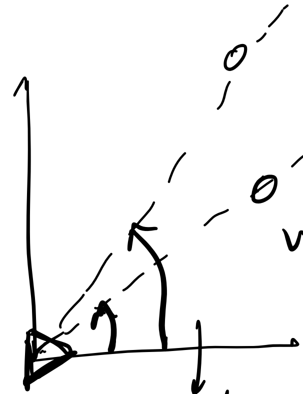
Exercise: Localisation w/ Kalman Filter: Bearing only

Obj: robot moves in a 2D plane

- bearing sensor → measures the relative angle
- controlled by translation + rotational velocities



landmarks location is known

only know in which direction the landmark is (not the distance)

Domains

- **Prior**: global location of the landmarks

$$\ell^{[i]} = \begin{bmatrix} x^{[i]} \\ y^{[i]} \end{bmatrix} \in \mathbb{R}^2 \rightarrow \text{IDENTIFIED w/ UNIQUE ID}$$

- **State**: location of the robot in the global 2D plane

$$X_t = \begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} \in SE(2) \qquad X_t = [R_t \mid t_t] \in SE(2)$$

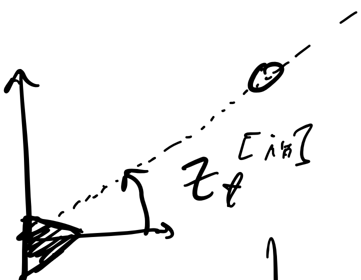
- **Control**: translational an rotational velocities

$$u_t = \begin{bmatrix} u_t^1 \\ u_t^2 \end{bmatrix} \in \mathbb{R}^2 \equiv \begin{matrix} \Delta\rho \\ \Delta\theta \end{matrix} \in \mathbb{R} \times SO(2)$$

- **Observation**: direction of the landmark relative to the robot

$$Z_t = \begin{bmatrix} z_t^{[i_1]} \\ \vdots \\ z_t^{[i_K]} \end{bmatrix} \in \mathbb{R}^K,$$

where we sense only about a subset of the landmarks



OR

$$Z_t^{[i]} = R_t^{[i]} \in SO(2)$$

this angle defines a transformation matrix

Transition Model

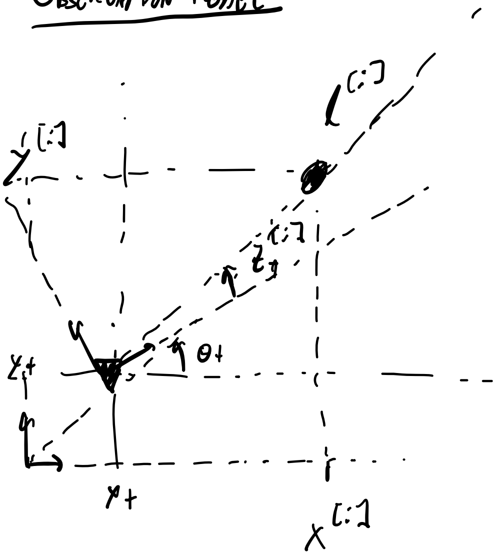
$$X_t = \begin{pmatrix} x_t \\ y_t \\ \theta_t \end{pmatrix} = \begin{pmatrix} x_{t-1} + u^1_{t-1} \cdot \cos(\theta_{t-1}) \\ y_{t-1} + u^1_{t-1} \cdot \sin(\theta_{t-1}) \\ \theta_{t-1} + u^2_{t-1} \end{pmatrix}$$

$$A_t = \begin{bmatrix} 1 & 0 & -u^1_{t-1} \cdot \sin(\theta_{t-1}) \\ 0 & 1 & u^1_{t-1} \cdot \cos(\theta_{t-1}) \\ 0 & 0 & 1 \end{bmatrix}$$

$$B_t = \begin{bmatrix} \cos(\theta_{t-1}) & 0 \\ \sin(\theta_{t-1}) & 0 \\ 0 & 1 \end{bmatrix}$$

$$R_t^T = \begin{bmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{bmatrix}$$

