Let

$$x_{t-1} \sim N(\mu_x, \Sigma_x) \quad , \quad w_{t-1} = N(\mu_w, \Sigma_w)$$

① Compute the parameters of the joint distribution $p(x_{t-1}, w_{t-1})$, assuming x and w are independent

$$\mu_{x,w} = \begin{pmatrix} ? \\ ? \end{pmatrix} \qquad \Sigma_{x,w} = \begin{pmatrix} ? & 0 \\ 0 & ? \end{pmatrix}$$

② Consider the following affine transformation

$$x_t = \begin{pmatrix} A_t & B_t \end{pmatrix} \begin{pmatrix} x_{t-1} \\ w_{t-1} \end{pmatrix} + c_t$$

Compute the parameters of $p(x_t)$

$$\mu_{x_{t|t-1}} = \qquad \Sigma_{x_t|t-1} =$$

③ Consider the following conditional density

$$z_t = N(C_t \cdot x_{t|t-1} + d_t ; \Sigma_z) \quad , \quad \text{and consider} \quad x_{t|t-1} = N\left(\mu_{x_{t|t-1}}, \Sigma_{t|t-1}\right)$$

apply the chain rule to compute the joint $p(x_{t|t-1}, z_t)$

$$\mu_{xz} = \begin{pmatrix} \mu_x \\ ? \end{pmatrix} \qquad \Sigma_{xz} = \begin{pmatrix} \Sigma_{xx} & \Sigma_{xz} \\ \Sigma_{zx} & \Sigma_{zz} \end{pmatrix} \qquad \begin{aligned} \Sigma_{xx} &= ? \\ \Sigma_{xz} &= \Sigma_{zx}^T = ? \\ \Sigma_{zz} &= ? \end{aligned}$$

④ Apply the conditioning on $z_t = \tilde{z}_t$ to the joint of the step before, and get the parameters of $p(x_t | z_t)$

$$\mu_{x_t} = ? \qquad \Sigma_{x_t} = ?$$