Probabilistic Robotics Course

Localization with Kalman Filters [Example Application]

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EKF: recap

- Estimate the current state distribution from
  - Previous state distribution
  - Sequence of observations $z_{0:t}$
  - Sequence of controls $u_{0:t-1}$
  - Transition model
  - Observation model

$$\begin{aligned}\mu_{t|t-1} &= \boxed{\mathbf{f}(\boxed{\mu_{t-1|t-1}}, \mathbf{u}_{t-1})} \\ \boldsymbol{\Sigma}_{t|t-1} &= \mathbf{A}_t \boxed{\boldsymbol{\Sigma}_{t-1|t-1}} \mathbf{A}_t^T + \mathbf{B}_t \boldsymbol{\Sigma}_u \mathbf{B}_t^T\end{aligned}$$

$$\begin{aligned}\mu_z &= \boxed{\mathbf{h}(\mu_{t|t-1})} \\ \boxed{\mu_{t|t}} &= \mu_{t|t-1} + \mathbf{K}_t \left(\mathbf{z}_t - \mu_z\right) \\ \boxed{\boldsymbol{\Sigma}_{t|t}} &= \left(\mathbf{I} - \mathbf{K}_t \mathbf{C}_t\right) \boldsymbol{\Sigma}_{t|t-1}\end{aligned}$$

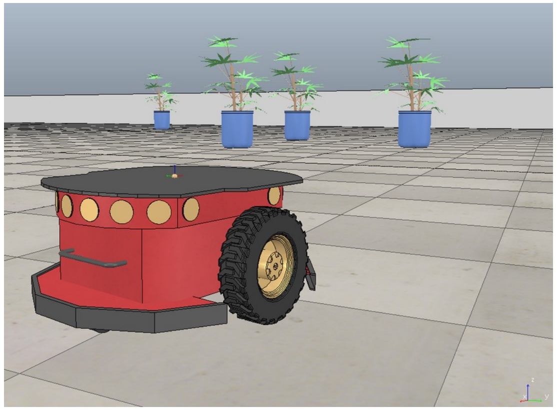
Outline

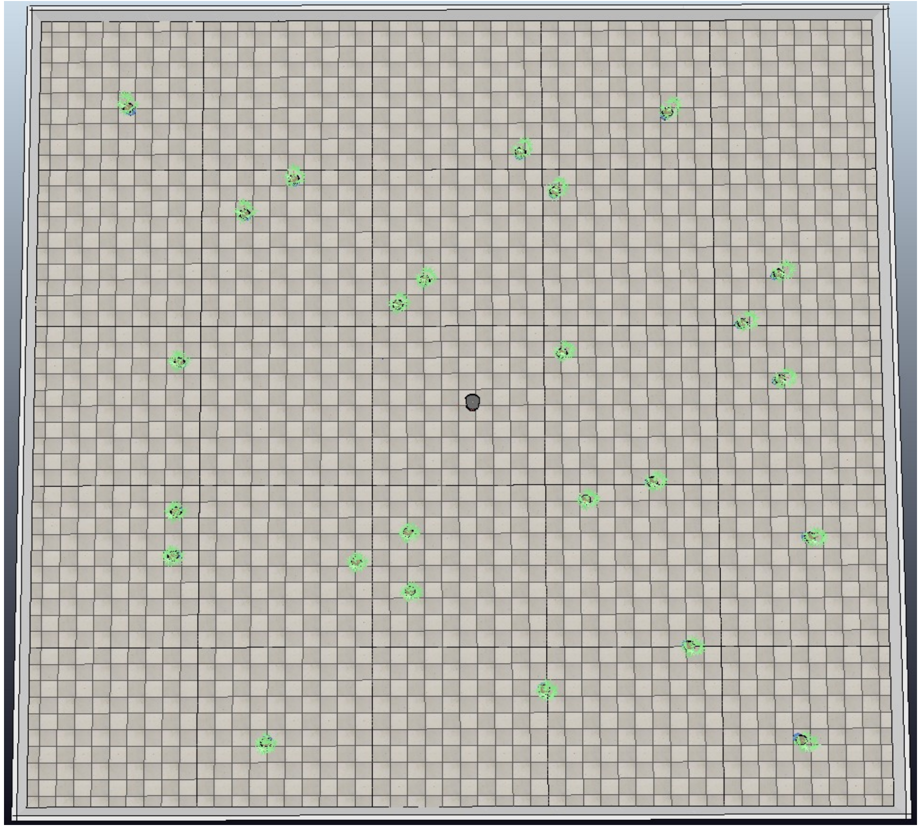
- Scenario
- Controls
- Observations
- Jacobians
- Implementation

