Linear programming problems

$\hookrightarrow$ Lpp

Linear programming is a mathematical programming technique to optimize performance [ex:- profit or cost] under a set of ~~ge~~ resource constraints [ex:- machine-hours, man-hours, money, materials, etc.] as specified by an ~~ea~~ organization.

form of Lpp :-

$$\begin{matrix}\text{Maximum} \\ \text{or} \\ \text{minimum}\end{matrix} \; [Z] = c_1x_1 + c_2x_2 + \cdots\cdots + c_nx_n$$

STC [subject to the constraints]

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \le = \ge b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \le = \ge b_2$$

$$\vdots \qquad\qquad\qquad\qquad \vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + \cdots\cdots + a_{nn}x_n \le = \ge b_n$$

where $x_1, x_2 \cdots\cdots x_n \ge 0$

Where $x_1, x_2 --- x_n$ are variables
$C_1, C_2 --- C_n$ are the Cost of variables
$a_{11}, a_{12} --- a_{1n}, a_{21}, a_{22}, -- a_{2n} --- a_{n1}, a_{n2} -- a_{nn}$
are the number of variables
$b_1, b_2 --- b_n$ are the basic feasible
Solutions

applications of Lpp :-

(1) product mix problem
(2) Diet planning problem
(3) Cargo loading problem
(4) Capital Budgeting problem
(5) Manpower planning problem

Models of Lpp :- 2 models

(1) formation of Lpp
(2) Solvation of Lpp

Methods for Solving of Lpp :-

(1) Graphical method
(2) Simplex method
↳ Artifical variable technique
Big-M-method   Two phase method
(3) Dual simplex method

Assumptions of linear programming

mainly four assumptions are their in LPP
(1) Linearity (2) Divisibility (3) Non-negativity
(4) Additivity

→ Linearity :- The amount of resource required for a given activity level is directly proportional to the level of that activity

Ex:- If the numbers of hours required on a particular machine* is 5 hours per unit of that activity, then the total number of hours required on that machine to produce 10 units of that activity is 50 hours.
[* given activity level]

→ ~~Divisibil~~ Divisibility :- This means that fractional values of decision variables are permitted.

→ Non-negativity :- This means that the decision variables are permitted to have only the values which are greater than ~~0~~ or equal to zero

→ Additivity :- This means that the total output for a given combination of activity levels is the algebraic sum of the output of each individual process.

Properties of linear programming Solutions :-

- → Feasible Solution
- → optimal Solution
- → Alternative optimal Solution
- → unbounded Solution
- → Infeasible Solution
- → Degenerate Solution

2.3 DEVELOPMENT OF LP MODELS

Modelling is an art. One can develop this expertise only by seeing more and more models. In this section, the concept of model building is demonstrated using some example problems.

Example 2.2 A company manufactures two types of products, P_1 and P_2. Each product uses lathe and milling machine. The processing time per unit of P_1 on the lathe is 5 hours and on the milling machine is 4 hours. The processing time per unit of P_2 on the lathe is 10 hours and on the milling machine, 4 hours. The maximum number of hours available per week on the lathe and the milling machine are 60 hours and 40 hours, respectively. Also the profit per unit of selling P_1 and P_2 are Rs. 6.00 and Rs. 8.00, respectively. Formulate a linear programming model to determine the production volume of each of the products such that the total profit is maximized.

Solution The data of the problem are summarized in Table 2.2.

Table 2.2 Details of Products

(in hour)

Machine	Machine hours/unit		Limit on machine hours
	Product P_1	Product P_2	
Lathe	5	10	60
Milling machine	4	4	40
Profit/unit (Rs.)	6	8	

Let X_1 and X_2 be the production volumes of the products P_1 and P_2 respectively. The corresponding linear programming model to determine the production volume of each of the products, such that the total profit is maximized, is presented below.

$$\text{Maximize } Z = 6X_1 + 8X_2$$

subject to

$$5X_1 + 10X_2 \le 60$$

$$4X_1 + 4X_2 \le 40$$

$$X_1 \text{ and } X_2 \ge 0$$

Example 2.7 **(Cargo loading problem)** Consider the cargo loading problem, where five items are to be loaded on a vessel. The weight (w_i) and volume (v_i) of each unit of the different items as well as their corresponding returns per unit (r_i) are tabulated in Table 2.7.

Table 2.7 Example 2.7

Item-i	w_i	v_i	r_i
1	5	1	4
2	8	8	7
3	3	6	6
4	2	5	5
5	7	4	4

The maximum cargo weight (W) and volume (V) are given as 112 and 109, respectively. It is required to determine the optimal cargo load in discrete units of each item such that the total return is maximized. Formulate the problem as an integer programming model.

Solution Let, X_i be the number of units of the ith item to be loaded in the cargo, where i varies from 1 to 5. A model to maximize the return is as follows:

$$\text{Maximize } Z = 4X_1 + 7X_2 + 6X_3 + 5X_4 + 4X_5$$

subject to

$$5X_1 + 8X_2 + 3X_3 + 2X_4 + 7X_5 \leq 112$$

$$X_1 + 8X_2 + 6X_3 + 5X_4 + 4X_5 \leq 109$$

$$X_1, X_2, X_3, X_4, \text{ and } X_5 \geq 0 \text{ and integers}$$

... three products P_1, P_2 and P_3. The machine hour

$X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8$ and $X_9 \geq 0$

Example 2.10 A computer company procures cabinets from three different suppliers (*A*, *B* and *C*) located in three different cities. The company has production plants (*P*, *Q* and *R*) in three other cities. The cost of transportation per cabinet for different combinations of supplier and production plant are summarized in Table 2.10. The purchase price per cabinet from different suppliers are also indicated in the same table. The weekly demand at different production plants and the weekly availability of cabinets at different suppliers are also given in the table. Formulate a linear programming model to find the optimal procurement plan for the cabinets such that the total cost is minimized.

Table 2.10 Details of Unit Transportation Cost and Purchase Cost in Rupees

		Production plant				
		P	*Q*	*R*	Supply	Price/Cabinet
Supplier	*A*	10	15	8	300	85
	B	20	21	15	500	90
	C	12	16	13	100	75
	Demand	100	200	300		

Solution Table 2.10 summarizes the transportation cost/cabinet and purchase price/cabinet. The purchase price per cabinet from each supplier is added to different cells of the row corresponding to that supplier. The modified totals of the cost of transportation per unit and purchase cost per unit, for different combinations of supplier and production plant are summarized in Table 2.11.

Table 2.11 Details of Totals of Transportation Cost per Unit and Purchase Cost per Unit

		Production plant			
		P	*Q*	*R*	Supply
Supplier	*A*	95	100	93	300
	B	110	111	105	500
	C	87	91	88	100
	Demand	100	200	300	

Let, Y_{ij} be the number of cabinets to be procured from the supplier i for the production plant j for $i = A, B, C$ and $j = P, Q, R$.

A linear programming model to find the optimal procurement plan for the cabinets to minimize the total cost of procurement is presented below.

$$\text{Minimize } Z = 95Y_{AP} + 100Y_{AQ} + 93Y_{AR} + 110Y_{BP} + 111Y_{BQ} + 105Y_{BR} + 87Y_{CP} + 91Y_{CQ} + 88Y_{CR}$$

subject to

$$Y_{AP} + Y_{AQ} + Y_{AR} \leq 300$$
$$Y_{BP} + Y_{BQ} + Y_{BR} \leq 500$$
$$Y_{CP} + Y_{CQ} + Y_{CR} \leq 100$$
$$Y_{AP} + Y_{BP} + Y_{CP} \geq 100$$
$$Y_{AQ} + Y_{BQ} + Y_{CQ} \geq 200$$
$$Y_{AR} + Y_{BR} + Y_{CR} \geq 300$$
$$Y_{AP},\ Y_{AQ},\ Y_{AR},\ Y_{BP},\ Y_{BQ},\ Y_{BR},\ Y_{CP},\ Y_{CQ}, \text{ and } Y_{CR} \geq 0$$

1 Maximize $Z = 20x_1 + 10x_2$

~~[illegible]~~ STC $10x_1 + 5x_2 \leq 50$

$6x_1 + 10x_2 \leq 60$

$4x_1 + 12x_2 \leq 48$

& $x_1 x_2 \geq 0$

Sol:- Step: 1 Equating the constraints

$\Rightarrow$ Max $Z = 20x_1 + 10x_2$

STC $10x_1 + 5x_2 = 50 \longrightarrow$ 1

$6x_1 + 10x_2 = 60 \longrightarrow$ 2

$4x_1 + 12x_2 = 48 \longrightarrow$ 3

Eq 1 can be written as.

$2x_1 + x_2 = 10 \longrightarrow$ A

Eq 2 can be written as

$3x_1 + 5x_2 = 30 \longrightarrow$ B

Eq 3 can be written as

$x_1 + 3x_2 = 12 \longrightarrow$ C

Step: 2 Construction of points from A, B & C

A $\rightarrow$ put $x_1 = 0$ in A then $x_2 = 10 \rightarrow (0,10)$

put $x_2 = 0$ in A then $x_1 = 5 \rightarrow (5,0)$

from A the points (5,0) (0,10)

