Probabilistic Robotics Exam Sample
Dec 12 2019

December 30, 2019

1 Question: Least Squares

Derive the Gauss-Newton algorithm, and motivate the fact that the coefficient matrix $\mathbf{H}$ at the equilibrium is the inverse covariance of the estimate.

2 Exercise: Bouncing Ball

Scenario: An easily detectable red ball bounces on an unlimited field, observed by a fixed pinhole camera. An algorithm processes the image, and returns the center and the radius of the ball. The radius r of the ball is known, and the ball motion is subject to air fricton $\alpha < 1$, and once it bounces against the floor represented as the $x - y$ plane, the velocity component along the z axis changes sign. In short, the motion of the ball is approximated by the following equation:

$$\begin{pmatrix} v_{x,t+1} \\ v_{y,t+1} \\ v_{z,t+1} \end{pmatrix} = \begin{cases} \alpha \begin{pmatrix} v_{x,t} \\ v_{y,t} \\ v_{z,t} \end{pmatrix} + \mathbf{g} \cdot \Delta T & \text{if } z > r \\ \alpha \begin{pmatrix} v_{x,t} \\ v_{y,t} \\ \|v_{z,t}\| \end{pmatrix} + \mathbf{g} \cdot \Delta T & \text{otherwise} \end{cases}$$

Here ΔT is the (small!) time interval of the discrete system.

2.1 Question 1

Identify the system by defining:

- State
- Controls (if any)
- Transition function
- Measurement function assuming the camera is at position $\mathbf{T}_{\text{cam}}$, and has known camera parameters $\mathbf{K}$.

Hint: assume the ball is far enough from the camera and its observed radius depends on the inverse of the depth.

2.2 Question 2

After having identified the system, approach the problem of estimating the ball position with a filter at your choice. Report the filter equations, for the specific problem and a sketch of filter implementation. If Jacobians are involved report the sizes.

2.3 Question 3

Discuss the pro and the contra of PF, KF and UKF in this particular problem.

3 Exercise: Fit The Circle

Scenario: Given a set of points on a plane, we want find a subset of these points that belong a single circle of unknown radius, and center.

3.1 Question 1

Approach the problem with RANSAC by

- Defining a the parameters to be fitted
- Defining a minimal solver
- Defining an error function to determine inliers and outliers

3.2 Question 2

Given a set of inliers and an initial estimate, formulate the least squares problem to optimally refine the parameters by considering only the inliers.

3.3 Question 3

Assuming that only 10% of the points belong to the circle, how many rounds of RANSAC are needed to compute a solution with probability $p = 0.99$?