Scenario

Orazio moves on a 2D plane

- It is controlled by translational and rotational velocities
- Senses a set of uniquely distinguishable landmarks through a “2D landmark sensors”
- The location of the landmarks in the world is known





Approaching the problem

We want to develop an EKF based algorithm to track the pose of Orazio as it moves

The inputs of our algorithms will be

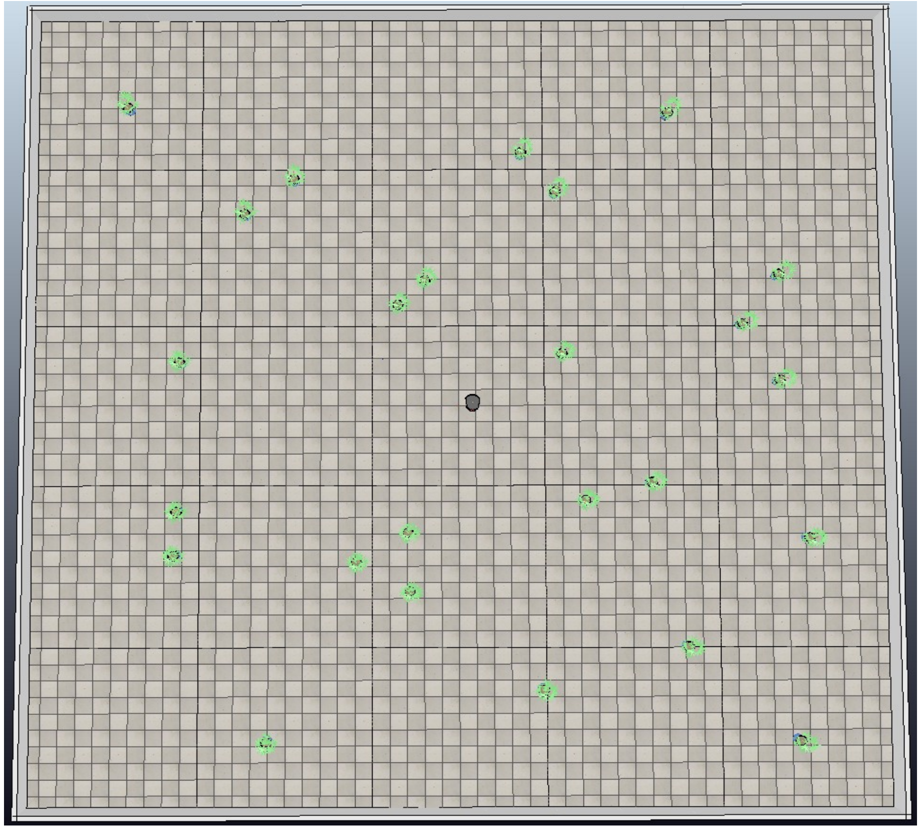
- velocity measurements
- landmark measurements