(B) $\rightarrow$ $3x_1 + 5x_2 = 30$

put $x_1 = 0$ then $x_2 = 6$ $\Rightarrow (0,6)$

put $x_2 = 0$ then $x_1 = 10$ $\Rightarrow (10,0)$

From (B) the points are $(0,6)$ & $(10,0)$

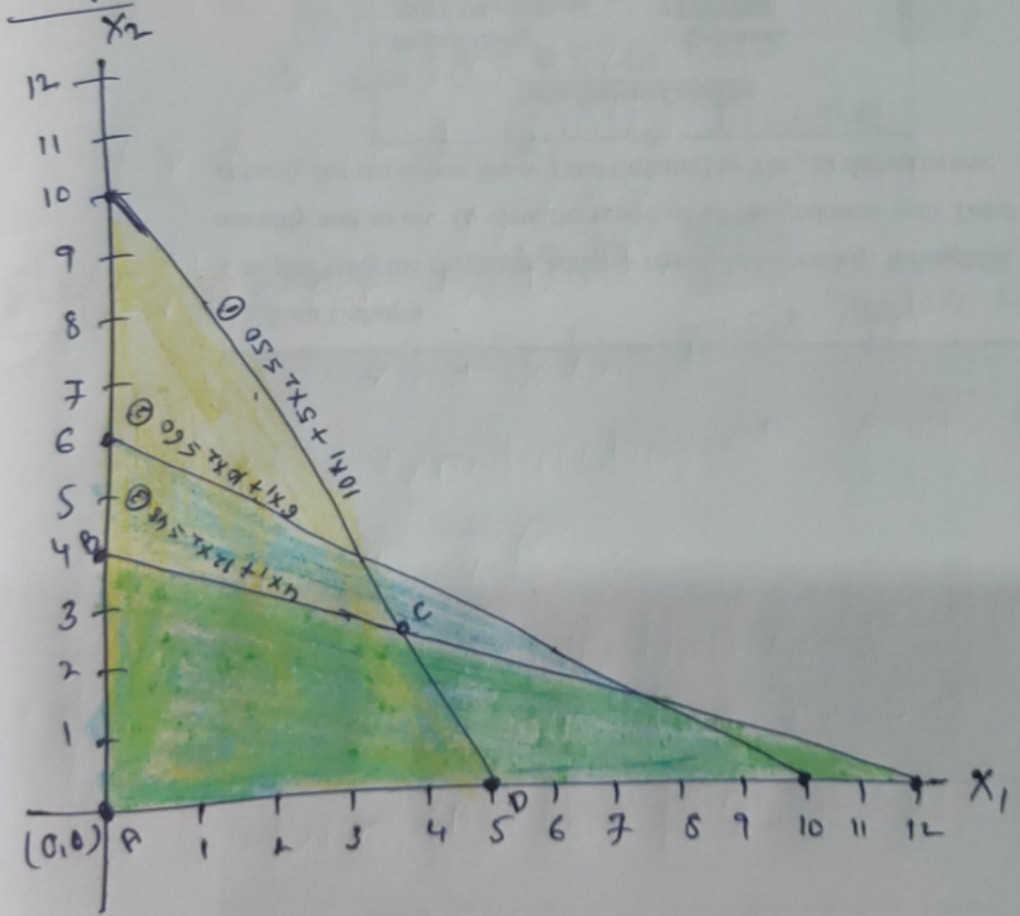
(C) $\rightarrow$ $x_1 + 3x_2 = 12$

put $x_1 = 0$ then $x_2 = 4$ $\Rightarrow (0,4)$

put $x_2 = 0$ then $x_1 = 12$ $\Rightarrow (12,0)$

from (C) the points are $(12,0)$ & $(0,4)$

Step: (3) Draw the Graph



Step: 4

Take Constrain 1 $10x_1 + 5x_2 \le 50$

put $x_1 = x_2 = 0$

$\Rightarrow 0 \le 50$ is true

Shaded is Inside

Take Constrain 2 $6x_1 + 10x_2 \le 60$

put $x_1 = x_2 = 0$

$\Rightarrow 0 \le 60$ is true

Shaded is Inside

Take Constrain 3 $4x_1 + 12x_2 \le 48$

put $x_1 = x_2 = 0$

$\Rightarrow 0 \le 48$ is true

Shaded is Inside

Step: 5 Profit Region is

Another name for profit region is Perimessible Region. Now the Extreme points of perimessible region is A, B, C, D

$A = (0,0)$; $B = (0,4)$; $C = ?$ By Solving 1 & 3

$D = (5,0)$

By solving ① & ③ ie Ⓐ & Ⓒ

$$2x_1 + x_2 = 10 \quad \text{— Ⓐ}$$

$$x_1 + 3x_2 = 12 \quad \text{— Ⓑ}$$

Ⓐ − 2Ⓑ

$$\Rightarrow \cancel{2x_1} + x_2 = 10$$

$$\cancel{2x_1} + 6x_2 = 24$$

$$(-) \quad (-) \quad (-)$$

$$-5x_2 = -14$$

$$x_2 = 14/5 \;\&$$

put x_2 value in Ⓐ

$$2x_1 = 10 - x_2$$

$$= 10 - 14/5 = \frac{50-14}{5} = 36/5$$

$$\cancel{2}x_1 = \overset{18}{\cancel{36}}/5$$

$$\therefore x_1 = 18/5$$

$$\therefore C = [18/5, 14/5]$$

Now substitute A, B, C, D values in the profit function

$$\text{Max } Z = 20x_1 + 10x_2$$

Step: 6 $Z = 20x_1 + 10x_2$

at $A(0,0) \Rightarrow Z = 20(0) + 10(0)$

$\Rightarrow Z = 0$

at $B(0,4) \Rightarrow Z = 20(0) + 10(4)$

$\Rightarrow Z = 40$

at $C(18/5, 14/5) \Rightarrow Z = 20(18/5) + 10(14/5)$

$= \cancel{20}^{4} \times \frac{18}{\cancel{5}_1} + \cancel{10}^{2} \times \frac{14}{\cancel{5}_1}$

$= [4 \times 18] + [2 \times 14]$

$= 72 + 28$

$= 100$

$\therefore Z = 100$

at $D(5,0) \Rightarrow Z = 20(5) + 10(0)$

$= 100 + 0$

$\therefore Z = 100$

$\therefore$ MAX $Z = 100$ &

M.F.S are $x_1 = 18/5$ & $x_2 = 14/5$

(or)

$x_1 = 5$ & $x_2 = 0$

For calculating of optimal solution two Alternative feasible solutions are their. So this type of problems will be called as Alternative optimal solution problem

② Max $Z = 100x_1 + 50x_2$
STC $4x_1 + 6x_2 \le 24$
$x_1 \le 4$
$x_2 \le 4/3$
& $x_1, x_2 \ge 0$

} Graphical method

Sol:- Step: ① Equating the Constraints
i.e., max $Z = 100x_1 + 50x_2$
STC $4x_1 + 6x_2 = 24$ — ①
$x_1 = 4$ — ②
$x_2 = 4/3$ — ③

Step: ② Take Eq ①

$4x_1 + 6x_2 = 24$
put $x_1 = 0 \Rightarrow \cancel{6}x_2 = \cancel{24}^{4}$
$\Rightarrow x_2 = 4$ (0,4)

put $x_2 = 0 \Rightarrow \cancel{4}x_1 = \cancel{24}\,6$
$\Rightarrow x_1 = 6$ (6,0)

$\boxed{\therefore \text{From Eq ①} \Rightarrow (6,0)\ (0,4)}$

Take Eq ② $x_1 = 4$ i.e. $x_2 = 0$
$\Rightarrow (4,0)$

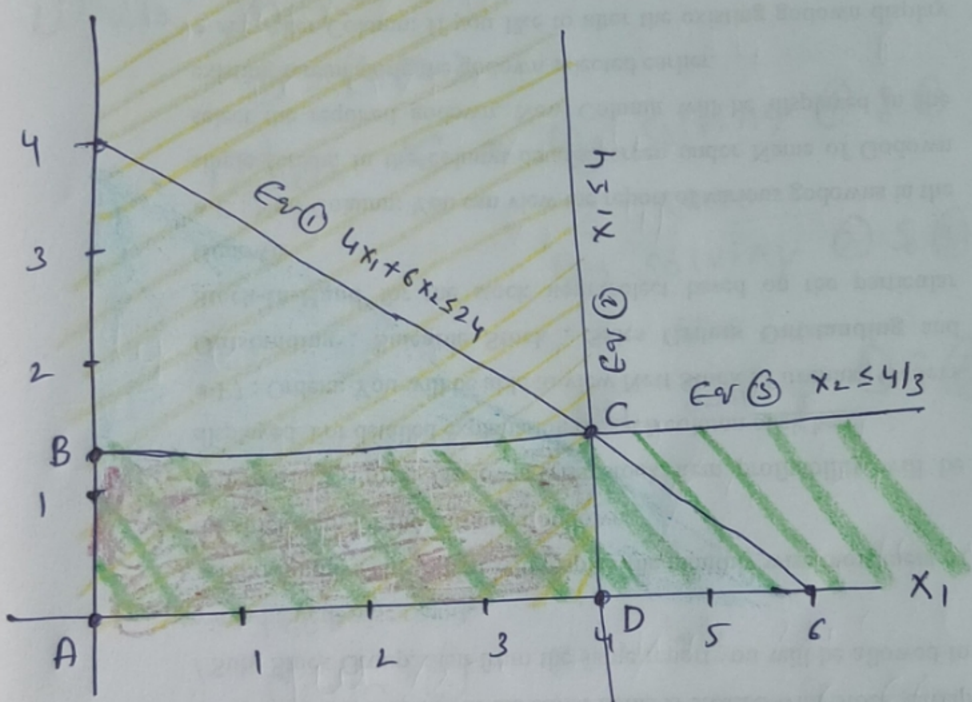
Take Eq ③ $x_2 = 4/3$ i.e. $x_1 = 0$
$\Rightarrow (0, 4/3)$

Step: ③ Now draw the Graph by using those points from ①, ②, ③

from ① ⇒ $(6,0), (4,0)$

from ② ⇒ $(4,0)$

from ③ ⇒ $(0, 4/3)$

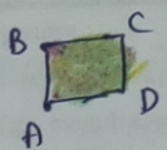


Eq ① ⇒ $4x_1 + 6x_2 \le 24$ ⇒ $0 \le 24$ is true

Eq ② ⇒ $x_1 \le 4$ ⇒ $0 \le 4$ is true

Eq ③ ⇒ $x_2 \le 4/3$ ⇒ $0 \le 4/3$ is true

Step: 4 Now the perimessible region is



Now the Extreme points of permessible region is A B C D

Where $A = (0,0)$ $B = (0, 4/3)$ $C = ?$

$D = (4,0)$

Where C = By Solving 1 & 2

(or)

By solving 2 & 3

(or)

By solving 1 & 3

By Solving 2 & 3

from 2 $x_1 = 4$

from 3 $x_2 = 4/3$

$\therefore$ $C = (x_1\ x_2) = (4, 4/3)$

For Calculating of 'C' is feasible point '3'-Chances are their.

So this type of problem is called as problem of Degeneracy.

