Beam Moment and Deflection with Point Load

Calculate the moment and deflection demands of a simply-supported steel beam due to a point load.

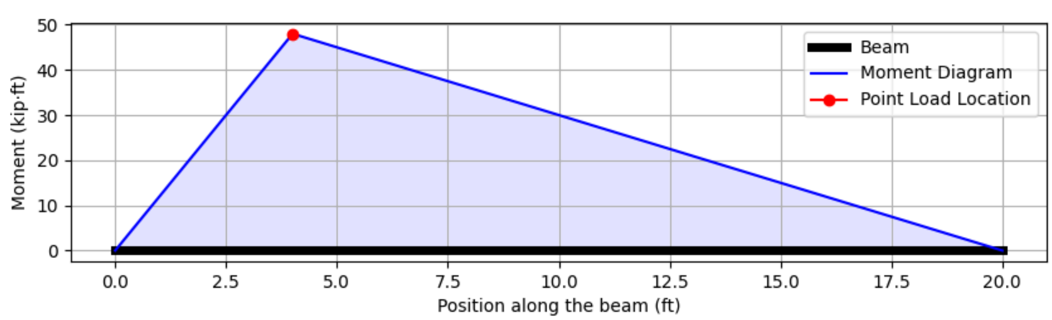
1. Inputs

Beam length;	$L = 20\ \mathrm{ft}$
Point load force;	$F = 15\ \mathrm{kips}$
Point load position along the beam;	$x = 4\ \mathrm{ft}$
Beam section size;	$\mathrm{size} = \mathrm{W12X40}$

2. Moment Demand

$$M_{max} = \frac{F \cdot x \cdot (L - x)}{L} = \frac{15\ \mathrm{kips} \cdot 4\ \mathrm{ft} \cdot (20\ \mathrm{ft} - 4\ \mathrm{ft})}{20\ \mathrm{ft}}$$

$$\therefore M_{max} = 48\ \mathrm{kip - ft}$$



3. Deflection

3.1. Beam and Load Dimensions

The longer distance from the location of the load to a beam support

$$a = \max(x, L - x) = \max(4\ \mathrm{ft}, 20\ \mathrm{ft} - 4\ \mathrm{ft})$$

$$\therefore a = 16\ \mathrm{ft}$$

The shorter distance from the location of the load to a beam support

$$b = L - a = 20 \text{ ft} - 16 \text{ ft}$$
$$\therefore b = 4 \text{ ft}$$

3.2. Section Properties

Modulus of elasticity for steel

$$E = 29000 \text{ ksi}$$

Beam moment of inertia

$$I = 307 \text{ in}^4$$

3.3. Deflection Calculation

$$\delta_{max} = \frac{F \cdot a \cdot b \cdot (a + 2 \cdot b) \cdot \sqrt{3 \cdot a \cdot (a + 2 \cdot b)}}{27 \cdot E \cdot I \cdot L} \cdot (12 \text{ in/ft})^3$$
$$= \frac{15 \text{ kips} \cdot 16 \text{ ft} \cdot 4 \text{ ft} \cdot (16 \text{ ft} + 2 \ \cdot 4 \text{ ft}) \cdot \sqrt{3 \ \cdot 16 \text{ ft} \cdot (16 \text{ ft} + 2 \ \cdot 4 \text{ ft})}}{27 \ \cdot 29000 \text{ ksi} \cdot 307 \text{ in}^4 \cdot 20 \text{ ft}} \cdot (12 \text{ in/ft})^3$$
$$\therefore \delta_{max} = 0.2811 \text{ in}$$