Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Monday 9 October 2023

Afternoon (Time: 1 hour 30 minutes) | **Paper reference** **WMA11/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 11 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

Pearson

1. Given that

$$y = 5x^3 + \frac{3}{x^2} - 7x \qquad x > 0$$

find, in simplest form,

(a) $\frac{dy}{dx}$ **(3)**

(b) $\frac{d^2y}{dx^2}$ **(2)**

(a) $y = 5x^3 + 3x^{-2} - 7x$

$\Rightarrow \frac{dy}{dx} = 5(3)x^2 + 3(-2)x^{-3} - 7$

$= 15x^2 - 6x^{-3} - 7$

(b) $\frac{dy}{dx} = 15x^2 - 6x^{-3} - 7$

$\Rightarrow \frac{d^2y}{dx^2} = 15(2)x - 6(-3)x^{-4}$

$= 30x + 18x^{-4}$

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Question 1 continued

(Total for Question 1 is 5 marks)

2. Given that

$$a = \frac{1}{64}x^2 \qquad b = \frac{16}{\sqrt{x}}$$

express each of the following in the form kx^n where k and n are simplified constants.

(a) $a^{\frac{1}{2}}$

(1)

(b) $\dfrac{16}{b^3}$

(1)

(c) $\left(\dfrac{ab}{2}\right)^{-\frac{4}{3}}$

(2)

(a) $a^{\frac{1}{2}}$

$= \left(\dfrac{x^2}{64}\right)^{\frac{1}{2}}$

$= \dfrac{x}{8}$

(b) b^3

$= \left(\dfrac{16}{x^{\frac{1}{2}}}\right)^3$

$= \dfrac{4096}{x^{3/2}}$

$\therefore \dfrac{16}{b^3}$

$= \dfrac{16}{4096/x^{3/2}}$

$= \dfrac{x^{3/2}}{256}$

(c) ab

$= \dfrac{x^2}{64} \times \dfrac{16}{x^{1/2}}$

$= \dfrac{x^{3/2}}{4}$

$\therefore \left(\dfrac{ab}{2}\right)^{-\frac{4}{3}}$

$= \left(\dfrac{\frac{x^{3/2}}{4}}{2}\right)^{-\frac{4}{3}}$

$= \left(\dfrac{x^{\frac{3}{2}}}{8}\right)^{-\frac{4}{3}}$

$= \dfrac{x^{-2}}{8^{-\frac{4}{3}}}$

$= \dfrac{16}{x^2}$

Question 2 continued

(Total for Question 2 is 4 marks)

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Write $\dfrac{8-\sqrt{15}}{2\sqrt{3}+\sqrt{5}}$ in the form $a\sqrt{3}+b\sqrt{5}$ where a and b are integers to be found.

(3)

(b) Hence, or otherwise, solve

$$\left(x+5\sqrt{3}\right)\sqrt{5} = 40 - 2x\sqrt{3}$$

giving your answer in simplest form.

(3)

(a) $\dfrac{8-\sqrt{15}}{2\sqrt{3}+\sqrt{5}} \times \dfrac{2\sqrt{3}-\sqrt{5}}{2\sqrt{3}-\sqrt{5}}$

$= \dfrac{16\sqrt{3}-8\sqrt{5}-2\sqrt{45}+\sqrt{75}}{(2\sqrt{3})^2-(\sqrt{5})^2}$

$= \dfrac{16\sqrt{3}-8\sqrt{5}-6\sqrt{5}+5\sqrt{3}}{12-5}$

$= \dfrac{21\sqrt{3}-14\sqrt{5}}{7}$

$= 3\sqrt{3}-2\sqrt{5}$

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Question 3 continued

(b) $(x+5\sqrt{3})\sqrt{5} = 40-2x\sqrt{3}$

$\Rightarrow x\sqrt{5} + 5\sqrt{15} = 40 - 2x\sqrt{3}$

$\Rightarrow x\sqrt{5} + 2x\sqrt{3} = 40 - 5\sqrt{15}$

$\Rightarrow x(\sqrt{5} + 2\sqrt{3}) = 40 - 5\sqrt{15}$

$\Rightarrow x = \dfrac{40-5\sqrt{15}}{2\sqrt{3}+\sqrt{5}}$

$= 5 \times \dfrac{8-\sqrt{15}}{2\sqrt{3}+\sqrt{5}}$

$= 5(3\sqrt{3} - 2\sqrt{5})$

$= 15\sqrt{3} - 10\sqrt{5}$

(Total for Question 3 is 6 marks)

4.

