Pearson Edexcel International GCSE

Tuesday 21 May 2024

Morning (Time: 2 hours)

Paper reference **4PM1/01R**

Further Pure Mathematics
PAPER 1R

Calculators may be used. | Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need*.
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question*.

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere $= 4\pi r^2$

Curved surface area of cone $= \pi r \times$ slant height

Volume of sphere $= \dfrac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$

Geometric series

Sum to n terms, $S_n = \dfrac{a(1 - r^n)}{(1 - r)}$

Sum to infinity, $S_\infty = \dfrac{a}{1 - r} \quad |r| < 1$

Binomial series

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule
In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\sin(A + B) = \sin A \cos B + \cos A \sin B$ $\qquad$ $\sin(A - B) = \sin A \cos B - \cos A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$ $\qquad$ $\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$ $\qquad$ $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

Logarithms

$\log_a x = \dfrac{\log_b x}{\log_b a}$

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1   Without using a calculator, solve the inequality $\sqrt{50}\,x - \sqrt{18} > 6x + 5$

Give your answer in an exact form with a rationalised denominator.
Show your working clearly.

(4)

(Total for Question 1 is 4 marks)

2 Given that

$$1 - \frac{1}{3}x + \frac{5}{36}x^2 + \dots$$

is the binomial expansion, in ascending powers of x, of $(1 + Ax)^n$

where A and n are rational numbers,

(a) find the value of A and the value of n

(6)

(b) Hence find the value of the coefficient of x^3

Give your answer in the form $-\frac{p}{q}$ where p is a prime number and q is an integer.

(2)

(Total for Question 2 is 8 marks)

B

Diagram **NOT**
accurately drawn

x cm

30°

A

C
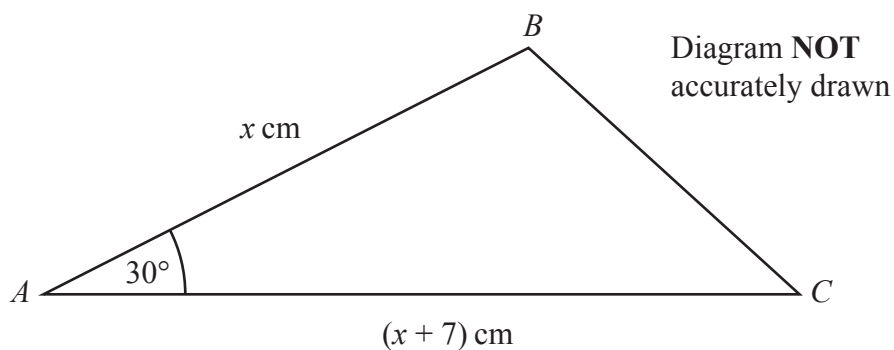
$(x + 7)$ cm

**Figure 1**

Figure 1 shows triangle ABC where

$AB = x$ cm $AC = (x + 7)$ cm $\angle BAC = 30º$

The area of triangle $ABC = 36$ cm^2

(a) Show that $x = 9$

(3)

(b) Find, in cm to 3 significant figures, the length of BC

(2)

(c) Find, in degrees to one decimal place, the size of

(i) $\angle ABC$

(ii) $\angle ACB$

(3)

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(Total for Question 3 is 8 marks)

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(21 − r) cm

P   Q   Diagram **NOT** accurately drawn

r cm

θ rad

O
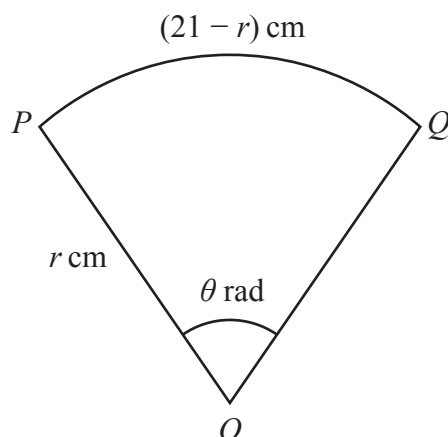
Figure 2

Figure 2 shows the sector OPQ of a circle with centre O and radius r cm.

