Probabilistic Robotics Course

Particle Localization

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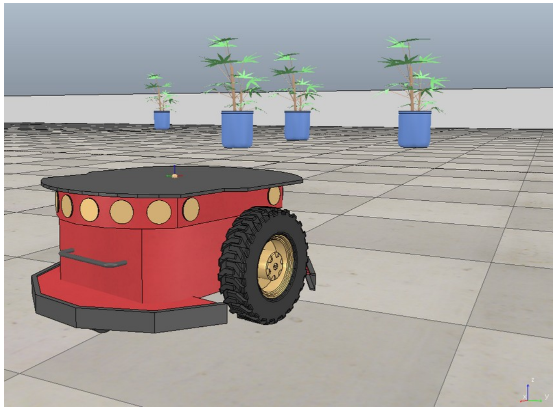
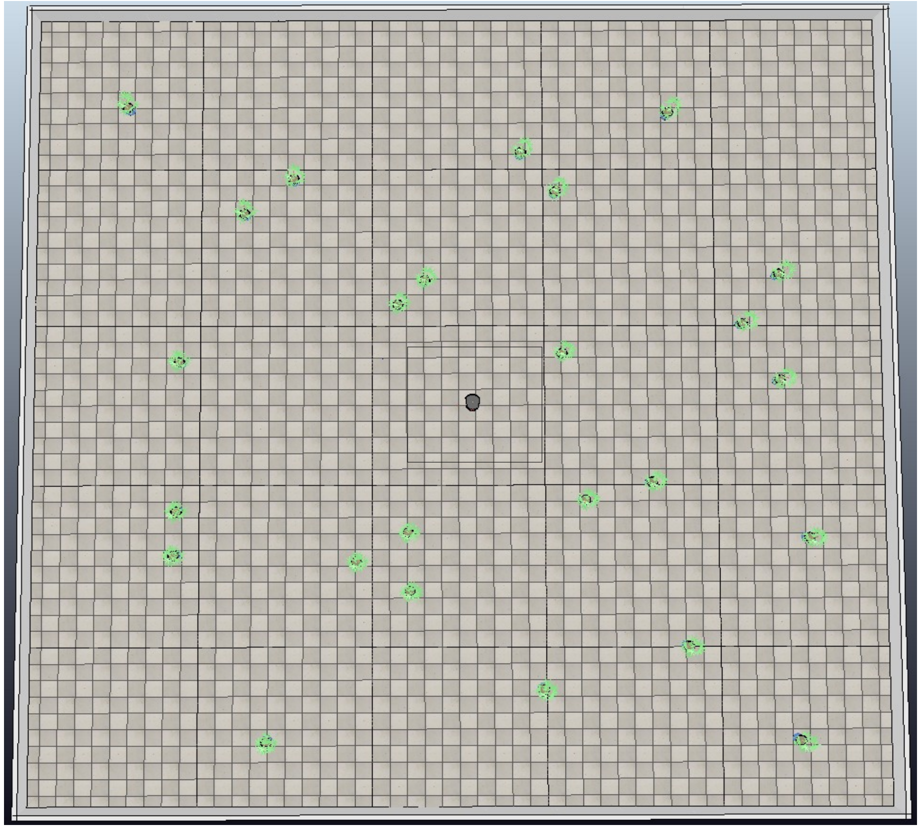
Scenario

Orazio moves on a 2D plane

- Is controlled by translational and rotational velocities
- Senses a set of indistinguishable landmarks through a "2D landmark sensors"
- The location of the landmarks in the world is known





Approaching the problem

We want to develop a Particle Filter based algorithm to track the position of Orazio as it moves