Red crosses
are marked intersections
for part d

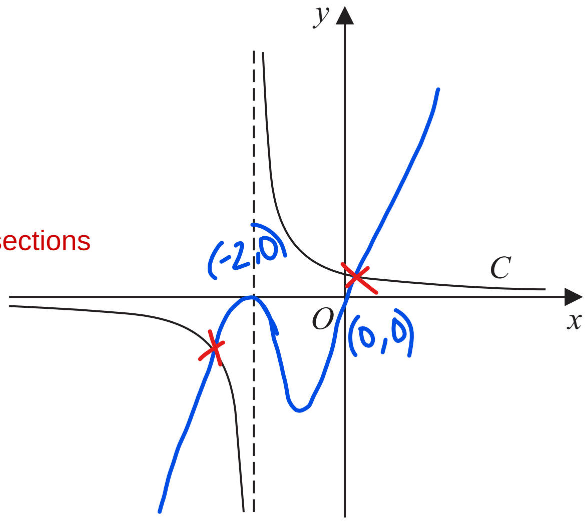


Figure 1

Figure 1 shows a sketch of part of the curve C with equation $y = \dfrac{1}{x+2}$

(a) State the equation of the asymptote of C that is parallel to the y-axis. **(1)**

(a) $x+2=0$
$\Rightarrow x=-2$

(b) Factorise fully $x^3 + 4x^2 + 4x$ **(2)**

(b) $x(x^2+4x+4)$

A copy of Figure 1, labelled Diagram 1, is shown on the next page.

(c) On Diagram 1, add a sketch of the curve with equation

$$y = x^3 + 4x^2 + 4x$$

On your sketch, state clearly the coordinates of each point where this curve cuts or meets the coordinate axes. **(3)**

(c) $y=0$
$\Rightarrow x(x^2+4x+4)=0$
$\Rightarrow x=0 \quad x^2+4x+4=0$
$\Rightarrow (x+2)^2=0$
$\Rightarrow x=-2$

y intercept
$y=(0)^3+4(0)^2+4(0)$
$=0 \quad \therefore (0,0)$

(d) Hence state the number of real solutions of the equation

$$(x+2)(x^3+4x^2+4x) = 1$$

giving a reason for your answer. **(1)**

(d)There are two real solutions since the graphs intersect each other twice.
(visible on diagram)

Question 4 continued

Note for part c

Normally, to draw cubic graph, solutions for x-axis and y intercept need to be found.
However, if one of the solutions is (0,0) such as the example in this question, then (0,0) is the y-intercept itself. In this case, there is no need to calculate y-intercept separately (to save time, also it's not needed at all)