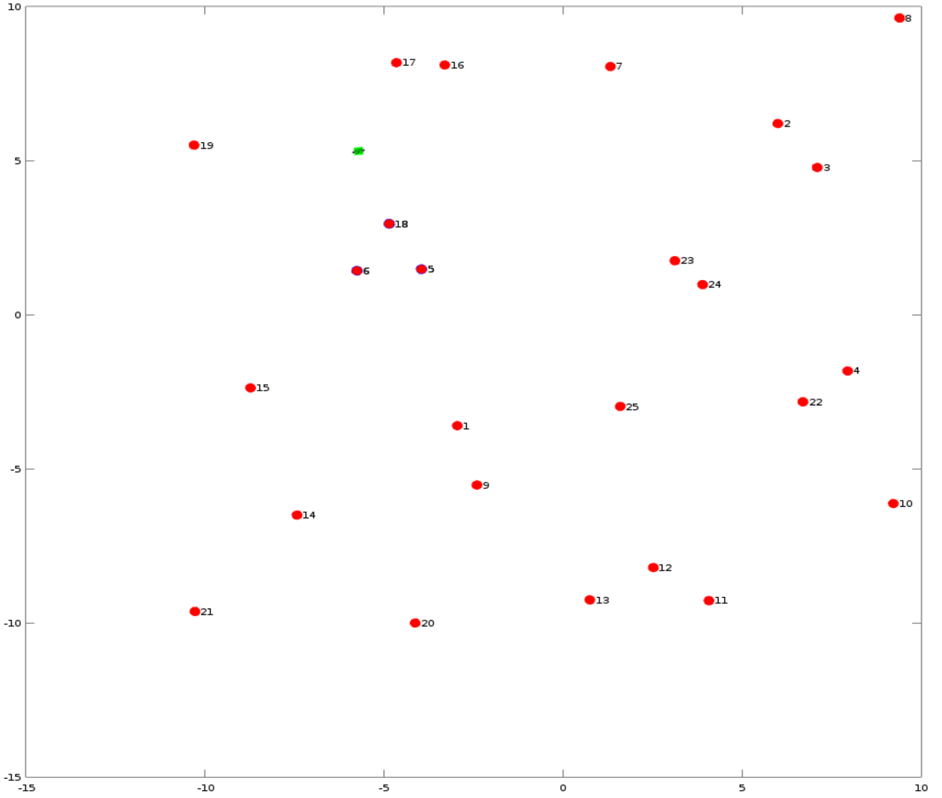
The prior knowledge about the map is represented by the location of each landmark in the world

Prior

The map is represented as a set of landmark coordinates

$$\mathbf{l}^{[i]} = \left(\begin{array}{c} x^{[i]} \\ y^{[i]} \end{array} \right) \in \Re^2$$





Domains

Define

- state space

$$\mathbf{X}_t = [\mathbf{R}_t | \mathbf{t}_t] \in SE(2)$$

Instead of considering rotational and translational velocities, we consider the integrated motion in the interval as input

This leads to a lighter notation

- space of controls (inputs)

$$\mathbf{u}_t = \begin{pmatrix} \Delta_t v_t \\ \Delta_t \omega_t \end{pmatrix} = \begin{pmatrix} u_t^1 \\ u_t^2 \end{pmatrix} \in \Re^2$$

- space of observations (measurements)

$$\mathbf{z}_t^{[i]} = \begin{pmatrix} x_t^{[i]} \\ y_t^{[i]} \end{pmatrix} \in \Re^2$$

Domains

Find a Euclidean parameterization of non-Euclidean spaces

poses are not Euclidean, we map them to 3D vectors

- state space

$$\mathbf{X}_t = [\mathbf{R}_t | \mathbf{t}_t] \in SE(2) \Rightarrow \mathbf{x}_t = \begin{pmatrix} x_t \\ y_t \\ \theta_t \end{pmatrix} \in \Re^3$$

- space of controls (inputs)