The inputs of our algorithms will be

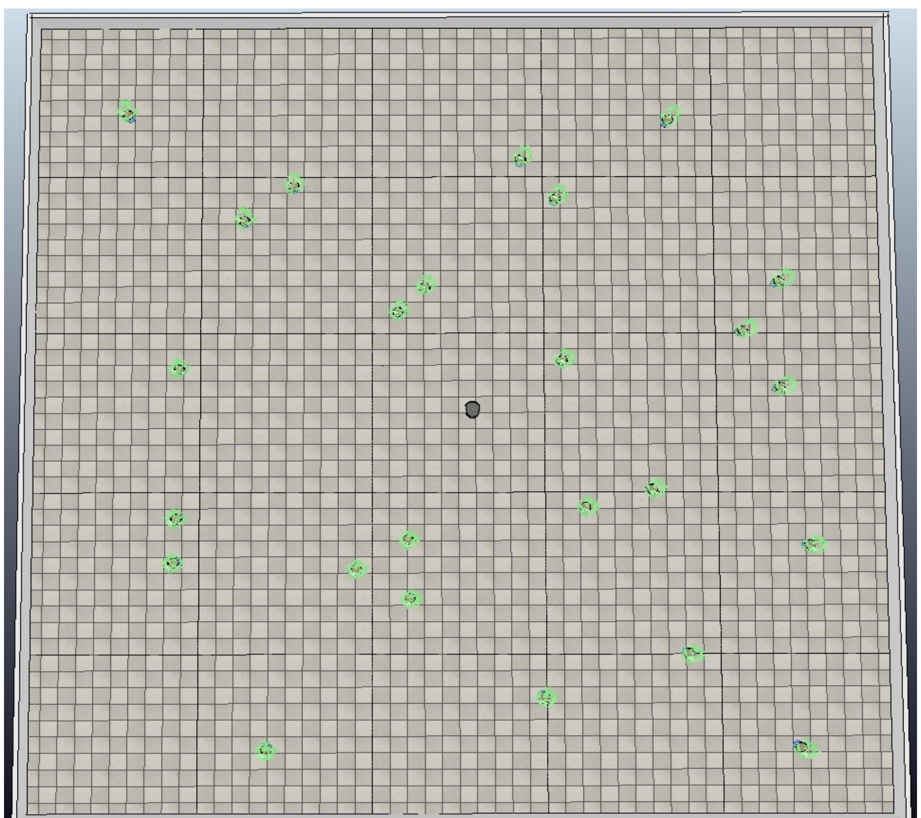
- velocity inputs
- landmark measurements

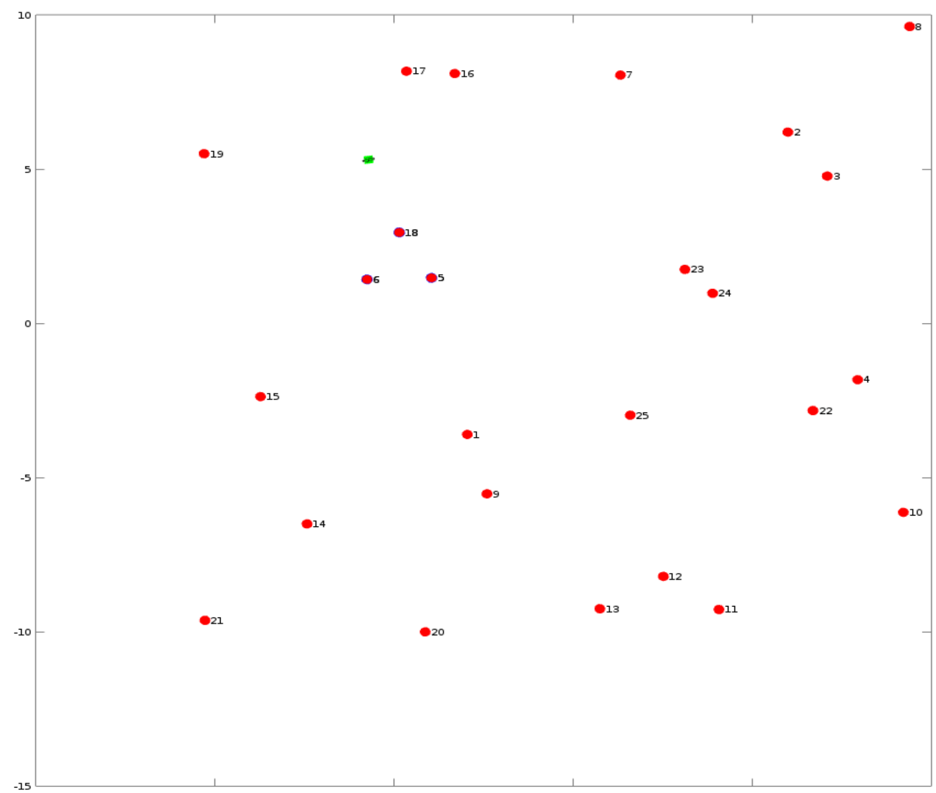
The prior knowledge about the map is represented by the location of each landmark in the world

Prior

The map is represented as a set of landmark coordinates

$$\mathbf{l}^{[i]} = \begin{pmatrix} x^{[i]} \\ y^{[i]} \end{pmatrix} \in \Re^2$$





