Rectangular HSS Compression Design

[ASSUME] Members are in pure compression

[ASSUME] AISC manual of steel (14th ed.) controls member design

[ASSUME] Torsional unbraced length does not exceed lateral unbraced length

1. Inputs

Ultimate compressive load;	$P_u = 10$ kips
Member length;	$L = 4$ ft
Member effective length factor;	$k_c = 1$
Member section size;	Shape $=$ HSS6X2X1/8
Material yield stress;	$F_y = 36$ ksi
Modulus of elasticity;	$E = 29000$ ksi
Resistance factor for compression;	$\phi_c = 0.9$

2. Section Properties

$A_g = 1.77 \text{ in}^2$

$b = 1.65$ in

$h = 5.65$ in

$t_{des} = 0.116$ in

$r_x = 2.05$ in

$r_y = 0.861$ in

$\mathrm{b/t} = 14.2$

$\mathrm{h/t} = 48.7$

3. Buckling Properties

Element compression slenderness limit

$$\lambda_r = 1.4 \cdot \sqrt{\frac{E}{F_y}} = 1.4 \ \cdot \sqrt{\frac{29000 \text{ ksi}}{36 \text{ ksi}}}$$ [AISC Table B4.1a]

$$\therefore \lambda_r = 39.74$$

Maximum element slenderness ratio

$$\lambda_{max} = \max\left(\text{b/t}, \text{h/t}\right) = \max\left(14.2\ , 48.7\ \right)$$

$$\therefore \lambda_{max} = 48.7$$

Member slenderness ratio

$$\text{KL/r} = \frac{k_c \cdot L \cdot 12 \text{ in/ft}}{\min\left(r_x, r_y\right)}$$

$$= \frac{1\ \ \cdot 4 \text{ ft} \cdot 12 \text{ in/ft}}{\min\left(2.05 \text{ in}, 0.861 \text{ in}\right)}$$

$$\therefore \text{KL/r} = 55.75$$

4. Compressive Strength

Elastic buckling stress

$$F_e = \frac{\left(\pi\right)^2 \cdot E}{\left(\text{KL/r}\right)^2}$$

$$= \frac{\left(3.142\ \right)^2\ \cdot 29000 \text{ ksi}}{\left(55.75\ \right)^2}$$ [AISC Eq. E3-4]

$$\therefore F_e = 92.09 \text{ ksi}$$

$$Check\ \lambda_{max}\ <\ \lambda_r$$

$$48.7\ \ <\ 39.74$$

$$\therefore Slender$$

4.1. Reduced Effective Widths of Slender Elements (AISC Eq. E7-18)

$\rightarrow \text{b/t} \; < \; \lambda_r$

Side B is not slender.

$b_e = 1.65 \text{ in}$

$\rightarrow \text{h/t} \; > \; \lambda_r$

Side Ht is slender.

$$h_e = 1.92 \cdot t_{des} \cdot \sqrt{\frac{E}{F_y}} \cdot \left(1 - \frac{0.38}{\text{h/t}} \cdot \sqrt{\frac{E}{F_y}}\right)$$

$$= 1.92 \;\cdot 0.116 \text{ in} \cdot \sqrt{\frac{29000 \text{ ksi}}{36 \text{ ksi}}} \cdot \left(1 \; - \frac{0.38}{48.7} \cdot \sqrt{\frac{29000 \text{ ksi}}{36 \text{ ksi}}}\right)$$

$$\therefore h_e = 4.921$$

Ineffective length on side B

$$b_i = \max\left(0, b - b_e\right) = \max\left(0 \; , 1.65 \text{ in} - 1.65 \text{ in}\right)$$

$$\therefore b_i = 0 \text{ in}$$

Ineffective length on side Ht

$$h_i = \max\left(0, h - h_e\right) = \max\left(0 \; , 5.65 \text{ in} - 4.921 \;\right)$$

$$\therefore h_i = 0.7286 \text{ in}$$

4.2. Reduction Factor for Slender Stiffened Elements (AISC Section E7.2)

Effective cross-sectional area

$$A_e = A_g - b_i \cdot t_{des} \cdot 2 - h_i \cdot t_{des} \cdot 2$$
$$= 1.77\ \text{in}^2 - 0\ \text{in} \cdot 0.116\ \text{in} \cdot 2\ - 0.7286\ \text{in} \cdot 0.116\ \text{in} \cdot 2$$
$$\therefore A_e = 1.601\ \text{in}^2$$

Reduction factor for cross-section

$$Q_a = \frac{A_e}{A_g} = \frac{1.601\ \text{in}^2}{1.77\ \text{in}^2}$$ [AISC Eq. E7-16]
$$\therefore Q_a = 0.9045$$

4.3. Nominal Compressive Strength (AISC Section E7)

$$\lambda_{crit} = 4.71 \cdot \sqrt{\frac{E}{Q_a \cdot F_y}} = 4.71\ \cdot \sqrt{\frac{29000\ \text{ksi}}{0.9045\ \cdot 36\ \text{ksi}}}$$
$$\therefore \lambda_{crit} = 140.6$$

$$\rightarrow \text{KL/r}\ \leq\ \lambda_{crit}$$

...Inelastic buckling controls.

Critical compressive stress

$$F_{cr} = Q_a \cdot F_y \cdot (0.658)^{\frac{Q_a \cdot F_y}{F_e}}$$
$$= 0.9045\ \cdot 36\ \text{ksi} \cdot (0.658\ )^{\frac{0.9045\ \cdot 36\ \text{ksi}}{92.09\ \text{ksi}}}$$ [AISC Eq. E7-2]
$$\therefore F_{cr} = 28.08\ \text{ksi}$$

Nominal member compressive strength

$$P_n = F_{cr} \cdot A_g = 28.08\ \text{ksi} \cdot 1.77\ \text{in}^2$$ [AISC Eq. E7-1]
$$\therefore P_n = 49.71\ \text{kips}$$

5. Member Demand vs. Capacity Check

Design member compressive capacity

$$\phi P_n = \phi_c \cdot P_n = 0.9 \ \cdot 49.71 \text{ kips}$$

$$\therefore \phi P_n = 44.74 \text{ kips}$$

$$Check\ P_u \ \le \ \phi P_n$$

$$10 \text{ kips} \ \le \ 44.74 \text{ kips}$$

$$\therefore OK$$