Step ⑤ Now substitute A, B, C, D valles in profit function $Z = 100x_1 + 50x_2$

at $A(0,0) \Rightarrow Z = 0$

at $B(0, 4/3) \Rightarrow Z = 0 + 50 \times \frac{4}{3} = 200/3$

at $C(4, 4/3) \Rightarrow Z = 100(4) + 50(4/3)$

$= 400 + \frac{200}{3} = \frac{1400}{3}$

at $D(4,0) \Rightarrow Z = 100(4) + 50(0)$

$= 400$

$\therefore$ Max $Z = \frac{1400}{3}$

& M.F.S are $x_1 = 4$ & $x_2 = 4/3$

This problem will be called as

Degeneracy

7. Solve the following LP problem graphically:

$$\text{Maximize } Z = 20X_1 + 80X_2$$

subject to

$$4X_1 + 6X_2 \le 90$$
$$8X_1 + 6X_2 \le 100$$
$$5X_1 + 4X_2 \le 80$$
$$X_1 \text{ and } X_2 \ge 0$$

8. Solve the following LP problem graphically:

$$\text{Minimize } Z = 20X_1 + 10X_2$$

subject to

$$X_1 + 2X_2 \le 40$$
$$3X_1 + X_2 \ge 30$$
$$4X_1 + 3X_2 \ge 60$$
$$X_1 \text{ and } X_2 \ge 0$$

9. Solve the following LP problem graphically:

$$\text{Maximize } Z = 60X_1 + 90X_2$$

subject to

$$X_1 + 2X_2 \le 40$$
$$2X_1 + 3X_2 \le 90$$
$$X_1 - X_2 \ge 10$$
$$X_1 \text{ and } X_2 \ge 0$$

10. Solve the following LP problem graphically:

$$\text{Minimize } Z = 45X_1 + 55X_2$$

subject to

$$X_1 + 2X_2 \le 30$$
$$2X_1 + 3X_2 \le 80$$
$$X_1 - X_2 \ge 8$$
$$X_1 \text{ and } X_2 \ge 0$$

11. Solve the following LP problem graphically:

$$\text{Maximize } Z = 3X_1 + 2X_2$$

subject to

$$-2X_1 + 3X_2 \le 9$$
$$X_1 - 5X_2 \ge -20$$
$$X_1 \text{ and } X_2 \ge 0$$

GRAPHICAL METHOD

(1) Solve the following LPP by using Graphical Method:-

Maximum $Z = 6x_1 + 8x_2$

STC $5x_1 + 10x_2 \leq 60$

$4x_1 + 4x_2 \leq 40$

& $x_1, x_2 \geq 0$

Sol:- step: ① Equating the Constraints

$\Rightarrow$ Max $Z = 6x_1 + 8x_2$

STC $5x_1 + 10x_2 = 60 \longrightarrow$ ①

$4x_1 + 4x_2 = 40 \longrightarrow$ ②

& $x_1\ x_2 \geq 0$

step: ② Take Equation ①

$5x_1 + 10x_2 = 60$

Put $x_1 = 0$ in above Equation

$\Rightarrow 5(0) + 10x_2 = 60$

$\Rightarrow 0 + \cancel{10}x_2 = \cancel{60}\ 6$

$\Rightarrow x_2 = 6$

If $x_1 = 0$ then $x_2 = 6$

The point will be $(x_1\ x_2)$ ie, $(0, 6)$

put $x_2 = 0$ in above Equation

$5x_1 + 10(0) = 60$

$\Rightarrow \cancel{5}x_1 + 0 = \cancel{60}\ 12$

$\Rightarrow x_1 = 12$

If $x_2 = 0$ then $x_1 = 12$

The point will be $(x_1\ x_2)$ ie, $(12, 0)$

∴ From Equation ① the points will be

(12, 0) & (0, 6)

Step: ③ Take Equation ②

$$4x_1 + 4x_2 = 40$$

put $x_1 = 0 \Rightarrow \cancel{4}x_2 = \overset{10}{\cancel{40}}$

$\Rightarrow x_2 = 10$ (0, 10)

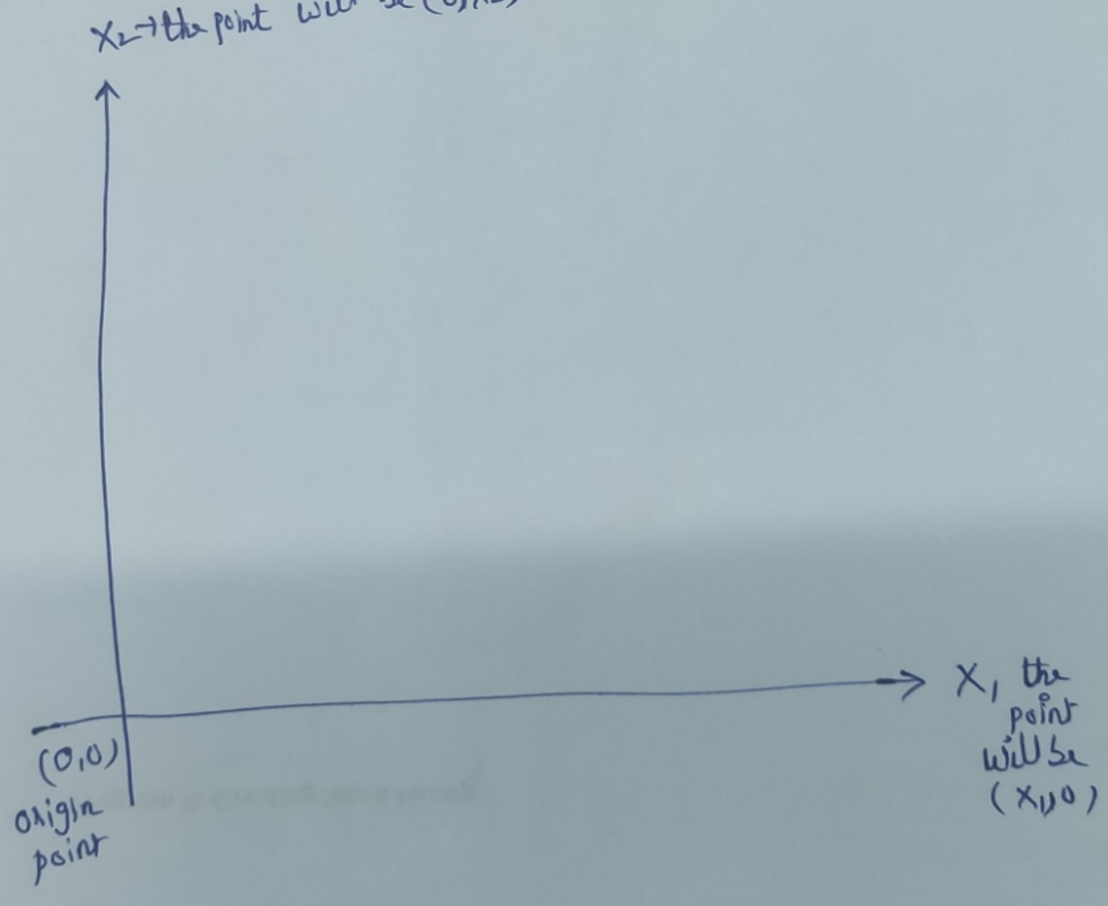
put $x_2 = 0 \Rightarrow 4x_1 = 40$

$\Rightarrow x_1 = 10$ (10, 0)

∴ From Equation ② the points will be

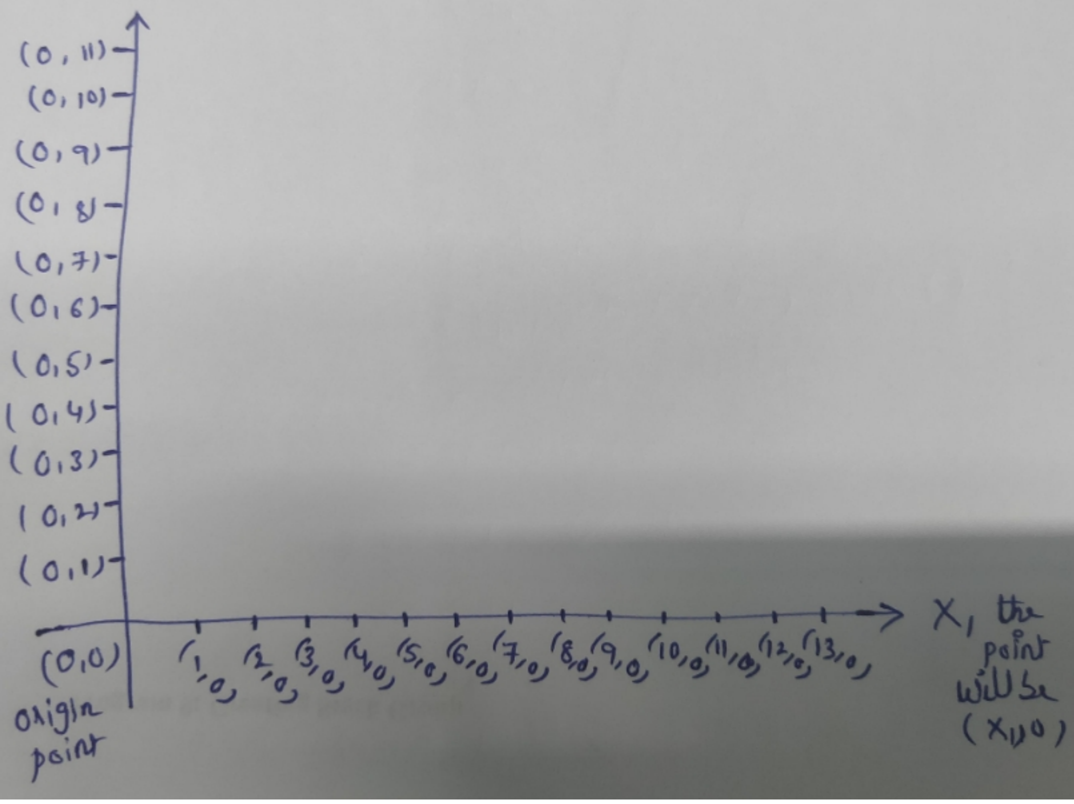
(10, 0) & (0, 10)