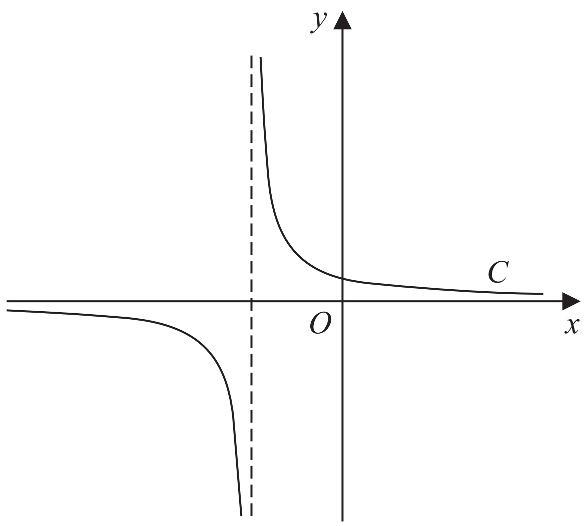
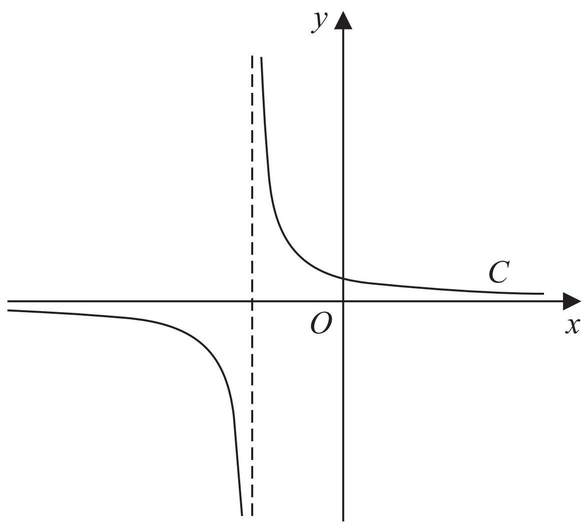


Diagram 1



copy of Diagram 1

Only use the copy of Diagram 1 if you need to redraw your answer to part (c).

(Total for Question 4 is 7 marks)

5.

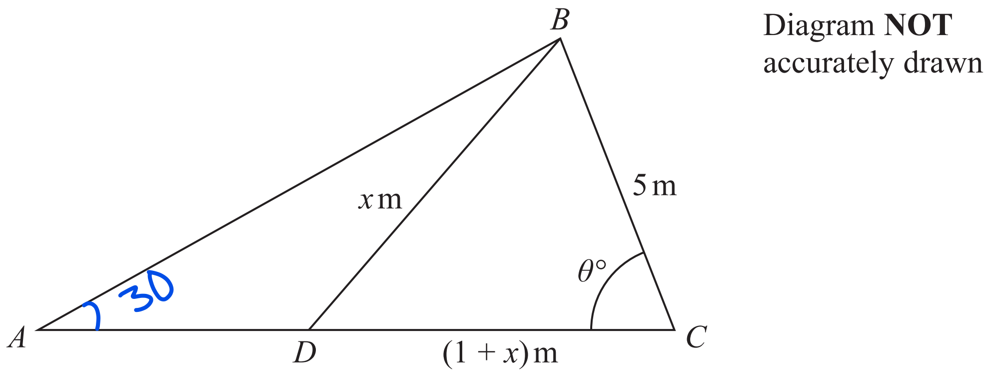


Figure 2

Figure 2 shows the plan view of a frame for a flat roof.

The shape of the frame consists of triangle ABD joined to triangle BCD.

Given that

- $BD = x$ m
- $CD = (1 + x)$ m
- $BC = 5$ m
- angle $BCD = \theta°$

(a) show that $\cos\theta° = \dfrac{13 + x}{5 + 5x}$ **(2)**

$$(a)\ \cos\theta = \frac{5^2 + (1+x)^2 - x^2}{2(5)(1+x)}$$

$$= \frac{25 + 1 + 2x + x^2 - x^2}{10 + 10x}$$

$$= \frac{2(13+x)}{2(5+5x)}$$

$$= \frac{13+x}{5+5x}$$

Given also that

- $x = 2\sqrt{3}$
- angle $BAC = 30°$
- ADC is a straight line

(b) find the area of triangle ABC, giving your answer, in m^2, to one decimal place.

