# Probabilistic Robotics Course

# Modeling Dependencies Bayesian Networks


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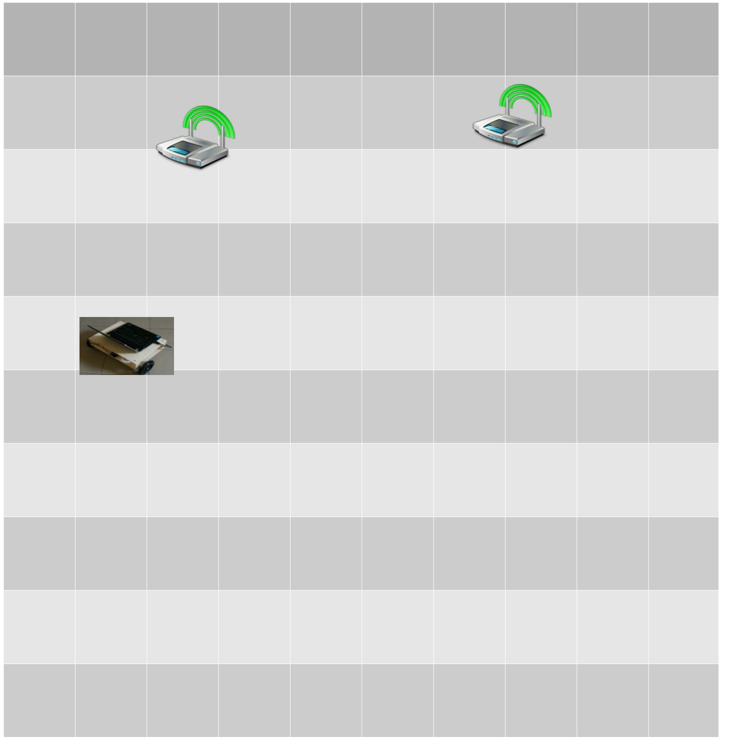
# Outline

- Considerations about dimensions
- Role of Conditional Independence
- Modeling Phenomena: Bayesian Networls
- Inference on Bayesian Networks

# Orazio!

Let us go back to our robot. The scene was

- 2 access points (AP)
- orazio moving
- one elevator



Quantities

- signal strengths from the 2 APs
- floor of the elevator
- location of Orazio

# Dimensionality

Having

- 4 possible levels of strength per access point
- 2 access points
- 6 floors
- 100 locations

How many possible disjoint outcomes of events do I have?

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4*4*6*100 = **9600**

# Dimensionality

What if we gift Orazio a monochromatic camera with resolution “poorVGA” (10x10), and 16 levels of gray

- number of disjoint events becomes:

  4*4*6*100*16^(10*10) = 2.4790e+124

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Enough to challenge some good computer