Domains

Instead of considering rotational and translational velocities, we consider the integrated motion in the interval as input

Define

- state space

$$\mathbf{X}_t = [\mathbf{R}_t | \mathbf{t}_t] \in SE(2)$$

- space of controls (inputs)

$$\mathbf{u}_t = \begin{pmatrix} \Delta_t v_t \\ \Delta_t \omega_t \end{pmatrix} = \begin{pmatrix} u_t^x \\ u_t^\theta \end{pmatrix} \in \Re^2$$

- space of observations (measurements)

$$\mathbf{z}_t^{[i]} = \begin{pmatrix} x_t^{[i]} \\ y_t^{[i]} \end{pmatrix} \in \Re^2$$

Domains

Find an Euclidean parameterization of non-Euclidean spaces

- state space

$$\mathbf{X}_t = [\mathbf{R}_t | \mathbf{t}_t] \in SE(2) \Rightarrow \mathbf{x}_t = \begin{pmatrix} x_t \\ y_t \\ \theta_t \end{pmatrix} \in \Re^3$$

poses are not Euclidean, we map them to 3D vectors

- space of controls (inputs)

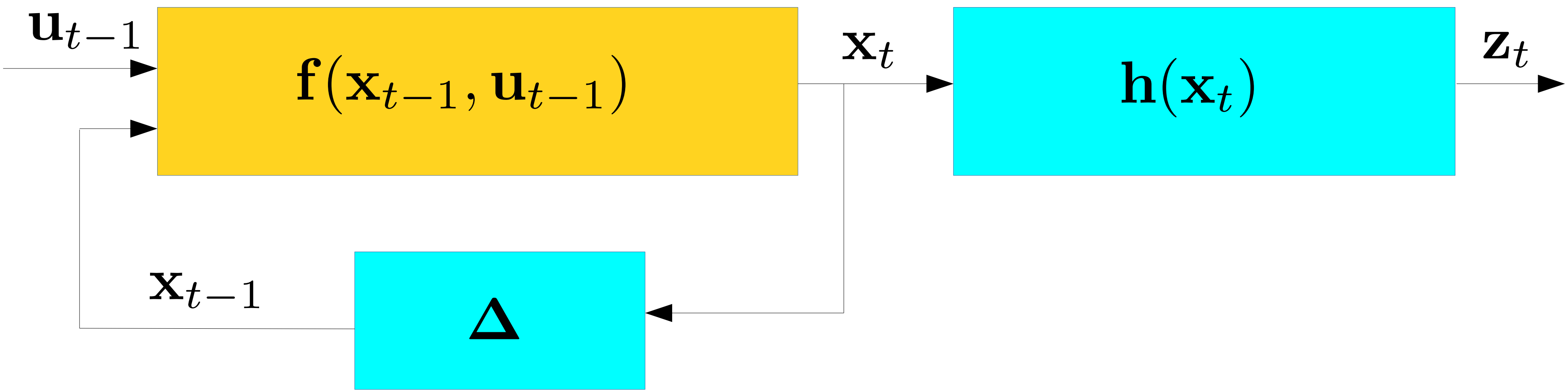
$$\mathbf{u}_t = \begin{pmatrix} u_t^x \\ u_t^\theta \end{pmatrix} \in \Re^2$$

measurement and control, in this problem are already Euclidean

- space of observations (measurements)

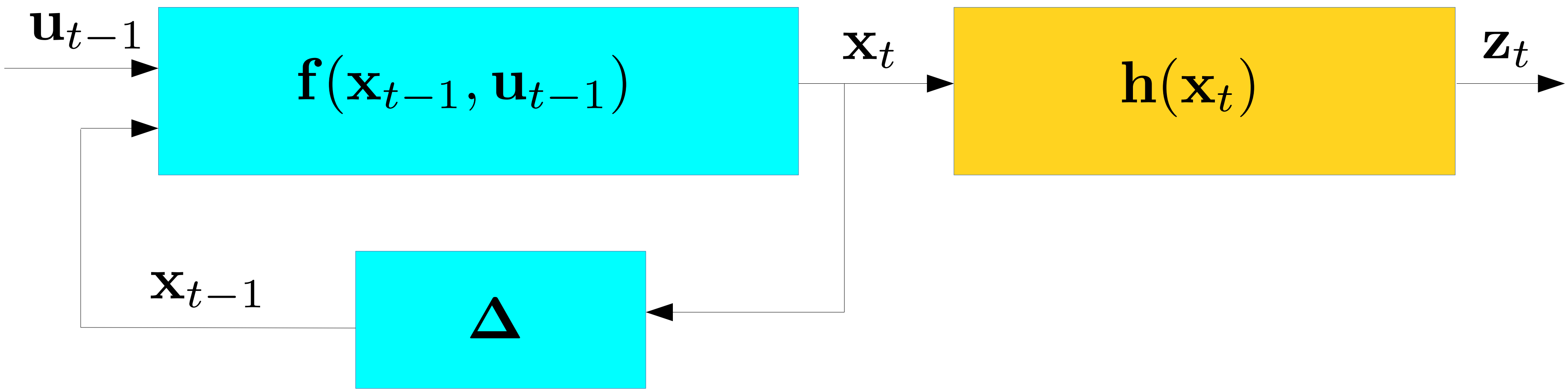
$$\mathbf{z}_t^{[n]} = \begin{pmatrix} x_t^{[n]} \\ y_t^{[n]} \end{pmatrix} \in \Re^2$$