(5)

$$(b)\ \cos\theta = \frac{13 + 2\sqrt{3}}{5 + 5(2\sqrt{3})}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{13+2\sqrt{3}}{5+10\sqrt{3}}\right)$$

$$= 42.47$$

In $\triangle ABC$,

$$\angle ABC = 180 - 30 - 42.47$$

$$= 107.53$$

Question 5 continued

In ΔABC,

$$\frac{\sin 30}{5} = \frac{\sin 107.53}{AC}$$

$$\Rightarrow AC = \frac{5\sin 107.53}{\sin 30}$$

$$= 9.536$$

$$\therefore \text{Area of } \Delta ABC = \frac{1}{2} \times AC \times BC \times \sin\theta$$

$$= \frac{1}{2} \times 9.536 \times 5 \times \sin 42.47$$

$$= 16.1 m^2$$

Question 5 continued

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Question 5 continued

(Total for Question 5 is 7 marks)

6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

The equation

$$4(p - 2x) = \frac{12 + 15p}{x + p} \qquad x \neq -p$$

where p is a constant, has two distinct real roots.

(a) Show that

$$3p^2 - 10p - 8 > 0$$

(3)

(b) Hence, using algebra, find the range of possible values of p

(3)

(a) $4(p-2x) = \frac{12+15p}{x+p}$

$\Rightarrow 4p - 8x = \frac{12+15p}{x+p}$

$\Rightarrow (4p-8x)(x+p) = 12+5p$

$\Rightarrow 4px + 4p^2 - 8x^2 - 8xp = 12+15p$

$\Rightarrow -8x^2 - 4px + 4p^2 - 15p - 12 = 0$

$\Rightarrow 8x^2 + 4px + 12 + 15p - 4p^2 = 0$

Since there are two distinct real roots,

$b^2 - 4ac > 0$

$\Rightarrow (4p)^2 - 4(8)(12+15p-4p^2) > 0$

$\Rightarrow 16p^2 - 384 - 480p + 128p^2 > 0$

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Question 6 continued

$\Rightarrow 144p^2 - 480p - 384 > 0$

$\Rightarrow 3p^2 - 10p - 8 > 0$ (shown)

(b) $3p^2 - 10p - 8 > 0$

$\Rightarrow 3p^2 - 12p + 2p - 8 > 0$

$\Rightarrow 3p(p-4) + 2(p-4) > 0$

$\Rightarrow (p-4)(3p+2) > 0$

critical values are

$p = 4, \quad p = -\frac{2}{3}$

$p > 4 \quad p < -\frac{2}{3}$

(Total for Question 6 is 6 marks)

7. The curve C has equation $y = \mathrm{f}(x)$ where $x > 0$

Given that

- $\mathrm{f}'(x) = \dfrac{4x^2 + 10 - 7x^{\frac{1}{2}}}{4x^{\frac{1}{2}}}$

- the point $P(4, -1)$ lies on C

(a) (i) find the value of the gradient of C at P

(ii) Hence find the equation of the normal to C at P, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(4)

(b) Find $\mathrm{f}(x)$.

(6)

(a)(i) $m_c = \dfrac{4(4)^2 + 10 - 7\sqrt{4}}{4\sqrt{4}}$

$= \dfrac{15}{2}$

(b) $f(x) = \int f'(x)\,dx$

$= \int \left(\dfrac{4x^2 + 10 - 7x^{\frac{1}{2}}}{4x^{\frac{1}{2}}} \right) dx$

$= \int \left(x^{\frac{3}{2}} + \frac{5}{2}x^{-\frac{1}{2}} - \frac{7}{4} \right) dx$

$= \dfrac{1}{5/2}x^{\frac{5}{2}} + \dfrac{5/2}{1/2}x^{\frac{1}{2}} - \dfrac{7}{4}x + c$

$= \frac{2}{5}x^{5/2} + 5x^{\frac{1}{2}} - \frac{7}{4}x + c$

$-1 = \frac{2}{5}(4)^{\frac{5}{2}} + 5(4)^{\frac{1}{2}} - \frac{7}{4}(4) + c$

$\Rightarrow \frac{79}{5} + c = -1 \Rightarrow c = -\frac{84}{5}$

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Question 7 continued

$$f(x) = \frac{2}{5}x^{\frac{5}{2}} + 5x^{\frac{1}{2}} - \frac{7}{4}x - \frac{84}{5}$$

