Probabilistic Robotics Course

EKF SLAM with unknown Data Association

Lorenzo De Rebotti
derebotti@diag.uniroma1.it

Department of Computer, Control, and Management Engineering
Sapienza University of Rome

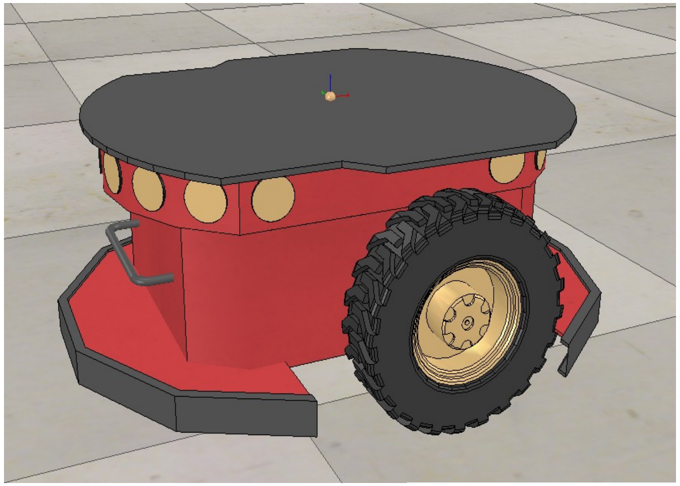
Outline

- Scenario
- Controls
- Observations
- Non-Linear Systems and Gaussian Noise
- Data Association

Scenario

Orazio moves on a 2D plane

- Is controlled by translational and rotational velocities
- Senses a set of **not distinguishable** landmarks through a “2D landmark sensors”
- The location of the landmarks in the world is **not known**



Approaching the problem

We want to develop a KF based algorithm to track the position of Orazio as it moves (localization) and, at the same time, the position of the observed plant-landmarks (mapping) while performing data association.

The inputs of our algorithms will be

- velocity measurements
- landmark measurements

We have no prior knowledge of the map.

EFK SLAM

1. Predict: incorporate new control

$$
\begin{aligned}
\mu_{t|t-1} &= \mathbf{f}(\mu_{t-1|t-1}, \mathbf{u}_{t-1}) \\
\mathbf{A}_t &= \left.\frac{\partial \mathbf{f}(\mathbf{x}, \mathbf{u})}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mu_{t-1|t-1}} \\
\mathbf{B}_t &= \left.\frac{\partial \mathbf{f}(\mathbf{x}, \mathbf{u})}{\partial \mathbf{u}}\right|_{\mathbf{u}=\mu_{u,t-1}} \\
\mathbf{\Sigma}_{t|t-1} &= \mathbf{A}_t \mathbf{\Sigma}_{t-1|t-1} \mathbf{A}_t^T + \mathbf{B}_t \mathbf{\Sigma}_u \mathbf{B}_t^T
\end{aligned}
$$

2. Correct: incorporate new measurement

$$
\begin{aligned}
\mathbf{C}_t &= \left.\frac{\partial \mathbf{h}(\mathbf{x})}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mu_{t|t-1}} \\
\mathbf{K}_t &= \mathbf{\Sigma}_{t|t-1} \mathbf{C}_t^T \left(\mathbf{\Sigma}_z + \mathbf{C}_t \mathbf{\Sigma}_{t|t-1} \mathbf{C}_t^T\right)^{-1} \\
\mu_{t|t} &= \mu_{t|t-1} + \mathbf{K}_t \left(\mathbf{z}_t - \mathbf{h}(\mu_{t|t-1})\right) \\
\mathbf{\Sigma}_{t|t} &= (\mathbf{I} - \mathbf{K}_t \mathbf{C}_t)\, \mathbf{\Sigma}_{t|t-1}
\end{aligned}
$$

innovation

3. Add: extend state with new landmarks

Domains

Define $\mathbf{X}_t^{[r]} = [\mathbf{R}_t|\mathbf{t}_t] \in SE(2)$

- state space

$$\mathbf{x}_t^{[r]} = \begin{pmatrix} x_t \\ y_t \\ \theta_t \end{pmatrix} \in \Re^3$$

landmarks in the state

$$\mathbf{x}_t^{[n]} = \begin{pmatrix} x_t^{[n]} \\ y_t^{[n]} \end{pmatrix} \in \Re^2 \quad {}_{n=1..N}$$

