(b) LEAST COST ENTRY METHOD

(or)

Lowest cost entry method

(or)

Matrix minimum method

	P_1	P_2	P_3	S
S_1	2	7	4	5
S_2	3	3	1 $\boxed{8}$	~~8~~
S_3	5	4	7	7
S_4	1	6	2	14
D	7	9	~~18~~ 10	$\boxed{34}$

(a) start the problem where the minimum cost is their. In this problem the M.C is '1'. But, '1' is in two places. Select any one

(b) LEAST COST ENTRY METHOD

(or)

Lowest Cost entry method

(or)

Matrix minimum method

	P_1	P_2	P_3	S
S_1	2	7	4	5
S_2	3	3	1 $\boxed{8}$	~~8~~
S_3	5	4	7	7
S_4	1	6	2	14
D	7	9	~~18~~ 10	$\boxed{34}$

(a) start the problem where the minimum cost is their. In this problem the M.C is '1'. But, '1' is in two places. Select any one

(b) LEAST COST ENTRY METHOD

(or)

Lowest Cost entry method

(or)

Matrix minimum method

	P_1	P_2	P_3	S
S_1	2	7	4	5
S_2	3	3	1 $\boxed{8}$	~~8~~
S_3	5	4	7	7
S_4	1 $\boxed{7}$	6	2	~~14~~ 7
D	~~7~~	9	~~18~~ 10	$\boxed{34}$

(a) start the problem where the minimum cost is their. In this problem the M.C is '1'. But, '1' is in two places. Select any one

(b) Second Row is deleted. Now Select min cost i.e. '1'

(b) LEAST COST ENTRY METHOD

(or)

Lowest cost entry method

(or)

Matrix minimum method

	P_1	P_2	P_3	S
S_1	2	7	4	5
S_2	3	3	1 $\boxed{8}$	~~8~~
S_3	5	4	7	7
S_4	1 $\boxed{7}$	6	2	~~14~~ 7
D	~~7~~	9	~~18~~ 10	$\boxed{34}$

(a) start the problem where the minimum cost is their. In this problem the M.C is '1'. But, '1' is in two places. Select any one

(b) Second Row is deleted. Now select min cost i.e. '1'

(b) LEAST COST ENTRY METHOD

(or)

Lowest Cost entry method

(or)

Matrix minimum method

	P_1	P_2	P_3	S
S_1	2	7	4	5
S_2	3	3	1 $\boxed{8}$	~~8~~
S_3	5	4	7	7
S_4	$\boxed{7}$ 1	6	$\boxed{7}$ 2	~~14~~ ~~7~~
D	~~7~~	9	~~18~~ ~~10~~ 3	$\boxed{34}$

(a) Start the problem where the minimum cost is their. In this problem the M.C is '1'. But, '1' is in two places. Select any one

(b) Second Row is deleted. Now Select min cost i.e., '1'

(c) Second Row & First Coloumn is deleted Now Select min cost i.e., '2'

(b) LEAST COST ENTRY METHOD

(or)

Lowest Cost entry method

(or)

Matrix minimum method

	P_1	P_2	P_3	S
S_1	2	7	4 $\boxed{3}$	$\cancel{5}$ 2
S_2	3	3	1 $\boxed{8}$	$\cancel{8}$
S_3	5	4	7	7
S_4	1 $\boxed{7}$	6	2 $\boxed{7}$	$\cancel{14}$ $\cancel{7}$
D	$\cancel{7}$	9	$\cancel{18}$ $\cancel{10}$ 3	$\boxed{34}$

(a) start the problem where the minimum cost is their. In this problem the M.C is '1'. But, '1' is in two places. Select any one

(b) Second Row is deleted. NOW Select min Cost i.e, '1'

(c) Second Row & First Coloumn is deleted NOW Select min Cost i.e, '2'

~~[illegible]~~

(d) 2nd Row, 1st Col, 4th Row is deleted NOW Select min cost i.e, '4'. But '4' is two places. Select any '4'

(b) LEAST COST ENTRY METHOD

(or)

Lowest Cost entry method

(or)

Matrix minimum method

	P_1	P_2	P_3	S
S_1	2	7	4 [3]	~~5~~ 2
S_2	3	3	1 [8]	~~8~~
S_3	5	4 [7]	7	~~7~~
S_4	1 [7]	6	2 [7]	~~14~~ ~~7~~
D	~~7~~	~~9~~ 2	~~18~~ ~~10~~ ~~3~~	34

(a) start the problem where the minimum cost is their. In this problem the M.C is '1'. But, '1' is in two places. Select any one

(b) Second Row is deleted. Now Select min cost i.e, '1'

(c) Second Row & First Coloumn is deleted Now Select min cost i.e, '2'

~~Hence select 1st~~

(d) 2nd Row, 1st Col, 4th Row is deleted Now Select min cost i.e, '4'. But '4' is two places. Select any '4'

(e) Select min cost i.e, '4'

(b) LEAST COST ENTRY METHOD

(or)

Lowest Cost entry method

(or)

Matrix minimum method

	P_1	P_2	P_3	S
S_1	2	7 $\boxed{2}$	4 $\boxed{3}$	~~5~~ ~~2~~
S_2	3	3	1 $\boxed{8}$	~~8~~
S_3	5	4 $\boxed{7}$	7	~~7~~
S_4	1 $\boxed{7}$	6	2 $\boxed{7}$	~~14~~ ~~7~~
D	~~7~~	~~9~~ ~~2~~	~~18~~ ~~10~~ ~~3~~	$\boxed{34}$

(a) start the problem where the minimum cost is their. In this problem the M.C is '1'. But, '1' is in two places. Select any one

(b) Second Row is deleted. NOW Select min Cost i.e, '1'

(c) Second Row & First Coloumn is deleted NOW Select min Cost i.e, '2'

~~Remove select, eyes~~

(d) 2nd Row, 1st Col, 4th Row is deleted NOW Select min cost ie, '4'. But '4' is two places. Select any '4'

(e) Select min cost i.e, '4'

$\therefore$ Min Cost $= 2\times7 + 3\times4 + 8\times1 + 7\times4 + 7\times1 + 7\times2$

$= 14 + 12 + 8 + 28 + 7 + 14$

$= 83$

$\boxed{\therefore M.C = 83/-}$

Step by step process :-

Step : 1

	P_1	P_2	P_3	S
S_1	~~7~~	7	4	5
S_2	~~3~~	3	1	8
S_3	~~5~~	4	7	7
S_4	$\boxed{7}$ 1	6	2	~~14~~ 7
D	~~7~~	9	18	$\boxed{34}$

→

Step : 2

	P_2	P_3	S
S_1	7	4	5
S_2	~~3~~	$\boxed{8}$ 1	~~8~~
S_3	4	7	7
S_4	6	2	7
D	9	~~18~~ 10	$\boxed{27}$

Step : 3

	P_2	P_3	S
S_1	7	4	5
S_3	4	7	7
S_4	~~6~~	$\boxed{7}$ 2	~~7~~
D	9	~~18~~ 3	$\boxed{19}$

→

Step : 4

	P_2	P_3	
S_1	7	$\boxed{3}$ 4	~~5~~ 2
S_3	4	~~7~~	7
	9	~~3~~	$\boxed{12}$

Step : 5

	P_2	
S_1	7	2
S_3	$\boxed{7}$ 4	~~7~~
	~~9~~ 2	$\boxed{9}$

→

Step : 6

	P_2	
S_1	$\boxed{2}$ 7	~~2~~
	~~2~~	$\boxed{2}$

$\therefore$ M.C = 7 + 8 + 14 + 12 + 28 + 14

$\boxed{\text{M.C} = 83/-}$

Travelling salesman :

The problem is to find the route shortest distance / time / cost.

Condition :- The salesman starts from his home city, he must visit every city exactly once and returns to his home city

Phase : 1

→ TSP can be first solved as AP by using HM to find optimal solution

→ Then check the TSP condition.

→ If the condition is satisified, then the AP solution will be the optimal solution. even for TSP solution

If not, optimal solution for TSP

Phase : 2

(*) The solution can be adjusted by inspection

(*) Form a single circuit

(*) The iterative procedure

→ Branch & Bound method

TRAVELLING SALESMAN PROBLEMS

CITY

	A	B	C	D	E
A	∞	2	5	7	1
B	6	∞	3	8	2
C	8	7	∞	4	7
D	12	4	6	∞	5
E	1	3	2	8	∞

CITY

Condition :-

The salesman starts from his home city, he must visit every city exactly once and returns to his home city

Sol:- step:① Row Reduction

∞	2	5	7	1	1
6	∞	3	8	2	2
8	7	∞	4	7	4
12	4	6	∞	5	4
1	3	2	8	∞	1

→

∞	1	4	6	0
4	∞	1	6	0
4	3	∞	0	3
8	0	2	∞	1
0	2	1	7	∞

step:② Coloumn Reduction

∞	1	4	6	0
4	∞	1	6	0
4	3	∞	0	3
8	0	2	∞	1
0	2	1	7	∞
0	0	1	0	0

→

∞	1	3	6	0
4	∞	0	6	0
4	3	∞	0	3
8	0	1	∞	1
0	2	0	7	∞

The problem is to find the route shortest distance / time / cost.