(a)(ii) $m_{normal} = -\frac{1}{15/2}$

$$= -\frac{2}{15}$$

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - (-1) = -\frac{2}{15}(x - 4)$$

$$\Rightarrow y = -\frac{2}{15}x + \frac{8}{15} - 1$$

$$\Rightarrow y = -\frac{2}{15}x - \frac{7}{15}$$

$$\Rightarrow \frac{2}{15}x + y + \frac{7}{15} = 0 \; [\times 15]$$

$$\Rightarrow 2x + 15y + 7 = 0$$

Question 7 continued

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Question 7 continued

(Total for Question 7 is 10 marks)

8. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

The curve C_1 has equation

$$xy = \frac{15}{2} - 5x \qquad x \neq 0$$

The curve C_2 has equation

$$y = x^3 - \frac{7}{2}x - 5$$

(a) Show that C_1 and C_2 meet when

$$2x^4 - 7x^2 - 15 = 0$$

(2)

Given that C_1 and C_2 meet at points P and Q

(b) find, using algebra, the exact distance PQ

(5)

(a) $xy = \frac{15}{2} - 5x$

$\Rightarrow y = \frac{15}{2x} - 5$

$C_1 = C_2$

$\Rightarrow \frac{15}{2x} - 5 = x^3 - \frac{7}{2}x - 5 \quad (\times 2x)$

$\Rightarrow 15 - 10x = 2x^4 - 7x^2 - 10x$

$\Rightarrow 2x^4 - 7x^2 - 15 = 0$

(b) $2x^4 - 7x^2 - 15 = 0$

$\Rightarrow 2x^4 - 10x^2 + 3x^2 - 15 = 0$

$\Rightarrow 2x^2(x^2 - 5) + 3(x^2 - 5) = 0$

$\Rightarrow (x^2 - 5)(2x^2 + 3) = 0$

$\Rightarrow x^2 = 5 \quad x^2 \neq -\frac{3}{2}$

$\Rightarrow x = \pm\sqrt{5}$

When $x = \sqrt{5}$

$y = \frac{15}{2 \times \sqrt{5}} - 5$

$= \frac{-10 + 3\sqrt{5}}{2}$

When $x = -\sqrt{5}$

$y = \frac{15}{-2\sqrt{5}} - 5$

$= \frac{-10 - 3\sqrt{5}}{2}$

Question 8 continued

$$PQ = \sqrt{\left(\sqrt{5} - -\sqrt{5}\right)^2 + \left(\frac{-10+3\sqrt{5}}{2} - \frac{-10-3\sqrt{5}}{2}\right)^2}$$

$$= \sqrt{65}$$

Question 8 continued

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Question 8 continued

(Total for Question 8 is 7 marks)

9.