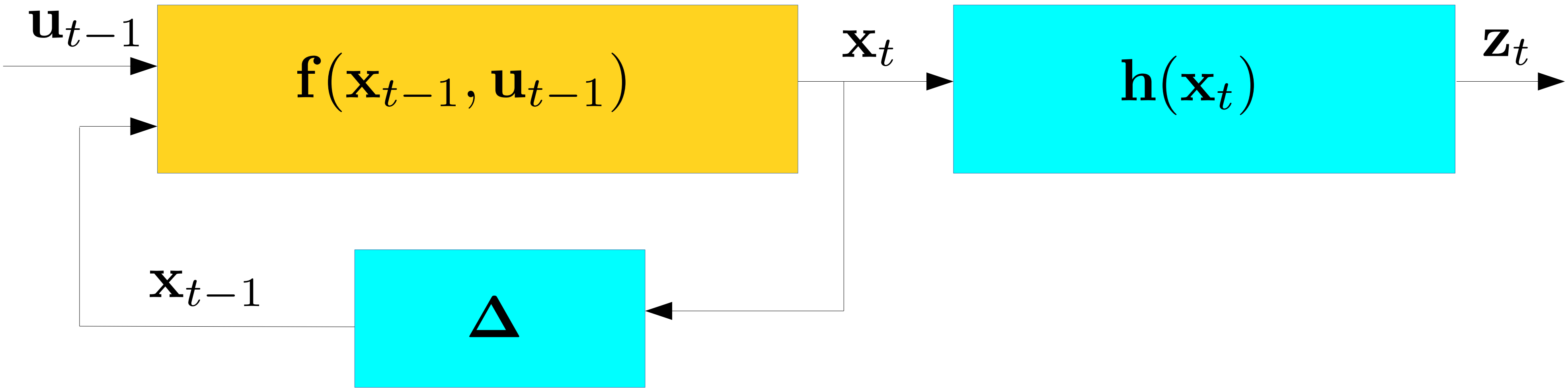
- space of controls (inputs)

$$\mathbf{u}_t = \begin{pmatrix} u_t^x \\ u_t^\theta \end{pmatrix} \in \Re^2$$

- space of observations (measurements)

$$\mathbf{z}_t^{[m]} = \begin{pmatrix} x_t^{[m]} \\ y_t^{[m]} \end{pmatrix} \in \Re^2 \quad {}_{m=1..M}$$

Transition Function

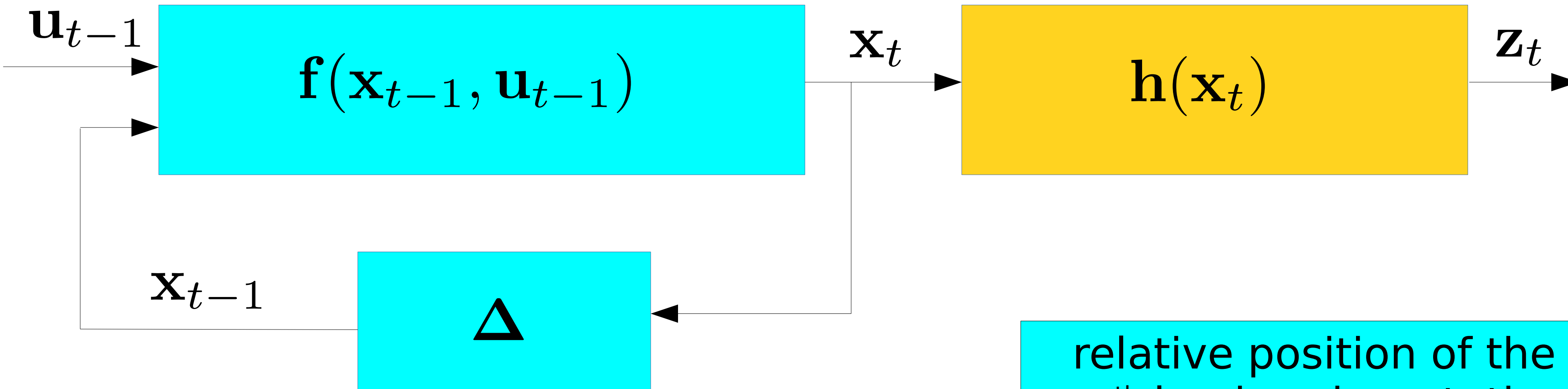


pose update like localization

$$\mathbf{x}_t = \mathbf{f}(\mathbf{x}_{t-1}, \mathbf{u}_{t-1}) = \begin{pmatrix} x_{t-1} + u^x_{t-1} \cdot \cos(\theta_{t-1}) \\ y_{t-1} + u^x_{t-1} \cdot \sin(\theta_{t-1}) \\ \theta_{t-1} + u^\theta_{t-1} \\ \mathbf{x}^{[1]}_t \\ \mathbf{x}^{[2]}_t \\ \dots \\ \mathbf{x}^{[N]}_t \end{pmatrix}$$

the landmarks don't move

Measurement Function



relative position of the m[th] landmark w.r.t. the robot at time t

$$
\begin{aligned}
\mathbf{z}_t^{[m]} &= \mathbf{h}^{[m]}(\mathbf{x}_t) \\
&= \mathbf{R}_t^T(\mathbf{x}_t^{[m]} - \mathbf{t}_t) \\
&= \begin{pmatrix} \cos\theta_t(x_t^{[m]} - x_t) + \sin\theta_t(y_t^{[m]} - y_t) \\ -\sin\theta_t(x_t^{[m]} - x_t) + \cos\theta_t(y_t^{[m]} - y_t) \end{pmatrix}
\end{aligned}
$$

