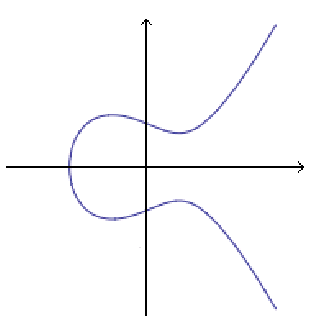
Birch and Swinnerton-Dyer conjecture

In mathematics, the **Birch and Swinnerton-Dyer conjecture** (often called the **Birch–Swinnerton-Dyer conjecture**) describes the set of rational solutions to equations defining an elliptic curve. It is an open problem in the field of number theory. It is one of the most challenging mathematical problems in active research.



An example of an elliptic curve

Background

An elliptic curve is a function of the form $y^2 = x^3 + ax + b$, where a and b are rational numbers.

The x, y values that satisfy the equation are called solutions of the elliptic curve. These solutions may be both rational, both irrational, or one rational and the other irrational. The set of rational solutions is an abelian group. This means that given rational points P and Q, there is a way to produce the sum $P + Q$, and this is another rational point.

In 1922, Louis Mordell proved that the set of rational solutions is a finitely generated abelian group. This means that any rational point P can be written as a finite combination of certain generating points. For example, the points $P_1 = (-2, 3)$ and $P_2 = (-1, 4)$ are generators of the rational points of $y^2 = x^3 + 17$. This means that all rational points on it, even very complicated points, can be written in terms of these two points. For example, we have:

$$\frac{17091468000743375527383814475801\,2}{1116938840470976473737667359544\,81}, -\frac{5355483650472998648137959899973372475907238039375}{1180440469197546185983425190055773135812433974871} = -5P_1 + 5P_2$$

References