Constraint Optimization Technique

$\boxed{x^2 + y^2 - 100 = 0}$

$\rightarrow f(x, y) = 3x + 4y \longrightarrow$ Objective Function

$\rightarrow \underline{x^2 + y^2 = 100} \longrightarrow$ Constraint

$\boxed{g(x, y) = x^2 + y^2 - 100}$

Lagrange Multiplier Method

$\rightarrow \underline{f(x, y)} - \lambda * \underline{g(x, y)} = 0$ —— (*)

$\Rightarrow 3x + 4y - \lambda * (x^2 + y^2 - 100) = 0$

$\Rightarrow 3x + 4y - \lambda x^2 - \lambda y^2 + 100\lambda = 0$

$$\underline{3x + 4y - \lambda x^2 - \lambda y^2 + 100\lambda = 0}$$

Partially derive by 'x', we get,

$3 + 0 - 2\lambda x - 0 + 0 = 0$

$\Rightarrow 3 = 2\lambda x$

$\therefore x = \dfrac{3}{2\lambda}$

'y', we get

$0 + 4 - 0 - 2\lambda y + 0 = 0$

$\Rightarrow 4 = 2\lambda y$

$\therefore y = \dfrac{4}{2\lambda} = \dfrac{2}{\lambda}$

$x^2 + y^2 = 100$

$\Rightarrow \left(\dfrac{3}{2\lambda}\right)^2 + \left(\dfrac{2}{\lambda}\right)^2 = 100$

$\Rightarrow \dfrac{9}{4\lambda^2} + \dfrac{4}{\lambda^2} = 100$

$\Rightarrow \dfrac{9 + 16}{4\lambda^2} = 100$

$\Rightarrow 25 = 400\lambda^2$

$\Rightarrow \lambda^2 = \dfrac{25}{400}$

$\Rightarrow \lambda^2 = \dfrac{1}{16}$

$\therefore \lambda = \pm \dfrac{1}{4}$

when, $\lambda = \frac{1}{4}$

$x = \frac{3}{2\lambda}$

$= \frac{3}{2 \cdot \frac{1}{4}}$

$= 6$

$y = \frac{2}{\lambda}$

$= 8$

when, $\lambda = -\frac{1}{4}$

$$\boxed{\begin{aligned} x &= -6 \\ y &= -8 \end{aligned}}$$

$\lambda = \frac{1}{4}$ $(x, y) = (6, 8)$

$\lambda = -\frac{1}{4}$ $(x, y) = (-6, -8)$

$\boxed{f(x, y) = 3x + 4y}$ —— $\boxed{x^2 + y^2 = 100}$

$f(6, 8) = 3 \cdot 6 + 4 \cdot 8$

$= 18 + 32$

$= 50$ (Max)

$f(-6, -8) = -50$ (Min)