In this case, we do not know the ID of the observations ← **EKF w/ UNKNOWN DATA ASSOCIATION**

ORAZIO → senses a set of NOT DISTINGUISHABLE landmarks through a "2D landmark sensor" ≡ give relative position of landmarks relative to the ROBOT FRAME but UNKNOWN ID

+

location of the landmarks in the world NOT KNOWN!!

↑ translational + rotational velocities

⇓

KF-based algorithm to track the position of Orazio + position of observed plant-landmarks, while performing data association

← But NO PRIOR KNOWLEDGE OF THE MAP

EKF SLAM

1. Predict: incorporate new control

$$\mu_{t|t-1} = f(\mu_{t-1|t-1}, u_{t-1})$$

$$A_t = \left.\frac{\partial f(x,u)}{\partial x}\right|_{x=\mu_{t-1|t-1}}$$

$$B_t = \left.\frac{\partial f(x,u)}{\partial u}\right|_{u=u_{t-1}}$$

$$\Sigma_{t|t-1} = \underbrace{A_t \Sigma_{t-1|t-1} A_t^T}_{\text{PROJECT THE PREVIOUS COVARIANCE OF THE STATE}} + \underbrace{B_t \Sigma_u B_t^T}_{\text{UNCERTAINTY OF THE CONTROLS}}$$

2. Correct: incorporate new measurement

$$C_t = \left.\frac{\partial h(x)}{\partial x}\right|_{x=\mu_{t|t-1}}$$

$$K_t = \Sigma_{t|t-1} C_t^T \left(\Sigma_z + C_t \Sigma_{t|t-1} C_t^T\right)^{-1}$$

$$\mu_{t|t} = \mu_{t|t-1} + K_t \underbrace{\left(z_t - h(\mu_{t|t-1})\right)}_{\text{INNOVATION}}$$

$$\Sigma_{t|t} = (I - K_t C_t)\, \Sigma_{t|t-1}$$

3. Add: extend state with new landmarks

Domains

state space:

$$x_t^{[R]} = \begin{pmatrix} x_t \\ y_t \\ \theta_t \end{pmatrix} \in \mathbb{R}^3$$

$$x_t^{[n]} = \begin{pmatrix} x_t^{[n]} \\ y_t^{[n]} \end{pmatrix} \in \mathbb{R}^2, \quad n = 1, \dots, N$$

$$X_t = \begin{pmatrix} x_t^{[R]} \\ x_t^{[1]} \\ \vdots \\ x_t^{[N]} \end{pmatrix} \in \mathbb{R}^{(3+2\cdot N)}$$

control inputs space:

$$u_t = \begin{pmatrix} u_t^{[x]} \\ u_t^{[\theta]} \end{pmatrix} \in \mathbb{R}^2$$

observation space:

$$z_t^{[m]} = \begin{pmatrix} x_t^{[m]} \\ y_t^{[m]} \end{pmatrix} \in \mathbb{R}^2, \quad m = 1, \dots, M$$

$$Z_t = \begin{pmatrix} z_t^{[1]} \\ z_t^{[2]} \\ \vdots \\ z_t^{[M]} \end{pmatrix} \in \mathbb{R}^{(2\cdot M)}$$

Transition Model

$$X_t = f(X_{t-1}, u_{t-1}) = \begin{pmatrix} x_{t-1} + u_{t-1}^{[x]} \cdot \cos(\theta_{t-1}) \\ y_{t-1} + u_{t-1}^{[x]} \cdot \sin(\theta_{t-1}) \\ \theta_{t-1} + u_{t-1}^{[\theta]} \\ x_{t-1}^{[1]} \\ \vdots \\ x_{t-1}^{[N]} \end{pmatrix}$$

Kinematic model of unicycle after 1st-order Euler integration

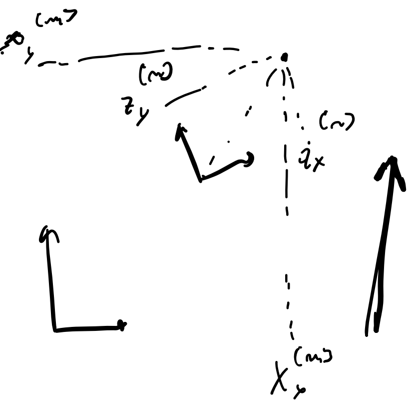
Landmarks do not move

$$A_t = \frac{\partial f(\cdot)}{\partial x}\bigg|_{x=\mu_{t-1}, u=u_{t-1}} = \begin{pmatrix} 1 & 0 & -u_{t-1}^{[x]} \cdot s_{\theta_{t-1}} & \\ 0 & 1 & u_{t-1}^{[x]} \cdot c_{\theta_{t-1}} & 0_{3\times(2\cdot N)} \\ 0 & 0 & 1 & \\ & 0_{(2\cdot N)\times 3} & & \begin{matrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{matrix} \end{pmatrix}_{N\times N}$$

$$B_t = \frac{\partial f(\cdot)}{\partial u}\bigg|_{u=u_{t-1}} = \begin{pmatrix} c_{\theta_{t-1}} & 0 \\ s_{\theta_{t-1}} & 0 \\ 0 & 1 \\ & 0_{(N\cdot 2)\times 2} \end{pmatrix}$$

Observation Model

$$z_t^{[m]} = h(x_t) = R_t^T \cdot \left(x_t^{[m]} - t_t\right) = \begin{pmatrix} c_{\theta_t} \cdot \left(x_t^{[m]} - x_t^{[r]}\right) + s_{\theta_t} \cdot \left(y_t^{[m]} - y_t^{[r]}\right) \\ -s_{\theta_t} \cdot \left(x_t^{[m]} - x_t^{[r]}\right) + c_{\theta_t} \cdot \left(y_t^{[m]} - y_t^{[r]}\right) \end{pmatrix}$$



$$\left.\frac{\partial h(x)}{\partial x^{[r]}}\right|_{x=\mu_{t|t-1}} = \begin{pmatrix} -c_{\theta_t} & -s_{\theta_t} & -s_{\theta_t} \cdot \left(x_t^{[m]} - x_t^{[r]}\right) + c_{\theta_t} \cdot \left(y_t^{[m]} - y_t^{[r]}\right) \\ s_{\theta_t} & -c_{\theta_t} & -c_{\theta_t} \cdot \left(x_t^{[m]} - x_t^{[r]}\right) - s_{\theta_t} \cdot \left(y_t^{[m]} - y_t^{[r]}\right) \end{pmatrix}$$

$$R_t z_t^{[m]} + t_t = x_t^{[m]}$$

$$\frac{\partial}{\partial x^{[r]}}\left(R_t^T \cdot \left(x_t^{[m]} - t_t\right)\right) = \left(\frac{\partial R_t^T}{\partial x}\right) \cdot \left(x_t^{[m]} - t_t\right) + R_t^T \cdot \frac{\partial}{\partial x}\left(x_t^{[m]} - t_t\right)$$

$$\frac{\partial}{\partial x} = \left(\frac{\partial}{\partial x} \; \frac{\partial}{\partial y} \; \frac{\partial}{\partial \theta}\right)$$

$\frac{\partial}{\partial x}, \frac{\partial}{\partial y} = \emptyset$ $\qquad$ $\frac{\partial}{\partial \theta} = \emptyset \; \oplus \; \frac{\partial}{\partial x} t_t = I$

$$= \left(-R_t^T \,,\; \left(\frac{\partial R_t^T}{\partial \theta}\right) \cdot \left(x_t^{[m]} - t_t\right)\right)$$

where $\frac{\partial R_t^T}{\partial \theta} = \frac{\partial}{\partial \theta}\begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} = \begin{pmatrix} -s_\theta & c_\theta \\ -c_\theta & -s_\theta \end{pmatrix}$

usually, $M < N$

measurement functions, one for each observed landmark that is part of the state (N)

$$C_t^{[m]} = \left(-R_t^T, \left(\frac{\partial R_t^T}{\partial \theta}\right) \cdot \left(x_t^{[m]} - t_t\right), \emptyset \dots \frac{\partial h^{[m]}}{\partial x_t^{[m]}} \dots, \emptyset\right)$$

Control Noise : constant part $\oplus$ proportional

...

Data Association

Now → - we do not observe landmark IDs
- when new landmark → IT IS our responsability to assign a new UNIQUE id

⇓

ID LAND. ⇒ IDX_STATE = id_to_state_map = (-1 -1, ... -1)

IDX_STATE ⇒ ID_LAND. = state_to_id_map = (-1 -1 .... -1)

IDX in the vector

At each time step, compute LIKELIHOOD of the associations for each measure @ landmark

$$a_{mn} = \left(z^{[m]} - \underbrace{h^{[n]}(x_t)}_{\text{predicted landmark}}\right)^T \cdot \Omega_{m,n} \cdot \left(z^{[m]} - h^{[n]}(x_t)\right)$$

~~MAHALANOBIS DISTANCE~~

seen like factor graph ....

INFORMATION MATRIX

$$\Omega_{m,n} = \Sigma_{m,n}^{-1}$$

$$\Sigma_{m,n} = C_t^{[m]} \Sigma \left(C_t^{[m]}\right)^T + \Sigma_{meas}$$

takes into consideration the uncertainty of our measurement

compute new information from the linearization

association cost matrix $A = \begin{pmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & & \vdots \\ a_{M1} & \cdots & a_{MN} \end{pmatrix}$

1^{st}-ORDER PROPAGATION = project the covariance into the measurement state!

BUT for which entry we have to invert the matrix...

Best friends = choose landmark closest to the measurement

measurement closer to the landmark

Lonely best friends :

BEST

2ND BEST

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1N} \\ a_{21} & a_{22} & a_{23} & \cdots & \\ a_{31} & a_{32} & a_{3} & \cdots & \\ \vdots & & & \ddots & \vdots \\ a_{M1} & \cdots & & \cdots & a_{MN} \end{pmatrix}$$

/ COST MATRIX

We want them, the distance, it's pretty high

↓

IF NOT, (AMBIGUOUS ASSOCIATION) it may be due to noise

IF TRUE,
then is redundant that the association