Transition Function



$$\mathbf{x}_t = \mathbf{f}(\mathbf{x}_{t-1}, \mathbf{u}_{t-1}) = \begin{pmatrix} x_{t-1} + u_{t-1}^1 \cdot \cos(\theta_{t-1}) \\ y_{t-1} + u_{t-1}^1 \cdot \sin(\theta_{t-1}) \\ \theta_{t-1} + u_{t-1}^2 \end{pmatrix}$$

Measurement Function



We have [i] measurement functions, one per landmark

$$
\begin{aligned}
\mathbf{z}_t^{[i]} &= \mathbf{h}^{[i]}(\mathbf{x}_t) \\
&= \boxed{\mathbf{R}_t^T}(\mathbf{l}^{[i]} - \mathbf{t}_t) \\
&= \begin{pmatrix} \cos\theta_t(x^{[i]} - x_t) + \sin\theta_t(y^{[i]} - y_t) \\ -\sin\theta_t(x^{[i]} - x_t) + \cos\theta_t(y^{[i]} - y_t) \end{pmatrix}
\end{aligned}
$$

relative position of the I[th] landmark w.r.t the robot at time t

$$
\boxed{\mathbf{R}_t = \begin{pmatrix} \cos\theta_t & -\sin\theta_t \\ \sin\theta_t & \cos\theta_t \end{pmatrix}}
$$

rotation matrix of theta

Measurement Function

At each point in time, our robot will sense only a subset of *K* landmarks in the map

The measurement is thus consisting of a stack of measurements

$$\mathbf{z}_t = \begin{pmatrix} \mathbf{z}^{[i_1]} \\ \mathbf{z}^{[i_2]} \\ \dots \\ \mathbf{z}^{[i_K]} \end{pmatrix} = \mathbf{h}(\mathbf{x}_t) = \begin{pmatrix} \mathbf{h}^{[i_1]}(\mathbf{x}_t) \\ \mathbf{h}^{[i_2]}(\mathbf{x}_t) \\ \dots \\ \mathbf{h}^{[i_K]}(\mathbf{x}_t) \end{pmatrix}$$

index of the landmark generating the measurement

Control Noise

Thanks to Particle Filter method, we can remove the Gaussian assumption.

E.g. we assume here that the velocity inputs are affected by a uniform noise whose amplitude grows with the speed.

Translational and rotational noise are assumed independent.

$$\mathbf{n}_{u,t} \sim \begin{pmatrix} a_1 ||u_t^1|| U(-0.5, 0.5) \\ a_2 ||u_t^2|| U(-0.5, 0.5) \end{pmatrix}$$

Measurement Noise and Observation model

We assume the measurement noise is zero mean, Gaussian and constant

$$\mathbf{n}_z \sim \mathcal{N}\left(\mathbf{n}_z; \mathbf{0}, \begin{pmatrix} \sigma_z^2 & 0 \\ 0 & \sigma_z^2 \end{pmatrix}\right)$$

With this noise the observation model becomes

$$p(\mathbf{z}_t^{[i]} \mid \mathbf{x}_t) \propto \exp\left(-(\mathbf{h}^{[i]}(\mathbf{x}_t) - \mathbf{z}_t^{[i]})^T \Sigma_z^{-1} (\mathbf{h}^{[i]}(\mathbf{x}_t) - \mathbf{z}_t^{[i]})\right)$$

$$p(\mathbf{z}_t \mid \mathbf{x}_t) = \prod_i p(\mathbf{z}_t^{[i]} \mid \mathbf{x}_t);$$

