y],orientation,[i,j],visited))
    action ← POP(plan)
  else action ← a randomly chosen move
return action
```

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