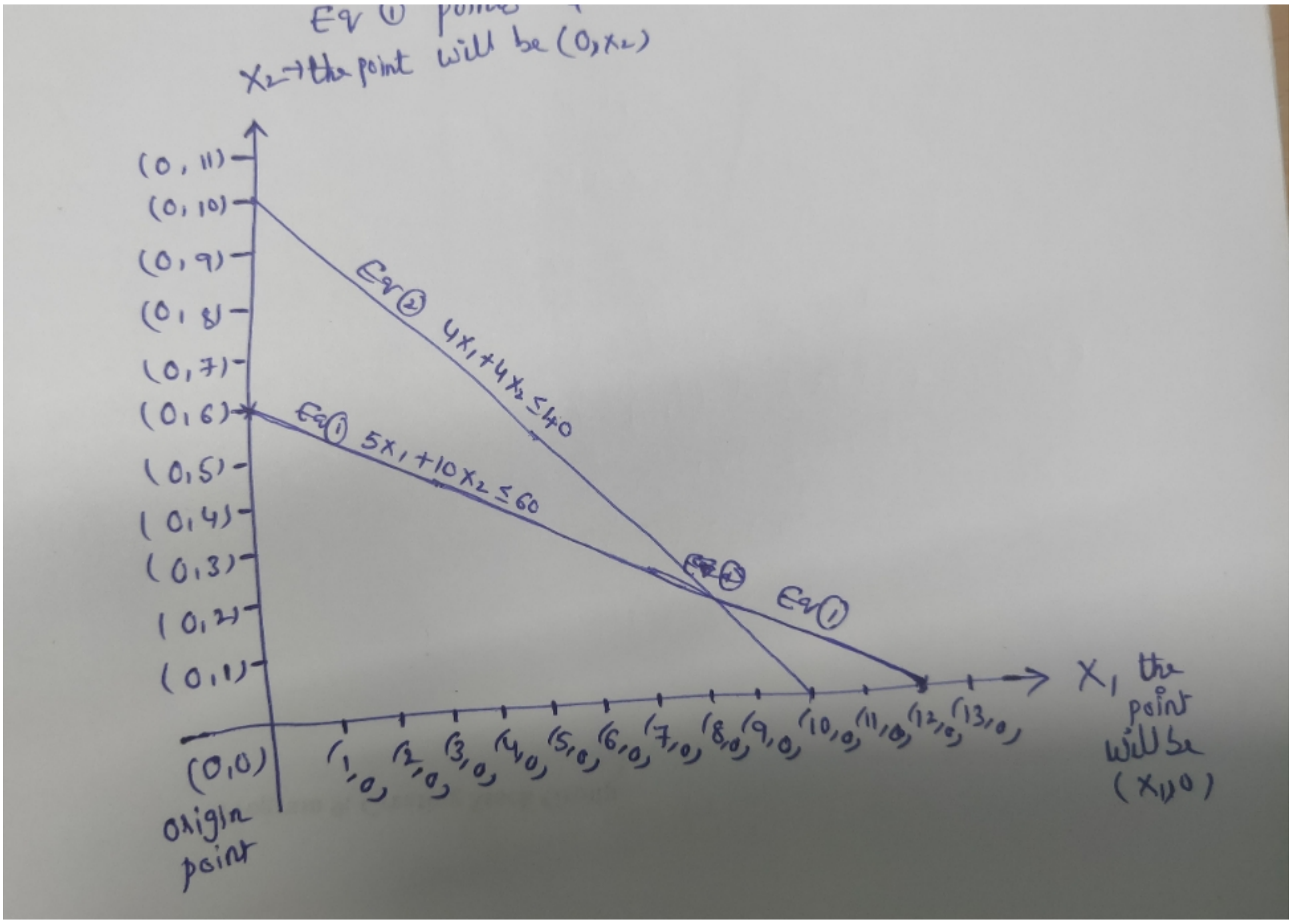
Step: ④ Now draw the Graph by using Eq ① points & Eq ② points



Eq ① points & Eq ② points

$X_2 \rightarrow$ the point will be $(0, X_2)$




Eq ① point
$X_2 \to$ the point will be $(0, x_2)$
(0, 11)
(0, 10)
(0, 9)
(0, 8)
(0, 7)
(0, 6)
(0, 5)
(0, 4)
(0, 3)
(0, 2)
(0, 1)
(0,0)
origin point
Eq ② $4x_1 + 4x_2 \le 40$
Eq ① $5x_1 + 10x_2 \le 60$
Eq ①
(1,0) (2,0) (3,0) (4,0) (5,0) (6,0) (7,0) (8,0) (9,0) (10,0) (11,0) (12,0) (13,0)
X_1 the point will be $(x_1, 0)$

Step: 5 Now check the profit region of Each Constraint is Either Inside the region or out side the region

How can we check ?

Take Constrain 1

ie, $5x_1 + 10x_2 \leq 60$

put $x_1 = x_2 = 0$ in above ~~for~~ Constrain 1

Now Constrain 1 becomes

$5(0) + 10(0) \leq 60$

$\Rightarrow 0 + 0 \leq 60$

$\Rightarrow 0 \leq 60$

Is it true or false.

$\therefore 0 \leq 60$ is True

→ If the Condition is True, shade the profit region Inside the scale

→ If the condition is false, Shade the profit region outside the scale

For this Constrain 1 the Condition is True
Now Constrain 1 shaded region is Inside

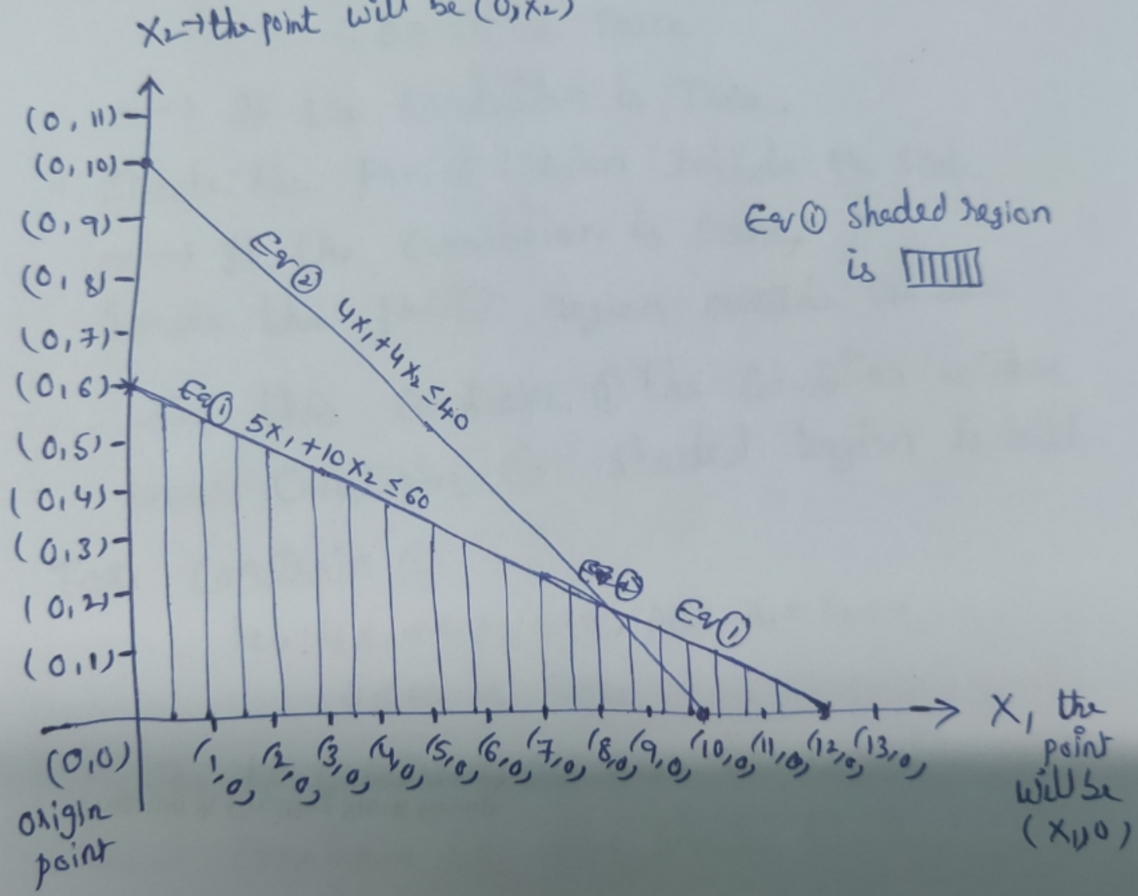
Take Constrain 2

ie, $4x_1 + 4x_2 \leq 10$, put $x_1 = x_2 = 0$

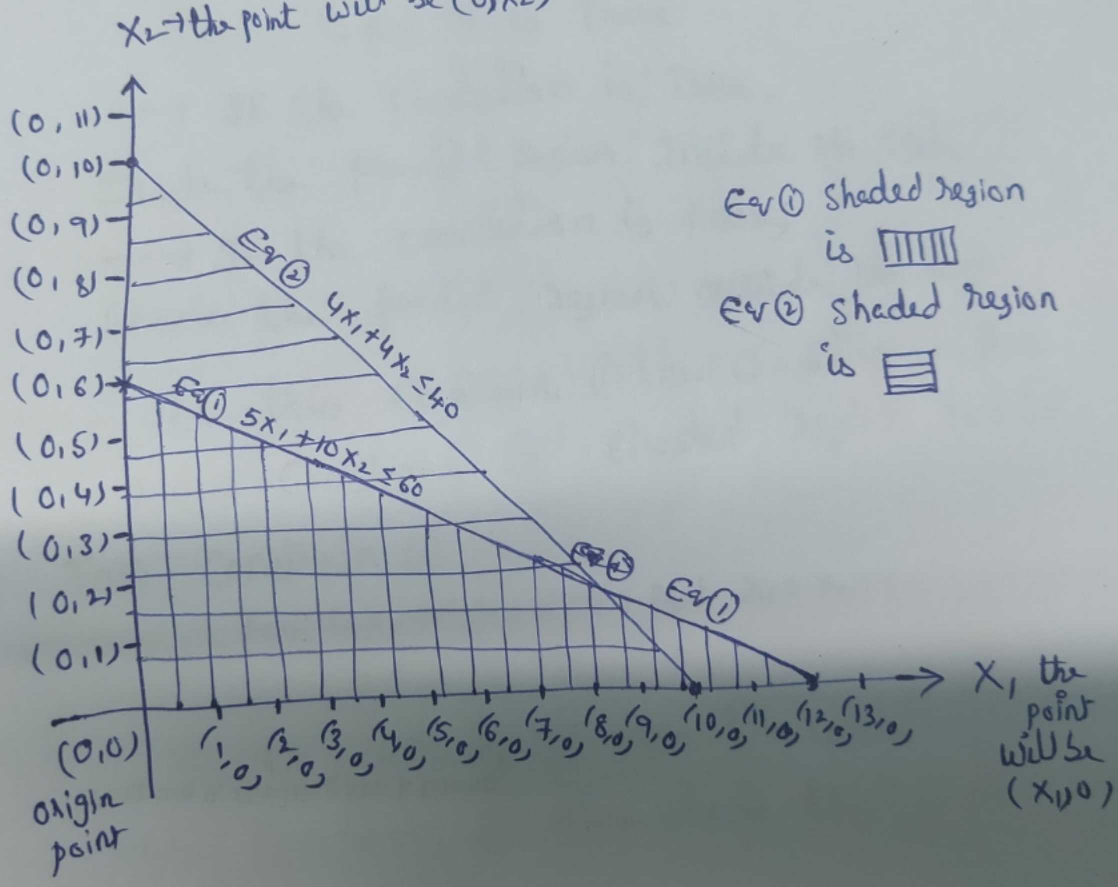
$0 \leq 10$ is True.