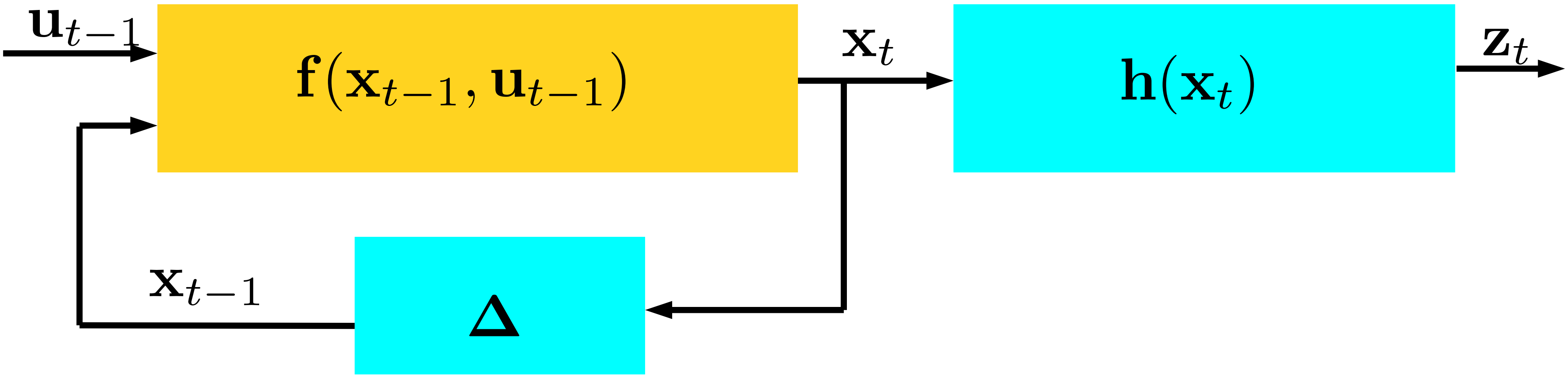
measurement and control, in this problem are already Euclidean

$$\mathbf{u}_t = \begin{pmatrix} u_t^1 \\ u_t^2 \end{pmatrix} \in \Re^2$$

- space of observations (measurements)

$$\mathbf{z}_t = \begin{pmatrix} x_t^{[i]} \\ y_t^{[i]} \end{pmatrix} \in \Re^2$$

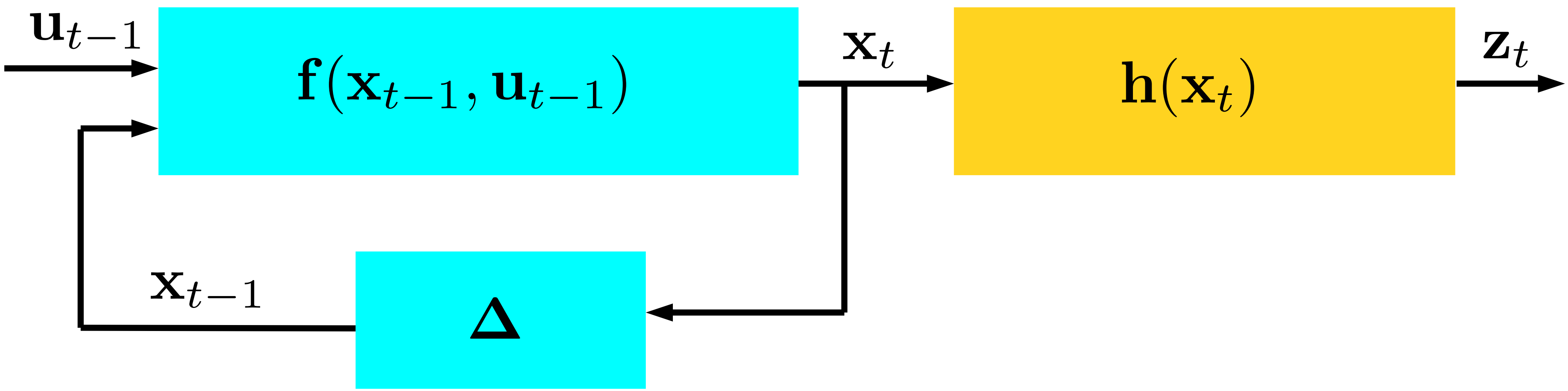
Transition Function



- Consider constant velocity in interval $[t_{t-1}, t_t]$
- State x_t is obtained by Euler integration