$OP = OQ = r$ cm   arc $PQ = (21 - r)$ cm $\angle POQ = \theta$ radians

The area of the sector is A cm^2

(a) Show that $A = \dfrac{r}{2}(21 - r)$

(3)

The area of the sector must be greater than or equal to 27 cm^2

(b) Find the set of possible values of r

(4)

(c) Hence write down the set of possible values of θ

(2)

P 7 4 0 9 9 A 0 8 3 2

Question 4 continued

(Total for Question 4 is 9 marks)

5 The sum of the first 10 terms of an arithmetic series A is $36k + 1$ where k is a constant.

The 6th term of A is $4k + 1$

(a) (i) Find an expression in terms of k for the common difference of A

    (ii) Show that the first term of A is -8

(5)

Given that the 4th term of A is 7

(b) show that $k = 4$

(2)

The sum of the first n terms of A is S_n and the nth term of A is U_n

(c) Find the value of n such that $S_n = 5U_{n+10} + 105$

(4)

P 7 4 0 9 9 A 0 1 1 3 2

P 7 4 0 9 9 A 0 1 2 3 2

Question 5 continued

(Total for Question 5 is 11 marks)

6 A particle P is moving in a straight line. The displacement s of P, in metres, at time t seconds, $t \geqslant 0$, is given by

$$s = e^{2t} \sin 3t + 2$$

At time $t = 0$, P is at the point A and at time $t = \dfrac{\pi}{6}$, P is at the point B

(a) Find the exact distance AB

(2)

(b) Find the exact velocity of P when $t = \dfrac{\pi}{3}$

(4)

Question 6 continued

**(Total for Question 6 is 6 marks)**

7   The curve C has equation $y = -\log_4(x + 4)$

   (a) Using the axes below, sketch the graph of C.
       Label the coordinates of the points of intersection of C with the coordinate axes
       and the equation of any asymptote to C.

       (4)