③ Row scanning:

	A	B	C	D	E
A	∞	1	3	6	$\boxed{0}$
B	4	∞	$\boxed{0}$	6	~~0~~
C	4	3	∞	$\boxed{0}$	3
D	8	$\boxed{0}$	1	∞	1
E	$\boxed{0}$	2	~~0~~	7	∞

5 Rows = 5 Assignments

Assignment problem is completed

A-E, B-C, C-D, D-B, E-A

A-E, E-A

B, C, D

→ B-C, C-D, D-B

Here A visits E and E visits A

But A does not visit to B, C, D.

So, it ~~doesn't~~ is not a travelling salesman

	A	B	C	D	E
A	∞	$\boxed{1}$	3	6	~~0~~
B	4	∞	$\boxed{0}$	6	~~0~~
C	4	3	∞	$\boxed{0}$	3
D	8	~~0~~	1	∞	$\boxed{1}$
E	$\boxed{0}$	2	~~0~~	7	∞

Here Zero is not ~~sui~~ suitable for calculating of optimal solution for ~~TSP~~ Travelling salesman problem. Now Select next maximum i.e 'i' in First row

A-B, B-C, C-D, D-E, E-A

∴ The optimal travelling cost is 15/-

$\boxed{\text{i.e, M.C} = 15/-}$

A-B → 2
B-C → 3
C-D → 4
D-E → 5
E-A → 1

15

VOGEL'S APPROXIMATION METHOD : VAM -

(OR)

PENALTY Method

	P_1	P_2	P_3	Sup
S_1	2	7	4	5
S_2	3	3	1	8
S_3	5	4	7	7
S_4	1	6	2	14
Dem	7	9	18	$\boxed{34}$

VOGEL'S APPROXIMATION METHOD : VAM -

(OR)

PENALTY Method

	P_1	P_2	P_3	Sup	Row Penality
S_1	2	7	4	5	
S_2	3	3	1	8	
S_3	5	4	7	7	
S_4	1	6	2	14	
Dem	7	9	18	$\boxed{34}$	
Col Penality					

Penality = 2^{nd} min − 1^{st} min

Step: ① Calculate the penality for Each and Every Row and Each and Every Coloumn

VOGEL'S APPROXIMATION METHOD : VAM -

(or)

PENALITY method

	P_1	P_2	P_3	Sup	Row Penality
S_1	2	7	4	5	2
S_2	3	3	1	8	2
S_3	5	4	7	7	1
S_4	1	6	2	14	1
Dem	7	9	18	$\boxed{34}$	
Col Penality					

Penality = 2^{nd} min − 1^{st} min

Step: ① Calculate the penality for Each and Every Row and Each and Every Coloumn

~~Firs~~ 1^{st} Row $= 4-2 = 2$

2^{nd} Row $= 3-1 = 2$

3^{rd} Row $= 5-4 = 1$

4^{th} Row $= 2-1 = 1$

VOGEL'S APPROXIMATION METHOD : VAM -

(or)

PENALTY method

	P_1	P_2	P_3	Sup	Row Penality
S_1	2	7	4	5	2
S_2	3	3	1	8	2
S_3	5	4	7	7	1
S_4	1	6	2	14	1
Dem	7	9	18	$\boxed{34}$	
Col Penality	1	1	1		

Penality = 2^{nd} min − 1^{st} min

Step: ① Calculate the penality for Each and Every Row and Each and Every Coloumn

S_1 ~~First~~ → 1^{st} Row = 4 − 2 = 2

S_2 → 2^{nd} Row = 3 − 1 = 2

S_3 → 3^{rd} Row = 5 − 4 = 1

S_4 → 4^{th} Row = 2 − 1 = 1

P_1 → 1^{st} col = 2 − 1 = 1

P_2 → 2^{nd} col = 4 − 3 = 1

P_3 → 3^{rd} col = 2 − 1 = 1

Step : 2

	P_1	P_2	P_3	S	Row Penality
S_1	2	7	4	5	2
S_2	3	3	1	8	2
S_3	5	4	7	7	1
S_4	1	6	2	14	1
D	7	9	18	34	
Col penality	1	1	1		

Select maximum penality in all the penality of Both Rows and coloumns

Step : 2

	P_1	P_2	P_3	S	Row Penality
S_1	2	7	4	5	2
S_2	3	3	1	8	2
S_3	5	4	7	7	1
S_4	1	6	2	14	1
D	7	9	18	34	
Col penality	1	1	1		

Select maximum penality in all the penality of Both Rows and coloumns

Here max penality '2'. But '2' is in 2 places i.e, In S_1 & S_2

Now Select the supply of Both S_1 & S_2 is 5 & 8.

Select minimum ~~that~~ of Both 5 & 8 i.e, 5 i.e, S_1

Now Select 'S_1' Row

Now Select min cost in S_1

Step : 2

	P_1	P_2	P_3	S	Row Penality
S_1	5 / 2	7	4	~~5~~	2
S_2	3	3	1	8	2
S_3	5	4	7	7	1
S_4	1	6	2	14	1
D	2 ~~7~~	9	18	34	
Col penality	1	1	1		

Select maximum penality in all the penality of Both Rows and Coloumns

Here max penality '2'. But '2' is in 2 places i.e, In S_1 & S_2

Now Select the supply of Both S_1 & S_2 is 5 & 8.

Select minimum ~~that~~ of Both 5 & 8 i.e, 5 i.e S_1

Now Select 'S_1' Row

Now Select min cost in S_1

Step: 3

				S	Row Penality
	~~3~~	3	1	8	2
	~~5~~	4	7	7	1
	1 $\boxed{2}$	6	2	~~14~~ 12	1
D	~~2~~	9	18	$\boxed{29}$	
Col Penality	2	1	1		

→ Select the max penality in Both Rows & Coloumns ie, 2. But '2' is in two places.

→ Now Select the Corresponding Supply & Demand for the max penality '2'
ie, supply = 2 & Demand = 8

→ Now Select minimum value in both 2 & 8 ie, 2 = supply

→ Select the 1st coloum then Select the min cost in that coloumn

Step: 4

	P_2	P_3	Sup	Row Penality
S_2	3	1	8	2
S_3	4	7	7	3
S_4	6	2 (12)	~~12~~	4
Rem	9	6 ~~18~~	$\boxed{27}$	
Col Penality	1	1		

→ out of '5' penalities select max penality i.e., 4. But '4' is 3rd row i.e., S_4.

→ Select S_4 then select min cost in S_4 i.e., 2.

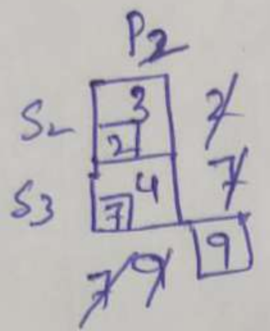
step : 5

	P_2	P_3	Sup	Row Penality
S_2	3	1 $\boxed{6}$	~~8~~ 2	2
S_3	4	~~7~~	7	3
Rem	9	~~6~~	$\boxed{15}$	
Col Penality	1	6		

→ out of '4' Penalities Select max Penality i.e, 6. But '6' is in 2nd Coloumn i.e, P_3.

→ In that P_3 select min cost

Step: 6



$$\therefore \text{Min Cost} = 5\times2 + 2\times1 + 12\times2 + 6\times1 + 2\times3 + 7\times4$$

$$= 10 + 2 + 24 + 6 + 6 + 28$$

$$= 76$$

$$\boxed{\therefore M.C = 76/-}$$

	P1	P2	P3	Supply	Row Penalty			
S1	2 $\boxed{5}$	~~7~~	~~4~~	~~5~~ ✓	2	*	*	*
S2	~~3~~	3 $\boxed{2}$	1 $\boxed{6}$	~~8~~ ~~2~~	2	2	2	2
S3	~~5~~	4 $\boxed{7}$	~~7~~	~~7~~	1	1	3	3
S4	1 $\boxed{2}$	~~8~~	2 $\boxed{12}$	~~14~~ ~~12~~	1	1	4	*
Demand	~~7~~ ~~2~~ ✓	~~9~~ ~~7~~	~~18~~ ~~6~~	$\boxed{34}$				

Col Penality	P1	P2	P3
	1	1	1
	2	1	1
	*	1	1
	*	1	6

$$M \cdot C = 10 + 6 + 6 + 28 + 2 + 24 = 76$$

$$\boxed{M \cdot C = 76/-}$$

ASSIGNMENT problems

Assignment problem :- choose the correct person for the correct JOB with in the minimum cost is called Assignment.