For ~~it~~ Constrain 2 also the Condition is True.
Now Constrain 2 also shade the region Inside only.

Step: (4) Now draw the Graph by using Eq (1) points & Eq (2) points

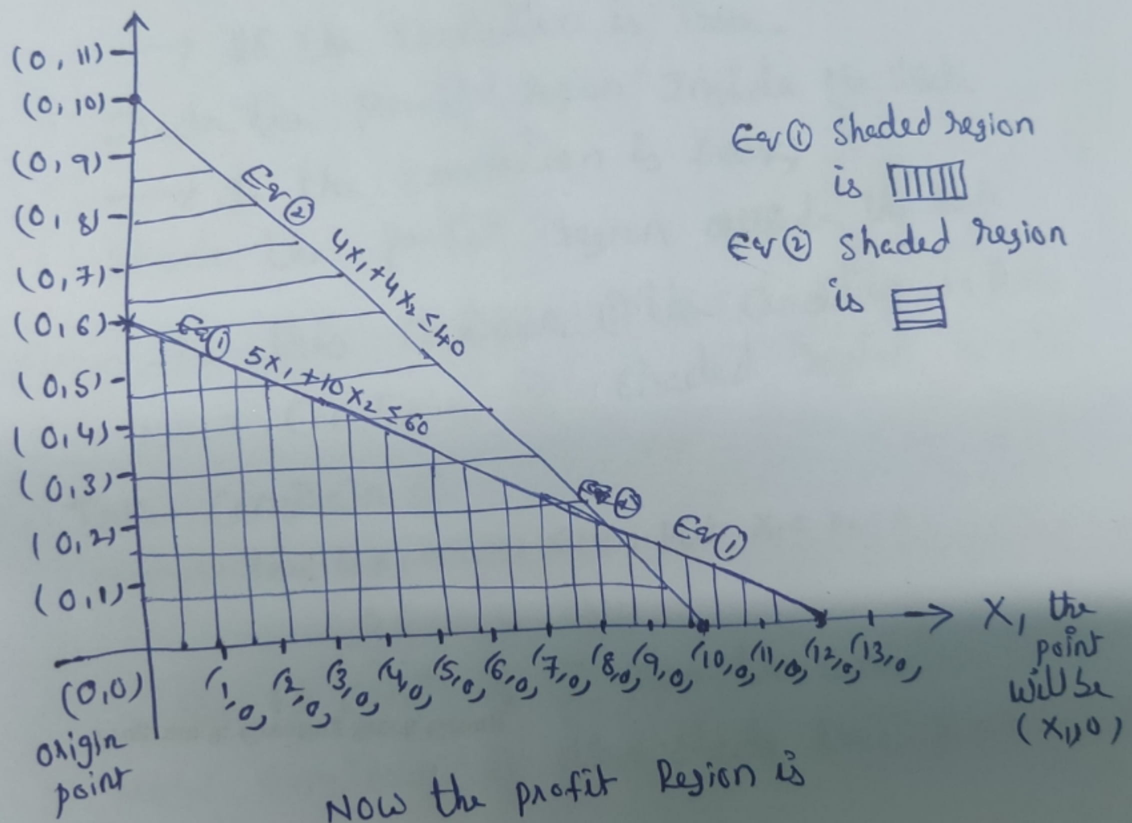


Step: 4 NOW draw the Graph by using Eq 1 points & Eq 2 points



Step: 4 Now draw the Graph by using Eq 1 points & Eq 2 points

$x_2 \rightarrow$ the point will be $(0, x_2)$



Now the profit Region is

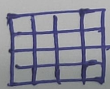
Step: 6 NOW shade the region through Horizontal lines & vertical lines.

→ shade the Eq 1 through vertical lines

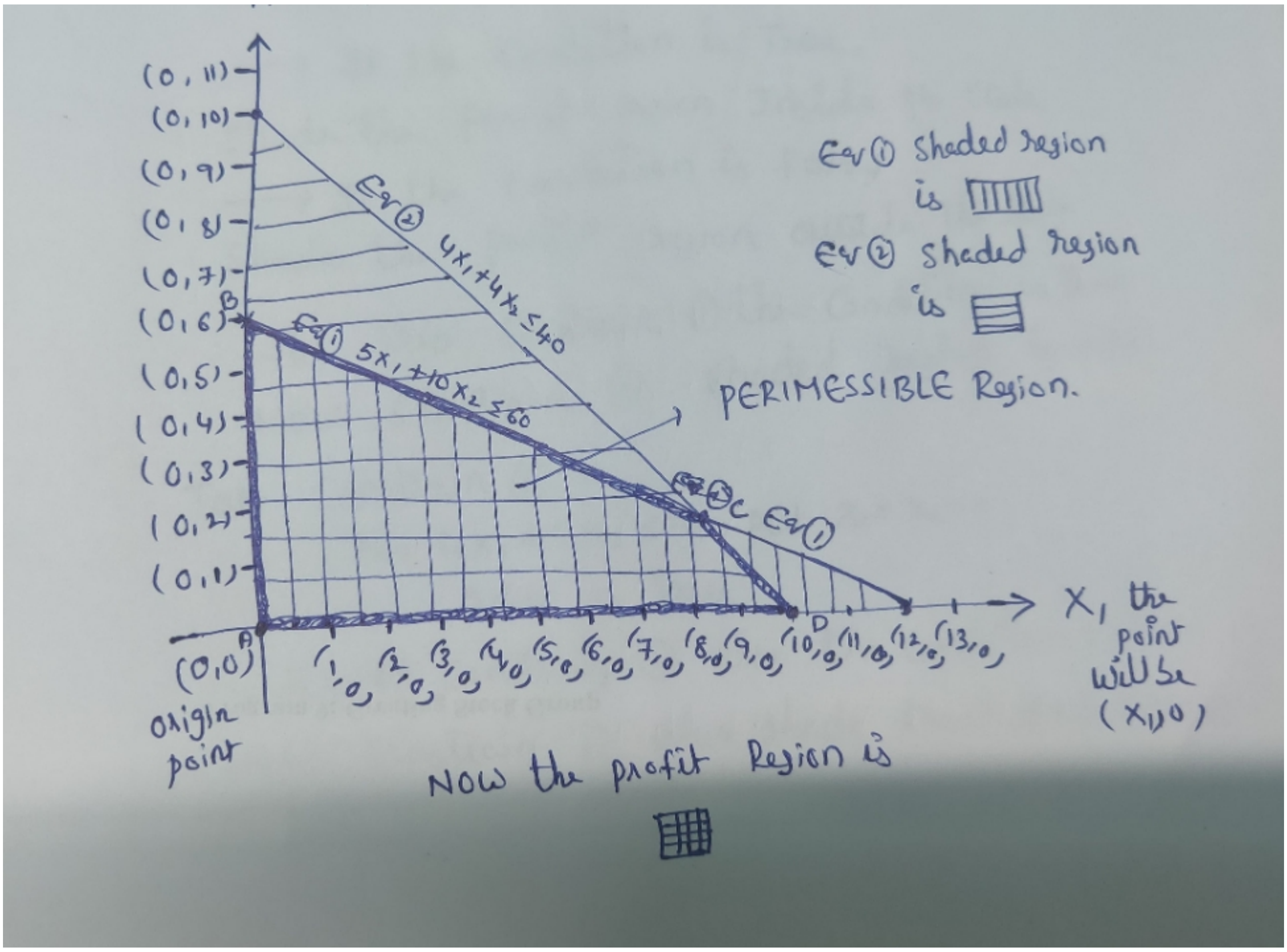
i.e., 

→ shade the Eq 2 through Horizontal lines

ie, 

NOW the ~~[illegible]~~ Intersecting of these two Shaded regions will be called as profit Area. ie, 

Another Name for profit Area is perimissible region


Eq ① Shaded region is
Eq ② Shaded region is
PERIMESSIBLE Region.
Eq ② 4x₁+4x₂≤40
Eq ① 5x₁+10x₂≤60
(0,0) Origin point
X₁ the point will be (X₁,0)
Now the profit Region is

perimessible region

step: 7 Now calculate the Extreme points of the perimessible region is A, B, C, D

where $A = (0,0)$ on origin

$B = (0,6)$ on X_2 Axis

$C = ?$ ~~Intersection~~ Intersecting the constraint 1 & 2

$D = (10,0)$ on X_1 Axis

Now By solving of Eq 1 & Eq 2 we get the value of 'C'

step : 8 By Solving of Eq 1 & Eq 2

$$5x_1 + 10x_2 = 60 \Rightarrow 5[x_1 + 2x_2] = 5(12)$$

$$4x_1 + 4x_2 = 40 \Rightarrow 4[x_1 + x_2] = 4(10)$$

Step : 8 By Solving of Eq 1 & Eq 2

$$5x_1 + 10x_2 = 60 \Rightarrow \cancel{5}[x_1 + 2x_2] = \cancel{5}(12)$$

$$4x_1 + 4x_2 = 40 \Rightarrow \cancel{4}[x_1 + x_2] = \cancel{4}(10)$$

$$\Rightarrow x_1 + 2x_2 = 12$$

$$x_1 + x_2 = 10$$

$$(-) \quad (-) \quad (-)$$

Step : 8 By Solving of Eq 1 & Eq 2

$$5x_1 + 10x_2 = 60 \Rightarrow \cancel{5}[x_1 + 2x_2] = \cancel{5}(12)$$

$$4x_1 + 4x_2 = 40 \Rightarrow \cancel{4}[x_1 + x_2] = \cancel{4}(10)$$

$$\Rightarrow \cancel{x_1} + 2x_2 = 12$$

$$\cancel{x_1} + x_2 = 10$$

$$(-) \quad (-) \quad (-)$$

$$x_2 = 2$$

Put $x_2 = 2$ in $x_1 + x_2 = 10$

$$\Rightarrow x_1 = 10 - x_2$$

$$= 10 - 2$$

$$= 8$$

$$\Rightarrow x_1 = 8$$

Now Take the point $C = (x_1\ x_2)$

then $C = (8, 2)$

Step:- (9) Now Take the profit Equation & Profit points A, B, C, D

Profit Equation is Max $Z = 6x_1 + 8x_2$

& profit points are $A = (0,0)$

$B = (0,6)$, $C = (8,2)$, $D = (10,0)$

Now substitute profit points in profit equation.