Observation Model



$l^{[i]}$ Global $\rightarrow$ Robot

$\Downarrow$

$$z_t^{[i]} = \operatorname{atan2}\left(l^{[i]\,robot} \dots\right)$$

$$l^{[i]\,global} = R_t \cdot l^{[i]\,robot} + t_t \Leftrightarrow$$

$$\Leftrightarrow l^{[i]\,robot} = R_t^{-1} \cdot \left(l^{[i]\,global} - t_t\right)$$

$$R_t^{-1} = R_t^T$$

$$l^{[i]\,robot} = \begin{pmatrix} (x^{[i]} - x_t) \cdot \cos(\theta_t) + (y^{[i]} - y_t) \cdot \sin(\theta_t) \\ -(x^{[i]} - x_t) \cdot \sin(\theta_t) + (y^{[i]} - y_t) \cdot \cos(\theta_t) \end{pmatrix}$$

$$\hat{z}_t^{[i]} = \operatorname{atan2}\left(-(x^{[i]} - x_t) \cdot \sin(\theta_t) + (y^{[i]} - y_t) \cdot \cos(\theta_t),\ (x^{[i]} - x_t) \cdot \cos(\theta_t) + (y^{[i]} - y_t) \cdot \sin(\theta_t)\right)$$

they use atan ...?

$l^{[i]}(x_t) = \operatorname{atan}\left(\hat{y}_t^{[i]} / \hat{x}_t^{[i]}\right)$, where $\begin{pmatrix} \hat{x}_t^{[i]} \\ \hat{y}_t^{[i]} \end{pmatrix}$ = pos estimated of the landmark relative to the robot frame

$$\frac{\partial l^{[i]}(\cdot)}{\partial x} = \left.\frac{\partial \operatorname{atan}(\hat{p}_t^{[i]})}{\partial \hat{p}_t^{[i]}}\right|_{\hat{p}_t^{[i]} = g(\check{x})} \cdot \left.\frac{\partial g_i(x)}{\partial x}\right|_{x = \check{x}_i}$$

$$= \frac{1}{1 + (\hat{y}_t / \hat{x}_t)^2} \cdot \left(-\frac{\hat{y}_t}{\hat{x}_t^{[i]\,2}},\ \frac{1}{\hat{x}_t^{[i]}}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{y}{x}\right) = y \cdot (-1) \cdot (1) \cdot x^{-2} = -\frac{y}{x^2}$$

$$\frac{\partial}{\partial y}\left(\frac{y}{x}\right) = \frac{1}{x} \cdot (1) \cdot (1) \cdot y^0 = \frac{1}{x}$$

$$g_i(x) = R_t^T \cdot \left(l^{[i]} - t_t\right) = R_t^T \cdot l^{[i]} - R_t^T \cdot t_t$$

$$\partial g / \partial t_t = -R_t^T$$

$$\partial g / \partial \theta_t = \frac{\partial R_t^T}{\partial \theta_t} \cdot \left(l^{[i]} - t_t\right)$$

$\downarrow \partial / \partial x$

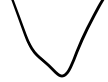
previous exercise: $\left(-R_t^T,\ \frac{\partial R_t^T}{\partial \theta_t}\left(l^{[i]} - t_t\right)\right)$

Control Noise = Previous Exercise $\Downarrow$

$$u = \frac{\hat{y}_t^{[i]}}{\hat{x}_t^{[i]}} = g\left(x_t, l_t^{[i]}\right)$$

Chain Rule:

$$\left.\frac{\partial f(g(x))}{\partial x}\right|_{x = \check{x}} = \left.\frac{\partial f(u)}{\partial u}\right|_{u = g(\check{x})} \left.\frac{\partial g(x)}{\partial x}\right|_{x = \check{x}} = \check{J}_f\, \check{J}_g$$



2 multi-variate functions

of compatible functions