Form of Assignment :-

	J_1	J_2	----	J_n
P_1	C_{11}	C_{12}	---	C_{1n}
P_2	C_{21}	C_{22}	---	C_{2n}
⋮	⋮	⋮		⋮
P_n	C_{n1}	C_{n2}	---	C_{nn}

Models of Assignment :- Two types

(a) Balanced ⇒ No of Rows = No of Coloumns

(b) UnBalanced ⇒ No of Rows ≠ No of Coloumns

Methods of Assignment :- Two types

(a) Maximization

(b) Minimization

Note :- ① Assignment problem can be solved when it is in the Form of Balanced & minimization.

② If problem is UnBalanced then Balanced it

③ If problem is maximization convert it into ~~mini~~ minimization.

where P_1 P_2 --- P_n are persons

J_1, J_2 --- J_n are JOBs

C_{11}, C_{12} --- .. C_{nn} are costs.

Another Name for the Assignment problem is Hungarian method.

① Solve the following Assignment problem and Find the minimum Cost.

	J_1	J_2	J_3	J_4
P_1	2	10	9	7
P_2	15	4	14	8
P_3	13	14	16	11
P_4	4	15	13	4

Sol:- Step: ① The Given problem is Balanced Assignment problem. Since No of Rows is Equal to No of Coloumns

$\Longrightarrow 4 = 4$

Step: ② Check the given problem is ~~Balanced~~ maximization & minimization.

In the above problem they asked minimum Cost. So the given problem is minimization problem.

Step: ③ Row Reduced :-

Select the minimum Element in Each and Every Row. Then Substract it in all the elements of the Corresponding Rows.

	J_1	J_2	J_3	J_4	
P_1	2	10	9	7	2
P_2	15	4	14	8	4
P_3	13	14	16	11	11
P_4	4	15	13	4	4

$\longrightarrow$

	J_1	J_2	J_3	J_4
P_1	0	8	7	5
P_2	11	0	10	4
P_3	2	3	5	0
P_4	0	11	9	0

Step: ④ Coloumn Reduction

Select the minimum Element in each and every Coloumn then Subtract it in all the Elements of corresponding Coloumns to reduced problem.

	J_1	J_2	J_3	J_4
P_1	0	8	7	5
P_2	11	0	10	4
P_3	2	3	5	0
P_4	0	11	9	0
	0	0	5	0

$\longrightarrow$

0	8	2	5
11	0	5	4
2	3	0	0
0	11	4	0

Step: ⑤ Row Scanning

Scane Each and Every Row, if their is a single zero '0' (or) more than one zero

Case: ① If there is a single zero

Assign that zero "$\boxed{0}$" as called as Assignment. and strike the corresponding coloumn zeros

Case: ② If more than one zero.

Jump to next row.

	J_1	J_2	J_3	J_4
P_1	$\boxed{0}$	8	2	5
P_2	11	$\boxed{0}$	5	4
P_3	2	3	$\boxed{0}$	$\cancel{0}$
P_4	$\cancel{0}$	11	4	$\boxed{0}$

$P_1 - J_1 - 2$
$P_2 - J_2 - 4$
$P_3 - J_3 - 16$
$P_4 - J_4 - 4$
$\overline{26/-}$

Each Row has an assignment "$\boxed{0}$".

No of Rows = 4 & No of Assignment = 4

The problem is completed

$\therefore$ The M.C = 26/-

HUNGARIAN METHOD :-

step (1) Check the given problem is Balanced (or) UnBalanced

Balanced → No of Rows = No of Coloumns

UnBalanced → No of Rows ≠ No of Coloumns

If UnBalanced is given then Convert it into Balanced then go to step:②

step (2) Check the given problem is either maximization (or) minimization.

If maximization convert it into minimization

If minimization go to step:③

step (3) Row Reduction

Select the minimum element in each and Every row then Subtract it in all the Elements of Corresponding Rows

step (4) Coloumn Reduction

Select the minimum Element in Each and Every Coloumn then Subtract it in all the elements of Corresponding Coloumns

step:(5) Row Scanning

Scan Each and Every row ~~the~~ if their is a Single Zero (or) more than one Zero.

→ If Single Zero is their

Assign that Zero '$\boxed{0}$' as called as assignment then strike of all the Corresponding Coloumn Zero's '$\cancel{0}$'

→ If more than one zero :-

leave that row then Jump to next row.

Step: ⑥ Coloumn Scanning

Scane Each and Every Coloumn if their is single zero (or) more than one zero.

→ If their is single zero :-

Assign that zero $\boxed{0}$ as called as a assignment and strike of the corresponding row zero's '⊠'

Step: ⑦ After completion of Row Scanning and Coloumn Scanning check all the rows have assignments '$\boxed{0}$'

→ If all the rows have assignments

i.e., No of Rows = No of assignments

then the problem is completed

then find the minimum cost.

→ If all the rows have no assignments

i.e., No of Rows ≠ No of assignments

then the problem is Not completed.

Now construct new assignment table then apply Row Scanning & Coloumn Scanning untill No of Rows is Equal to No of asisnments.

Step: ⑧ Construction of New assignment by using of marking process & stricking process.

→ Marking process:-

(a) Mark the row's which have no assignments $\boxed{0}$

(b) Marks the coloumn's which have cancelled zero's '⊠' in the marked row's

(c) Mark the row's which have assignments in the marked coloumn's

(d) Repeate (b) & (c)

→ Striking process:-

Strike unmarked rows and marked coloumns.

Then Construct the new assignment table

Operators

Ex: ①

JOBS	1	2	3	4	5
1	10	12	15	12	8
2	7	16	14	14	11
3	13	14	7	9	9
4	12	10	11	13	10
5	8	13	15	11	15

Solution:

(1) Step: ① The given problem is a Balanced assignment problem since No of Rows is Equal to No of Coloumns

$\implies$ 5 = 5

(2) Step: ② The given problem is a minimization problem. since they did not given for Calculating of min Cost (or) max cost.

(3) Step: ③ Row Reduction

					Row min
10	12	15	12	8	8
7	16	14	14	11	7
13	14	7	9	9	7
12	10	11	13	10	10
8	13	15	11	15	8

$\rightarrow$

2	4	7	4	0
0	9	7	7	4
6	7	0	2	2
2	0	1	3	0
0	5	7	3	7

Step: ④ Coloumn Reduction

2	4	7	4	0
0	9	7	7	4
6	7	0	2	2
2	0	1	3	0
0	5	7	3	7
0	0	0	2	0

→

2	4	7	2	0
0	9	7	5	4
6	7	0	0	2
2	0	1	1	0
0	5	7	1	7

Step: ⑤ Row scanning

2	4	7	2	$\boxed{0}$
$\boxed{0}$	9	7	5	4
6	7	0	0	2
2	$\boxed{0}$	1	1	$\cancel{0}$
$\cancel{0}$	5	7	1	7

Step: ⑥ Coloumn scanning

2	4	7	2	$\boxed{0}$
$\boxed{0}$	9	7	5	4
6	7	$\boxed{0}$	$\cancel{0}$	2
2	$\boxed{0}$	1	1	$\cancel{0}$
$\cancel{0}$	5	7	1	7

Step: ⑦ Now check No of Rows is Equal to No of Assignments

$\Rightarrow 5 \neq 4$

The problem is Not Completed

Step: 8 Construction of New assignment

$$\begin{bmatrix} 2 & 4 & 7 & 2 & \boxed{0} \\ \boxed{0} & 9 & 7 & 5 & 4 \\ 6 & 7 & \boxed{0} & \cancel{0} & 2 \\ 2 & \boxed{0} & 1 & 1 & \cancel{0} \\ \cancel{0} & 5 & 7 & 1 & 7 \end{bmatrix}$$

$\downarrow$

$$\begin{array}{c|ccccc|} & O_1 & O_2 & O_3 & O_4 & O_5 \\ \hline J_1 & 3 & 4 & 7 & 2 & \boxed{0} \\ J_2 & \boxed{0} & 8 & 6 & 4 & 3 \\ J_3 & 7 & 7 & \boxed{0} & \cancel{0} & 2 \\ J_4 & 3 & \boxed{0} & 1 & 1 & \cancel{0} \\ J_5 & \cancel{0} & 4 & 6 & \boxed{0} & 6 \\ \hline \end{array}$$

Now check: No of Rows is equal to No of Assignments

$\therefore$ The problem is completed

$J_1 - O_5 \rightarrow 8$

$J_2 - O_1 \rightarrow 7$

$J_3 - O_3 \rightarrow 7$

$J_4 - O_2 \rightarrow 10$

$J_5 - O_4 \rightarrow 11$

$\overline{\quad 43 \quad}$

$\boxed{\therefore M.C = 43/-}$

— x —

06/09/20

(1) Solve the following Assignment problem and find Maximum Cost.

