Robust Estimators

Gauss-Newton → minimize function which is the sum little function (usually, Ω-norm) / ... as weight matrix

$$e^{[i]}(x) = e^{[i]}(x)^T \Omega^{[i]} e^{[i]}(x)$$

Mat Ximer

Ω not change in the optimization

Check L2 norm ——— $= \|e^{[i]}(x)\|^2_{\Omega^{[i]}}$

Ω → LL^T → Cholesky decomposition

↓

positive definite

$v' = L^T \cdot v$ ——→ Why?... optimized computation?

L1-norm ——— squared-root of L_2-norm (≡ euclidean distance) ——— grows quadratically w/ length of the error

⇓

outlier will push away the optimal solution gradually

/ L overweight correction of the solution

they are few but the error ......

M estimator / robust estimation ⇒ having a good enough INITIAL GUESS

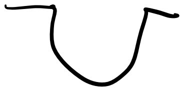
↓

minimize $F(x) = \sum_i \rho\left(\underbrace{\|e^{[i]}(x)\|_{\Omega^{[i]}}}\right)$

↑ $u^{[i]}(x) \equiv$ L1-norm

/ robustifier → MONOTONICALLY INCREASING of L1-norm

if $\rho(u) = u^2$ ⇒ $u^{[i]}(x)^2 \equiv$ TRADITIONAL GAUSSIAN NEWTON



outlier

TRUNCATED — if error too big, not then consider the much...

$$e^{[i]}(x) = e^{[i]}(x)^T \Omega^{[i]} e^{[i]}(x) = \|e^{[i]}(x)\|^2_{\Omega^{[i]}}$$

$$\frac{\partial \|e\|^2_\Omega}{\partial x} = \frac{\partial e^T}{\partial x}\Omega e(x) + e^T \Omega \frac{\partial e}{\partial x} = e^T \underbrace{\Omega^T}_{= \Omega} \frac{\partial e}{\partial x} + e^T \Omega \frac{\partial e}{\partial x} = 2 e^T \Omega \frac{\partial e}{\partial x}$$

$$\frac{\partial \sqrt{\|e\|^2_\Omega}}{\partial x} = \frac{1}{2} \cdot \frac{\partial \|e\|^2_\Omega}{\partial x} \cdot \frac{1}{\sqrt{\|e\|^2_\Omega}} = \frac{1}{2\underbrace{\|e\|_\Omega}_{u(x)}} \cdot \frac{\partial \|e\|^2_\Omega}{\partial x}$$

$u(x)$ → scalar

$u^n = n u' u^{n-1}$

chain rule!

$$\frac{\partial \rho(u(x))}{\partial x} = \frac{\partial \rho(u)}{\partial u}\Big|_{u=u(x)} \cdot \frac{\partial u}{\partial x}$$

$\frac{\partial u}{\partial x}$ → = L2-norm

$$= \frac{\partial \rho(u)}{u} \cdot \frac{1}{u(x)} \cdot \frac{\partial \|e\|^2_\Omega}{\partial x}$$

$$\gamma(x) = \frac{\partial \rho(u)}{\partial u}\Big|_{u=u(x)} \cdot \frac{1}{u(x)}$$

a

incorporates "all the ugly things" that multiply the derivative of L2-norm

∂e → γ is a number

$\nabla \xi$ look similar

$\frac{\partial e}{\partial x}$

γ computed in previous iteration and use it in the next one

⇓

current iteration

ITERATIVE RE-WEIGHTED LEAST-SQUARES

γ can be embedded into Ω

↓

Only a detail

scale down or up the information matrix to modulate the error

↓

apply this trick to get rid of the outliers

ROBUST GAUSS-NEWTON

Exercise : apply robustifier to classical ICP