# Queries

To perform a query of the type

$$p(S^A|S^B_2)$$

we can start from a joint probability distribution

$$P(S^A,S^B_2,I^{1,1},..I^{10,10},L,E)$$

eliminate all variables that are non relevant through *marginalization*

$$P(S^A,S^B_2,\cancel{I^{1,1},..I^{10,10},L,E})$$

and use the *Bayes*' *Rule* to get the answer

# Conditional Independence

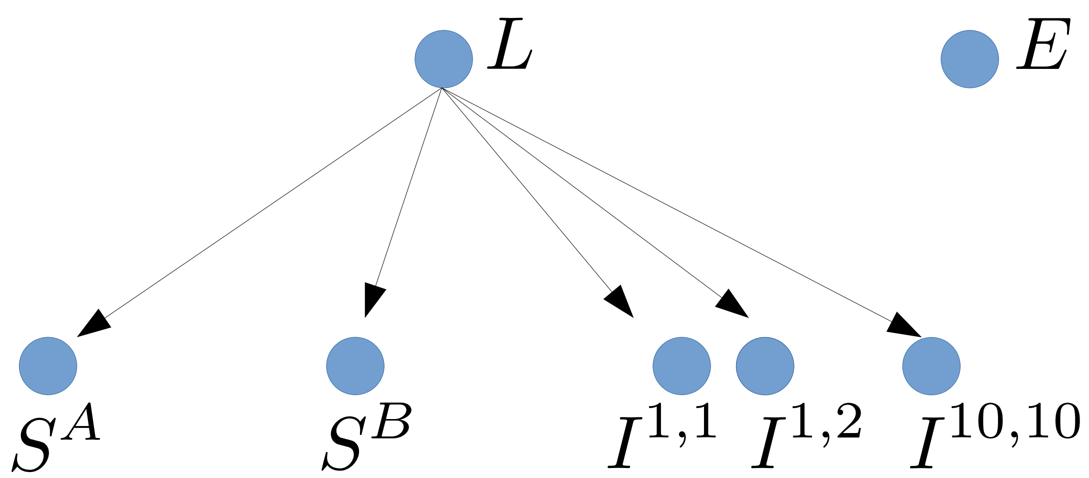
Using our wisdom we can notice some astonishing facts:

- The signal strength of an access point depends only on the location
- The pixels in the image depend only on the location
  - if nothing but Orazio move
  - if the light conditions do not change
- The elevator lives in his own world (as long as Orazio does not take it)

# Making Order

We can represent this phenomena as a *Bayesian Network*

- One per variable
- One if the source influences the destination
- No loops, it's a directed acyclic graph!



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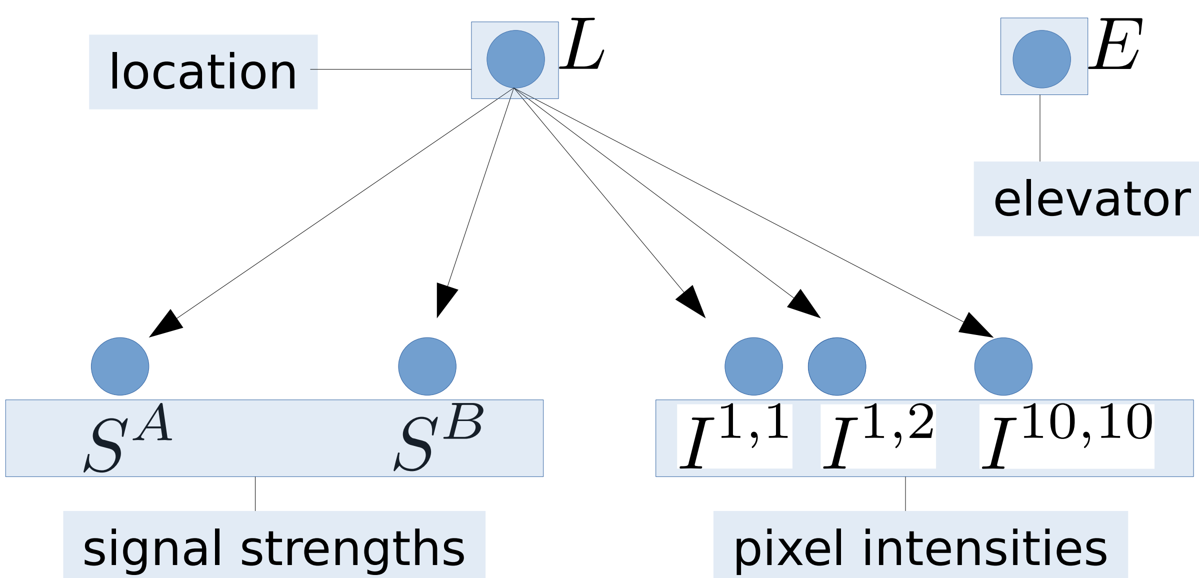


# Making Order

For storing the conditional probabilities we need:

- 100 numbers for robot locations
- 4*100 numbers for signal strength A
- 4*100 numbers for signal strength B
- 16*100 numbers for each pixel, as they are independent GIVEN the location
- 6 numbers for the elevator

