$f(x, y) = y^2 - 4x^2$ — Objective

$x^2 + 2y^2 = 4$ — Constraint

$g(x, y) = x^2 + 2y^2 - 4$

$f(x, y) - \lambda * g(x, y) = 0$

$\Rightarrow y^2 - 4x^2 - \lambda * (x^2 + 2y^2 - 4) = 0$

$\Rightarrow y^2 - 4x^2 - \lambda x^2 - 2\lambda y^2 + 4\lambda = 0$

$y^2 - 4x^2 - \lambda x^2 - 2\lambda y^2 + 4\lambda = 0$

$0 - 8x - 2\lambda x - 0 + 0 = 0$

$\Rightarrow -8x - 2\lambda x = 0$

$\Rightarrow 2\lambda x + 8x = 0$

$\Rightarrow \underline{2x}\,\underline{(\lambda + 4)} = 0$

$2x = 0 \qquad \lambda + 4 = 0$

$x = 0 \qquad \boxed{\lambda = -4}$

$2y - 0 - 0 - 4\lambda y + 0 = 0$

$\Rightarrow 2y - 4\lambda y = 0$

$\Rightarrow \underline{2y}\,(\underline{1 - 2\lambda}) = 0$

$y = 0 \qquad \lambda = \frac{1}{2}$

when. $\boxed{\lambda = -4}$

$x = 0$

$x^2 + 2y^2 = 4$

$\Rightarrow 0 + 2y^2 = 4$

$\Rightarrow 2y^2 = 4$

$\Rightarrow y^2 = 2$

$\therefore y = \pm\sqrt{2}$

$$\left.\begin{array}{l}(x,y) = (0, \sqrt{2}) \\ (x,y) = (0, -\sqrt{2})\end{array}\right\}$$

when. $\lambda = \frac{1}{2}$

$y = 0$

$x^2 + 2y^2 = 4$

$\Rightarrow x^2 + 0 = 4$

$\Rightarrow x = \pm 2$

$$\left\{\begin{array}{l}(x,y) = (2,0) \\ (x,y) = (-2,0)\end{array}\right.$$

$f(x,y) = y^2 - 4x^2$

$\underline{\underline{(0, \sqrt{2})}}$

$f(0, \sqrt{2})$

$= (\sqrt{2})^2 - 0$

$= 2$ —— (Max)

$\underline{\underline{(0, -\sqrt{2})}}$

$f(0, -\sqrt{2})$

$= (-\sqrt{2})^2 - 0$

$= 2$ — (Max)

$\underline{\underline{(2,0)}}$

$f(2,0) = 0^2 - 4(2)^2$

$= 0 - 16$

$= -16$ (Min)

$f(-2,0) = 0^2 - 4(-2)^2$

$= 0 - 16$

$= -16$ (Min)