

# **A Simple Technique to Passively Gravity-Balance Articulated Mechanisms**

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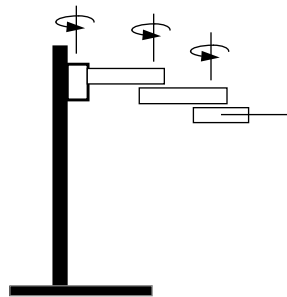
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## **Abstract**

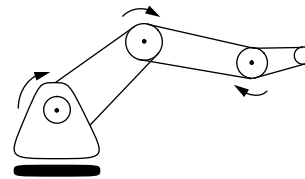
A simple method to counter the effects of gravity in articulated mechanisms is proposed. The scheme uses kinematics and linear springs to produce a non-linear restoring force to oppose the gravitational moment. The method equilibrates a rotational mechanism for all postures. A solution for one link is obtained then general equations for  $n$  links are derived. The method is simpler than previous schemes and has applications in robotics, orthotics and a host of everyday mechanisms.

## Introduction

One of the advantages of a SCARA (Selective Compliance Robotic Arm for Assembly), figure 1, configuration robot is that three of the links operate in the horizontal plane thereby freeing the actuators of gravitational loading. This considerably reduces the size of the actuators needed thereby allowing precise movements. This may be contrasted with robots that have an anthropomorphic configuration. Anthropomorphic robots have all rotary joints with movements similar to a person's arm. Robots with this configuration include the PUMA. The two main links of the PUMA operate in the vertical as well as the horizontal planes as shown in figure 1. This places a large and configuration dependent torque requirement on the PUMA motors to account for the varying gravitational torque.



SCARA



ANTHRPOMORPHIC

This paper describes a method to passively negate the effect of the varying torque due to gravity. This is accomplished using linear springs, and the solution proposed is exact for all configurations of an articulated mechanism. The solution offers a simple method that can be applied to any number of links and has applications in other articulated mechanisms such as equipoised lamps, dentists lights, orthoses, excavators, and cranes.

The problem of gravity balancing is not new. A number of solutions have been proposed. An

active mechanism like a robot applying torque to a joint through the existing actuator, accomplishes the job but requires energy input to the system. This is the conventional way to account for gravity in robots and requires larger actuators. The advantage of an active system, however, is that dynamic balancing may be accomplished.

The alternative is to employ passive balancing, which offers only static balancing but nevertheless reduces load requirements on the actuators considerably (Huisson and Wang, 1991). Passive balancing can be achieved in two ways.

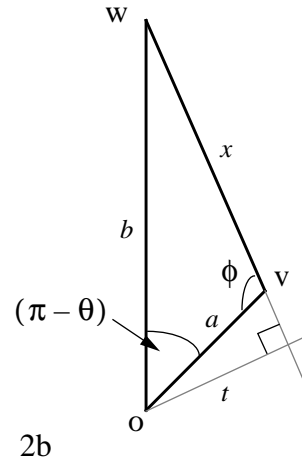
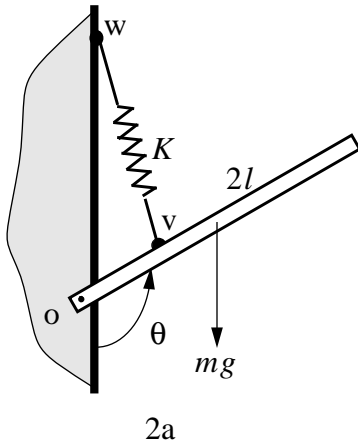
1. Adding a counterweight so that the mass center is coincident with the pivot point.
2. Using the stored energy in springs to counter the effects of gravity.

The first approach does provide a system that is balanced for all positions, however, this is achieved at the expense of weight and inertia. The spring approach appears to be more attractive since no undue energy is added to the system. Since the moment due to gravity is configuration dependent, it is non-linear. A perfectly balanced system would use non-linear springs, however, construction of customized non-linear springs is complex and the results may not be compact enough. The alternative is to use off-the-shelf linear springs and create a non-linear restoring moment through geometrical variation of the moment arm. This is the approach taken by Ulrich and Kumar (Ulrich and Kumar, 1991) who used linear springs and cams to achieve a weightless feeling, however this method required the fabrication of specific cam shapes. Herve (1986) also used linear springs to exactly balance a link for all positions, however, the geometry is complex and impractical to use for more than one link. The method proposed in this paper offers a much simpler way of achieving gravity balancing.