   (b) Solve the equation $\log_{(x+4)} 256 - \log_4(x + 4) = 0$

       (5)
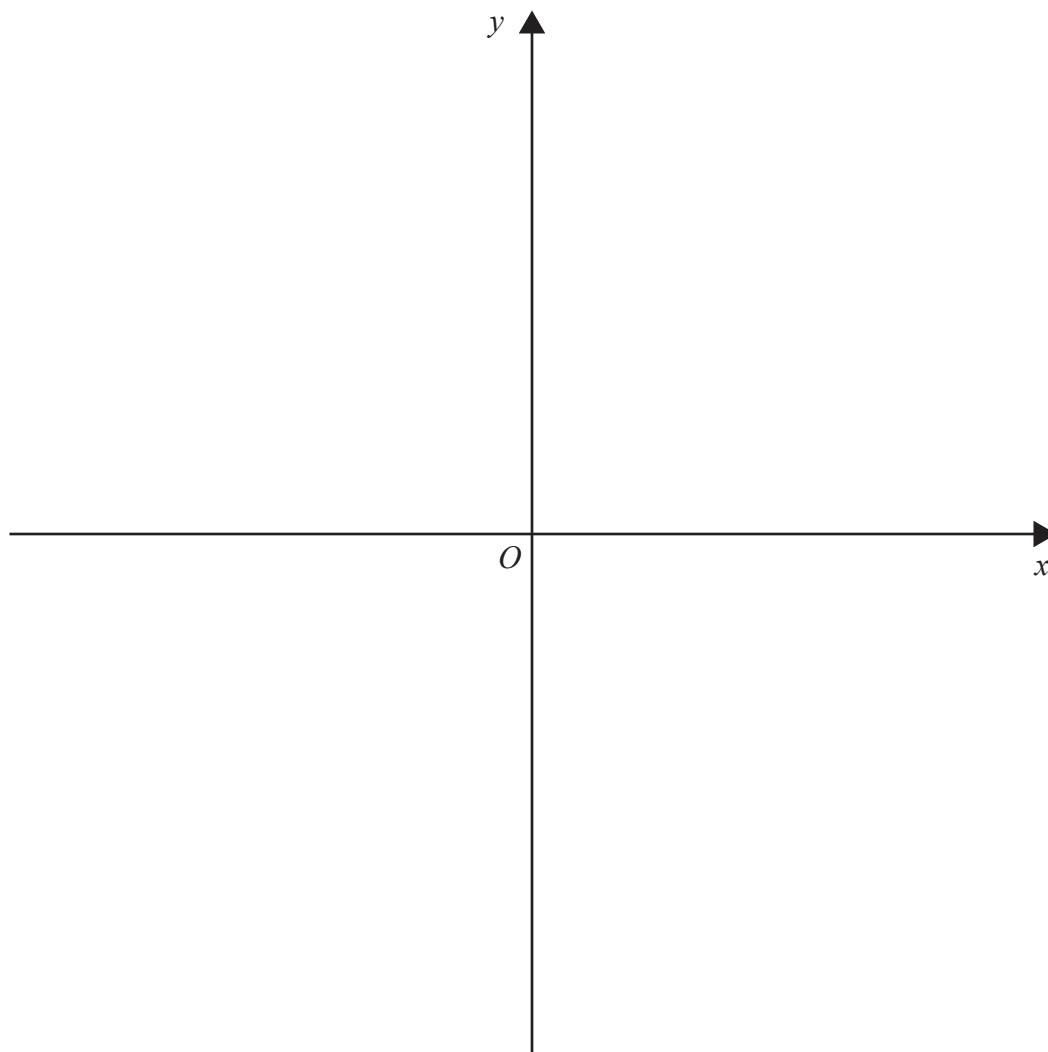
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Question 7 continued

(Total for Question 7 is 9 marks)
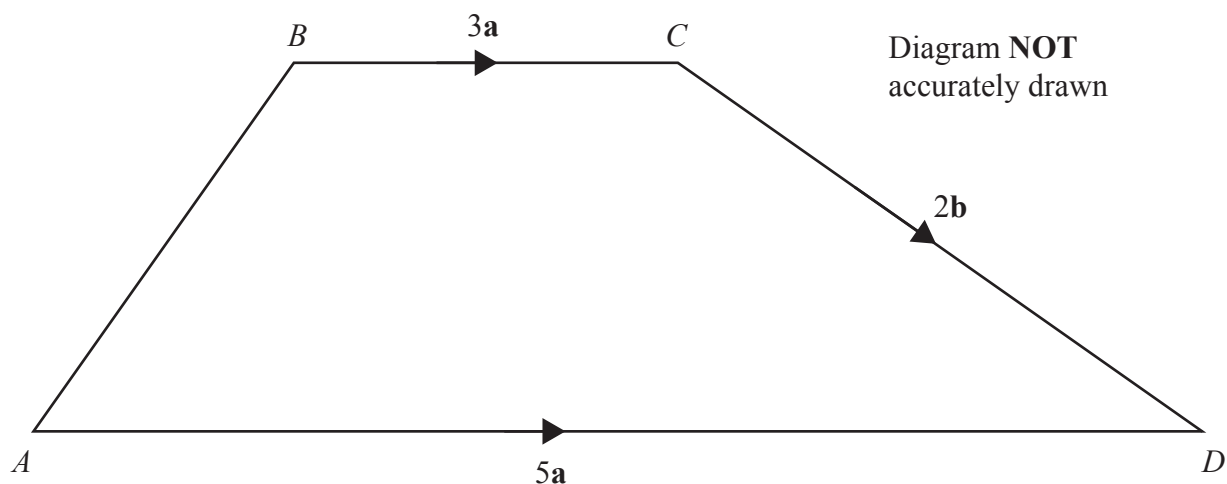
Figure 3

