Localization w/ Kalman Filters

EKF → estimate current state distribution from:
- previous state distribution
- $z_{1:t}$
- $u_{0:t-1}$
- transition model
- observation model

Scenario - Orazio (differential robot) → controlled by translational + rotational velocities

$\Delta\rho \qquad \Delta\theta$

- sensor set of uniquely distinguishable landmarks!
- known location of landmarks

⇓

Domain

- **State**: $X \in SE(2)$

  $X = (x, y, \theta)^T$ or $X = [R|t]$ = transformation matrix

- **Controls**: $U \in \mathbb{R}^2$ (in the case of velocity measurements of the wheels)

  $u = (\omega_L, \omega_R)^T$

  or $U \in \mathbb{R} \times SO(2)$, $u = (\Delta\rho, \Delta\theta)^T$ → $u_t = \begin{pmatrix} u_t^1 \\ u_t^2 \end{pmatrix} \in \mathbb{R}^2$

- **Prior**: map represented as a set of landmarks

  $l^{[i]} \in \mathbb{R}^2$, $l^{[i]} = (x^{[i]}, y^{[i]})$

Observations: measurements

$$z_t^{[i]} = \begin{pmatrix} x_t^{[i]} \\ y_t^{[i]} \end{pmatrix} \in \mathbb{R}^2$$

→ relative measurement of the landmarks relative to the robot

Transition Model:

$$X_t = f(X_{t-1}; u_{t-1}) = \begin{pmatrix} x_{t-1} + u^1_{t-1} \cdot \cos(\theta_{t-1}) \\ y_{t-1} + u^1_{t-1} \cdot \sin(\theta_{t-1}) \\ \theta_{t-1} + u^2_{t-1} \end{pmatrix}$$

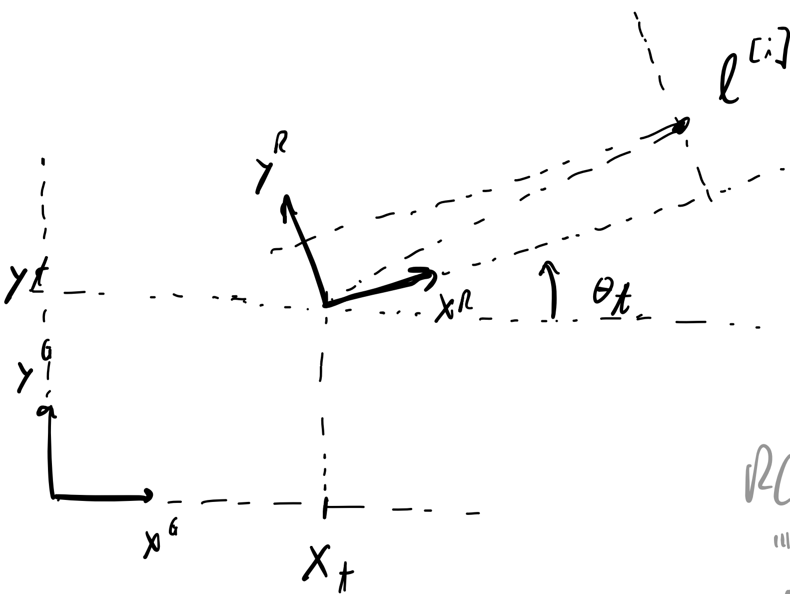
$$B_t = \frac{\partial f}{\partial u} = \begin{pmatrix} \cos(\theta_{t-1}) & 0 \\ \sin(\theta_{t-1}) & 0 \\ 0 & 1 \end{pmatrix}$$

$$A_t = \left.\frac{\partial f(x_{t-1}; u_{t-1})}{\partial x}\right|_{x_{t-1}} = \begin{pmatrix} 1 & 0 & -u^{1}_{t-1} \cdot \text{sen}(\theta_{t-1}) \\ 0 & 1 & u^{1}_{t-1} \cdot \cos(\theta_{t-1}) \\ 0 & 0 & 1 \end{pmatrix}$$

Observation Model:

$z_t^{[i]} = h(x_t)$ ⟶ relative measurement of landmark (expressed in the frame of the robot / local)

$$l^{[i]} = \begin{pmatrix} x^{[i]} \\ y^{[i]} \end{pmatrix}$$



$$z_t^{[i]} = H^{ROBOT}_{GLOBAL}(t) \cdot l^{[i]}$$

$$R(-\theta) = R(\theta)^T = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix}$$

$$H^{GLOBAL}_{ROBOT} = \left[\begin{pmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{pmatrix} \middle| \begin{pmatrix} x_t \\ y_t \end{pmatrix} \right] \longrightarrow X^{GLOBAL} = R(\theta) \cdot X^{ROBOT} + t$$

$$R(\theta)^{-1} \cdot (X^{GLOBAL} - t) = X^{ROBOT}$$

$$z_t^{[i]} = \begin{pmatrix} (x_t - x^{[i]}) \cdot \cos(\theta_t) + (y_t - y^{[i]}) \cdot \text{sen}(\theta_t) \\ -(x_t - x^{[i]}) \cdot \text{sen}(\theta_t) + (y_t - y^{[i]}) \cdot \cos(\theta_t) \end{pmatrix}$$

is mixed
$x_t \rightleftarrows x^{[i]}$
y_t ...

$$C_t^{[i]} = \begin{pmatrix} \cos(\theta_t) & \text{sen}(\theta_t) & -(x_t - x^{[i]}) \,\text{sen}(\theta_t) + (y_t - y^{[i]}) \cdot \cos(\theta_t) \\ -\text{sen}(\theta_t) & \cos(\theta_t) & -(x_t - x^{[i]}) \cos(\theta_t) - (y_t - y^{[i]}) \cdot \text{sen}(\theta_t) \end{pmatrix}$$

$$z_t = \begin{pmatrix} z^{[i_1]} \\ z^{[i_2]} \\ \vdots \end{pmatrix}$$

Stack the observation in a vector

$(K \times 2!) \times 1$

Only a subset of K landmarks

$\#$ rows

other effects on the platform

meter

CONSTANT TERM

Control Noise

velocity ~ Gaussian noise

$$n_{u,t} \sim \mathcal{N}\left(n_{u,t};\ \emptyset,\ \begin{pmatrix} (u_t^{i})^2 + \sigma_u^2 & \emptyset \\ \emptyset & (u_t^{2})^2 + \sigma_\omega^2 \end{pmatrix}\right)$$

mean

DESIGN CHOICE

Observation Noise

$$n_z \sim \mathcal{N}\left(n_z;\ \emptyset,\ \begin{pmatrix} \sigma_z^2 & \emptyset \\ \emptyset & \sigma_z^2 \end{pmatrix}\right)$$

usually constant, plus usually given in the datasheet

May be simplified to

$$C_t^{[i]} = \left(-R(\theta_t) \quad \frac{\partial R}{\partial \theta} \cdot [l^{[i]} - t_t] \right)$$