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Mark Scheme (Results)

June 2015

International GCSE
Further Pure Mathematics
4PM0/02

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)
- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers may score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.
- **With working**
  Always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.
  Any case of suspected misread loses 2A (or B) marks on that part, but can gain the M marks.
  If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded in another.

General Principles for Further Pure Mathematics Marking
(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:
$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$ leading to $x=....$
$(ax^2+bx+c)=(mx+p)(nx+q)$ where $|pq|=|c|$ and $|mn|=|a|$ leading to $x=....$

2. Formula:
Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=....$

3. Completing the square:
$x^2+bx+c=0$: $(x\pm\frac{b}{2})^2\pm q\pm c=0$, $q\neq 0$ leading to $x=....$

Method marks for differentiation and integration:

1. Differentiation
Power of at least one term decreased by 1. $(x^n\to x^{n-1})$

2. Integration:
Power of at least one term increased by 1. $(x^n\to x^{n+1})$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required.
(Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Question Number	Scheme	Marks
1.(a)	$S_n = \frac{n}{2}\left(2a+(n-1)d\right)$	
	$\sum = \frac{n}{2}\left(2+(n-1)\times 1\right)$ or $S_n = \frac{n}{2}(a+l)$	M1
	$\sum = \frac{n}{2}(1+n)$ * $\frac{n}{2}(1+n)$	A1 cso (2)
(b)	$\sum_1^{100} r = 50(100+1) = 5050$	M1(either sum using result in (a))
	Multiples of 7 $S_{14} = \frac{14}{2}(7+98) = 735$	A1(both sums correct)
	Reqd sum $= 5050 - 735 = 4315$	A1cso (3) **[5]**

(a) M1 This is a "show" question, so the formula used must be quoted first if using $S_n = \frac{n}{2}(a+l)$.

For $S_n = \frac{n}{2}\left(2a+(n-1)d\right)$, the unsimplified substitution is sufficient.

A1cso Obtain the GIVEN result

(b) M1 Attempt the sum of all integers up to 100 (100 terms) **or** the multiples of 7 (14 terms)
A1 **Both** sums correct
A1cso Subtract to obtain 4315

ALT List the terms needed and add: Question says "Hence", so the result must be used at least once, probably seen in the sum of all the integers up to 100. The multiples of 7 may be listed and added before subtracting from the sum up to 100.

2. (a)	Missing values: 7, 5.09, 3.5, 3.46	B1(any 2) B1(all corr.) (2)
(b)	Plot points Smooth curve through points plotted	B1ft B1 (2)
(c)	$x-3+\frac{3}{x^2}=0$	
	$x+\frac{6}{x^2}=6-x$	M1A1
	Draw $y=6-x$	dM1
	$x=1.3$ or 1.4, 2.5 or 2.6	A1 (4) **[8]**

