UNIVERSITY OF DHAKA

Department of Applied Mathematics
Program: B.S. (Honours) in Applied Mathematics
Year: 4th , Academic Session: 2021 – 2022
Course No: AMTH 450, Course Title: MATH LAB IV
Assignment – 1: Solving Problems on Multivariate and Vector Calculus with its Real-life Applications

Instruction: Write programming code using Mathematica/MATLAB to get the outputs and visualize the results of the following problems.

Name: **Roll:** **Group:**

Day – 1

1. (a) Use **PolarPlot**, **ParametricPlot, and ParametricPlot3D** to sketch the following curves/parametric curves:

$(i)\, r = e^{\cos\theta} - 2\cos 4\theta + \sin^5 \dfrac{\theta}{12}, 0 \le \theta \le 24\pi$ [Butterfly]

$(ii)\, \vec{r} = \langle 2\cos t + \cos 2t, 2\sin t - \sin 2t \rangle, 0 \le t \le 4\pi$ [Tricuspoid]

$(iii)\, x = \int_0^t \cos\left(\dfrac{\pi u^2}{2}\right) du, y = \int_0^t \sin\left(\dfrac{\pi u^2}{2}\right) du, -10 \le t \le 10$ [Cornu spiral]

$(iii)\, \vec{r} = \langle a\cos t, a\sin t, ct \rangle, \dfrac{\pi}{3} \le t \le 8\pi, a = 1, c = \dfrac{1}{3}$ [Circular Helix]

(b) Use **Plot3D** to sketch the curve of intersection of the surfaces, and hence obtain two different views.

$(i)\, z = x^2 + y^2, y = 12 - z, (ii)\, 9x^2 + y^2 + 9z^2 = 81, y = x^2 \ (z > 0)$

$(iii)\, \vec{r} = \langle t\sin t, tc\,\mathrm{os}\, t, t^2 \rangle, z = x^2 + y^2$

(c) Sketch the contour plots of

$(i)\, f(x,y) = 4x^2 + y^2, (ii)\ f(x,y,z) = z^2 - x^2 - y^2$ using level curves of height $k = 1,4,9,16,26,36$.

2. (a) Consider the functions:

$(i)\, f(x,y) = \left|\sin x \sin y\right|, 0 \le x \le 2\pi, 0 \le y \le 2\pi$

$(ii)\, f(x,y) = xye^{-\frac{1}{4}(x^2+y^2)}, -5 \le x \le 5, -5 \le y \le 5$

Draw the three dimensional figures, and the level curves using **Plot3D, DensityPlot** in the neighborhoods of the origin. Also, try to use **Axes, PlotPoints, PlotStyle, ColorFunction** options to get a better graph.

(b) The equations $(i) r = 1 - 2\sin(n\theta)$, and $(ii) r = 2 - 3\cos^2(n\theta)$, where n is a positive integer that represent a family of polar curves. Investigate the behavior of this family and form a conjecture about how the number of loops is related to n.

3. (a) Find the volume of the solid obtained by revolving the region bounded by the graph of $y = x\cos^2 x, x = 0, x = \pi$, and x-axis about (i) the x-axis and (ii) the y-axis.

(b) Find the equation of the tangent plane and normal line to $f(x,y) = e^x \sin(\pi y)$ at the point $P(0,1,0)$. Confirm your result graphically.

Day – 2

4. (a) The temperature (in degrees Celsius) at a point (x, y) on a metal plate in the xy-plane is $T(x, y) = 3x^2y$. Compute the gradient of $T(x, y)$ at the point $\left(-1, \frac{3}{2}\right)$, and the directional derivative of $T(x, y)$ at the point $\left(-1, \frac{3}{2}\right)$ in the direction $\left(-1, -\frac{1}{2}\right)$. Also, plot of the directional derivative with $-2 \le x \le 0, 0 \le y \le 2$, and visualize directional derivative over a surface.

(b) Locate all relative extrema and saddle points of
$(i) f(x, y) = 4xy - x^4 - y^4, (ii) f(x, y) = x^4 + y^4 - 20x^2 - 10xy - 25,$
$(iii) 3xe^y - x^3 - e^{3y}, (iv) f(x, y) = x^3 + y^3 - 3x - 3y, (v) f(x, y) = 4x^2e^y - 2x^4 - e^{4y}$
Confirm that your obtained results are consistent with graphs.

5. (a) An electric charge is spread over the half-disk described by $x^2 + y^2 = 9, y \ge 0$. Draw the region graphically and calculate the total charge on if the charge density at any point in (measured in coulombs per square meter) is $\sigma(x, y) = \sqrt{x^2 + y^2}$.

(b) Sketch the solid in the first octant that is enclosed by the planes $x = 0$, $z = 0$, $x = 5$, $z - y = 0$, and $z = -2y + 6$. Find the volume of the solid by breaking it into two parts.

(c) Sketch the solid bounded between the surfaces $z = 4(x^2 + y^2)$ and $z = 16 - 4(x^2 + y^2)$ on the rectangular domain $[-1, 1] \times [-1, 1]$. Compute the volume of solid.

6. (a) Suppose that the temperature in degrees Celsius at a point (x, y) on a flat metal plate is $T(x, y) = 10 - 8x^2 - 2y^2$, where x and y are in meters. Find the average temperature of the rectangular portion of the plate for which $0 \le x \le 1$ and $0 \le y \le 2$.

(b) Find the area of the region inside the circle $x^2 + y^2 = 4$ and to the right of the line $x = 1$. Sketch the resultant region.

(c) Find the volume of the solid inside the surface $r^2 + z^2 = 4$ and outside the surface $r = 2 \cos \theta$. Sketch the resultant volume of the solid.

(d) Suppose that a geyser, centered at the origin of a polar coordinate system, sprays water in a circular pattern in such a way that the depth D of water that reaches a point at a distance of r feet from the origin in 1 hour is $D = ke^{-r}$. Find the total volume of water that the geyser sprays inside a circle of radius R centered at the origin. Take k = 1.5, R = 13.

7. (a) Find the volume of the solid within the cylinder $x^2 + y^2 = 9$ and between the planes $z = 1$ and $x + z = 5$.

(b) Compute the mass of the solid region W bounded between the planes $z = 1 - x - y$ and $z = 1 + x + y$ situated over the triangular domain D bounded by x = 0, y = 0, and y = 1-x. Assume the density of W is given by $\rho(x, y, z) = 1 + x^2 + y^2$.

(c) Find the volume of the solid lying under the graph of the surface $z = x^3 + 4y$ and above the region in the xy-plane bounded by the line y = 2x and the parabola $y = x^2$. Also draw the solid region.

(d) Verify that the force field $\vec{F}(x, y) = e^y\hat{i} + xe^y\hat{j}$ is conservative on the entire xy-plane. Find the work done by the field on a particle that moves from $(1, 0)$ to $(-1, 0)$. Take the potential function $\phi = xe^y + K$ for $\vec{F}(x, y)$.

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