	I	II	III	IV	V
1	9	8	4	8	4
2	7	3	2	3	5
3	9	9	5	9	6
4	8	7	5	2	5
5	9	8	8	5	2

Sol:- Step:① The given problem is Balanced assignment problem sinc No of Rows ~~of~~ is Equal to No of Coloumns

$\Rightarrow$ $5 = 5$

Step:② The given problem is maximization assignment problem sinc the given problem is maximum cost.

Now convert maximization into minimization

Conversion of Max into Min:-

Select the maximum cost in the above problem. i.e, '9'. Now subtract all elements in that maximum element. Now the Reduced matrix is

0	1	5	1	5
2	6	7	6	4
0	0	4	0	3
1	2	4	7	4
0	1	1	4	7

Step: 3 Row Reduction

$$\begin{array}{|c|c|c|c|c|}\hline 0 & 1 & 5 & 1 & 5 \\ \hline 2 & 6 & 7 & 6 & 4 \\ \hline 0 & 0 & 4 & 0 & 3 \\ \hline 1 & 2 & 4 & 7 & 4 \\ \hline 0 & 1 & 1 & 4 & 7 \\ \hline \end{array}\begin{array}{c} 0 \\ 2 \\ 0 \\ 1 \\ 0 \end{array} \longrightarrow \begin{array}{|c|c|c|c|c|}\hline 0 & 1 & 5 & 1 & 5 \\ \hline 0 & 4 & 5 & 4 & 2 \\ \hline 0 & 0 & 4 & 0 & 3 \\ \hline 0 & 1 & 3 & 6 & 3 \\ \hline 0 & 1 & 1 & 4 & 7 \\ \hline \end{array}$$

Step: 4 Coloumn Reduction

$$\begin{array}{c} \begin{array}{|c|c|c|c|c|}\hline 0 & 1 & 5 & 1 & 5 \\ \hline 0 & 4 & 5 & 4 & 2 \\ \hline 0 & 0 & 4 & 0 & 3 \\ \hline 0 & 1 & 3 & 6 & 3 \\ \hline 0 & 1 & 1 & 4 & 7 \\ \hline \end{array} \\ \begin{array}{ccccc} 0 & 0 & 1 & 0 & 2 \end{array} \end{array} \longrightarrow \begin{array}{|c|c|c|c|c|}\hline 0 & 1 & 4 & 1 & 3 \\ \hline 0 & 4 & 4 & 4 & 0 \\ \hline 0 & 0 & 3 & 0 & 1 \\ \hline 0 & 1 & 2 & 6 & 1 \\ \hline 0 & 1 & 0 & 4 & 5 \\ \hline \end{array}$$

Step: 5 Row Scanning

Step: 6 Coloumn Scanning

$$\begin{array}{|c|c|c|c|c|}\hline \boxed{0} & 1 & 4 & 1 & 3 \\ \hline \cancel{0} & 4 & 4 & 4 & \boxed{0} \\ \hline \cancel{0} & 0 & 3 & 0 & 1 \\ \hline \cancel{0} & 1 & 2 & 6 & 1 \\ \hline \cancel{0} & 1 & \boxed{0} & 4 & 5 \\ \hline \end{array} \longrightarrow \begin{array}{|c|c|c|c|c|}\hline \boxed{0} & 1 & 4 & 1 & 3 \\ \hline \cancel{0} & 4 & 4 & 4 & \boxed{0} \\ \hline \cancel{0} & \boxed{0} & 3 & \cancel{0} & 1 \\ \hline \cancel{0} & 1 & 2 & 6 & 1 \\ \hline \cancel{0} & 1 & \boxed{0} & 4 & 5 \\ \hline \end{array}$$

No of Rows ≠ No of Assignments

⇒ The problem is Not Completed

Construction of New Assignments:-

marking process :- & Sticking process

$\boxed{0}$	1	4	1	3
~~0~~	4	4	4	$\boxed{0}$
~~0~~	$\boxed{0}$	3	~~0~~	1
~~0~~	1	2	6	1
~~0~~	1	$\boxed{0}$	4	5

New Assignment & Row Scanning & Coloumn Scanning

0	0	3	0	2
1	4	4	4	$\boxed{0}$
1	0	3	0	1
0	0	1	5	~~0~~
1	1	$\boxed{0}$	4	5

There is No Chance for Assigning the Assignment since parallel zero's are their. This type of Assignment problem is called Branch & Bound Technique Assignment problem.

Now Apply Assuming process for Selecting Assignments in Branch & Bound Technique.

0	0	3	0	2
1	4	4	4	$\boxed{0}$
1	0	3	0	1
0	0	1	5	~~0~~
1	1	$\boxed{0}$	4	5

Case ①

Case ②

Now the above problem is split into two case:

Case ①

	I	II	III	IV	V
1	$\boxed{0}$	~~0~~	3	~~0~~	2
2	1	4	4	4	$\boxed{0}$
3	1	~~0~~	3	$\boxed{0}$	1
4	~~0~~	$\boxed{0}$	1	5	~~0~~
5	1	1	$\boxed{0}$	4	5

Case ②

	I	II	III	IV	V
1	~~0~~	$\boxed{0}$	3	~~0~~	2
2	1	4	4	4	$\boxed{0}$
3	1	~~0~~	3	$\boxed{0}$	1
4	$\boxed{0}$	~~0~~	1	5	~~0~~
5	1	1	$\boxed{0}$	4	5

In the Both cases ~~the~~

NO of Rows = NO of Assignments

The problem is completed

Case: ①

1-I = 9
2-V = 5
3-IV = 9
4-II = 7
5-III = 8

38

Case: ②

1-II = 8
2-V = 5
3-IV = 9
4-I = 8
5-II = 8

38

∴ Maximum Cost is 38/-

UNBALANCED ASSIGNMENT problem

Find the Max cost ?

	J_1	J_2	J_3
O_1	2	7	4
O_2	3	3	1
O_3	5	4	7
O_4	1	6	2

Sol:- step: ① The given problem is UnBalanced Assignment problem since No of Rows is Not Equal to No of Coloumns

$\Rightarrow 4 \neq 3$

Now we have to Balance it by adding one dummy Coloumn with cost zero's.

Now the Reduced problem is

	J_1	J_2	J_3	J_4
O_1	2	7	4	0
O_2	3	3	1	0
O_3	5	4	7	0
O_4	1	6	2	0

Balanced since No of Rows is Equal to No of Coloumns

Step:② The Given problem is Maximization Assignment problem since they asked maximum cost. Now convert max into min

Select the max cost in the Reduced problem then subtract it in all the cost's

Now the Reduced problem is

	J_1	J_2	J_3	J_4
O_1	2	7	4	0
O_2	3	3	1	0
O_3	5	4	7	0
O_4	1	6	2	0

$\xrightarrow{\text{Select '7'}}$

	J_1	J_2	J_3	J_4
O_1	5	0	3	7
O_2	4	4	6	7
O_3	2	3	0	7
O_4	6	1	5	7

Step:③ Row Reduction

5	0	3	7	0
4	4	6	7	4
2	3	0	7	0
6	1	5	7	1

$\longrightarrow$

5	0	3	7
0	0	2	3
2	3	0	7
5	0	4	6

Step:④ Coloumn Reduction

5	0	3	7
0	0	2	3
2	3	0	7
5	0	4	6
0	0	0	3

$\longrightarrow$

5	0	3	4
0	0	2	0
2	3	0	4
5	0	4	3

Step: 5 Row Scanning   Step: 6 Coloumn Scanning

5	$\boxed{0}$	3	4
0	$\cancel{0}$	2	0
2	3	$\boxed{0}$	4
5	$\cancel{0}$	4	3

$\longrightarrow$

5	$\boxed{0}$	3	4
$\boxed{0}$	$\cancel{0}$	2	$\cancel{0}$
2	3	$\boxed{0}$	4
5	$\cancel{0}$	4	3

No. of Rows $\neq$ No. of assignments

The problem is Not Completed.

Construction New assignment

Marking process & Sticking process

5	$\boxed{0}$	3	4	✓ 3
$\boxed{0}$	$\cancel{0}$	2	$\cancel{0}$	
2	3	$\boxed{0}$	4	
5	$\cancel{0}$	4	3	✓ 1
	✓ 2			

2	0	0	1
0	3	2	0
2	6	0	4
2	0	1	0

Apply Row Scanning & Coloumn Scanning

	J_1	J_2	J_3	J_4
O_1	2	$\boxed{0}$	$\cancel{0}$	1
O_2	$\boxed{0}$	3	2	$\cancel{0}$
O_3	2	6	$\boxed{0}$	4
O_4	2	$\cancel{0}$	1	$\boxed{0}$

$O_1 - J_2 \rightarrow 7$
$O_2 - J_1 \rightarrow 3$
$O_3 - J_3 \rightarrow 7$
$O_4 - J_4 \rightarrow 0$

13

$\boxed{\therefore \text{Max cost} = 13/-}$

4. Discuss the steps of Hungerian method.
5. Consider the assignment problem as shown below. In this problem, five different jobs are to be assigned to five different operators such that the total processing time is minimized. The matrix entries represent processing times in hours.