Other methods that use linear springs to balance articulated mechanisms have been proposed (Mahalingam and Sharan, 1986, Gopalswamy et al. 1992, Fisher, 1991) but these do not offer exact solutions for all postures. The proposed method uses a single linear spring for each rotational link and offers a solution that is in equilibrium for the entire range of motion.

### Single Link Solution

The following describes how a single link rotating in the vertical (and horizontal) plane can be balanced by a linear spring so that it is in equilibrium in any position despite the effect of gravity. The one link case will then be extended to one with  $n$  links. Figure 2a describes a rigid link pinned at  $o$  and held by a linear spring attached to a vertical wall at  $w$ . The question is, under what conditions will the link be in equilibrium for  $0^\circ < \theta < 180^\circ$ ?



For the system to be in equilibrium  $M_o$ , the moment about  $O$ , must be 0. From figure 2

$$M_o = mgl \sin \theta - K(x - x_0)t = 0$$

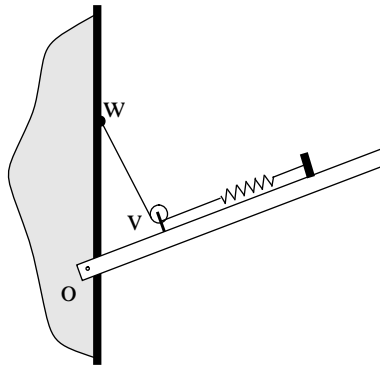
For  $\theta \neq 0$  this reduces to

$$mgl = \frac{K}{x} (x - x_0) ab$$

If  $x_0 = 0$  the equation further reduces to,

$$K = mgl/ab \quad (1)$$

Equation (1) shows that the stiffness  $K$  becomes a constant and independent of the angle  $\theta$  of the link. This is achievable only if the unstretched length of the spring  $x_0$  is chosen to be 0. This condition may be physically realized if the tension spring were placed outside the line connecting  $wv$  (figure 3).

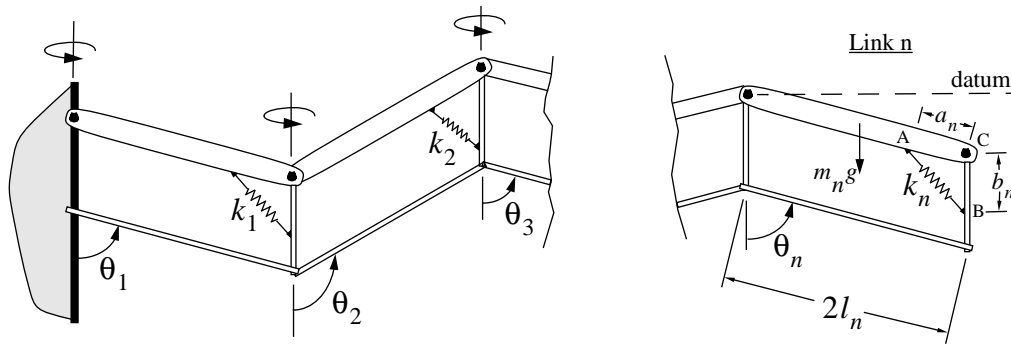


Therefore by choosing a spring of stiffness  $K$  according to equation (1) and placing the spring outside of line  $wv$  connecting the link and the fixed reference, the link can be perfectly balanced for all positions.

### N-link Solution

The one-link solution may be applied to  $n$  links connected in series with each joint having 2 dof , one about the vertical axis which is unaffected by gravity, and the other about the horizontal axis. Each link, however, must consist of a four bar mechanism to ensure that vertical members exist at

the end of each link (figure 4).



