# 3D Point Registration

Bearing-only
- Know location of a set of points in the world
- Know angle (bearing)

points
+
gravity
+
length data association

→ find transformation of the world in respect to the sensor

SENSOR — x, y, z of each individual point → $z = (x \;\; y \;\; z)$

STATE — $x \in SE(3)$

$$x = (\underbrace{x \;\; y \;\; z}_{t} \;\; \underbrace{\alpha_x \;\; \alpha_y \;\; \alpha_z}_{\alpha})^T$$

3D euclidean vector

$t$ — translation

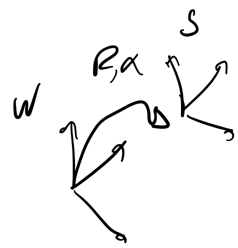
$\alpha$ — rotation around the 3 axes

MEASUREMENTS → $z \in \mathbb{R}^3$ → observation from the sensor coordinate frame

$$h^{[i]}(x) = R(\alpha)\, p^{[i]} + t$$

$h^{[i]}(x)$ — PREDICTION FUNCTION

$p^{[i]}$ — point in the world frame




$$R(\alpha) = R_x(\alpha_x)\, R_y(\alpha_y)\, R_z(\alpha_z)$$

$R_x$ — rotation along the x-axis

ERROR — $e^{[i]} = h^{[i]} - z^{[i]}$

$\partial e^i = \partial h^i$ — $z^{[i]}$ appears as constant!

$$h^{[i]}(x) = R_x(\alpha_x)\, R_y(\alpha_y)\, R_z(\alpha_z) \cdot p^{[i]} + t$$

$$\frac{\partial h}{\partial \alpha_x} = R_x' R_y R_z\, p^{[i]}$$

$$\frac{\partial h}{\partial \alpha_y} = R_x R_y' R_z\, p^{[i]}$$

$$\frac{\partial h}{\partial \alpha_z} = R_x R_y R_z'\, p^{[i]}$$

Least Squares exactly the same !

$$H\ t = J' * J$$

$$b\ t = J' * e$$

$$chi\ t = e' * e$$

→ Removed information matrix for clarity / simplicity

$$J^{*} * J \;=\; J^{T} \cdot \underline{I} \cdot J \;=\; \Omega$$

Start by initial guess = ground-truth

⇓

should return $\emptyset$ error

## 4) Linear Relaxation

MANIFOLD → space which is not Euclidean (space of the angles)

- previous implementation leads to many iterations .....
- condition made each least square converges in 1 iteration ⇒ when solution does not change between iterations

↓

but we need to change the error function....

↓

relax some constraints:

- instead of being a rotation matrix, $R \longrightarrow$ generic matrix (12 parameters)

$$X^{T} = \left( r_1^{T}\ r_2^{T}\ r_3^{T}\ t^{T} \right)$$

1st row of the rot matrix ... 3rd row translation vector

$$h^{[i]}(x) = R\,p^{[i]} + t$$

dot product → no, can be transposed!

$$= \begin{pmatrix} r_1^{T} \\ r_2^{T} \\ r_3^{T} \end{pmatrix} p^{[i]} + t$$

$$= \underbrace{\begin{pmatrix} p^{[i]T} & & \\ & p^{[i]T} & \\ & & p^{[i]T} \end{pmatrix}}_{M} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix} + t$$

$$\frac{\partial h^{[i]}}{\partial x} = \left( \underset{3\times 9}{M^{[i]}} \,\middle|\, \underset{3\times 3}{I} \right)$$

$$\underset{3\times 9}{M^{[i]}} \quad 9\times 1$$

$$\Updownarrow$$

$$h^{[i]}(x) = M^{[i]} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix} + t$$

in this case, initial guess is IRRELEVANT

initial transform = Identity

$\Omega$ = Identity (info mtx)

$H = \dots$

$b = \dots$

**PROBLEM**

relaxation on the orig. formula will found a solution but DOES NOT GUARANTEE $R$ will be a rotation matrix

$$A = \begin{pmatrix} r_1^T \\ r_2^T \\ r_3^T \end{pmatrix} = U D V^T$$

diagonal matrix (singular values) magnitude of the

if identity $\Rightarrow$ $A$ rotation matrix

orthogonal matrices

in general, does not represent a rotation matrix

SVD

$\Downarrow$

in other circumstances, other techniques may allow forcing the constraints of your problem!

$$\Updownarrow$$

$$R = UV^T$$

but if $D$ far from $I$, solution found through linear relaxation NOT GOOD

NOISE

OR

DATA ASSOCIATION (outliers)

many destroy solution by linear relaxation!

do linear relaxation by formula in the slides!