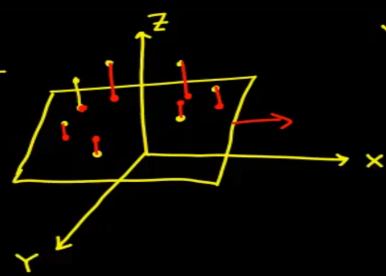
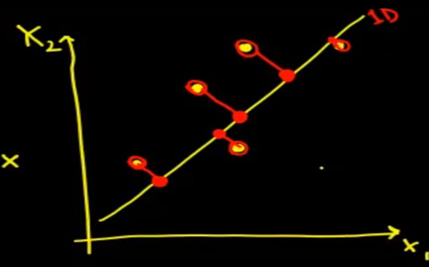
Dimension Reduction





PCA

Principle
Component
Analysis

(i) Dataset

x_1	4	8	13	7
x_2	11	4	5	14

(ii) Calculating Mean from the features

$$\bar{x}_1 = \frac{1}{4}(4+8+13+7) = 8$$

$$\bar{x}_2 = \frac{1}{4}(11+4+5+14) = 8.5$$

$\boxed{x_1, x_2}\ x_3$

(iii) Covariance Matrix

$$C = [2\times 2] = \begin{bmatrix} \underset{14}{Cov(x_1,x_1)} & \underset{-11}{Cov(x_1,x_2)} \\ \underset{-11}{Cov(x_2,x_1)} & \underset{23}{Cov(x_2,x_2)} \end{bmatrix}$$

$$C = [3\times 3]$$

$$C = \begin{bmatrix} x_1x_1 & x_1x_2 & x_1x_3 \\ x_2x_1 & x_2x_2 & x_2x_3 \\ x_3x_1 & x_3x_2 & x_3x_3 \end{bmatrix}$$

$$Cov(x_1,x_1) = \frac{1}{N-1}\sum_{k=1}^{N}(x_{1k}-\bar{x}_1)^2$$

$$= \frac{1}{3}\left\{(4-8)^2 + (8-8)^2 + (13-8)^2 + (7-8)^2\right\}$$

$$= \frac{1}{3}\left\{16+0+25+1\right\} = 14$$

$$Cov(x_1,x_2) = \frac{1}{N-1}\sum_{k=1}^{N}(x_{1k}-\bar{x}_1)(x_{2k}-\bar{x}_2)$$

$$= \frac{1}{3}\left\{(4-8)(11-8.5) + (8-8)(4-8.5) + (13-8)(5-8.5) + (7-8)(14-8.5)\right\}$$

$$= -11$$

$$Cov(x_2,x_1) = \frac{1}{N-1}\sum_{k=1}^{N}(x_{2k}-\bar{x}_2)(x_{1k}-\bar{x}_1)$$

$$= -11$$

$$Cov(x_2,x_2) = \frac{1}{N-1}\sum_{k=1}^{N}(x_{2k}-\bar{x}_2)^2$$

$$= \frac{1}{3}\left\{(11-8.5)^2 + (4-8.5)^2 + (5-8.5)^2 + (14-8.5)^2\right\} = 23$$

(iv) Calculating eigenvalues

$$\det(\underbrace{S}_{\text{Covariance Matrix}} - \overset{\text{eigen values}}{\lambda} I) = 0 \qquad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} 14-\lambda & -11 \\ -11 & 23-\lambda \end{vmatrix} = 0$$

$$\boxed{\begin{aligned} &ax^2+bx+c=0 \\ &x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \end{aligned}}$$

$$\Rightarrow (14-\lambda)(23-\lambda) - (-11)(-11) = 0$$

$$\Rightarrow \lambda^2 - 37\lambda + 201 = 0$$

$$\lambda = 30.38 \, , \, 6.61 \qquad \lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 30.38 \\ 6.61 \end{bmatrix}$$

(v) Calculating eigen vectors

$$(S - \lambda_1 I)U = 0$$

$$\Rightarrow \begin{bmatrix} 14-\lambda_1 & -11 \\ -11 & 23-\lambda_1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} (14-\lambda_1)u_1 - 11u_2 \\ -11u_1 + (23-\lambda_1)u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$(14-\lambda_1)u_1 - 11u_2 = 0$$

$$\Rightarrow (14-\lambda_1)u_1 = 11u_2$$

$$\Rightarrow \frac{u_1}{u_2} = \frac{11}{14-\lambda_1}$$

$$\|\bar{U}\| = \sqrt{(11)^2 + (14-\lambda_1)^2} = \boxed{19.73}$$

(vi)

$$e_1 = \begin{bmatrix} u_1 / \|\bar{U}\| \\ u_2 / \|\bar{U}\| \end{bmatrix} = \begin{bmatrix} 11/19.73 \\ (14-30.38)/19.73 \end{bmatrix} = \begin{bmatrix} 0.55 \\ -0.83 \end{bmatrix}$$

$$(S - \lambda_2 I) V = 0$$

$$\Rightarrow \begin{bmatrix} 14-\lambda_2 & -11 \\ -11 & 23-\lambda_2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} (14-\lambda_2)v_1 - 11v_2 \\ -11v_1 + (23-\lambda_2)v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \longrightarrow \frac{v_1}{v_2} = \frac{11}{14-\lambda_2}$$

$$\|\bar{v}\| = \sqrt{(11)^2 + (14-6.61)^2} = 13.25$$

$$e_2 = \begin{bmatrix} v_1 / \|\bar{v}\| \\ v_2 / \|\bar{v}\| \end{bmatrix} = \begin{bmatrix} 11 / 13.25 \\ (14-6.61)/13.25 \end{bmatrix} = \begin{bmatrix} 0.83 \\ 0.55 \end{bmatrix}$$

(vi) Calculating Principle Component

$$e_1 = \begin{bmatrix} 0.55 \\ -0.83 \end{bmatrix}$$

$$e_1^T = \begin{bmatrix} 0.55 & -0.83 \end{bmatrix}$$

$$e_1^T \begin{bmatrix} 4 - \bar{x}_1 \\ 11 - \bar{x}_2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.55 & -0.83 \end{bmatrix} \begin{bmatrix} 4-8 \\ 11-8.5 \end{bmatrix}$$

$$= \begin{bmatrix} -4.30 \end{bmatrix}$$