		Operator				
		1	2	3	4	5
Job	1	5	6	8	6	4
	2	4	8	7	7	5
	3	7	7	4	5	4
	4	6	5	6	7	5
	5	4	7	8	6	8

Develop a zero-one programming model for the above problem.
6. Solve the following assignment problem using Hungerian method. The matrix entries are processing times in hours.

		Operator				
		1	2	3	4	5
	1	20	22	35	22	18
	2	4	26	24	24	7
Job	3	23	14	17	19	19
	4	17	15	16	18	15
	5	16	19	21	19	25

7. A college is having an undergraduate programme for which the effective semester time available is very less and the degree course requires field work. Hence, the savings in the total number of class hours handled can be utilized for such field work. Based on past experience, the college has established the number of hours required by each faculty to teach each subject. The course in its present semester has 4 subjects and the college has considered 6 existing faculty members to teach these courses. The objective is to assign the best 4 teachers, out of these 6 faculty to teach 4 different subjects such that the total number of class hours required is minimized. The data for this problem is summarized below. Solve and optimize the assignment problem.

		Subject			
		1	2	3	4
	1	25	44	33	35
	2	33	40	40	43
Faculty	3	40	35	33.	30
	4	44	45	28	35
	5	45	35	38	40
	6	40	49	40	46

8. Consider the problem of assigning four sales persons to four different sales regions as shown below such that the total sales is maximized.

		Sales region			
		1	2	3	4
	1	5	11	8	9
Salesman	2	5	7	9	7
	3	7	8	9	9
	4	6	8	11	12

The cell entries represent annual sales figures in crores of rupees. Find the optimal allocation of the sales persons to different regions.

Step: 8 Construction of New assignment

(2)

2	4	7	2	$\boxed{0}$	
$\boxed{0}$	9	7	5	4	(3) ✓
6	7	$\boxed{0}$	$\cancel{0}$	2	
2	$\boxed{0}$	1	1	$\cancel{0}$	
$\cancel{0}$	5	7	1	7	(1) ✓

↓

	O1	O2	O3	O4	O5
J1	3	4	7	2	$\boxed{0}$
J2	$\boxed{0}$	8	6	4	3
J3	7	7	$\boxed{0}$	$\cancel{0}$	2
J4	3	$\boxed{0}$	1	1	$\cancel{0}$
J5	$\cancel{0}$	4	6	$\boxed{0}$	6

Now check: No of Rows is equal to

No of Assignments

∴ The problem is completed

$J_1 - O5 \rightarrow 8$

$J_2 - O_1 \rightarrow 7$

$J_3 - O_3 \rightarrow 7$

$J_4 - O2 \rightarrow 10$

$J5 - O4 \rightarrow 11$

$\quad\quad\quad\quad\quad 43$

$\boxed{\therefore M.C = 43/-}$

— x —

Step: ④ Coloumn Reduction

Select the minimum Element in each and every Coloumn then Subtract it in all the Elements of corresponding Coloumns to reduced problem.

	J_1	J_2	J_3	J_4
P_1	0	8	7	5
P_2	11	0	10	4
P_3	2	3	5	0
P_4	0	11	9	0
	0	0	5	0

→

0	8	2	5
11	0	5	4
2	3	0	0
0	11	4	0

Step: ⑤ Row Scanning

Scane Each and Every Row, if their is a single zero '0' (or) more than one zero

Case: ① If there is a single zero

Assign that zero "$\boxed{0}$" as called as Assignment. and stike the corresponding Coloumn zeros

Case: ② If more than one zero.

Jump to next row.

	J_1	J_2	J_3	J_4
P_1	$\boxed{0}$	8	2	5
P_2	11	$\boxed{0}$	5	4
P_3	2	3	$\boxed{0}$	$\cancel{0}$
P_4	$\cancel{0}$	11	4	$\boxed{0}$

$P_1 - J_1 - 2$
$P_2 - J_2 - 4$
$P_3 - J_3 - 16$
$P_4 - J_4 - 4$
26/-

Each Row has an assignment '$\boxed{0}$'.
No of Rows = 4 & No of Assignment = 4
The problem is Completed
∴ The M.C = 26/-

HUNGARIAN METHOD :-

Step (1) Check the given problem is Balanced (or) UnBalanced

Balanced → No of Rows = No of Coloumns

UnBalanced → No of Rows ≠ No of Coloumns

If UnBalanced is given then Convert it into Balanced then go to step: ②

Step (2) Check the given problem is either maximization (or) minimization.

If maximization convert it into minimization

If minimization go to step: ③

Step (3) Row Reduction

Select the minimum element in each and Every row then Subtract it in all the Elements of Corresponding Rows

Step (4) Coloumn Reduction

Select the minimum Element in Each and Every Coloumn then Subtract it in all the elements of Corresponding Coloumns

Step (5) Row Scanning

Scan Each and Every row ~~the~~ if their is a Single Zero (or) more than one Zero.

→ If Single Zero is their

Assign that Zero '$\boxed{0}$' as called as assignment then strike of all the Corresponding Coloumn Zero's '⊗'

ASSIGNMENT problems

Assignment problem :- choose the Correct person For the Correct JOB with in the minimum Cost is Called Assignment.

Form of Assignment :-

	J_1	J_2	---	J_n
P_1	C_{11}	C_{12}	---	C_{1n}
P_2	C_{21}	C_{22}	---	C_{2n}
⋮	⋮			⋮
P_n	C_{n1}	C_{n2}	---	C_{nn}

Models of Assignment :- Two types

(a) Balanced ⇒ NO of Rows = NO of Coloumns

(b) unBalanced ⇒ NO of Rows ≠ NO of Coloumns

Methods of Assignment :- Two types

(a) Maximization

(b) Minimization

Note :- ① Assignment problem can be solved when it is in the Form of Balanced & minimization.

② If problem is unBalanced then Balanced it

③ If problem is maximization convert it into minimization.

where P_1 P_2 --- P_n are persons

J_1, J_2 --- J_n are JOBs

C_{11}, C_{12} ... .. C_{nn} are costs.

Another Name for the Assignment problem is Hungarian method.

① Solve the following Assignment problem and Find the minimum cost.

	J_1	J_2	J_3	J_4
P_1	2	10	9	7
P_2	15	4	14	8
P_3	13	14	16	11
P_4	4	15	13	4

Sol:- Step: ① The Given problem is Balanced Assignment problem. Since No of Rows is Equal to No of Coloumns

$\Longrightarrow 4 = 4$

Step: ② Check the given problem is ~~Balanced~~ maximization & minimization. In the above problem they asked minimum cost. So the given problem is minimization problem.

Step: ③ Row Reduced :-

Select the minimum Element in each and Every Row. Then substract it in all the elements of the Corresponding Rows.

	J_1	J_2	J_3	J_4	
P_1	2	10	9	7	2
P_2	15	4	14	8	4
P_3	13	14	16	11	11
P_4	4	15	13	4	4

$\longrightarrow$

	J_1	J_2	J_3	J_4
P_1	0	8	7	5
P_2	11	0	10	4
P_3	2	3	5	0
P_4	0	11	9	0

→ If more than one zero :-
leave that row then Jump to next row.

Step: ⑥ Coloumn scanning

Scane Each and Every Coloumn if their is single zero (or) more than one zero.

→ If their is single zero :-
Assign that zero $\boxed{0}$ as called as assignment and strike of the corresponding row zero's '⊗'

Step: ⑦ After completion of Row scanning and Coloumn scanning check all the rows have assignments '$\boxed{0}$'

→ If all the rows have assignments
i.e. No of Rows = No of assignments
then the problem is completed
then Find the minimum cost.

→ If all the rows have no assignments
i.e. No of Rows ≠ No of assignments
then the problem is Not Completed.
then the

Now construct new assignment table then apply Row scanning & Coloumn scanning untill No of Rows is Equal to No of assisnments.

Step: ⑧ Construction of New assignment by using of marking process & stricking process.

→ Marking process:-

(a) Mark the row's which have No assignments $\boxed{0}$

(b) Marks the coloumn's which have in the marked row's Cancelled zero's '$\otimes$' in the marked row's

(c) Mark the row's which have assignments in the marked coloumn's

(d) Repeate (b) & (c)

→ Striking process:-

Strike un marked rows and marked Coloumns.

