DATA ASSOCIATION

WHICH $\Rightarrow$ $j(m)$ functions!

Until now, landmarks UNIQUELY IDENTIFIABLE! (we assume we know observation $\Leftrightarrow$ LANDMARK)

what if we do not know to which it corresponds this?

- we must deal with distributions having multiple maxima $\longrightarrow$ [deal with multi-modal! / no EKF/UKF
- choose best possible association $\longrightarrow$ $\downarrow$ robustness (e.g., if we do a wrong association (but we can still use EKF/UKF)

$\Downarrow$

current state estimation $\longrightarrow$ DATA ASSOCIATION $\longrightarrow$ determine which state would be responsible for the measurement

history of the measurements $\longrightarrow$ DATA ASSOCIATION

$\Downarrow$ e.g., if we want to use the history of measurement

$$\#\text{ combinations} = \frac{n!}{m!\,(n-m)!} = \binom{n}{m}$$

(↑↑ # combinations .....)

n landmarks

m measurements

$(n > m)$

ASSOCIATION PROBLEM

$$j^* = \arg\max_j \; p\left(\hat{z}^{[j(1)]} = z^{[1]}, \dots, \hat{z}^{[j(M)]} = z^{[M]}\right)$$

↑ probability that landmark corresponding to landmark 1 is measurement 1

....

$j(m) \Rightarrow$ association vector

$\longrightarrow$ predictions of landmarks $\sim$ distributions

$j(i) \in [\emptyset, 1, \dots, N] \longrightarrow j(3) = 5$ = measurement 3 originates from landmark 5

$j(m) = \emptyset \equiv$ NO CORRESPONDENCE

$$j^* = \underset{j}{\operatorname{argmax}}\; p\left(\hat{z}^{[j(1)]} = z^{[1]}, \dots, \hat{z}^{[j(M)]} = z^{[M]} \right)$$

$\hat{z}^{[j(1)]}$: observation estimated based on the current state and originated from the landmark $j(1)$

$z^{[1]}$: the same as the measurement 1 itself

$j(i) \in [\emptyset, 1, \dots, N]$

$i = 1, \dots, M$

$Z = z^{[1]}, \dots, z^{[M]} \longrightarrow$ MEASUREMENTS $(1, \dots, M)$

$\hat{Z} = \hat{z}^{[1]}, \dots, \hat{z}^{[N]} \longrightarrow$ PREDICTIONS $(1, \dots, N)$

Naive algorithm $\longrightarrow$ generate all possible associations $\binom{N}{M}$

$\downarrow$ evaluate the formula for each associations

$$p\left(\hat{z}^{[j(1)]} = z^{[1]}, \dots, \hat{z}^{[j(M)]} = z^{[M]} \right) =$$

chain rule + marginalization

JOINT DISTRIBUTION OF ALL PREDICTIONS GIVEN AN ASSIGNMENT

IF WE KNOW THE STATE....

$$= \int_x p\left(\hat{z}^{[j(1)]} = z^{[1]}, \dots, \hat{z}^{[j(M)]} = z^{[M]} \mid x \right) p(x)\, dx =$$ (CHAIN RULE)

$$= \int_{X \in \Omega} \prod_{m=1}^{M} p\left(\hat{z}^{[j(m)]} = z^{[m]} \mid x^{r}, x^{[j(m)]} \right) \cdot p\left(x^{r}, x^{[j(m)]} \right)$$ (MARGINALIZATION)

MEASUREMENT INDEPENDENCE

Knowing X^r makes the landmarks conditionally independent of each other

$\Downarrow$

Can we do something more smart? YES — We can compute the distribution of the prediction only **ONCE**!

$$p(\hat{z}) = \int_x p(\hat{z} \mid x)\, p(x)$$

$$= \int_{x \in \Omega} \prod_{n=1}^{N} p(\hat{z}^{[n]} \mid x^{[n]}, x^r)\, p(x)$$

N blocks

(as many as the landmarks)

$\equiv$

PREDICTED MEASUREMENT VECTOR $\hat{z}$

Inverse Assignment

$$j(m) = n$$

$$j^{-1}(n) = m$$

returns invalid value if landmark does not appear in the measurements

(∅ in Octave
-1 in C/C++)

$$z^{[j]} = \begin{pmatrix} z^{[j^{-1}(1)]} \\ z^{[j^{-1}(2)]} \\ \vdots \end{pmatrix}$$

reshuffle measuring vector

by ordering the measures according to the inverse

$z^{[j^{-1}(N)]}$

alignment j^{-1}

$\Downarrow$

but it would have invalid values.....