Predict

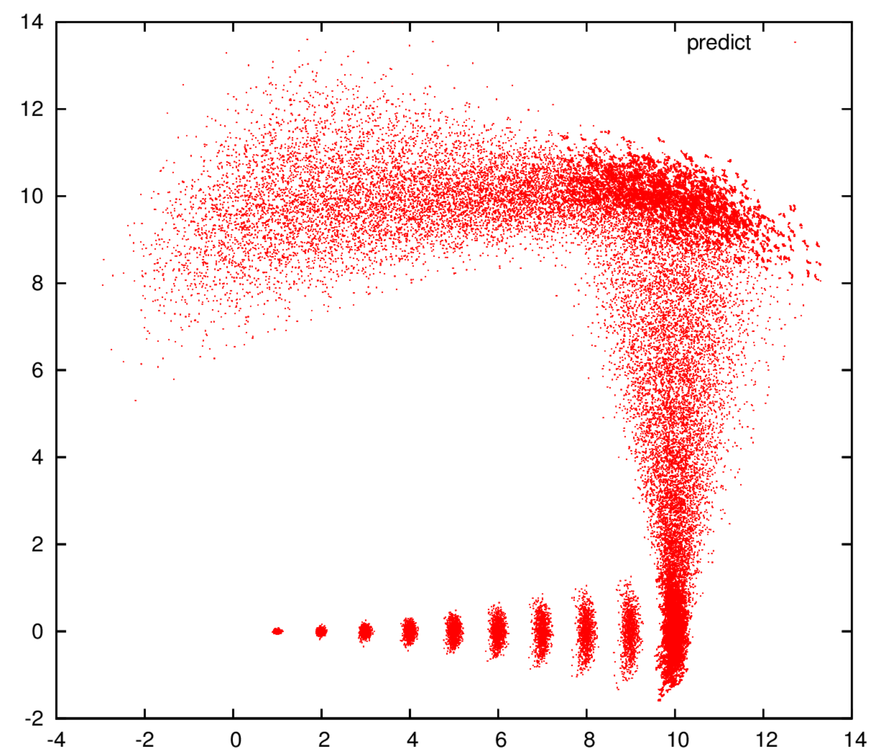
When a new control u_{t-1} becomes available, we compute the location by

- generating an independent noise sample for each particle
- computing the location of the new sample through the transition function, evaluated at the particle, at the control and at the noise sample

$$\mathbf{n}_u^{(i)} \sim p(\mathbf{n}_u)$$

$$\mathbf{x}_{t|t-1}^{(i)} = \mathbf{f}(\mathbf{x}_{t-1|t-1}^{(i)}, \mathbf{u}_{t-1} + \mathbf{n}_u^{(i)})$$

$$w_{t|t-1}^{(i)} = w_{t-1|t-1}^{(i)}$$



evolution of the particles when the robot moves on a square

Prediction: code

```
function samples = prediction(samples, transition)

 dim_particles = size(samples,2);

 u = transition.v;
 %it returns u = [ux, uy, utheta]. simply not consider uy
 u_x = u(1);
 u_theta = u(3);

 a1 = abs(u_x);
 a2 = abs(u_theta);
 %apply transition to every particle
 for i=1:dim_particles
  % sample noise from uniform between
  % [-0.5; 0.5]
  noise_x = (rand() - 0.5)*a1;
  noise_theta = (rand() - 0.5)*a2;

  samples(:,i) = %TODO;
 end
endfunction
```

Apply motion model to each particle

$$\mathbf{x}^{(i)}_{t|t-1} = \mathbf{f}(\mathbf{x}^{(i)}_{t-1|t-1}, \mathbf{u}_{t-1} + \mathbf{n}^{(i)}_{u})$$

Update

To compute the update we need to address the data association

We solve it in a greedy manner, for each particle

- given a particle, we evaluate the likelihood of all combinations landmarks/measurements

sample

$\mathbf{x}_t^{(i)}$

negative log likelihood of measurement m w.r.t landmark n seem by particle i

$$l_{mn}^{(i)} = (\mathbf{h}^{[n]}(\mathbf{x}_t^{(i)}) - \mathbf{z}_t^{[m]})^T \mathbf{\Sigma}_z^{-1} (\mathbf{h}^{[n]}(\mathbf{x}_t^{(i)}) - \mathbf{z}_t^{[m]})$$

$$l_m^{(i)} = \min_n l_{mn}^{(i)}$$

negative log likelihood is the one of the “closest” landmark

$$p(\mathbf{z}_t \mid \mathbf{x}_t^{(i)}) \propto \exp\left(-k^{(i)} l_{\text{miss}} - \sum_{m/\text{misses}} l_m^{(i)} \right)$$