Then Constuct the new assignment Table

Operators

Ex: 1

JOBS	1	2	3	4	5
1	10	12	15	12	8
2	7	16	14	14	11
3	13	14	7	9	9
4	12	10	11	13	10
5	8	13	15	11	15

Solution:

(1) Step: 1 The given problem is a Balanced assignment problem since No of Rows is Equal to No of Coloumns

$\Rightarrow 5 = 5$

(2) Step: 2 The given problem is a minimization problem. since they did not given for calculating of min cost (or) max cost.

(3) Step: 3 Row Reduction

					Row min
10	12	15	12	8	8
7	16	14	14	11	7
13	14	7	9	9	7
12	10	11	13	10	10
8	13	15	11	15	8

→

2	4	7	4	0
0	9	7	7	4
6	7	0	2	2
2	0	1	3	0
0	5	7	3	7

step: ④ Coloumn Reduction

$$\begin{bmatrix} 2 & 4 & 7 & 4 & 0 \\ 0 & 9 & 7 & 7 & 4 \\ 6 & 7 & 0 & 2 & 2 \\ 2 & 0 & 1 & 3 & 0 \\ 0 & 5 & 7 & 3 & 7 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & 4 & 7 & 2 & 0 \\ 0 & 9 & 7 & 5 & 4 \\ 6 & 7 & 0 & 0 & 2 \\ 2 & 0 & 1 & 1 & 0 \\ 0 & 5 & 7 & 1 & 7 \end{bmatrix}$$

0 0 0 2 0

step: ⑤ Row scanning

$$\begin{bmatrix} 2 & 4 & 7 & 2 & \boxed{0} \\ \boxed{0} & 9 & 7 & 5 & 4 \\ 6 & 7 & 0 & 0 & 2 \\ 2 & \boxed{0} & 1 & 1 & \cancel{0} \\ \cancel{0} & 5 & 7 & 1 & 7 \end{bmatrix}$$

step: ⑥ Coloumn scanning

$$\begin{bmatrix} 2 & 4 & 7 & 2 & \boxed{0} \\ \boxed{0} & 9 & 7 & 5 & 4 \\ 6 & 7 & \boxed{0} & \cancel{0} & 2 \\ 2 & \boxed{0} & 1 & 1 & \cancel{0} \\ \cancel{0} & 5 & 7 & 1 & 7 \end{bmatrix}$$

step: ⑦ NOW check NO of Rows is Equal to NO of Assignments

$\Longrightarrow 5 \neq 4$

The problem is NOT Completed

Operations Research

Dynamic Programming

Part 3

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Part 3

Solution of Linear Programming Problem by Dynamic Programming

Consider a general linear Programming Problem

Maximize $z = c_1x_1 + c_2x_2 + \ldots\ldots + c_nx_n$ subject to the constraints :

$$a_{i1}x_1 + a_{i2}x_2 + \ldots\ldots + a_{in}x_n \le b_i \qquad i = 1, 2, \ldots\ldots, m$$

and $$x_j \ge 0; \qquad j = 1, 2, \ldots\ldots, n$$

This problem can be formulated by dynamic programming as follows:

Let the general linear programming problem be considered as a multi stage problem with each activity j (1, 2,....., n) as individual stage. Then this is an n-stage problem and the decision variables (alternatives) are the levels of activities x_j ($\ge$0) at stage j. As x_j is continuous, each activity has an infinite number of alternatives within the feasible region. Let (B_{1j}, B_{2j}, B_{3j},......, B_{mj})be the state of the system at j th stage and f_j (B_{1j}, B_{2j}, B_{3j},......, B_{mj}) be the optimum value of the objective function at jth stage for the stages B_{1j}, B_{2j}, B_{3j},......, B_{mj}.

We know that the allocation problems are the particular types of linear programming problems. These problems require the allocation of available resources to the activities. Each constraints represents the limitation of different resources and b_1, b_2, ...,, b_m are the amounts of available resources. Since there are m resources, the state must be represented by an m-component vector s = (b_1, b_2, ..., b_m).

Let f_n (b_1, b_2, ..., b_m) be the maximum value of general linear programming problem defined above for the stages x_1, x_2, ... x_m for states b_1, b_2, ..., b_m.

Using forward computational method, the recursive equation is:

$$f_j(b_1, b_2, \ldots, b_m) = \max_{0 \le x_j \le b} \{c_j x_j + f_{j-1}(b_1 - a_{1j}x_j, b_2 - a_{2j}x_j, \ldots, b_m - a_{mj}x_j)$$

$$f_j\left(B_{1j}, B_{2j}, \ldots, B_{mj}\right) = \max_{0 \le x_j \le b} \{c_j x_j + f_{j-1}(b_1 - a_{1j}x_j, b_2 - a_{2j}x_j, \ldots, b_m - a_{mj}x_j)$$

The maximum value of b that x_j can assume is

$$b = \min.\left\{\frac{b_1}{a_{1j}}, \frac{b_2}{a_{2j}}, \dots, \frac{b_m}{a_{mj}}\right\}$$

because the minimum value satisfies the set of constraints simultaneously.

Q: Solve the L.P.P. by dynamic Programming method

Maximize $z = 3x_1 + 7x_2$ subject to the constraints

$$x_1 + 4x_2 \le 8; \quad x_2 \le 2 \quad \text{and} \quad x_1 \ge 0, x_2 \ge 0$$

Solution: The problem consist of two resources and two decision variable. The states of the equivalent dynamic programming, therefore are

$\therefore$ (B_{1j}, B_{2j}) for j = 1, 2

Thus $f_2(B_{12}, B_{22}) = \max. \{7x_2\}$

where maximum is taken over $0 \le 4x_2 \le 8$ and $0 \le x_2 \le 2$ that is

$$\begin{aligned} f_2(B_{12}, B_{22}) &= 7 \text{ x } \max(x_2) \\ &= 7 \text{ x } \min\{\tfrac{8}{4}, 2\} \\ &= 7 \text{ x } \min\{2, 2\} \\ &= 7 \text{ x } 2 = 14 \end{aligned}$$

since the maximum of x_2 satisfying the restrictions $x_2 \le \frac{8}{4}$ and $x_2 \le 2$ is the minimum of $\frac{8}{4}$ and 2.

$f_2(B_{12}, B_{22}) = 14$

that is $x_2^* = \min\left(\frac{8}{4}, 2\right) = 2$

Now,

$f_1(B_{11}, B_{21}) = \max\{3x_1 + f_2(B_{11}-x_1, B_{21}-0)\}$

$= \max\{3x_1 + f_2(\frac{8-x_1}{4}, 2)\}$

$= \max\{3x_1 + 7\min(\frac{8-x_1}{4}, 2)\}$

It is the last stage, we substitute the values of B_{11}, B_{21} as 8 and 2 respectively

Now $\min\{(\frac{8-x_1}{4}, 2)\}$

$$= \begin{cases} \frac{8-x_1}{4} & 0 \le x_1 \le 8 \\ 2 & x_1 = 0 \end{cases}$$

$$7\min\left(\frac{8-x_1}{4}, 2\right) = \begin{cases} 7(2-\frac{x_1}{4}) & 0 \le x_1 \le 8 \\ 7 \text{ x } 2 & x_1 = 0 \end{cases}$$

$$7\min\left(\frac{8-x_1}{4}, 2\right) = \begin{cases} 7(2-\frac{x_1}{4}) & 0 \le x_1 \le 8 \\ 14 & x_1 = 0 \end{cases}$$

$$\therefore\ 3x_1 + 7\min\left(\frac{8-x_1}{4}, 2\right) = \begin{cases} 3x_1 + 7(2-\frac{x_1}{4}) & 0 \le x_1 \le 8 \\ 3x_1 + 14 & x_1 = 0 \end{cases}$$

Hence the maximum of both $3x_1 + 7(2 - \frac{x_1}{4})$ and $3x_1 + 14$ is at $x_1 = 8$

.

$f_1(8,2) = 3 \text{ x } 8 + 7\min(\frac{8-8}{4}, 2)$

$= 3 \text{ x } 8 + 7 \text{x } 0$

$= 24$ at $x_1^0 = 8$

and $x_2^0 = \min(\frac{8-x_1}{4}, 2)$

$= \min\left(\frac{8-8}{4}, 2\right)$

$= \min(0,2) = 0.$

$x_2^0 = 0$

Max z = 3x8 + 7x0 = 24

Hence the optimum solution is $x_1^0 = 8$, $x_2^0 = 0$ and max z=24.

Q. Maximize $z = x_1 + 9x_2$ subject to the constraints

$$2x_1 + x_2 \le 25, \quad x_2 \le 11 \text{ and } x_1 \ge 0, x_2 \ge 0$$

The problem consists of the two resources and two decision variables therefore.