$100+2*4*100+100*16*100+6=160906$

Much less than before

# Using the Order

By storing the conditional probability we need much less numbers

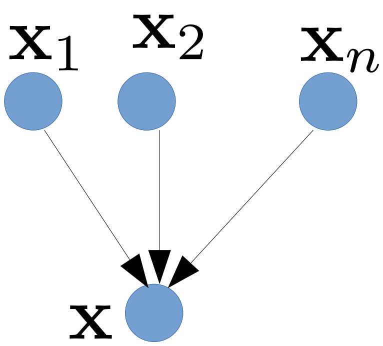
We can anyway recover the joint distribution and perform *inference* on that, by using the chain rule

Bayesian Networks are a formalism to highlight independence between variables, without loss of information

## Each node

- represents a variable $\mathbf{x}$
- stores a conditional probability:

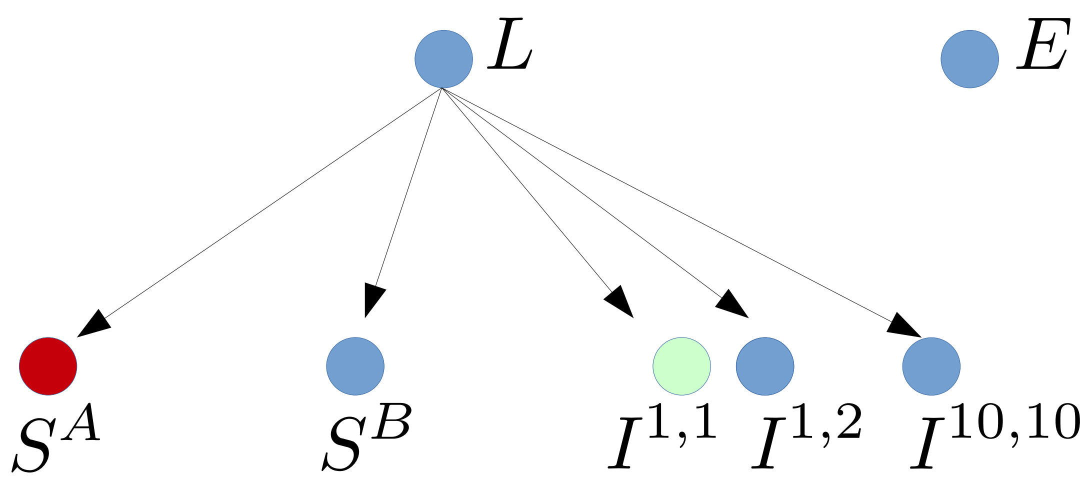
$$p(\mathbf{x} \mid \mathbf{x}_1, \mathbf{x}_2, \dots \mathbf{x}_n)$$



# Example of Inference

What is the probability distribution of $S^A$,

given the pixel 1,1 has intensity 7 [$I^{1,1} = 7$] ?



# Example of Inference

We want $\mathrm{P}(\mathrm{S}^A | I_7^{1,1}) = ?$

- compute the joint

it is the product of all distributions

$$P(S^A, S^B, I_7^{1,1}, \ldots I^{10,10}, L, E) =$$
$$P(S^A|L)P(S^B|L)P(I_7^{1,1}|L) \cdots P(I^{10,10}|L)P(L)P(E)$$

- marginalize out the variables we don't need

$$P(S^A, S^B, I_7^{1,1}, \ldots I^{10,10}, L, E)$$

to get a joint distribution over $\mathrm{S}^A, I_7^{0,0}$

$$P(S^A, I_7^{1,1}) = \sum_{S^B} \sum_{I^{0,1}} \cdots \sum_{I^{10,10}} \sum_L \sum_E P(S^A, S^B, I_7^{1,1}, \ldots I^{10,10}, L, E)$$