at $A(0,0) \Rightarrow Z = 6(0) + 8(0) = 0$

$$\boxed{\therefore Z = 0 \text{ at } A(0,0)}$$

at $B(0,6) \Rightarrow Z = 6(0) + 8(6)$

$= 0 + 48 = 48$

$$\boxed{\therefore Z = 48 \text{ at } (0,6)}$$

at $C(8,2) \Rightarrow Z = 6(8) + 8(2)$

$= 48 + 16 = 64$

$$\boxed{\therefore Z = 64 \text{ at } (8,2)}$$

at $D(10,0) \Rightarrow Z = 6(10) + 8(0)$

$= 60 + 0 = 60$

$$\boxed{\therefore Z = 60 \text{ at } (8,2)}$$

Step : 10 NOW Select the maximum profit in $Z=0$, $Z=48$, $Z=64$, $Z=60$

$\therefore$ Maximum profit $Z=64$

ie, Max $Z=64$

& Maximum feasible solution are

$X_1=8$ & $X_2=2$

$\therefore$ The optimal solution is 64

& feasible Solutions are $X_1=8$

$X_2=2$

—x—

Infeasible Solution

Solve the following LPP & Graphically

$$\text{Maximize } Z = 10x_1 + 3x_2$$

$$\text{STC} \quad 2x_1 + 3x_2 \leq 18$$
$$6x_1 + 5x_2 \geq 60$$
$$\& \ x_1 \ x_2 \geq 0$$

Sol:- Step: ① Equating the Constraints

$$\text{Max } Z = 10x_1 + 3x_2$$
$$\text{STC} \quad 2x_1 + 3x_2 = 18 \longrightarrow ①$$
$$6x_1 + 5x_2 = 60 \longrightarrow ②$$
$$\& \ x_1 \ x_2 \geq 0$$

Step: ② Take Equation ①

$$2x_1 + 3x_2 = 18$$
$$\text{put } x_1 = 0 \Rightarrow \cancel{3}x_2 = \cancel{18}\ 6$$
$$\Rightarrow x_2 = 6 \ (0, 6)$$
$$\text{put } x_2 = 0 \Rightarrow \cancel{2}x_1 = \cancel{18}\ 9$$
$$\Rightarrow x_1 = 9 \ (9, 0)$$

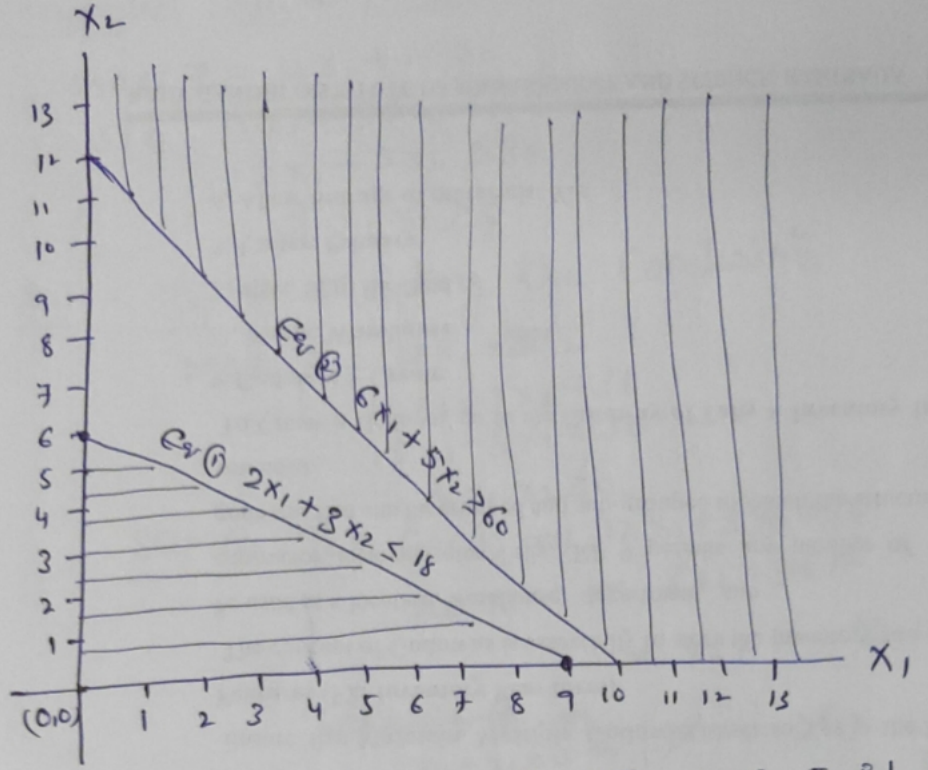
$\therefore$ From Eq ① the points are (9,0) & (0,6)

Step: ③ Take Equation ②

$$6x_1 + 5x_2 = 60$$
$$\text{put } x_1 = 0 \Rightarrow \cancel{5}x_2 = \cancel{60}\ 12 \ (0, 12)$$
$$\Rightarrow x_2 = 12$$
$$\text{put } x_2 = 0 \Rightarrow \cancel{6}x_1 = \cancel{60}\ 10$$
$$x_1 = 10 \ (10, 0)$$

$\therefore$ from Eq ② the points are (10,0) & (0,12)

Step: ④ Now Draw a Graph by using Eq ① points & Eq ② points & Shade it by using Shaded region for Calculating of profit Region



Eq ① Shaded region is ▤ since Inside the ~~region~~ and origin scale

Take Constrain ① $2x_1 + 3x_2 \leq 18$

put $x_1 = x_2 = 0 \Rightarrow 0 \leq 18$ is True

Eq ② Shaded region is ▥ since outside the origin scale

Take Constrain ② $6x_1 + 5x_2 \geq 60$

put $x_1 = x_2 = 0 \Rightarrow 0 \geq 60$ is false.

In the above graph there is No intersecting profit Area. i.e., No perimessible Region

∴ This type of problems Can be called as Infeasible Solution problems. since there is No feasible solution