(MAPPING NOT INVERTIBLE)

PREDICTION VECTOR

(PREDICTION OF ALL POSSIBLE LANDMARKS)

SAME SHAPE

MEASURING VECTOR !

cross-correlation between measurements

$$\Sigma_z = C \Sigma_x C^T + \Sigma_{z|x} = \begin{pmatrix} \Sigma_z^{[1]} & \cdots \\ \cdots & \Sigma_z^{[2]} \\ & & \vdots \end{pmatrix}$$

global C of all landmarks

w/ previous operation, all these blocks will become correlated

measurements of a subset of landmarks

$$z^{[j]} = \begin{pmatrix} z^{[j^{-1}(1)]} \\ \cancel{\quad} \\ z^{[j^{-1}(3)]} \\ \vdots \\ z^{[j^{-1}(N)]} \end{pmatrix}$$

$\equiv$ SAME SIZE AS LANDMARKS

suppress all holes

$\equiv$

marginalization

$\Downarrow$

COMPRESSED

we have ~~complement~~ the JOINT DIST. OVER ALL POTENTIAL MEASUREMENTS

$\tilde{z}^{[j]}, \tilde{\mu}_z, \tilde{\Sigma}$

to match the size of real measurement

EVALUATE THE GAUSSIAN

IN THE EVENT OF THE MEASUREMENT

BUT,

WE HAVE AN ISSUE → in all possible data associations, there will a possible association that none of landmarks in our state are producing that measurement

↓

will have a probability associated with it

⇓

will have the higher probability (almost is never the case that $\tilde{z}^{[j]}$ perfectly matches the mean $\tilde{\mu}_z$)

ASSIGN PROBABILISTIC PENALTY TERM of not observing a landmark ⇐

|||

event of missing a landmark

$$p(\hat{z} \ldots) \propto \frac{1}{\ldots} \exp\left(-\left(\tilde{z}^{[j]}-\tilde{\mu}_z\right)^T \tilde{\Sigma}_z^{-1}\left(\tilde{z}^{[j]}-\tilde{\mu}_z\right)\right) \cdot p_{miss}^{K}$$

unmatched measurements (w/ event of mismatch INDEPENDENT)

by playing with p_{miss} we can

- favor / prefer unmatched landmarks

OR

- explain measurement with old landmarks e.g. if $P_{new} = 1.0 \Rightarrow$ favor creation of new landmark every time

PARAMETER OF OUR DATA ASSOCIATION ENGINE !

if is not anymore exact !

1^a ASSUMPTION: non-independence of measurements is $\emptyset$

(as the landmarks are far from each other, indeed would be $\to \emptyset$)

FAR

$\Downarrow$

measurement independent from each other (no correlation between them)

$$p(\hat{z} = z^{[j]}) = \prod_{m=1}^{M} p(z^{[m]} = \hat{z}^{[j(m)]})$$

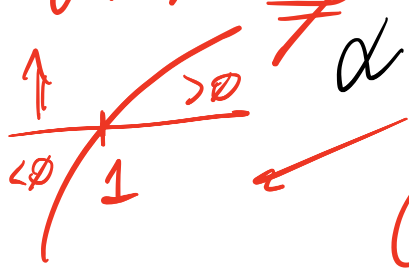
$\leftarrow$ MAXIMIZE

$\Downarrow$

$$\log\left(p(\hat{z} = z^{[j]})\right) = \sum_{m=1}^{M} \log\left(p\left(z^{[m]} = \hat{z}^{[j(m)]}\right)\right)$$

MINIMIZE

$-\log(p(\cdot)) > \emptyset$



$$\propto -\sum_{m=1}^{M} \left(z^{[m]} - \tilde{\mu}_z\right)^T \left(\Sigma^{[j(m)j(m)]}\right)^{-1} \left(z^{[m]} - \tilde{\mu}_z\right)$$

↳ much more efficient

If we have m and n (association $n \to m$) $\Rightarrow$

$$\Rightarrow a_{mn} = \left(z^{[m]} - \mu_z^{[n]}\right)^T \Omega^{[n,m]} \left(z^{[m]} - \mu_z^{[n]}\right)$$

↳ we can immediately compute the log-likelihood of an association

ASSOCIATION $\sum_m a_{m, j(m)}$ (compute the likelihood)

FOR EACH LANDMARK → Find the best one

EACH MEASURE

compute how good will be the association

↓

ASSIGNMENT PROBLEM: could be solved by the Hungarian algorithm

$$A: \begin{pmatrix} a_{11} & \cdots & a_{1N} \\ a_{21} & & \vdots \\ \vdots & & \vdots \\ a_{M1} & & a_{MN} \end{pmatrix}$$ usually $M < N$

how to make it square?

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & & \vdots \\ a_{M1} & & a_{MN} \\ l_{min} & & l_{min} \\ l_{min} & & l_{min} \end{pmatrix}$$

- sum of all costs minimized
- not more than 1 column in assignment

= assign the measurement to NO MEASURE

$l_{min} = -\log(p_{min})$

⇓

in many circumstances → $\boxed{\text{NEAREST NEIGHBOR}}$

pick my measurement w/ closest prediction if measurements covariances eigenvalues equal ⇒ greedy association degenerates nearest neighbour strategy

EVEN NEAREST NEIGHBOUR → can be too expensive w/ very large M, N

A → supplement Λ → FOR EACH COLUMN SELECT ROW w/ SMALLER VALUE

but many measurements may be associated to landmark

PRUNING HEURISTIC

↓

remove potential wrong associations!

↳ Gating: – cost – proportional to distance

↳ best friends ⇒ best ~~measu~~ measurement (by row)

⊕

best measurement by landmark (by col)

get rid of ambiguities !!!

↳ lonely best friends: 1 measurement ⇔ 1 landmark