$\therefore f_2(B_{1j}, B_{2j})$ for j = 1, 2

thus $f_2(B_{12}, B_{22}) = \max\{9x_2\}$

where maximum $0 \le x_2 \le 25$ and $0 \le x_2 \le 11$

$f_2(B_{12}, B_{22}) = 9 \text{ x } \min(x_2)$

$= 9 \text{ x } \min\{25, 11\}$

$= 9 \text{ x } 11 = 99$

Since $0 \le x_2 \le 25$ is satisfying the restriction

$x_2^0 = \min\{25, 11\} = 11$

$f_1(B_{12}, B_{21}) = \max\{x_1 + f_2\{(25-2x_1), 11\}\}$

$= \max\{x_1 + 9\min\{(25-2x_1), 11\}\}$

It is the last stage, we substitute the values of B_{11}, B_{21} as 25 and 11 respectively.

Now min $(25 - 2x_1, \; 11) = \begin{Bmatrix} 25 - 2x_1 & 0 \le x_1 \le 7 \\ 11 & x_1 = 0 \end{Bmatrix}$

$$x_1 + 9\,min\ \{(25 - 2x_1), 11\} = \begin{Bmatrix} x_1 + 9(25 - 2x_1) & 0 \le x_1 \le 7 \\ 11 & x_1 = 0 \end{Bmatrix}$$

Since the maximum of both

$x_1 + 9(25–2x_1)$ and 11 is at $x_1 = 7$, we have

$f_1(25, 11) = 7 + 9 \min \{25 – 2*7, 11\}$

$= 7 + 9*11$

$= 106$ at $x_1^0 = 7$

and $x_2^0 = \min(25 - 2x_1^0, 11)$

$= 11$ at $x_1^0 = 7$

Hence the optimum solution is $x_1^0 = 7, x_2^0 = 11$ and

Max $z = 7 + 9*11 = 106$

References:

1. Operations Research by Kanti Swarup, P.K.Gupta, Man Mohan.
2. Operations Research by P. Sankara, Tata McGraw-Hill Publishing Company Limited, New Delhi.
3. Nptel lectures.
4. Operations Research by Er. Prem Kumar Gupta and Dr. D.S. Hira.

Unit - 1

Introduction to OR → Method used in OR

1 LPP → Simplex & Artifical & Graphical Method.

unit - 2

2 Assignment

3 Transportation

4 Travelling Salesman

Unit - 3

DPP & IPP

5 Dyanamic programming Problem

6 Integee Programming Problem

Unit - 4

7 Game Theory

8 Simulation

Unit - 5

9 Networking - Analysis (or) Project Manager

&

10 Replacement Models.

Unit - 3

DPP - Dynamic Programming Problem

- → Capital budgeting
- → Production Allocation
- → LPP by DPP

Production Allocation:-

Q) The owner of a chain of Four Grossery stores has purchased 6 crades of fresh strawberries The estimated probabilities of Potential sales of the strawberries spilage befor amount the 4 stores. The following table gives the estimated total expected profit at each store when it is allocated various number of crades

store:-

No. of boxes/6	1	2	3	4
0	0	0	0	0
1	4	2	6	2
2	6	4	8	3
3	7	6	8	4
4	7	8	8	4
5	7	9	8	4
6	7	10	8	4

For administrate reasons the owner doest wish to spilt crades b/w stores however his is walling to zero crades to any of his stores.

i) find the allocation of 6 crades to 4 stores so as to maximum the expected profit

Sol:- Method:- 1) forward Recrussion method

2) Backward Recrussion method.

Forward Method:

stage-①

	store ①	Max
0	0	0
1	4	4
2	6	6
3	7	7
4	7	7
5	7	7
6	7	7

Max profit for store-1 = 7

stage - 2

No. of boxes	0	1	2	3	4	5	6	Max
0	0+0	-	-	-	-	-	-	0
1	0+4	2+0	-	-	-	-	-	4
2	0+6	2+4	4+0	-	-	-	-	6
3	0+7	2+6	4+4	6+0	-	-	-	8
4	0+7	2+7	4+6	6+4	8+0	-	-	10
5	0+7	2+7	4+7	6+6	8+4	9+0	-	12
6	0+7	2+7	4+7	6+7	8+6	9+4	18+0	14

Max - 14

stag - 3

No. of Boxes	0	1	2	3	4	5	6	Max
0	0+0	-	-	-	-	-	-	0
1	0+4	6+0	-	-	-	-	-	6
2	0+6	6+4	8+0	-	-	-	-	10
3	0+8	6+6	8+4	8+0	-	-	-	12
4	0+10	6+8	8+6	8+4	8+0	-	-	14
5	0+12	6+10	8+8	8+6	8+4	8+0	-	16
6	0+14	6+12	8+10	8+8	8+6	8+4	8+0	18

Max - 18

Stage - 4

No. of boxes	0	1	2	3	4	5	6	Max
0	0+0	-	-	-	-	-	-	0
1	0+6	2+0	-	-	-	-	-	6
2	0+10	2+6	3+0	-	-	-	-	10
3	0+12	2+10	3+6	4+0	-	-	-	12
4	0+14	2+12	3+10	4+6	4+0	-	-	14
5	0+16	2+14	3+12	4+10	4+6	4+0	-	16
6	0+18	2+16	3+14	4+12	4+10	4+6	4+0	18

Max - 18

Capital budgeting :-

An organization is planning two diversity its business with a maximum outlay of 5cr. It has identified 3 different locations to install plants. The organization can invest in one or more of these plants subject to the availability of the fund the different possible alternatives & their investment (in crores of rupees) and present worth of returns during the useful life (in crores of rupees) of each of these plants are sunrise in the following table. The first row of in the table has zero cost & zero return for all the plants. Hence it is known as do noting alternative find the optimal allocation of the capital to different plants which will maximize the corresponding sum of the present worth of returns.

Alternative	Plant-4		Plant-2		Plant-3	
	Cost	Return	Cost	Return	Cost	Return
1	0	0	0	0	0	0
2	1	15	2	14	1	3
3	2	18	3	18	2	7
4	4	28	4	21		

8

plant-1

Altenative	C-R 0-0	C-R 1-15	C-R 2-18	C-R 4-28	Max profit
0	0	–	–	–	0
1	0	15	–	–	15
2	0	15	18	–	18
3	0	15	18	~~28~~ –	18
4	0	15	18	28	28
5	0	15	18	28	28

Plant - 2.

Alternative	C-R 0-0	C-R 2-14	C-R 3-18	C-R 4-21	Max profit.
0	~~0+0~~ 0+0	-	-	-	0
1	0+15	-	-	-	15
2	0+18	14+0	-	-	18
3	0+18	14+5	18+0	-	29
4	0+28	14+18	18+15	21+0	33
5	0+28	14+18	18+18	21+15	36.

Plant - 3.

Alternative	C-R 0-0	C-R 1-3	C-R 2-7		Max profit
0	0+0	-	-		0
1	0+15	3+0	-		15
2	0+18	3+15	7+0		18
3	0+29	3+18	7+15		29
4	0+33	3+29	7+18		33
5	0+36	3+33.	7+29		36

IPP - Integer Programming Problem.

↳ Branch & Bound Technique.