$$\mathbf{x}_t = \mathbf{f}(\mathbf{x}_{t-1}, \mathbf{u}_{t-1}) = \begin{pmatrix} x_{t-1} + u_{t-1}^1 \cdot \cos(\theta_{t-1}) \\ y_{t-1} + u_{t-1}^1 \cdot \sin(\theta_{t-1}) \\ \theta_{t-1} + u_{t-1}^2 \end{pmatrix}$$

Measurement Function



We have [i] measurement functions, one per landmark

$$\begin{aligned}
\mathbf{z}_t^{[i]} &= \mathbf{h}^{[i]}(\mathbf{x}_t) \\
&= \boxed{\mathbf{R}_t^T}(\mathbf{l}^{[i]} - \mathbf{t}_t) \\
&= \begin{pmatrix} \cos\theta_t(x^{[i]} - x_t) + \sin\theta_t(y^{[i]} - y_t) \\ -\sin\theta_t(x^{[i]} - x_t) + \cos\theta_t(y^{[i]} - y_t) \end{pmatrix}
\end{aligned}$$

relative position of the I$^{\text{th}}$ landmark w.r.t the robot at time t

$$\mathbf{R}_t = \begin{pmatrix} \cos\theta_t & -\sin\theta_t \\ \sin\theta_t & \cos\theta_t \end{pmatrix}$$

rotation matrix of theta

Measurement Function

At each point in time, our robot will sense only a subset of *K* landmarks in the map

The measurement is thus consisting of a stack of measurements

$$\mathbf{z}_t = \begin{pmatrix} \mathbf{z}^{[i_1]} \\ \mathbf{z}^{[i_2]} \\ \dots \\ \mathbf{z}^{[i_K]} \end{pmatrix} = \mathbf{h}(\mathbf{x}_t) = \begin{pmatrix} \mathbf{h}^{[i_1]}(\mathbf{x}_t) \\ \mathbf{h}^{[i_2]}(\mathbf{x}_t) \\ \dots \\ \mathbf{h}^{[i_K]}(\mathbf{x}_t) \end{pmatrix}$$

index of the landmark generating the measurement

Control Noise

We assume the velocity measurements are affected by a Gaussian noise resulting from the sum of two aspects

- a term with constant standard deviation
- a velocity dependent term whose standard deviation grows with the speed

Translational and rotational noise are assumed independent

$$\mathbf{n}_{u,t} \sim \mathcal{N}\left(\mathbf{n}_{u,t}; \mathbf{0}, \begin{pmatrix} (u_t^1)^2 + \sigma_v^2 & 0 \\ 0 & (u_t^2)^2 + \sigma_\omega^2 \end{pmatrix}\right)$$

Measurement Noise

We assume it is zero mean with constant standard deviation

$$\mathbf{n}_z \sim \mathcal{N}\left(\mathbf{n}_z; \mathbf{0}, \begin{pmatrix} \sigma_z^2 & 0 \\ 0 & \sigma_z^2 \end{pmatrix}\right)$$

Noise affecting the x- and y- components of the landmark position are assumed to be independent

Jacobians!