sum over all values of variables to suppress

- use conditioning to get the answer

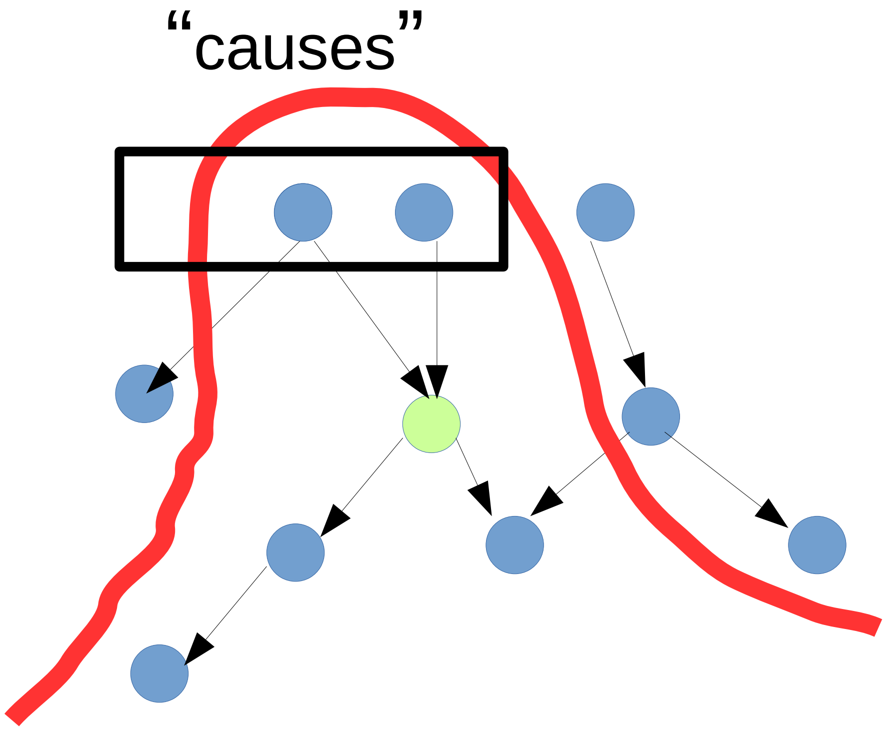
$$P(S^A \mid I_7^{1,1}) = \frac{P(S^A, I_7^{1,1})}{\sum_{S^A} P(S^A, I_7^{1,1})}$$

# Considerations

- Would we get the same answer if the elevator was not in the domain?
- Knowing the intensity of another pixel would provide us with more information?
- Does the same hold if we know the location?

# Local Semantics

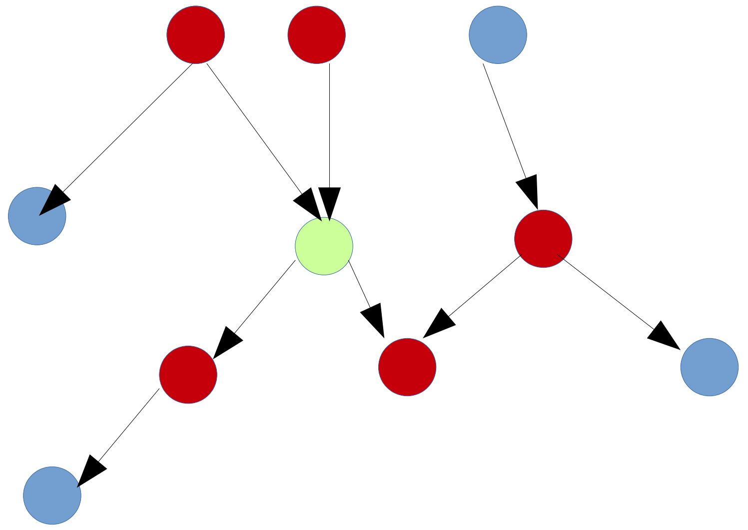
Each node is conditionally independent from its non-descendants given its parents