Example:-

Find the optim integer solution of the following LPP

$$\text{Max } Z = 5x_1 + 8x_2$$

STC $\Rightarrow x_1 + 2x_2 \leq 8$

$4x_1 + x_2 \leq 10$

& $x_1 x_2 \geq 0$ and are integers

Sol: Step ① Equation the Constraints

$$\text{Max } Z = 5x_1 + 8x_2$$

STC $\quad x_1 + 2x_2 = 8$ —— ①

$4x_1 + x_2 = 10$ —— ②

& $x_1 x_2 \geq 0$

Take Eq ① $x_1 + 2x_2 = 8$

Put $x_1 = 0$ in ① $\Rightarrow \cancel{2}x_2 = \cancel{8}4 \Rightarrow x_2 = 4$

$\therefore (0, 4)$

Put $x_2 = 0$ in ① $\Rightarrow x_1 = 8$ $(8, 0)$

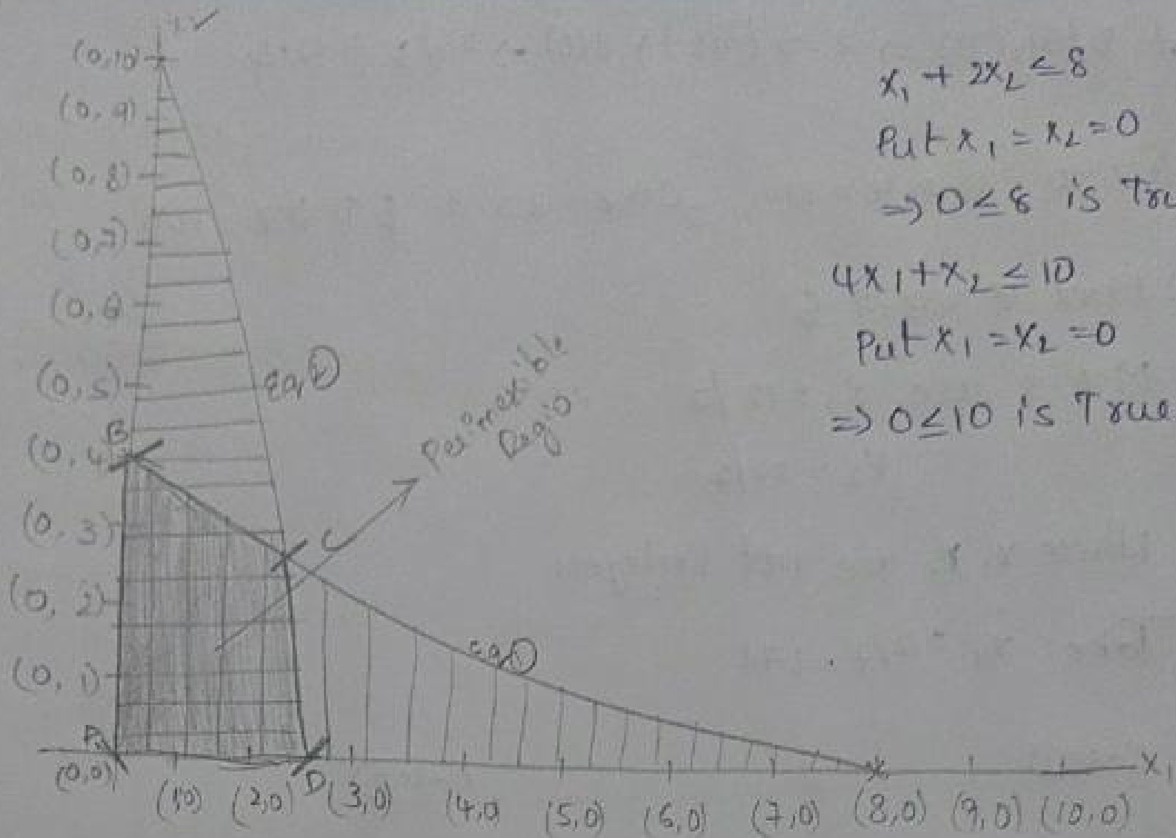
from eq ① $(8,0)$ & $(0,4)$

Take Eq ② $4x_1 + \cancel{x_2} = 10$

Put $x_1 = 0 \Rightarrow x_2 = 10 \Rightarrow (0, 10)$

Put $x_2 = 0 \Rightarrow 4x_1 = 10 \Rightarrow (5/2, 0)$

$\therefore$ from Eq ② $(5/2, 0)$ & $(0, 10)$



$x_1 + 2x_2 \leq 8$

Put $x_1 = x_2 = 0$

$\Rightarrow 0 \leq 8$ is True

$4x_1 + x_2 \leq 10$

Put $x_1 = x_2 = 0$

$\Rightarrow 0 \leq 10$ is True.

The extreme points of Permissible Region is

A(0,0), B(0,4) D(5/2,0), C = ?

By solving ① & ② $\Rightarrow$ Eq ① − 2Eq ②

$$x_1 + 2x_2 = 8$$
$$8x_1 + 2x_2 = 20$$
$$(-)\quad(-)\quad(-)$$
$$\cancel{-}7x_1 = \cancel{-}12$$

$\Rightarrow \boxed{x_1 = 12/7}$

$$x_2 = 10 - 4x_1$$
$$= 10 - 4\left[\frac{12}{7}\right]$$
$$= \frac{70-48}{7} = \frac{22}{7}$$

$\boxed{\therefore x_2 = 22/7}$

∴ Given profit condition

$$z = 5x_1 + 8x_2$$

at A(0,0) $\Rightarrow z = 5(0) + 8(0) = 0$

at B(0,4) $\Rightarrow z = 5(0) + 8(4) = 32$

at C (12/7, 22/7) $\Rightarrow z = 5(12/7) + 8(22/7)$

$= 60/7 + 176/7 = 236/7 \Rightarrow 33.7$

at $D(5/2, 0) \Rightarrow Z = 5(5/2) + 8(0) \Rightarrow 25/2 \Rightarrow 12.5$.

$A \Rightarrow Z = 0$; $B \Rightarrow Z = 32$, $C \Rightarrow Z = 33.7$ & $D \Rightarrow Z = 12.5$

$\therefore$ Max $Z = 33.7$ &

M.F.S are $x_1 = 12/7$

$x_2 = 22/7$

Where x_1 x_2 are not Integers

Take $x_1 = 12/7 = 1.71$

Case ①

$\underline{x_1 \leq 1}$

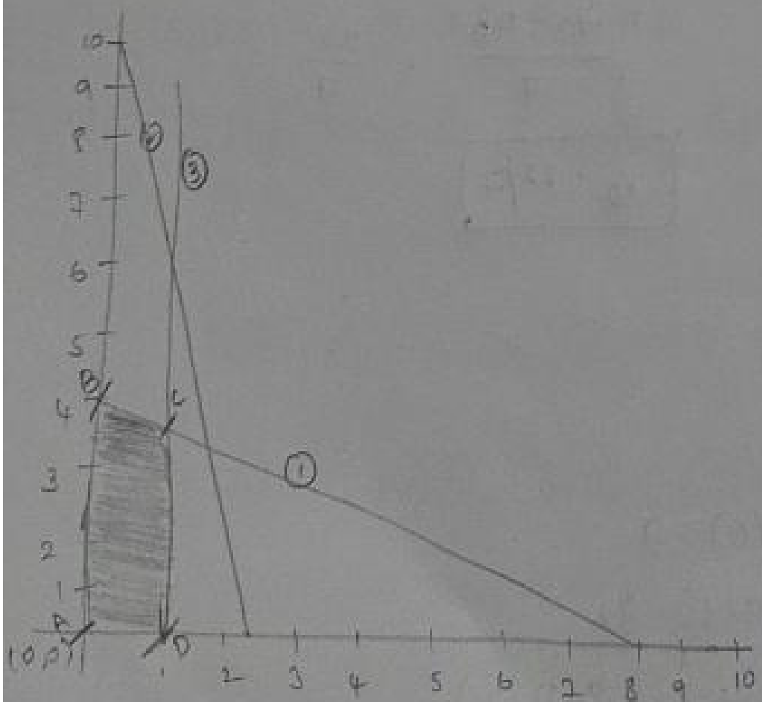
Max $Z = 5x_1 + 8x_2$

STC $x_1 + 2x_2 \leq 8 \rightarrow (8,0)\ (0,4)$

$4x_1 + x_2 \leq 10 \rightarrow (5/2, 0)\ (0,10)$

$x_1 \leq 1 \quad \rightarrow (1,0)$

& $x_1 x_2 \geq 0$ and are integer



Case ②

$\underline{x_1 \geq 2}$

Max $Z = 5x_1 + 8x_2$

STC $x_1 + 2x_2 \leq 8$

$4x_1 + x_2 \leq 10$

$x_1 \geq 2$

$x_1 x_2 \geq 0$ and are integer.

$A = (0,0) \Rightarrow z = 0$

$B = (0,4) \Rightarrow z = 32$

$D = (1,0) \Rightarrow z = 5$

$C = ? \Rightarrow (1, 7/2) \Rightarrow z = 33$

By solving ① & ③

$x_1 + 2x_2 = 8$

$x_1 = 1$

$\Rightarrow 2x_2 = 8 - 1 = 7$

$x_2 = 7/2$

$(1, 7/2)$

$\therefore$ Max $z = 33$ & M.F.S are

$x_1 = 1$ & $x_2 = 7/2 = 3.5$

Where $x_1, x_2 \geq 0$ & x_2 is not a integer.

Take $x_2 = 7/2 = 3.5$

$x_2 \leq 3$
Case 1 (a)

$x_2 \geq 4$
Case 1 (b)

Max $z = 5x_1 + 8x_2$

STC $x_1 + 2x_2 \leq 8 \rightarrow (8,0)\ (0,4)$

$4x_1 + x_2 \leq 10 \rightarrow (5/2, 0)(0,10)$

$x_1 \leq 1 \rightarrow (1,0)$

$x_2 \leq 3 \rightarrow (0,3)$

$x_1 x_2 \geq 0$

Max $z = 5x_1 + 8x_2$

STC $x_1 + 2x_2 \leq 8$

$4x_1 + x_2 \leq 10$

$x_1 \leq 1$

$x_2 \geq 4$

& $x_1, x_2 \geq 0$