Denavit-Hartenberg Parameters

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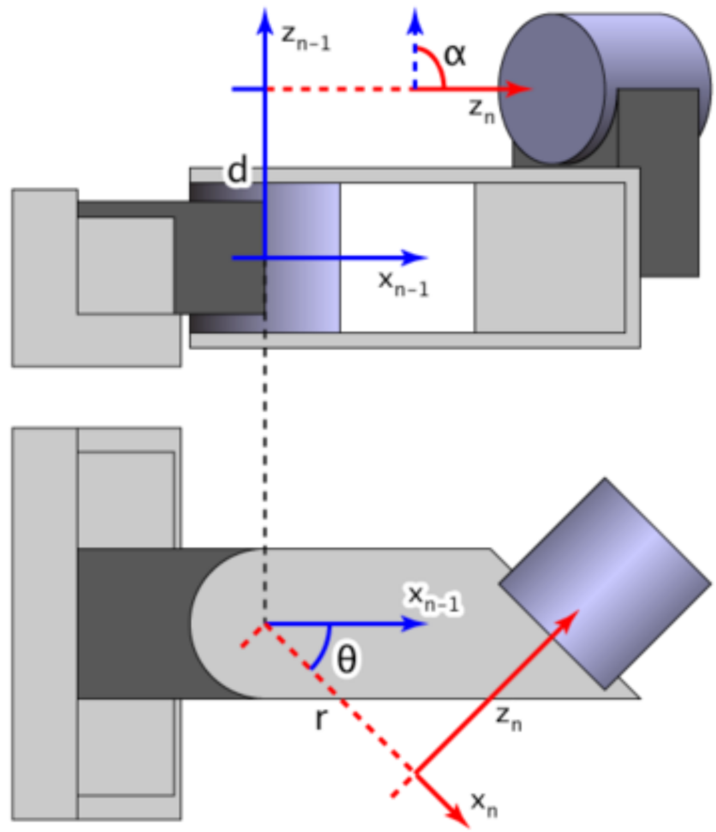

A commonly used convention for selecting frames of reference in robotics applications is the Denavit and Hartenberg (D-H) convention which was introduced by Jaques Denavit and Richard S. Hartenberg. In this convention, each homogeneous transformation is represented as a product of four basic transformations. The common normal between two lines was the main geometric concept that allowed Denavit and Hartenberg to find a minimal representation. The reference frames are laid out as follows:

1. the z-axis is in the direction of the joint axis
2. the x-axis is parallel to the common normal: $x_n = z_{n-1} \times z_n$
   If there is no unique common normal (parallel z axes), then d (below) is a free parameter. The direction of x_n is from $z_n - 1$ to z_n, as shown in the video below.
3. the y-axis follows from the x- and z-axis by choosing it to be a right-handed coordinate system.

The transformation is then described by the following four parameters known as **D-H Parameters**:[1].

- d: offset along previous z to the common normal
- θ: angle about previous z, from old x to new x
- r: length of the common normal (aka a, but if using this notation, do not confuse with α). Assuming a revolute joint, this is the radius about previous z.
- α: angle about common normal, from old z axis to new z axis

A visualization of D-H parameterization. Also available: YouTube (http://www.youtube.com/watch?v=rA9tm0gTln8) , 1280x720 MPEG-4 (http://tekkotsu.no-ip.org/movie/dh-hd.mp4) , 640x360 MPEG-4 (http://tekkotsu.no-ip.org/movie/dh-sd.mp4)

There is some choice in frame layout as to whether the previous x axis or the next x points along the common normal. The latter system allows branching chains more efficiently, as multiple frames can all point away from their common ancestor, but in the alternative layout the ancestor can only point toward one successor. Thus the commonly used notation places each down-chain x axis collinear with the common normal, yielding the transformation calculations shown below.

We can note constraints on the relationships between the axes:

- the x_n-axis is perpendicular to both the z_{n-1} and z_n axes
- the x_n-axis intersects both z_{n-1} and z_n axes
- the origin of joint n is at the intersection of x_n and z_n
- y_n completes a right-handed reference frame based on x_n and z_n

Every link/joint pair can be described as a coordinate transformation from the previous coordinate system to the next coordinate system.

$$^{n-1}T_n = \mathrm{Trans}_{z_{n-1}}(d_n) \cdot \mathrm{Rot}_{z_{n-1}}(\theta_n) \cdot \mathrm{Trans}_{x_n}(r_n) \cdot \mathrm{Rot}_{x_n}(\alpha_n)$$

Note that these are 2 screws after oneanother. See Screw (motion).

The matrices mentioned above are as follows:

$$\mathrm{Trans}_{z_{n-1}}(d_n) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_n \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{Rot}_{z_{n-1}}(\theta_n) = \begin{pmatrix} \cos\theta_n & -\sin\theta_n & 0 & 0 \\ \sin\theta_n & \cos\theta_n & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{Trans}_{x_n}(r_n) = \begin{pmatrix} 1 & 0 & 0 & r_n \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathrm{Rot}_{x_n}(\alpha_n) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_n & -\sin\alpha_n & 0 \\ 0 & \sin\alpha_n & \cos\alpha_n & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

This gives:

$$^{n-1}T_n = \begin{pmatrix} \cos\theta_n & -\sin\theta_n\cos\alpha_n & \sin\theta_n\sin\alpha_n & r_n\cos\theta_n \\ \sin\theta_n & \cos\theta_n\cos\alpha_n & -\cos\theta_n\sin\alpha_n & r_n\sin\theta_n \\ 0 & \sin\alpha_n & \cos\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

See also

- Robotics conventions
- J. Denavit and R.S. Hartenberg. A kinematic notation for lower-pair mechanisms based on matrices. Trans ASME J. Appl. Mech, 23:215-221,1955
- R.S. HartenBerg and J. Denavit *Kinematic synthesis of linkages* McGraw-Hill, New York, NY, 1964 (http://kmoddl.library.cornell.edu/bib.php?m=23)

References

1. ^ Spong, M., M. Vidyasagar, "Robot Dynamics and Control", John Wiley and Sons, 1989, ISBN 047161243X