Sample likelihood

number of dropped measurements

Update(1): code

```
function weights = update(samples, weights, landmarks, observations)

 % init useful stuff
 [...]

 sigma_z_noise = 10;
 Sigma_z_noise = [sigma_z_noise^2 0;
     0 sigma_z_noise^2];
 Omega_z_noise = inv(Sigma_z_noise);

 l_miss = 30; %this is a threshold

 for i=1:num_particles
  %init association matrix
  l_mn = zeros(num_landmarks_seen, num_landmarks);
  particle = samples(:,i); %current particle

  for n=1:num_landmarks
    landmark = landmarks(n); %current landmark
    land_x = landmark.x_pose;
    land_y = landmark.y_pose;
    [curr_h, _] = measurement_function(particle, [land_x; land_y]);

    for m=1:num_landmarks_seen
      %current measurement
      measurement = observations.observation(m);
      delta = %TODO;
      l_mn(m,n) = %TODO;
    endfor
  endfor
```

We have access to measurement.x_pose and .y_pose

Update

Being able to evaluate the likelihood of each predicted sample we can perform the conditioning

$$w_{t|t}^{(i)} = w_{t|t-1}^{(i)} p(\mathbf{z}_t \mid \mathbf{x}_{t|t-1}^{(i)})$$

$$\mathbf{x}_{t|t}^{(i)} = \mathbf{x}_{t|t-1}^{(i)}$$

After conditioning we NEED to resample

$$\{j^{(i)}\} = \text{uniformSample}(\{w_{t|t}^{(i)}\})$$

this will be the distribution after resampling

$$\boxed{\tilde{\mathbf{x}}_{t|t}^{(i)}} = \mathbf{x}_{t|t}^{(j(i))}$$

$$\boxed{\tilde{w}_{t|t}^{(i)}} = \frac{1}{\#\text{samples}}$$

Update(2): code

```
for i=1:num_particles
 %still in the particle cycle
  [...]
 %once the association matrix of
 %the current particle is filled

        negative_log_likelihood = min(l_mn');
        dropped_measurements = 0;
        for k=1:num_landmarks_seen
                if(negative_log_likelihood(k) > l_miss)
                        negative_log_likelihood(k) = 0;
                        dropped_measurements++;
                endif;
        endfor

        sum_of_negative_log_likelihood = sum(
    negative_log_likelihood);
        weights(i) *= %TODO;

endfor
endfunction
```

We can update the weights as:

$$p(\mathbf{z}_t \mid \mathbf{x}_t^{(i)}) \propto \exp\left(-k^{(i)} l_{\text{miss}} - \sum_m l_m^{(i)}\right)$$

$$w_{t|t}^{(i)} = w_{t|t-1}^{(i)} p(\mathbf{z}_t \mid \mathbf{x}_{t|t-1}^{(i)})$$

UniformSample: code

```
function sampled_indices = uniformSample(weights,
    num_desired_samples)

dim_weights = size(weights,1);
%normalize the weights (if not normalized)
normalizer = 1./sum(weights);
%resize the indices
step = 1./num_desired_samples;

y0 = rand()*step;   %sample between 0 and 1/num_desired_sample;
yi = y0;    %value of the sample in the y space
cumulative = 0;   %this is our running cumulative distribution

for weight_index=1:dim_weights
 cumulative += normalizer*weights(weight_index); %update cumulative
 % fill with current_weight_index
 % until the cumulative does not become larger than yi
 while(cumulative > yi)
  sampled_indices(end+1,1) = weight_index;
  yi += step;
 endwhile

endfor

endfunction
```

Resampling: code

```
function [new_samples, new_weights] = resample(samples, weights,
    dim_samples)

 indices = uniformSample(weights', dim_samples);

 new_samples = samples;
 new_weights = ones(1,dim_samples)/dim_samples;

 for i=1:dim_samples
  new_samples(:,i) = %TODO;
 endfor

endfunction
```

Call uniformSample

$$\{j^{(i)}\} = \text{uniformSample}(\{w_{t|t}^{(i)}\})$$

$$\tilde{\mathbf{x}}_{t|t}^{(i)} = \mathbf{x}_{t|t}^{(j(i))}$$

Reset weights

$$\tilde{w}_{t|t}^{(i)} = \frac{1}{\#\text{samples}}$$