At each time step our system will need to compute the derivatives of transition and measurement functions

$$f(x,u) = \begin{pmatrix} x_{t-1} + u^1_{t-1}\cos(\theta_{t-1}) \\ y_{t-1} + u^1_{t-1}\sin(\theta_{t-1}) \\ \theta_{t-1} + u^2_{t-1} \end{pmatrix}$$

$$\mathbf{A}_t = \frac{\partial \mathbf{f}(\cdot)}{\partial \mathbf{x}} = \begin{pmatrix} 1 & 0 & -u^1_{t-1}\sin(\theta_{t-1}) \\ 0 & 1 & u^1_{t-1}\cos(\theta_{t-1}) \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{B}_t = \frac{\partial \mathbf{f}(\cdot)}{\partial \mathbf{u}} = \begin{pmatrix} \cos(\theta_{t-1}) & 0 \\ \sin(\theta_{t-1}) & 0 \\ 0 & 1 \end{pmatrix}$$

Jacobians (cont)

We will have *K* measurement functions, one for each landmark

$$\mathbf{h}^{[i]}(\mathbf{x}_t) \quad = \quad \mathbf{R}_t^T(\mathbf{l}^{[i]} - \mathbf{t}_t)$$

this is a column vector!!!

$$\mathbf{C}_t^{[i]} = \frac{\partial \mathbf{h}^{[i]}(\cdot)}{\partial \mathbf{x}} = \begin{pmatrix} -\mathbf{R}_t^T & \boxed{\frac{\partial \mathbf{R}_t^T}{\partial \theta_t}(\mathbf{l}^{[i]} - \mathbf{t}_t)} \end{pmatrix}$$

derivative of rotation matrix w.r.t. theta

$$\boxed{\frac{\partial \mathbf{R}_t}{\partial \theta_t} = \begin{pmatrix} -\sin\theta_t & -\cos\theta_t \\ \cos\theta_t & -\sin\theta_t \end{pmatrix}}$$

Jacobians (cont)

The total Jacobian of the measurement will be the stack of the individual measurement functions

$$\mathbf{C}_t = \frac{\partial \mathbf{h}}{\partial \mathbf{x}} = \begin{pmatrix} \frac{\partial \mathbf{h}^{[i_1]}}{\partial \mathbf{x}} \\ \frac{\partial \mathbf{h}^{[i_2]}}{\partial \mathbf{x}} \\ \vdots \\ \frac{\partial \mathbf{h}^{[i_K]}}{\partial \mathbf{x}} \end{pmatrix} = \begin{pmatrix} \mathbf{C}_t^{[i_1]} \\ \mathbf{C}_t^{[i_2]} \\ \vdots \\ \mathbf{C}_t^{[i_K]} \end{pmatrix}$$

Hands on!

g2o Wrapper

Load your Vrep acquired dataset

```
[land, pose, transition, obs] = loadG2o('my_dataset.g2o');
```

It returns 4 Struct-Arrays(Landmark, Poses, Transitions, Observations), *i.e.* :

```
land =

  1x25 struct array containing the fields:

    id
    x_pose
    y_pose
```

```
pose =

  1x137 struct array containing the fields:

    id
    x
    y
    theta
```

```
transition =

  1x136 struct array containing the
fields:

    id_from
    id_to
    v
```

```
obs =

  1x136 struct array containing the fields:

    pose_id
    observation
```

EKF Localization

```
% load your own dataset dataset
[landmarks, poses, transitions, observations] = loadG2o('dataset.g2o');
mu = rand(3,1)*20-10; % init mean
mu(3) = normalizeAngle(mu(3));

sigma = eye(3)*0.001; % init covariance

%simulation cycle
for i=1:length(transitions)
    % predict with transitions
    [mu, sigma] = ekf_prediction(mu, sigma, transitions(i));
    % correct with observations
    [mu, sigma] = ekf_correction(mu, sigma, landmarks, observations(i));

    plot_state(landmarks, mu, sigma, observations(i));
endfor
```

EKF Localization

```
% load your own dataset dataset
[landmarks, poses, transitions, observations] = loadG2o('dataset.
    g2o');
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for i=1:length(transitions)
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    observations(i));

    plot_state(landmarks, mu, sigma, observations(i));
endfor
```