— x —

Unbounded solution problem

$\text{Max } Z = 12x_1 + 25x_2$

STC $12x_1 + 3x_2 \geq 36$

$15x_1 - 5x_2 \leq 30$

$\& \; x_1, x_2 \geq 0$

Sol:- Step: ① Equating the Constraints

$\text{Max } Z = 12x_1 + 25x_2$

STC $12x_1 + 3x_2 = 36$

$15x_1 - 5x_2 = 30$

$\& \; x_1, x_2 \geq 0$

Step: ② Take Eq ① $\Rightarrow 12x_1 + 3x_2 = 36$

put $x_1 = 0 \Rightarrow \cancel{3}x_2 = \cancel{36}\,12$

$\Rightarrow x_2 = 12 \quad (0, 12)$

~~Take Eq ①~~ $\Rightarrow \cancel{12}x_1 = \cancel{36}\,3$

put $x_2 = 0 \Rightarrow$

$\Rightarrow x_1 = 3 \quad (3, 0)$

$\therefore$ from Eq ① the points are

$(3, 0) \; \& \; (0, 12)$

Step: ③ Take Eq ② $\Rightarrow 15x_1 - 5x_2 = 30$

put $x_1 = 0 \Rightarrow -\cancel{5}x_2 = \cancel{30}\,6$

$\Rightarrow x_2 = -6 \quad (0, -6)$

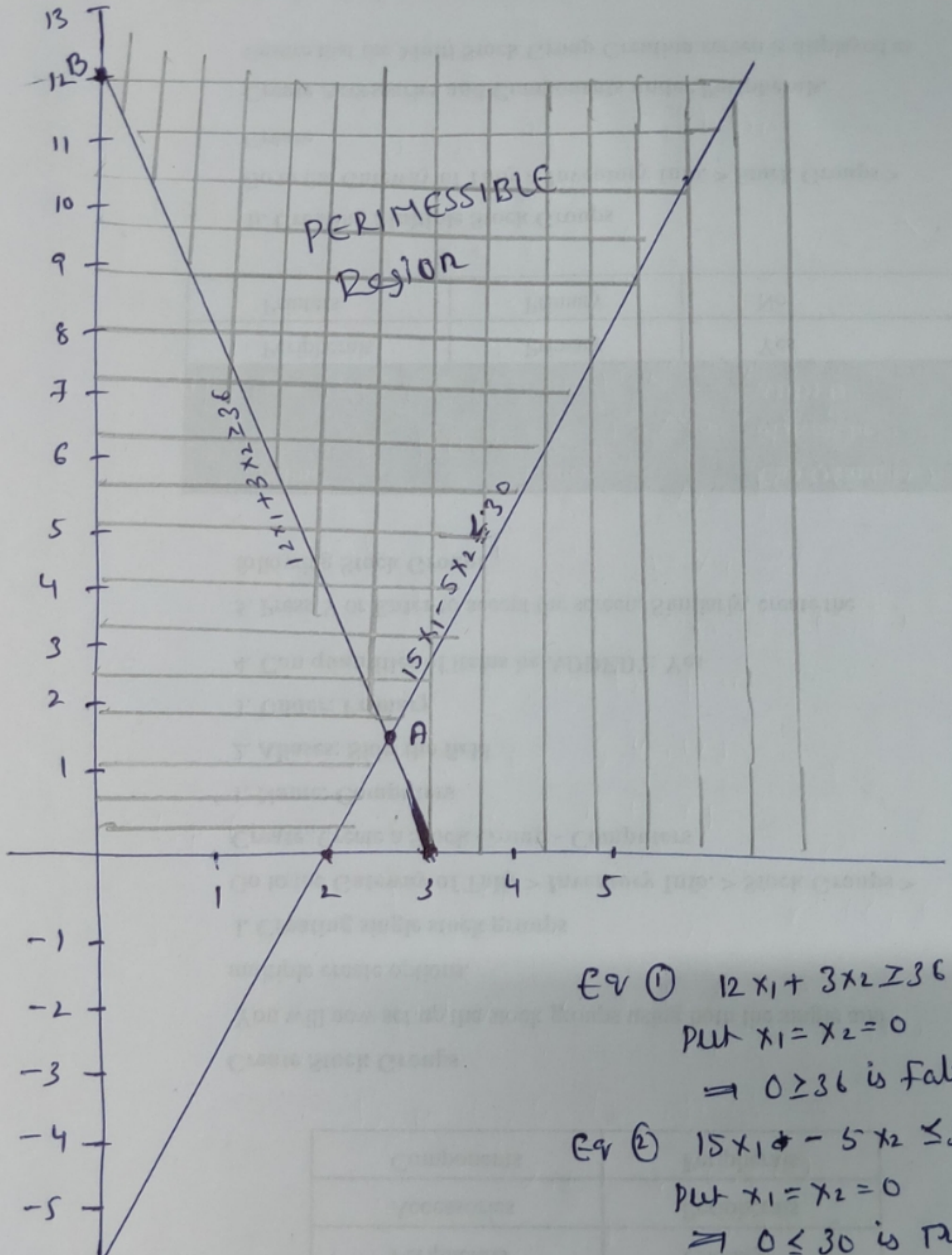
put $x_2 = 0 \Rightarrow 15x_1 = 30$

$\Rightarrow x_1 = 2 \quad (2, 0)$

$\therefore$ from Eq ② the points are

$(2, 0) \; \& \; (0, -6)$

Step: ④ Now draw the Graph by using Eq ① points & Eq ② points and shade it



Eq ① $12x_1 + 3x_2 \geq 36$

put $x_1 = x_2 = 0$

$\Rightarrow 0 \geq 36$ is False

Eq ② $15x_1 - 5x_2 \leq 30$

put $x_1 = x_2 = 0$

$\Rightarrow 0 \leq 30$ is True.

Now the perimissible has No boundries for Calculating of Maximum profit
Now this type of problems is called unbounded solution problem

SIMPLEX METHOD

(1) Slack variable :- If a constrain is less than or Equal to zero [≤ 0] then in order to make it an Equality we have add something positive of LHS. Then that positive variable is called as slack variable.

Ex :- Max $Z = 4x_1 + 7x_2$

STC $x_1 + x_2 \leq 6$

$2x_1 + x_2 \leq 5$

& $x_1 x_2 \geq 0$

Equating the constraints by add slack variables on LHS

$\Rightarrow$ Max $Z = 4x_1 + 7x_2 + 0S_1 + 0S_2$

STC $x_1 + x_2 + S_1 = 6$

$2x_1 + x_2 + S_2 = 5$

& $x_1 x_2 S_1 S_2 \geq 0$

where S_1 & S_2 are slack variables

(2) Surplus variable :- If a constrain is greater than or Equal to zero [≥ 0] then in order to make it an Equality we have add something negative on LHS. Then that negative variable is called as surplus variable

Ex; Max $Z = 4x_1 + 7x_2$

STC $x_1 + x_2 \geq 6$

$2x_1 + x_2 \geq 5$

$x_1 + 3x_2 \leq 7$

$x_1 x_2 \geq 0$

Equating the constraints by adding slack & surplus variables on LHS

$\Rightarrow$ Max $Z = 4x_1 + 7x_2 + 0S_1 + 0S_2 + 0S_3$

STC $x_1 + x_2 - S_1 = 6$

$2x_1 + x_2 - S_2 = 5$

$x_1 + 3x_2 + S_3 = 7$

when $x_1 x_2 S_1 S_2 S_3 \geq 0$

where S_1 S_2 are surplus variables & S_3 is slack variable.

Simplex Table:-

			$C_J = C_1 \ C_2 \cdots C_n$	$0 \ 0 \ 0$	$\begin{matrix}-M & -M & -M \\ -1 & -1 & -1\end{matrix}$	
C_B	B	X_B	$x_1 \ x_2 \cdots x_n$	$S_1 \ S_2 \cdots S_n$	$A_1 \ A_2 \cdots A_n$	$\theta = \min\left\{\frac{X_{Bi}}{x_{ik}}, \text{ where } X_{ik} > 0\right\}$
$Z_B = \Sigma C_B X_B$			$Z_J = \Sigma C_B X_J$ where $J = 1\ 2 \ldots n$			

where $Z_B = \Sigma C_B X_B$

Z_B = profit

C_B = Cost of Basic variable

X_B = Basic Feasible Solution

B = Basic variables

optamility test : $Z_J - C_J \geq 0$

where Z_J is construction part

C_J is given problem

ie, Cost of variables

$Z_J = \Sigma C_B X_J$

$\theta = \min\left\{\frac{X_{Bi}}{X_{ik}}, \ x_{ik} > 0\right\}$

In this simplex two types of variables are their

Incoming variable (or) Incoming vector

out going variable (or) outgoing vector

The Intersection of Incoming vector and out going vector is called as Key Element

Key Element plays main role for Construction of Next simplex tables.

Problem: ① Solve the following LPP by using Simplex method

$$\text{Max } Z = 6x_1 + 8x_2$$
$$\text{STC } 5x_1 + 10x_2 \le 60$$
$$4x_1 + 4x_2 \le 40$$
$$\& \ x_1, x_2 \ge 0$$

Sol:- Step: ① Equating the Constraints by adding Either slack (or) surplus variables. Here we have to add slack variables only since in the ~~max~~ constraints they given "$\le$"

$$\text{Max } Z = 6x_1 + 8x_2 + 0S_1 + 0S_2$$
$$\text{STC } 5x_1 + 10x_2 + S_1 = 60 \quad \text{— ①}$$
$$4x_1 + 4x_2 + S_2 = 40 \quad \text{— ②}$$

where $x_1, x_2, S_1, S_2 \ge 0$
and $S_1 \& S_2$ are slack variables

Step: ② write the Equations in matrix form

$$\begin{array}{cccc} x_1 & x_2 & S_1 & S_2 \end{array}$$
$$\begin{bmatrix} 5 & 10 & 1 & 0 \\ 4 & 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} 60 \\ 40 \end{bmatrix}$$

In the above matrix form their is a Basis matrix is their.. [Basic matrix means Identity matrix]

Note: - ① If the Basis matrix is their then start the simplex table

② If the Basis matrix is not their.. Then construct the Basis matrix Table by using Artificial variables

Step: ③ Initial Simplex table

		C_j:					
C_B	B	X_B	x_1	x_2	S_1	S_2	θ

Initial Simplex table :-

		C_j	6	8	0	0	
C_B	B	X_B	x_1	x_2	S_1	S_2	θ
0	S_1	60	5	$\boxed{10}$	1	0	$60/10=6$
0	S_2	40	4	4	0	1	$40/4=10$
$Z_B=\Sigma C_B X_B$			-6	-8	0	0	

$\theta = \min\left\{\frac{X_B}{x_2}, x_2>0\right\}$

$\Rightarrow [0\times 60]+[0\times 40]$

$\Rightarrow 0$

$\boxed{\therefore Z_B=0}$

Here the optamility test condition

$Z_j - C_j \geq 0$ is Not Satisified

$\Rightarrow \{-6,-8,0,0\} \not\geq 0$

Here most $-ve$ is -8 ie, in x_2

$\therefore x_2$ is Incoming vector

Min 'θ' is 6 ie in S_1

$\therefore S_1$ is outgoing vector

'10' is Key element.

Second Simplex Table :-

			6	8	0	0	
C_B	B	X_B	x_1	x_2	S_1	S_2	$\theta = \min\left\{\frac{X_B}{x_1}, x_1>0\right\}$
8	$\cancel{S_1}$ x_2	6	$1/2$	1	$1/10$	0	$6/1/2=12$
0	S_2	16	$\boxed{2}$	0	$-2/5$	1	$16/2=8$
			-2	0	$4/5$	0	

$Z_B = \Sigma X_B C_B$

$\Rightarrow \boxed{8(6)+0(16)=48}$

$NR_1 \rightarrow OR_1/10$	$NR_2 \rightarrow OR_2 - 4NR_1$
$60 \rightarrow 60/10 = 6$	$40 \rightarrow 40-4(6)=16$
$5 \rightarrow 5/10 = 1/2$	$4 \rightarrow 4-4(1/2)=2$
$10 \rightarrow 10/10 = 1$	$4 \rightarrow 4-4(1)=0$
$1 \rightarrow 1/10 = 1/10$	$0 \rightarrow 0-4(1/10) = -2/5$
$0 \rightarrow 0/10 = 0$	$1 \rightarrow 1-4(0) = 1$

Here the optamility test condition

$Z_J - C_J \geq 0$ is Not salisified

$= \{-2, 0, 4/5, 0\} \ngeq 0$

most negative is -2 i.e, in x_1

$\therefore x_1$ is Incoming vector

min 'θ' is 8 i.e, S_2 is

$\therefore S_2$ is out going vector

$\therefore$ '2' is key element

Third simplex table :.

C_B	B	X_B	x_1 (6)	x_2 (8)	S_1 (0)	S_2 (0)	θ
8	x_2	2	0	1	1/5	-1/4	
6	x_1	8	1	0	-1/5	1/2	
			0	0	2/5	1	

$Z_B = \sum C_B X_B$
$= 8(2) + 6(8)$
$= 16 + 48 = 64$

$NR_2 \to OR_2/2$

$16 \to 16/2 = 8$
$2 \to 2/2 = 1$
$0 \to 0/2 = 0$
$-2/5 \to -2/5/2 = -1/5$
$1 \to 1/2 = 1/2$

$NR_1 \to OR_1 - 1/2\, NR_1$

$6 \to 6 - 1/2(8) = 6 - 4 = 2$
$1/2 \to 1/2 - 1/2(1) = \frac{1}{2} - \frac{1}{2} = 0$
$1 \to 1 - 1/2(0) = 1 - 0 = 1$
$1/10 \to 1/10 - 1/2(-1/5) = \frac{1}{10} + \frac{1}{10} = 1/5$
$0 \to 0 - 1/2(1/2) = -1/4$

Here the optamility test condition

$Z_J - C_J \geq 0$ is satisified

$\{0, 0, 2/5, 1\} \geq 0$

$\therefore$ Max $Z = 64$ & M.F.S are $x_1 = 8$
$x_2 = 2$

TRANSPORTATION

Def:- Transporting the goods from one plac to another plac with in the minimum cost is called Transportation.

FORM OF TRANSPORTATION :-

$$
\begin{array}{c|cccc|c}
 & & & \text{To} & & \text{Supply} \\
\hline
 & C_{11} & C_{12} & \cdots & C_{1n} & S_1 \\
 & C_{21} & C_{22} & \cdots & C_{2n} & S_2 \\
\text{FROM} & \vdots & & & \vdots & \vdots \\
 & C_{n1} & C_{n2} & \cdots & C_{nn} & S_n \\
\hline
\text{Demand} \rightarrow & D_1 & D_2 & \cdots & D_n &
\end{array}
$$

TYPES OF TRANSPORTATION :- 2 Types

(a) Balanced $\Rightarrow$ Total supply = Total Demand

$$\Longrightarrow S_1 + S_2 + \cdots + S_n = D_1 + D_2 + \cdots + D_n$$

(b) unBalanced $\Rightarrow$ Total supply $\neq$ Total Demand

$$\Longrightarrow S_1 + S_2 + \cdots + S_n \neq D_1 + D_2 + \cdots + D_n$$

TYPES OF TRANSPORTATION SOLUTIONS :- 2 Types

(a) Initial Basic Feasible Solution [IBFS]

METHODS FOR CALCULATING OF IBFS

→ NORTH-WEST Corner method

→ LEAST COST Entry method

→ VOGEL'S APPROXIMATION Method

(b) OPTIMAL SOLUTION (OS)

Methods for Calculating OF OS

(a) MODI - Method

Modified Distribution method

(OR)

UV - method

(b) stepping stone method

— x —

PROBLEMS :-

(1) Solve the Following Transportation and find the IBFS

	P_1	P_2	P_3	Supply
S_1	2	7	4	5
S_2	3	3	1	8
S_3	5	4	7	7
S_4	1	6	3	14
Demand	7	9	18	

Sol:- Step: 1 The given problem is a Balanced Transportation problem

i.e., Total supply = Total Demand

$\Longrightarrow 5+8+7+14 = 7+9+18$

$\Longrightarrow 34 = 34$

$\Longrightarrow$ Total supply = Total Demand

(a) North-West Corner method :-

			Supply
2	7	4	5
3	3	1	8
5	4	7	7
1	6	2	14
Demand 7	9	18	$\boxed{34}$

/34

/34

(a) North-West Corner method :-

	P_1	P_2	P_3	Supply
S_1	2 $\boxed{5}$	$\boxed{7}$	$\boxed{4}$	$\cancel{5}$
S_2	3	3	1	8
S_3	5	4	7	7
S_4	1	6	2	14 /34
Demand	$\cancel{7}$ 2	9	18 /34	$\boxed{34}$

(a) start the problem (1,1) Cell

(a) North-West Corner method :-

	P_1	P_2	P_3	Supply
S_1	2 $\boxed{5}$	7	4	$\cancel{5}$
S_2	3 $\boxed{2}$	3	1	$\cancel{8}$ 6
S_3	5	4	7	7
S_4	1	6	2	14
Demand	$\cancel{7}$ $\cancel{7}$	9	18	$\boxed{34}$

/34

/34

(a) start the problem (1,1) Cell

(b) Here First Row Completed
Now start the problem (2,1) Cell

(a) North-West Corner method :-

	P_1	P_2	P_3	Supply
S_1	2 [5]	7	4	~~5~~
S_2	3 [2]	3 [6]	1	~~8~~ ~~6~~
S_3	5	4	7	7
S_4	1	6	2	14
Demand	~~7~~ ~~7~~	~~9~~ 3	18 / 34	$\boxed{34}$ / 34

(a) start the problem (1,1) cell

(b) Here first Row completed
Now start the problem (2,1) cell

(c) Here first Row & first Coloumn also completed. Now start (2,2) cell

(a) North-West Corner method :-

	P_1	P_2	P_3	Supply	
S_1	2 $\boxed{5}$	7	4	$\cancel{5}$	
S_2	3 $\boxed{2}$	3 $\boxed{6}$	1	$\cancel{8}$ $\cancel{6}$	
S_3	5	4 $\boxed{3}$	7	$\cancel{7}$ 4	
S_4	1	6	2	14	/34
Demand	$\cancel{7}$ / $\cancel{7}$	$\cancel{9}$ / $\cancel{3}$	18 / 34	$\boxed{34}$	

(a) start the problem (1,1) Cell

(b) Here first Row completed
Now start the problem (2,1) Cell

(c) Here first Row & first Coloumn also Completed. Now start (2,2) Cell

(d) Now start (3,2) Cell

(a) North-West Corner method:-

	P_1	P_2	P_3	Supply
S_1	2 $\boxed{5}$	7	4	~~5~~
S_2	3 $\boxed{2}$	3 $\boxed{6}$	1	8 ~~6~~
S_3	5	4 $\boxed{3}$	7 $\boxed{4}$	~~7~~ ~~4~~
S_4	1	6	2 $\boxed{14}$	~~14~~ /34
Demand	~~7~~ ~~7~~	~~9~~ ~~3~~	~~18~~ ~~14~~ /34	$\boxed{34}$

(a) start the problem (1,1) Cell

(b) Here first Row completed
Now start the problem (2,1) Cell

(c) Here first Row & first Column also Completed. Now start (2,2) Cell

(d) Now start (3,2) Cell

(e) Now start (3,3) Cell

(a) <u>North-West Corner method</u>:-

	P_1	P_2	P_3	<u>Supply</u>
S_1	$\boxed{5}$ 2	7	4	$\cancel{5}$
S_2	$\boxed{2}$ 3	$\boxed{6}$ 3	1	$\cancel{8}$ $\cancel{6}$
S_3	5	$\boxed{3}$ 4	$\boxed{4}$ 7	$\cancel{7}$ $\cancel{4}$
S_4	1	6	$\boxed{14}$ 2	$\cancel{14}$
Demand	$\cancel{7}$ $\cancel{7}$	$\cancel{9}$ $\cancel{3}$	$\cancel{18}$ $\cancel{14}$ /34	$\boxed{34}$

/34

(a) start the problem (1,1) Cell

(b) Here first Row completed
Now start the problem (2,1) Cell

(c) Here first Row & first Coloumn also Completed. Now start (2,2) Cell

(d) Now start (3,2) Cell

(e) Now start (3,3) Cell

(a) North-West Corner method :-

	P_1	P_2	P_3	Supply
S_1	2 $\boxed{5}$	7	4	~~5~~
S_2	3 $\boxed{2}$	3 $\boxed{6}$	1	~~8~~ ~~6~~
S_3	5	4 $\boxed{3}$	7 $\boxed{4}$	~~7~~ ~~4~~
S_4	1	6	2 $\boxed{14}$	~~14~~ / 34
Demand	~~7~~ ~~7~~	~~9~~ ~~3~~	~~18~~ ~~14~~ / 34	$\boxed{34}$

(a) start the problem (1,1) Cell

(b) Here first Row completed
Now start the problem (2,1) Cell

(c) Here first Row & first Coloumn also Completed. Now start (2,2) Cell

(d) Now start (3,2) Cell

(e) Now start (3,3) Cell

Now the Total Cost is

$\Rightarrow 5\times2 + 2\times3 + 6\times3 + 3\times4 + 4\times7 + 14\times2$

$\Rightarrow 10 + 6 + 18 + 12 + 28 + 28$

$\Rightarrow 102$

$\therefore$ The Minimum cost for N-W method is 102

(a) North-West Corner method :-

			Supply
2	7	4	5
3	3	1	8
5	4	7	7
1	6	2	14
7	9	18	$\boxed{34}$

Demand

/34

/34

N-W method by step by step process

Step: 1

	P_1	P_2	P_3	S
S_1	$\boxed{5}$ 2	~~7~~	~~4~~	~~5~~
S_2	3	3	1	8
S_3	5	4	7	7
S_4	1	6	2	14
D	~~7~~ 2	9	18	$\boxed{34}$

$\longrightarrow$

Step: 2

	P_1	P_2	P_3	S
S_2	$\boxed{2}$ 3	3	1	~~8~~ 6
S_3	~~5~~	4	7	7
S_4	~~1~~	6	2	14
D	~~7~~	9	18	$\boxed{29}$

Step: 3

	P_2	P_3	S
S_2	$\boxed{6}$ 3	~~1~~	~~6~~
S_3	4	7	7
S_4	6	2	14
D	~~9~~ 3	18	$\boxed{27}$

$\longrightarrow$

Step: 4

	P_2	P_3	S
S_3	$\boxed{3}$ 4	7	~~7~~ 4
S_4	~~6~~	2	14
D	~~3~~	18	$\boxed{21}$

Step: 5

	P_3	S
S_3	$\boxed{4}$ 7	~~4~~
S_4	2	14
D	~~18~~ 14	$\boxed{18}$

$\longrightarrow$

Step: 6

	P_3	S
S_4	$\boxed{14}$ 2	~~14~~
D	~~14~~	$\boxed{14}$

$\therefore$ Min Cost $= 10 + 6 + 18 + 12 + 28 + 28 = 102/-$

— x —

(b) LEAST COST ENTRY METHOD

(or)

Lowest cost entry method

(or)

Matrix minimum method

(b) LEAST COST ENTRY METHOD

(or)

Lowest Cost entry method

(or)

Matrix minimum method

	P_1	P_2	P_3	S
S_1	2	7	4	5
S_2	3	3	1	8
S_3	5	4	7	7
S_4	1	6	2	14
D	7	9	18	34

(a) Start the problem where the minimum cost is their. In this problem the M.C is '1'. But, '1' is in two places. Select any one

(or)

Matrix minimum method

	P_1	P_2	P_3	S
S_1	2	7	4	5
S_2	3	3	1 $\boxed{8}$	$\cancel{8}$
S_3	5	4	7	7
S_4	1	6	2	14
D	7	9	$\cancel{18}$ 10	$\boxed{34}$

(a) start the problem where the minimum cost is their. In this problem the M.C is '1'. But, '1' is in two places. Select any one