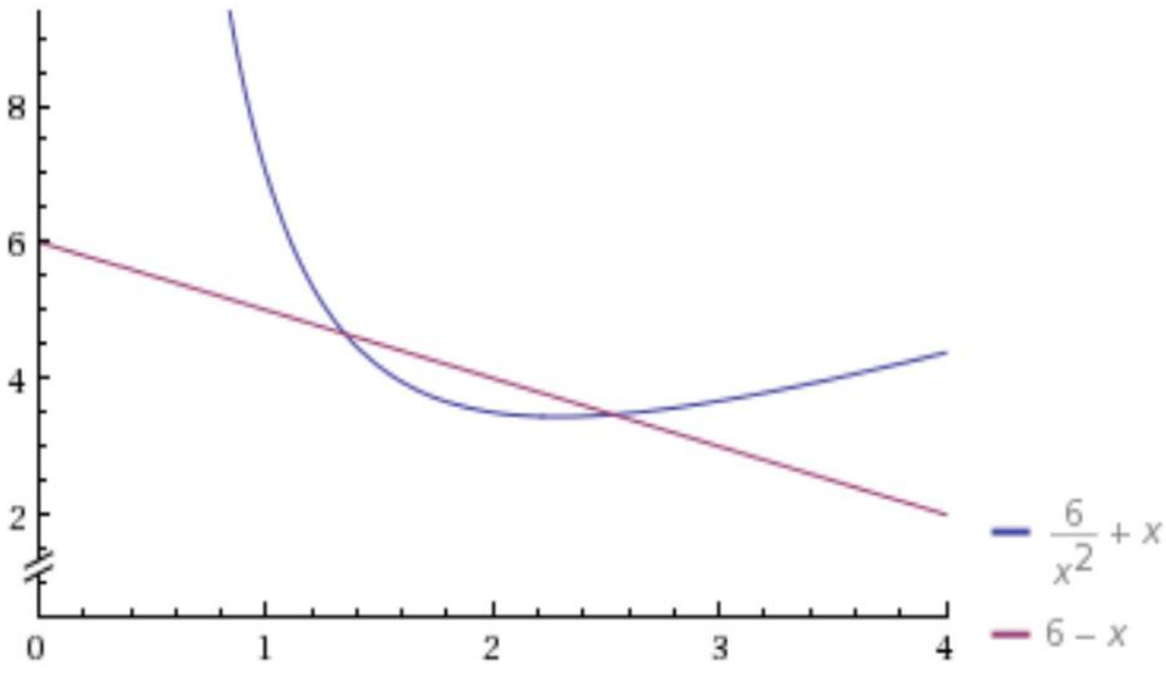
(a) B1 Any 2 values correct
B1 Remaining 2 values correct

(b) B1ft Plot *their* points correctly
B1 Draw a smooth curve through *their* plotted points.

(c) M1 Attempt to re-arrange the given equation to the form $x+\frac{6}{x^2}=ax+b$ (a,b to be numbers now or later)
A1 Correct RHS
dM1 Draw *their* line Must be of form $y=ax+b$ with $a\neq 0$
A1 $x=1.3$ or 1.4, 2.5 or 2.6 Must be 1 dp unless more figures are shown and a mark has been lost in (a) because of failing to round

4PM0/02: Q2 GRAPH PLOT

Plot:



Points of intersection:

x = 1.347 y = 4.653 (4sf)

x = 2.532 y = 3.468 (4sf)

3. (a)	$ar^2 - ar^3 = 2ar^4$ (or $ar^3 - ar^2 = 2ar^4$)	M1
	$ar^2\left(2r^2 + r - 1\right) = 0$	
	$ar^2\left(2r-1\right)\left(r+1\right) = 0$	dM1
	$r = \frac{1}{2}$ *	A1 cso (3)
(b)	$400 = \frac{a}{1-\frac{1}{2}}$	M1
	$a = 200$	A1 (2)
(c)	$S_{10} = \frac{a\left(1-r^n\right)}{1-r} = 400 \times \left(1 - \left(\frac{1}{2}\right)^{10}\right)$	M1
	$= 399.609375$ or $\frac{25575}{64}$ or $399\frac{39}{64}$	A1cao (2) **[7]**

(a) M1 Attempt to form an equation connecting the 3rd, 4th and 5th terms

depM1 Attempt to factorise their quadratic (usual rules). May divide by ar^2 first.

A1cso Obtain the GIVEN answer from correct working. Both solutions for the quadratic must be shown and $r = -1$ rejected

(b) M1 Use the formula for the sum to infinity with $r = \frac{1}{2}$

A1 Obtain the correct value of a

(c) M1 Use the formula for the sum of the first 10 terms, $r = \frac{1}{2}$ and their a

A1cao Obtain the correct sum (min 3 decimal places or exact answer)

(a) If done by verification: Scores M1M0A0 (max) as $r = \frac{1}{2}$ not shown to be the **only** solution.

4(a)	$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$	M1
	$= \mathbf{i} - 8\mathbf{j}$	A1 (2)
(b)	$\lvert\overrightarrow{PQ}\rvert = \sqrt{1^2 + 8^2} = \sqrt{65}$	M1
	Unit vector $= \pm\dfrac{1}{\sqrt{65}}(\mathbf{i} - 8\mathbf{j})$ oe	A1ft(on $\overrightarrow{PQ}$) (2)
(c)	$\lvert\overrightarrow{OP}\rvert = \sqrt{3^2 + 6^2} = \sqrt{45}$	M1either length
	$\lvert\overrightarrow{OQ}\rvert = \sqrt{4^2 + 2^2} = \sqrt{20}$	A1 (both corr.)
	$\lvert\overrightarrow{OP}\rvert^2 + \lvert\overrightarrow{OQ}\rvert^2 = \lvert\overrightarrow{PQ}\rvert^2$	M1
	$\therefore$ right angle at O ALT: Use gradients	A1 cso (4) [8]

Vector arrows may be missing and **i** and **j** need not be made bold/underlined. Column vectors allowed apart from (a)

(a) M1 A correct statement as shown, or equivalent to that
A1 For $\mathbf{i} - 8\mathbf{j}$
(b) M1 Attempting the modulus of *their* $\overrightarrow{PQ}$ Pythagoras statement must have a + sign
A1ft Giving a unit vector, follow through their $\overrightarrow{PQ}$. Can be in direction P to Q or reverse.

(c) M1 Attempting the length of either $\overrightarrow{OP}$ or $\overrightarrow{OQ}$ numbers to be added in the Pythagoras statement
A1 **Both** lengths correct, as given or equivalent exact answers
M1 Attempt to show their lengths fit Pythagoras
A1cso A correct solution and a concluding statement. (Accept eg "shown" or an appropriate symbol

ALT using gradients:
M1 Attempting the gradient of either vector (numbers needed)
A1 Both gradients correct grad $OP = 2$; grad $OQ = -\dfrac{1}{2}$
M1 Forming the product of their gradients
A1cso Product to be -1 **and** a concluding statement given

Special case: If both gradients are found as $\dfrac{"x"}{"y"}$, award M1A1M1A0 (max)

If one gradient found as $\dfrac{"x"}{"y"}$ and the other as $\dfrac{"y"}{"x"}$ only M marks are available

By Scalar Product: M1A1Correct product M1 Multiply out A1 All correct and a conclusion

5(a)	$t^3 - 5t^2 + 6t = 0 \quad t^2 - 5t + 6 = 0$	M1
	$(t-3)(t-2) = 0$	dM1
	$t = 2,\ t = 3$	A1 (3)
(b)	$v = \frac{\mathrm{d}s}{\mathrm{d}t} = 3t^2 - 10t + 6$	M1A1
	$t = 1 \quad v = 3\times1^2 - 10\times1 + 6 = -1, \quad v$ or speed $= 1$ m/s	A1,A1ft (4)
(c)	$\frac{\mathrm{d}v}{\mathrm{d}t} = 6t - 10$	M1
	$t = 2 \quad a = 6\times2 - 10 = 2$ m/s^2	M1 (A1 on ePEN)
	ft on values of t	
	$t = 3 \quad a = 6\times3 - 10 = 8$ m/s^2	A1 (3) **[10]**

(a) M1 Equate the given expression to zero and divide by (or take out) the common factor t
dM1 Factorise *their* resulting quadratic or use another method to solve (usual rules).
A1 Both (correct) values of t. Ignore $t = 0$ if included.

(b) M1 Attempt to differentiate the given expression
A1 Correct differentiation
A1 Obtain $v = -1$ by substituting $t = 1$ in a correct expression for v
A1ft Change a negative answer to a *positive* one. Follow through their v

(c) M1 Differentiate again to obtain an expression for the acceleration
M1 Substitute one of *their* answers from (a) in their expression for the acceleration
A1 For 2 and 8 (Ignore $a = \pm10$ from $t = 0$)

6 (a)	$S = \frac{1}{2}\times 10^2\theta - \frac{1}{2}\times 6^2\theta$	M1A1
	$= 32\theta$ *	A1cso (3)
(b)	$\frac{dS}{dt} = 32\frac{d\theta}{dt}$	M1
	$\frac{dS}{dt} = 32\times 0.2 = 6.4$	A1 (2)
(c)	$20 = 32\theta$	M1
	$\theta = \frac{20}{32}$ oe inc 0.625	A1
	Perim $= 10\times\frac{20}{32} + 6\times\frac{20}{32} + 2\times 4$	M1A1ft (their θ)
	$= 18$ cm	A1 (5) **[10]**
	ALT: Perim $= 10\theta + 6\theta + 8 = 16\theta + 8 = \frac{1}{2}S + 8 = 18$	

Can be worked in degrees but final answers must be exact and correct.

(a) M1 Using Area $= \frac{1}{2}r^2\theta$ to obtain an expression for S by subtracting 2 areas

A1 Correct numbers in the expression

A1cso Obtaining the GIVEN answer from correct working

(b) M1 Differentiate $S = 32\theta$ with respect to t (inc use of chain rule) **Evidence of differentiation needed.** $32\times 0.2 = 6.4$ alone is **not** sufficient

A1 Substitute $\frac{d\theta}{dt} = 0.2$ and obtain $\frac{dS}{dt} = 6.4$

(c) M1 Form an equation for θ

A1 Solve the equation to obtain the value of θ (any equivalent fraction)

M1 Forming an expression for the perimeter of the shaded area. Must have 2 arcs and 2 straight lines all added. Award with θ or their value.

A1ft All numbers correct, follow through their value for θ

A1 Correct perimeter

ALT:

M1 Form an expression for the perimeter in terms of θ Must be evidence of all 4 parts.

A1 Collect terms (may be awarded by implication)

M1 Substitute $\theta = \frac{S}{32}$ to obtain an expression for the perimeter in terms of S

A1 Correct expression

A1 Substitute $S = 20$ to obtain the correct perimeter.

7(a)	$\frac{1}{3}\times h\times 48+48h=256$	M1
	$h=4$ *	A1 (2)
(b)	$FH=10$ or $\frac{1}{2}FH=5$	B1
	$VF^2=8^2+\left(\frac{1}{2}\text{ their } FH\right)^2$	M1
	$VF=9.43$ cm	A1 (3)
(c)	$\tan A=\frac{4}{5}$	M1A1
	$A=38.7°$	A1 (3)
(d)	$\tan\phi=\frac{4}{3}$ $\tan\theta=\frac{4}{6}$	M1 (either)
	$\phi=53.13$	A1
	$\theta=33.69$	A1
	Reqd angle $=86.8°$	A1 cao (4) **[12]**

(a) M1 Forming an equation for the volume of the solid by adding the volumes of the two parts and equating to 256
A1 Solving to get the GIVEN answer
ALT Form an expression for the volume and substitute $h=4$ (M1) obtain 256 (A1)

(b) B1 Correct length of the diagonal or half-diagonal of the base (or top) of the cuboid
M1 Using Pythagoras (with height 8 and their $\frac{1}{2}FH$ and a + sign) to find VF^2
A1 Correct value for *VF* **must** be 3 sf

(c) M1 Finding an expression for the tan of the **required** angle (or cos or sin)
A1 Correct numbers in their chosen trig function
A1 Obtain $A=38.7°$ **must** be to nearest 0.1°

(d) M1 Obtaining a trig function for either of the two angles needed
A1 Correct value for either angle
A1 Correct value for the other angle
These two marks can be awarded by implication if all work done on a calculator and final answer is correct
A1cao Correct result when their angles are added. **must** be to nearest 0.1° unless penalised in (c)
ALT M1 Use cosine rule either form in the correct triangle. Lengths must be found
A1,A1 Deduct 1 per error ie A1A0 for 1 error seen
A1 correct answer

Correct cosine rule: $\cos\theta=\dfrac{5^2+52-73}{2\times 5\sqrt{52}}$ oe

8(a)	$x=0 \Rightarrow y=0 \;\therefore 0=0+0+0+c$	B1 (1)
(b)	$\left[\mathrm{f}(2)=\right] 0=8+4a+2b \qquad \left[\mathrm{f}(4)=\right] 0=64+16a+4b$	M1
	$a=-6, \quad b=8$	A1 A1 (3)
	ALT: Use factors and multiply out $a=-6, \quad b=8$	M1 A1, A1
(c)	$x=3 \;\; y=3^3-6\times3^2+8\times3=-3$	B1
	Grad $l=-\dfrac{3}{3}=-1$	M1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=3x^2-12x+8$	M1
	$x=3 \;\; \dfrac{\mathrm{d}y}{\mathrm{d}x}=3\times9-12\times3+8=-1 \qquad \therefore$ l is the tangent at P	A1 cso (4)
(d)	Eqn of l: $\quad y=-x$	
	Area $=\int\left(\mathrm{f}(x)-(-x)\right)\mathrm{d}x=\int\left(x^3-6x^2+8x+x\right)\mathrm{d}x$	M1
	$=\left[\dfrac{1}{4}x^4-2x^3+\dfrac{9}{2}x^2\right]_0^3$	M1A1ft
	$=\dfrac{1}{4}\times3^4-2\times3^3+\dfrac{9}{2}\times3^2 \; (-0)$	dM1
	$=6\dfrac{3}{4} \quad \left(\text{accept } 6.75 \text{ or } \dfrac{27}{4}\right)$	A1cao (5)**[13]**

(a) B1 Show $c=0$ by substituting $x=0$ in the curve equation or state $x=0 \Rightarrow y=0$
Or $x(x-2)(x-4)=0$ (Must be evidence for $c=0$)

(b) M1 **Either** use the factor theorem with $x=2$ **and** $x=4$ and attempt to solve the resulting equations **Or** use factors $(x-2)(x-4)$ and multiply out

A1 Either a or b correct

A1 Other value correct If factor method used values need not be shown explicitly.
$y=x^3-6x^2+8x$ scores A1A1

(c) B1 Correct value for y coordinate of P obtained using the equation of C

M1 Attempt the gradient or equation of l or equation of tangent

M1 Differentiate *their* curve equation

A1cso Substitute $x=3$ in the derivative to obtain the correct gradient of the tangent at P **and** give a concluding statement - "shown" is sufficient.

ALT B1 y coordinate M1 Equation of l M1 Solve with C A1 $x(x-3)^2=0$ and conclusion

(d) M1 Attempting the area integral, $\pm(C-l)$, with *their* equation of l Correct limits are needed.

M1 Attempting the integration. Limits not needed. Integrand need not be simplified

A1ft Correct integration, ignore limits. No simplification needed. ft their equation of l

dM1 Correct use of limits for their method. Dependent on both M marks above.

A1 Correct answer, any form. Must be positive.

NB: If the area is split into parts, 3 integrals are needed (C 0 to 2, 2 to 3 and l) or 2 integrals and a triangle. Award M1 if an appropriate expression using these 3 parts is seen Correct limits are needed. M1 Integrate **both** functions in separate integrals or integrate C and find the area of the triangle. Rest as above