Diagram **NOT** accurately drawn

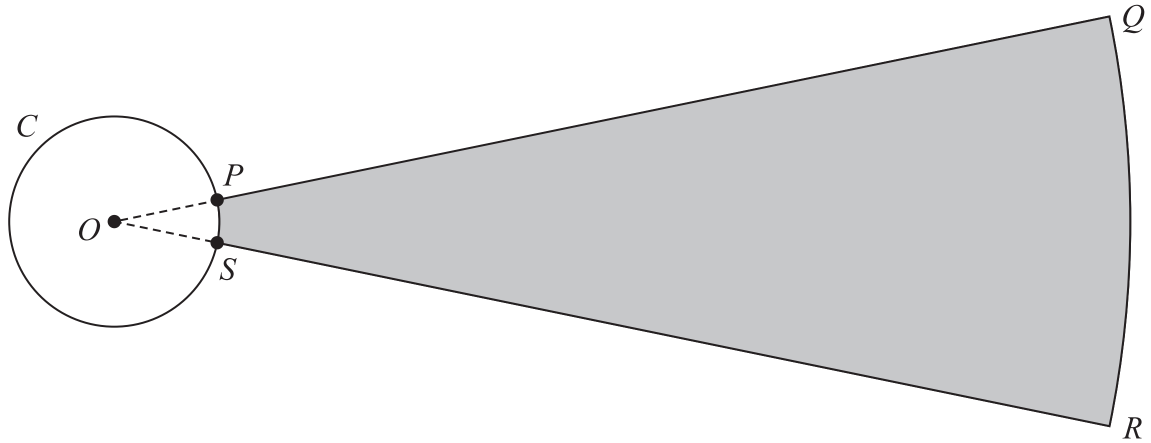

Figure 3

Figure 3 shows the plan view of the area being used for a ball-throwing competition.

Competitors must stand within the circle C and throw a ball as far as possible into the target area, $PQRS$, shown shaded in Figure 3.

Given that

- circle C has centre O
- P and S are points on C
- $OPQRSO$ is a sector of a circle with centre O
- the length of arc PS is 0.72 m
- the size of angle POS is 0.6 radians

(a) show that $OP = 1.2$ m

(1)

(a) arc PS = 0.72
$\Rightarrow 0.6 \times OP = 0.72$
$\Rightarrow OP = 1.2$m (shown)

Given also that

- the target area, $PQRS$, is $90\,\text{m}^2$
- length $PQ = x$ metres

(b) show that

$$5x^2 + 12x - 1500 = 0$$

(3)

(c) Hence calculate the total perimeter of the target area, $PQRS$, giving your answer to the nearest metre.

(3)

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Question 9 continued

(b) PQRS = 90

$\Rightarrow$ OPQRSO − OPS = 90

$\Rightarrow \left\{\frac{1}{2} \times 0.6 \times (x+1.2)^2\right\} - \left(\frac{1}{2} \times 0.6 \times 1.2^2\right) = 90$

$\Rightarrow 0.3(x^2+2.4x+1.44) - 0.432 = 90$

$\Rightarrow 0.3x^2 + 0.72x + 0.432 - 0.432 = 90$

$\Rightarrow 0.3x^2 + 0.72x - 90 = 0 \quad (\times 50)$

$\Rightarrow 15x^2 + 36x - 4500 = 0$ [shown]

(c) $15x^2 + 36x - 4500 = 0$

$\Rightarrow x = \dfrac{-36 \pm \sqrt{(36)^2 - 4(15 \times -4500)}}{2(15)}$

$= 16.162$ or -18.562

$x = 16.162$

P = PS + QR + PQ + SR

$= 0.72 + \{0.6 \times (1.2 + 16.162)\} + 16.162 + 16.162$

$= 43.46$

$= 43$ m

Question 9 continued

Note for part c
Perimeter=44m is also accepted in ms. Value for perimeter being 43 or 44 depends on the number of significant value taken for x.
If value of x is taken 16.2, perimeter comes out to be 43.56 which is 44m to the nearest meter.
In short, P=43m and P=44m are both correct.

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Question 9 continued

(Total for Question 9 is 7 marks)

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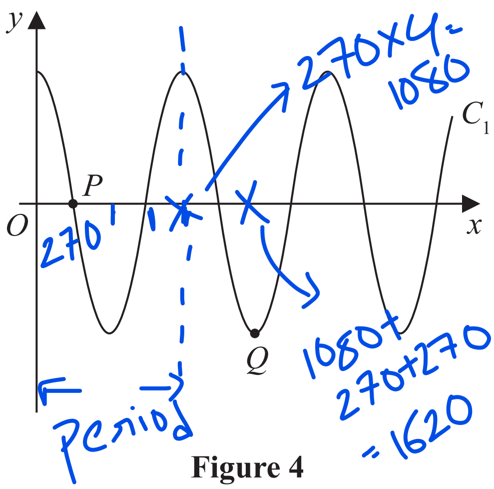