# Global Semantics

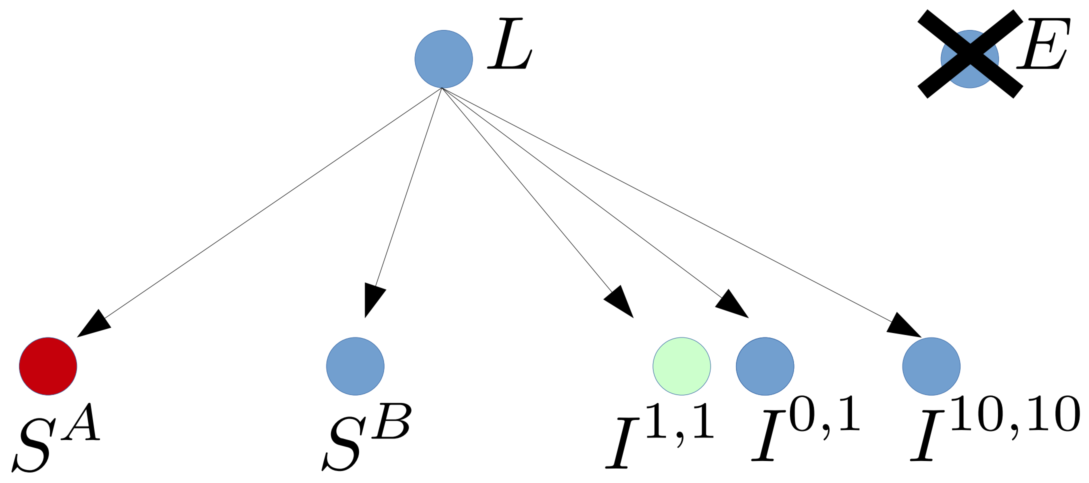
Each node is independent from the rest, given

- its parents
- its children
- the parents of its children



# Using Semantics

Elevator is disconnected, thus independent

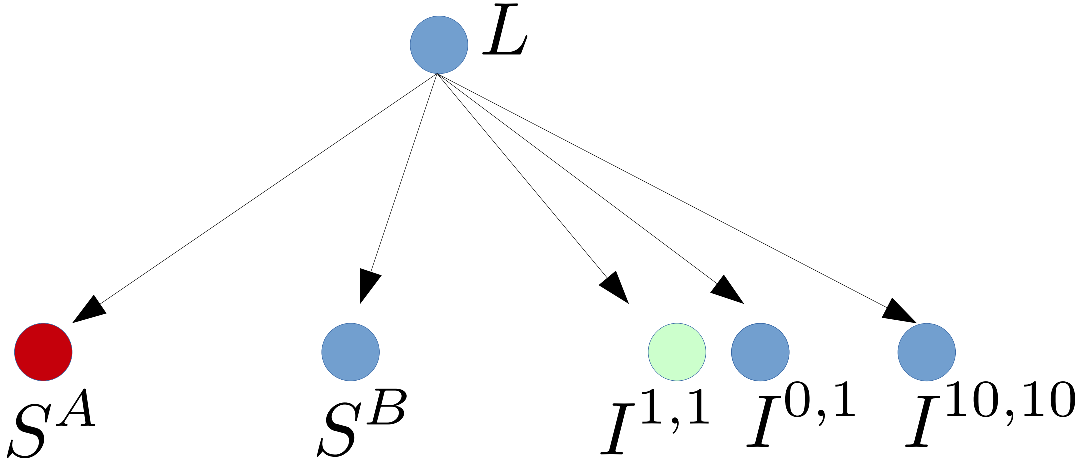


# Using Semantics

Elevator is disconnected, thus independent

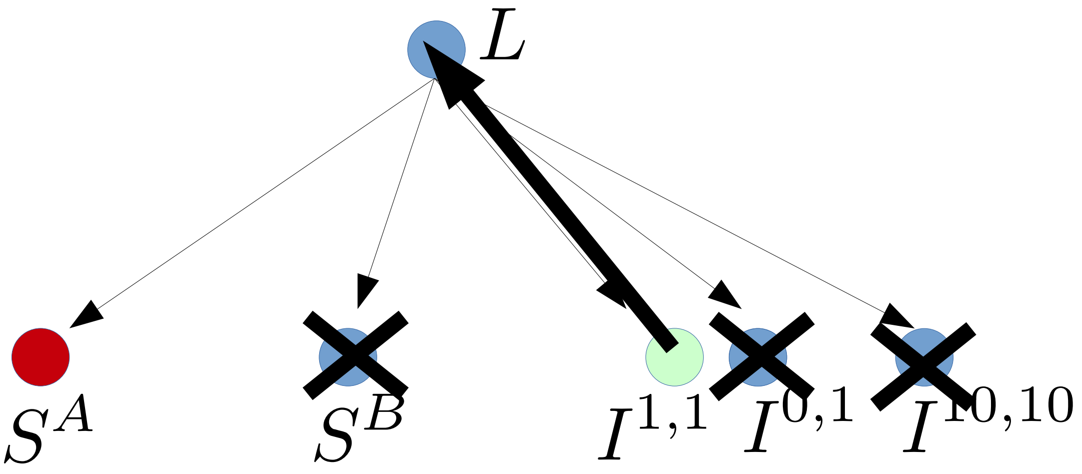
If we would know the location, the task would be easy

The only consequence of the location that we can observe is $I^{1,1}$



# Using Semantics

We can compute $P(L \mid I_7^{1,1})$ ignoring all the rest



# Using Semantics

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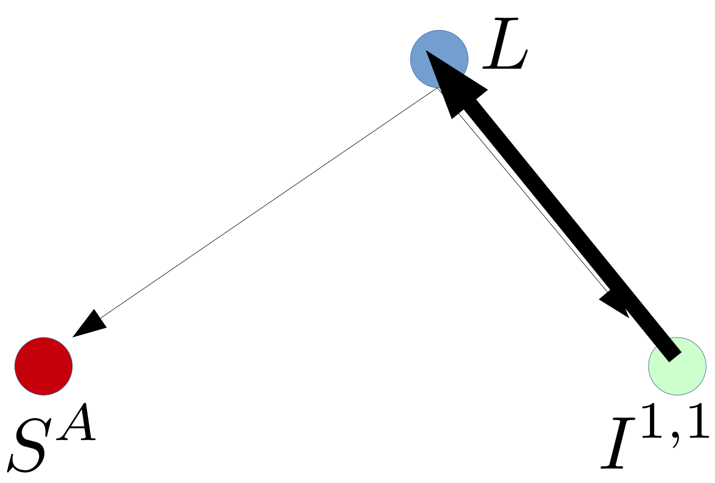
This will alter our prior about the location, incorporating the knowledge of the intensity




HOW:
- chain rule on $P(I^{1,1}|L)$and P(L) to get P(L,I)
- conditioning on P(L,I), to get P(L|I)

# Using Semantics

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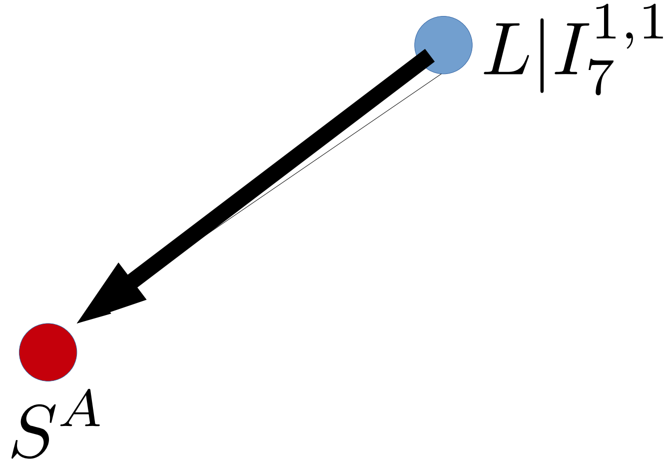
Finally, we determine the signal strength, from the improved location estimate




HOW:
- chain rule on $P(S^A|L)$and P(L|I)

# References

An excellent tutorial on Bayes networks and graphical models:

http://www.cs.ubc.ca/~murphyk/Bayes/bnintro.html