9 (a)	$(6,6)$	B1B1 (2)
(b)	$AB^2 = 8^2 + 6^2$	M1
	$AB = \sqrt{100} = 10$	A1 (2)
(c)	Grad $AB = -\frac{6}{8}$	M1
	Grad $\perp = \frac{8}{6}$ oe	A1
	Eqn $\perp$ bisector: $y - 6 = \frac{4}{3}(x - 6)$	M1
	$3y = 4x - 6$ or any integer multiple of this	A1 (4)
(d)	$y = 2 \quad 6 = 4d - 6$	M1
	$d = 3$	A1 (2)
(e)	$(12,14)$	B1B1 (2)
(f)	$DE = \sqrt{9^2 + 12^2} = 15$	M1
	Area $= \frac{1}{2} \times AB \times DE = \frac{1}{2} \times 10 \times 15$	M1A1ft (their lengths)
	$= 75$ (units^2)	A1 (4) **[16]**
ALT	Area $= \frac{1}{2}\begin{vmatrix} 2 & 3 & 10 & 12 & 2 \\ 9 & 2 & 3 & 14 & 9 \end{vmatrix} = 75$ units^2	M2A1ft(their coords) A1

(a) B1B1 B1B1 Both coords correct; B1B0 One correct, or recognisable method but neither correct

(b) M1 Attempt Pythagoras with + sign

A1 Correct length

(c) M1 Attempt the gradient of AB Must be $\frac{"y"}{"x"}$.

A1 Correct gradient of the perpendicular

M1 Attempt the equation of the perp bisector with their coordinates of M.. Gradient must be $\frac{-1}{\textit{their}\ \text{grad}\ AB}$.

A1 Correct equation, **must** be in the required form

(d) M1 Substitute $y = 2$ in *their* equation for l

A1 Obtain $d = 3$ or $x = 3$

(e) B1B1 B1B1 Both coords correct; B1B0 One correct.

(f) M1 Attempt the length of DE. Numbers to be added in Pythagoras. (Or DM and ME if using triangles ABE and ADE)

M1 Any complete method for the area of the kite - kite formula or by triangles

A1ft Correct lengths substituted follow through their lengths

A1cao Area = 75 (units^2)

ALT: M2 for a correct "determinant" with $\frac{1}{2}$ and 4 different pairs of coordinates, first repeated as fifth;
A1ft *their* coords used correctly (ie clockwise or anticlockwise round the kite from any vertex) A1 correct area

10(a)	$\log_3 9 = 2$	B1 (1)
(b)	$\log_9 4 = \dfrac{\log_3 4}{\log_3 9}$	M1
	$= \dfrac{1}{2}\log_3 4 \quad k = \dfrac{1}{2}$	A1 (2)
(c)	$2x\log_3 x - 3\log_3 x - 4x\log_9 4 + 6\log_9 4 = (2x-3)\log_3 x - 2(2x-3)\log_9 4$	M1
	$= (2x-3)(\log_3 x - 2\log_9 4)$	A1
	$= (2x-3)(\log_3 x - \log_3 4)$	M1
	$= (2x-3)\log_3\left(\dfrac{x}{4}\right)$	M1
	$= \log_3\left(\dfrac{x}{4}\right)^{(2x-3)}$ *	A1 cso (5)
(d)	$\log_3\left(\dfrac{x}{4}\right)^{(2x-3)} = 0$	
	$2x - 3 = 0$ or $\dfrac{x}{4} = 1$	M1 (either)
	$x = \dfrac{3}{2}, \quad x = 4$	A1A1 (3)
		[11]

(a) B1	Correct answer
(b) M1	Use change of base formula. (Can change both sides to any consistent base)
A1	Obtain $k = \dfrac{1}{2}$
(c) M1	Attempt to move to two log terms $\pm 2(2x-3)$ or $\pm(4x-6)$ with $\log_9 4$
A1	Complete to two correct brackets
M1	Change base of $\log_9 4$ to 3 Use their value of *k* from (b) for this
M1	Combine the two logs to form a single log
A1	Obtain the GIVEN answer - brackets not needed in the index May work in reverse: **M1** Bring down power; **A1** obtain 2 logs from the single log, must be a difference; **M1** change $\log_3 4$ to integer$\times\log_3 4$; **M1** Multiply out brackets; **A1** GIVEN answer There are variations on this - look for the 3 M marks and award the first A mark after first M mark and second A mark at end if all correct.
(d) M1	Obtain either of the two linear equations shown
A1	One correct answer
A1	Second correct answer