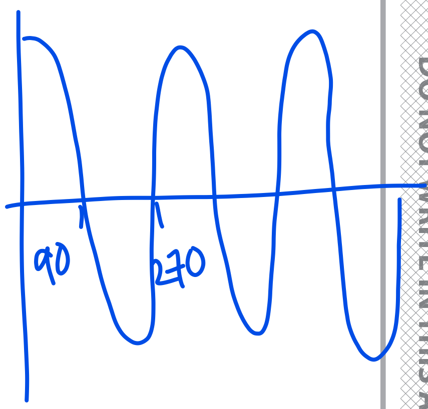

Figure 4

Figure 4 shows a sketch of part of the curve C_1 with equation

$$y = 3\cos\left(\frac{x}{n}\right)^\circ \qquad x \geqslant 0$$

where n is a constant.

The curve C_1 cuts the positive x-axis for the first time at point $P(270, 0)$, as shown in Figure 4.

(a) (i) State the value of n $n=3$

(ii) State the period of C_1 $\rightarrow 1080$

(2)

The point Q, shown in Figure 4, is a minimum point of C_1

(b) State the coordinates of Q. $(1620, -3)$

(2)

The curve C_2 has equation $y = 2\sin x^\circ + k$, where k is a constant.

The point $R\left(a, \frac{12}{5}\right)$ and the point $S\left(-a, -\frac{3}{5}\right)$, both lie on C_2

Given that a is a constant less than 90

(c) find the value of k.

(2)

$2\sin a + k = \frac{12}{5}$ — (1)

$-2\sin a + k = -\frac{3}{5}$ — (11)

$2k = \frac{12}{5} - \frac{3}{5}$

$\Rightarrow k = \frac{9}{10}$

Question 10 continued

(Total for Question 10 is 6 marks)

11.

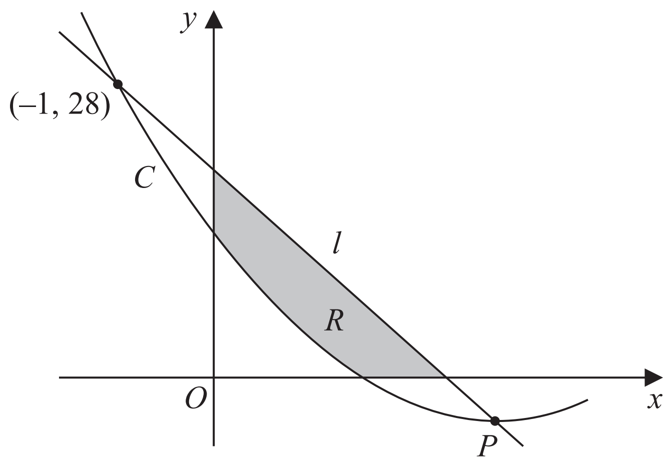

Figure 5

Figure 5 shows part of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 2x^2 - 12x + 14$$

(a) Write $2x^2 - 12x + 14$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

Given that C has a minimum at the point P

(b) state the coordinates of P

(1)

The line l intersects C at $(-1, 28)$ and at P as shown in Figure 5.

(c) Find the equation of l giving your answer in the form $y = mx + c$ where m and c are constants to be found.

(3)

The finite region R, shown shaded in Figure 5, is bounded by the x-axis, l, the y-axis, and C.

(d) Use inequalities to define the region R.

(3)

Question 11 continued

(a) $2x^2-12x+14$

$=2(x^2-6x)+14$

$=2\{(x-3)^2-(3)^2\}+14$

$=2(x-3)^2-18+14$

$=2(x-3)^2-4$

$(-1,28)$

(b) $P \rightarrow (3,-4)$

(c) $m=\dfrac{28-(-4)}{-1-3}$

$=-8$

$y-y_1=m(x-x_1)$

$\Rightarrow y-28=-8(x--1)$

$\Rightarrow y=-8x-8+28$

$=-8x+20$ //

(d) $y\leq -8x+20$

$y\geqslant 2x^2-12x+14$

$x\geqslant 0 \quad y\geqslant 0$

P 7 4 3 1 6 A 0 3 1 3 2

Question 11 continued

(Total for Question 11 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS