Discrete Filtering

$$b(x_t) = \eta_t \cdot p(z_t | x_t) \cdot \sum_{x_{t-1}} p(x_t | x_{t-1}, u_{t-1}) \cdot b(x_{t-1})$$

$$b(x_t) \equiv p(x_t | u_{0:t-1}, z_{1:t})$$

- η_t → normalization ⇒ to ensure that it represents a probabilistic distribution
- $p(z_t | x_t)$ → OBSERVATION MODEL
- $p(x_t | x_{t-1}, u_{t-1})$ → TRANSITION MODEL
- $b(x_{t-1})$ → previous state belief

Implementing a Bayes Filter

Map ≡ $\text{matrix}_{M \times N}$ (M = # rows, N = # cols)

where $\text{map}(\text{row}, \text{col}) = \begin{cases} 1 : \text{occupied} \\ 0 : \text{free} \end{cases}$

"maps/map.txt" = 10×10 matrix

→ Scenario: map, grid - orazio

→ Modeling the problem:
- Transition model [wo/noise, w/noise] —— $p(x_t | x_{t-1}, u_{t-1})$
- Observation model —— $p(z_t | x_t)$

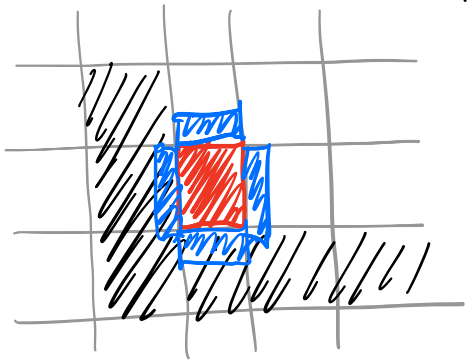
→ Building the filter
- Predict belief —— $p(x_t | u_{0:t-1}, z_{1:t-1})$ — no observation
- Update belief —— $p(x_t | u_{0:t-1}, z_{1:t})$ — w/ observation

→ SCENARIO : GRID_ORAZIO

4 commands / control inputs : UP / DOWN / LEFT / RIGHT

free cells – white
occupied – black

sensor w/ 4 bumpers (blue) mounted at its 4 sides



→ TRANSITION MODEL:

transition_probability_matrix = transitionModel (map_, row_from_, col_from_, control_input_)

matrix w/ SAME size as map_ !

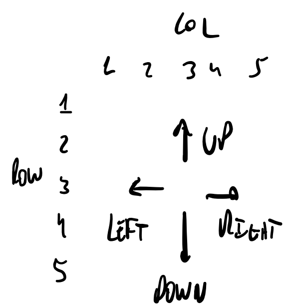
returns the probability of moving to any cell in the map from the start state = [row_from_ , col_from_]

- consider the constraints:
  - only adjacent cells → continue (probability remains zero in that cell)
  - invalid motions → if obstacle encountered
    → outside of the map

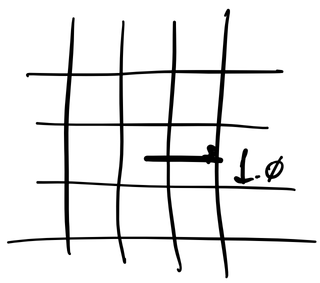
IF NULL translation → probability = 1 !
ELSE Ø

- evaluate in 4 directions

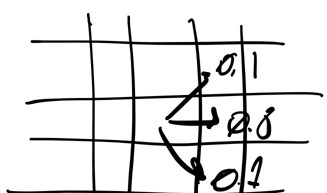
MOVE_UP → implements



w/o noise



w/ noise



switch (control_input_)

$$\boxed{p(x_t \mid x_{t-1}, u_{t-1})}$$

|||

	transition_rows	transition_cols		
MOVE_UP	-1	0	↑↑	0.8
	-1	1	↑↗	0.1
	-1	-1	↖↑	0.1
MOVE_DOWN	1	0	↓↓	0.8
	1	1	↓↘	0.1
	1	-1	↙↓	0.1

...

and, at each steps, we evaluate all possible configurations for the next step considering the previous state and the control input

4) OBSERVATION MODEL: observation_probability = observationModel(map_,
row_,
col_,
observation_)

array of 4 ≡ ← observation_
(≡ 4 bumpers)

⇓

16 possible configurations for 4 bumpers

that is why you see

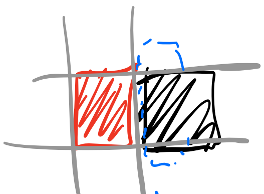
observability_prob * = 0.8;

$p(z_{RIGHT}) = 0.8$

|||

0.8 True Positive

0.2 False Positive



0.2 probability of wrong measurement

Assuming 4 bumpers independent observations ...

$$p(z_t \mid x_t) = p(z_{t,UP} \mid x_t)\, p(z_{t,DOWN} \mid x_t)\, p(z_{t,LEFT} \mid x_t)\, p(z_{t,RIGHT} \mid x_t)$$

Belief:

→ **Predict**: $b_{t|t-1} = p(x_t \mid u_{1:t-1}, z_{1:t-1}) = \sum_{x_{t-1}} p(x_t, x_{t-1} \mid u_{1:t-1}, z_{1:t-1})$

$$= \sum_{x_{t-1}} \underbrace{p(X_t \mid x_{t-1}, u_{t-1})}_{\text{TRANSITION MODEL}} \underbrace{p(x_{t-1} \mid u_{1:t-2}, z_{1:t-1})}_{b_{t-1} \,=\, \text{state\_belief\_previous}}$$

evaluate over all possible configurations of x_{t-1}

(combination of MARGINALISATION + CHAIN RULE)

$p(x_t \mid u_{1:t-1}, z_{1:t-1}) \rightarrow \sum_{x_{t-1}}(\dots)$

↓

joint x_t, x_{t-1}

→ **Update**: $b(x_t) = \eta_t \, p(z_t \mid x_t) \sum_{x_{t-1}} p(x_t \mid x_{t-1}, u_{t-1}) \, b(x_{t-1})$

where $\eta_t = \dfrac{1}{\sum_{x_t} p(x_t \mid x_{t-1}, u_{t-1}) \, b(x_{t-1})}$

RECAP

```
for (x_i : X) {
    b[x_i] = b_pred[x_i] x observationModel(z, x_i)
    normalizer += b[x_i]        (inverse normalizer!)
```