Figure 3 shows a trapezium *ABCD*

$$\overrightarrow{BC} = 3\mathbf{a} \qquad \overrightarrow{AD} = 5\mathbf{a} \qquad \overrightarrow{CD} = 2\mathbf{b}$$

(a) Find $\overrightarrow{AB}$ as a simplified expression in terms of **a** and **b**

(1)

The diagonals *BD* and *AC* intersect at point *X* where $\overrightarrow{BX} = k\,\overrightarrow{BD}$

(b) Using a vector method, find the value of *k*

(5)

(c) Find the ratio of the area of triangle *CXD* : area of the trapezium *ABCD*

(4)

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P 7 4 0 9 9 A 0 1 9 3 2

Question 8 continued

Question 8 continued

(Total for Question 8 is 10 marks)

9 The point A has coordinates $(-4, 3)$ and the point B has coordinates $(6, 8)$
The points A and B lie on the line k

(a) Find an equation of k

(2)

The point C, on k, is such that $AC : CB = 4:1$

(b) Find the coordinates of point C

(2)

The point D with coordinates (p, q), where $p < 0$, lies on the line l through C that is perpendicular to k

The length of CD is $8\sqrt{5}$

(c) Find the coordinates of D

(6)

(d) Find the area of triangle ACD

(2)

P 7 4 0 9 9 A 0 2 3 3 2

P 7 4 0 9 9 A 0 2 4 3 2

(Total for Question 9 is 12 marks)

10 The quadratic equation $2x^2 + kx + 4 = 0$ has roots α and β such that

$$k < 0 \text{ and } \alpha > \beta$$

Given that $\alpha^2 - \beta^2 = \dfrac{7\sqrt{17}}{4}$

(a) show that $k = -7$

(8)

(b) Hence form a quadratic equation that has roots

$$(\alpha - \beta) \text{ and } (\alpha + \beta)$$

(4)

Question 10 continued

Question 10 continued

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Question 10 continued

(Total for Question 10 is 12 marks)

11 $f(\theta) = (2\cos\theta - \sin\theta)(2\sin\theta + \cos\theta)$

(a) Show that $f(\theta) = \dfrac{3}{2}\sin 2\theta + 2\cos 2\theta$

(3)

Diagram **NOT** accurately drawn
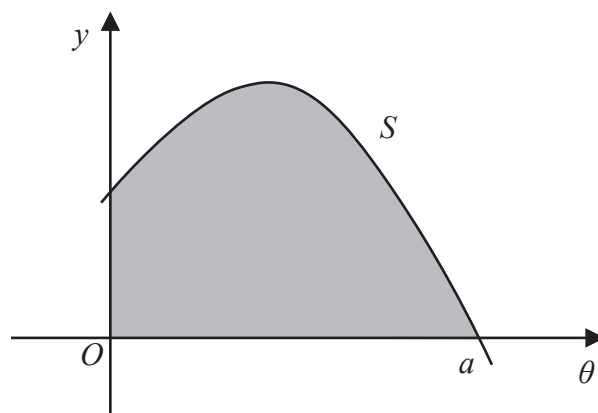
Figure 4

Figure 4 shows part of the curve S with equation $y = f(\theta) + 2$

Given that S intersects with the θ-axis at the point with coordinates $(a, 0)$

(b) using $\sin^2\theta + \cos^2\theta = 1$, or otherwise, show that $a = \dfrac{\pi}{2}$

(5)

(c) Using algebraic integration, find the exact area bounded by S, the positive θ-axis and the positive y-axis shown shaded in Figure 4

(3)

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Question 11 continued

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(Total for Question 11 is 11 marks)

TOTAL FOR PAPER IS 100 MARKS

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