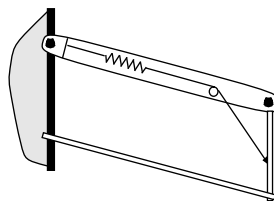
The analysis that results in a balanced system will use the method of conservation of energy. Since this is a static analysis the kinetic energy is zero and the potential energy must be constant for all configurations if balancing is to be achieved. Therefore

$$\frac{\partial}{\partial \theta_n} (PE) = 0$$

Writing the potential energy of the last link, link n, while fixing the other links (assuming the supporting links are massless).

$$PE = -m_n g l_n \cos \theta_n + (K_n/2) (x_n - x_0)^2 \quad (2)$$

As shown in the one link case if  $x_0 = 0$  a solution exists. This condition can be physically realized if the spring is placed outside of line AB, as shown in figure 5.



But from figure 4,  $x_n^2 = a_n^2 + b_n^2 + 2a_nb_n\cos\theta_n$  and  $x_0 = 0$ . Substituting this into equation (2) yields

$$PE = -m_n g l_n \cos\theta_n + (K_n/2) \left( a_n^2 + b_n^2 + 2a_nb_n\cos\theta_n \right)$$

$$\text{therefore } \frac{\partial}{\partial\theta_n}(PE) = m_n g l_n \sin\theta_n - K_n a_n b_n \sin\theta_n = 0$$

which leads to  $K_n = m_n g l_n / a_n b_n$  for  $\theta \neq 0$

Next we derive the expression for the stiffness  $K$  for link n-1. Writing the potential energy of link n-1 while holding all the links to the left fixed,

$$PE = -m_{n-1} g l_n \cos\theta_{n-1} + (K_{n-1}/2) \left( a_{n-1}^2 + b_{n-1}^2 + 2a_{n-1}b_{n-1}\cos\theta_{n-1} \right) - m_n g 2l_n \cos\theta_{n-1}$$

$$\text{Therefore } \frac{\partial}{\partial\theta_{n-1}}(PE) = m_{n-1} g l_{n-1} \sin\theta_{n-1} + m_n g 2l_n \sin\theta_{n-1} - K_{n-1} a_{n-1} b_{n-1} \sin\theta_{n-1} = 0$$

$$\text{so, } m_{n-1} g l_{n-1} + 2m_n g l_n = K_{n-1} a_{n-1} b_{n-1} \text{ for } \theta \neq 0$$

$$\text{Therefore } K_{n-1} = (g/a_{n-1}b_{n-1}) (m_{n-1}l_{n-1} + 2m_n l_n) \quad (3)$$

The term for  $m_n$  in equation (3) is included as a point mass at the end, since spring  $K_n$  has balanced link n therefore no moments exist for all  $\theta_n$ .

Equation (3) may be generalized for any link t as

$$K_t = (g/(a_t b_t)) \left( m_t l_t + \sum_{s=t+1}^n 2m_s l_s \right) \quad (4)$$



where  $1 \leq t \leq n$ . Equation (4), which represents the stiffness of a spring that balances link  $t$ , must however be applied recursively starting from the last link of a serial chain linkage and working backwards to the first link. This method applies only to open kinematic chains.

## **Application**

One application that has readily found use for the above methodology is in the area of human rehabilitation. In particular the development of an arm orthosis. An orthosis is an exoskeletal device that is attached to flail or weakened limbs to augment strength deficiency. An orthosis based on the anti-gravity technique has been built for the arm. The populations which will benefit from these devices are people with spinal muscular atrophy, Duchenne's muscular dystrophy, polio and Lou Gehrig disease. The target population is characterized by degeneratively weakening muscles, but no associated loss of sensation. Typically, the muscles of these individuals are so weak, that they cannot support their arms against gravity.

The designed device consists of two four bar linkages as seen in figure 4, compensated passively by the use of linear springs for the weight of the person's arm and associated hardware. Attached to the lower four bar linkage is a splint to support the person's arm. The modularity of the technique discussed can be exploited to advantage in this application, due to the nature of the disability being progressive in the target population. First a four bar linkage can be used for the upper arm, and as the disability progresses another module can be added for the lower or forearm. Next when the individual loses more muscle strength, powered actuators can be added. Since the devices are compensated for gravity, the motors will draw less power. Using only standard off-the-shelf linear springs, the values of  $a_t$  and  $b_t$  in equation (4) can be adjusted to obtain the neces-

sary compensation for varying arm weights. The value of  $l_t$  in equation (4) can also be varied to suit various arm lengths. This demonstrates the adaptability of the technique to accommodate for variations in any given application.

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