We have M < N measurement functions, one for each observed landmark that is part of the state (N)

Control Noise

We assume the velocity measurements are effected by a Gaussian noise resulting from the sum of two aspects

- a constant noise
- a velocity dependent term whose standard deviation grows with the speed
- Translational σ_v and rotational noise σ_ω which are assumed to be independent

$$\mathbf{n}_{u,t} \sim \mathcal{N}\left(\mathbf{n}_{u,t}; \mathbf{0}, \begin{pmatrix} (u_t^x)^2 + \sigma_v^2 & 0 \\ 0 & (u_t^\theta)^2 + \sigma_\omega^2 \end{pmatrix}\right)$$

Measurement Noise

We assume it is zero mean, constant

$$\mathbf{n}_z \sim \mathcal{N}\left(\mathbf{n}_z; \mathbf{0}, \begin{pmatrix} \sigma_z^2 & 0 \\ 0 & \sigma_z^2 \end{pmatrix}\right)$$

Jacobian: Transitions

$$\mathbf{f}(\mathbf{x}_{t-1}, \mathbf{u}_{t-1}) = \begin{pmatrix} x_{t-1} + u^x_{t-1} \cdot \cos(\theta_{t-1}) \\ y_{t-1} + u^x_{t-1} \cdot \sin(\theta_{t-1}) \\ \theta_{t-1} + u^\theta_{t-1} \\ \mathbf{x}_t^{[1]} \\ \mathbf{x}_t^{[2]} \\ \dots \\ \mathbf{x}_t^{[N]} \end{pmatrix}$$

Our usual Jacobians:

$$\mathbf{A}_t = \frac{\partial \mathbf{f}(\cdot)}{\partial \mathbf{x}} = \begin{pmatrix} \frac{\partial \mathbf{f}(\cdot)}{\partial \mathbf{x}^r} & \frac{\partial \mathbf{f}(\cdot)}{\partial \mathbf{x}^{[1]}} & \frac{\partial \mathbf{f}(\cdot)}{\partial \mathbf{x}^{[2]}} & \dots & \frac{\partial \mathbf{f}(\cdot)}{\partial \mathbf{x}^N} \end{pmatrix}$$

$$\mathbf{B}_t = \frac{\partial \mathbf{f}(\cdot)}{\partial \mathbf{u}}$$

Jacobian: Measurements

Our landmark sensor perceives points, thus our measurement function will be:

$$\mathbf{h}^{[m]}(\mathbf{x}_t) \quad = \quad \mathbf{R}_t^T(\mathbf{x}_t^{[m]} - \mathbf{t}_t)$$

Consequently, the Jacobian can be computed as:

$$\frac{\partial \mathbf{h}^{[m]}(\cdot)}{\partial \mathbf{x}_t^r} = \left(\begin{array}{cc} -\mathbf{R}_t^T & \frac{\partial \mathbf{R}_t^T}{\partial \theta_t}\left(\mathbf{x}_t^{[m]} - \mathbf{t}_t\right) \end{array} \right) \qquad \frac{\partial \mathbf{h}^{[m]}(\cdot)}{\partial \mathbf{x}_t^{[m]}} = \mathbf{R}_t^T$$

$$\mathbf{C}_t^{[m]} = \frac{\partial \mathbf{h}^{[m]}(\cdot)}{\partial \mathbf{x}_t} = \left(\begin{array}{cccccc} \underbrace{\frac{\partial \mathbf{h}^{[m]}}{\partial \mathbf{x}_t^r}}_{\text{pose block}} & \mathbf{0} & \cdots & \underbrace{\frac{\partial \mathbf{h}^{[m]}}{\partial \mathbf{x}_t^{[m]}}}_{\text{landmark block}} & \cdots & \mathbf{0} \end{array} \right)$$

Data Association

We do **not** observe the landmark ids.

When a new landmark appears, it's our duty to assign it a new, unique id.

For convenience, we keep unchanged the state-id mapping structure seen in the previous lesson:

$$\mathbf{id\_to\_state\_map} = \begin{pmatrix} -1 & -1 & \dots & \dots & -1 & \end{pmatrix}$$

$$\mathbf{state\_to\_id\_map} = \begin{pmatrix} -1 & -1 & \dots & \dots & -1 & \end{pmatrix}$$

Data Association

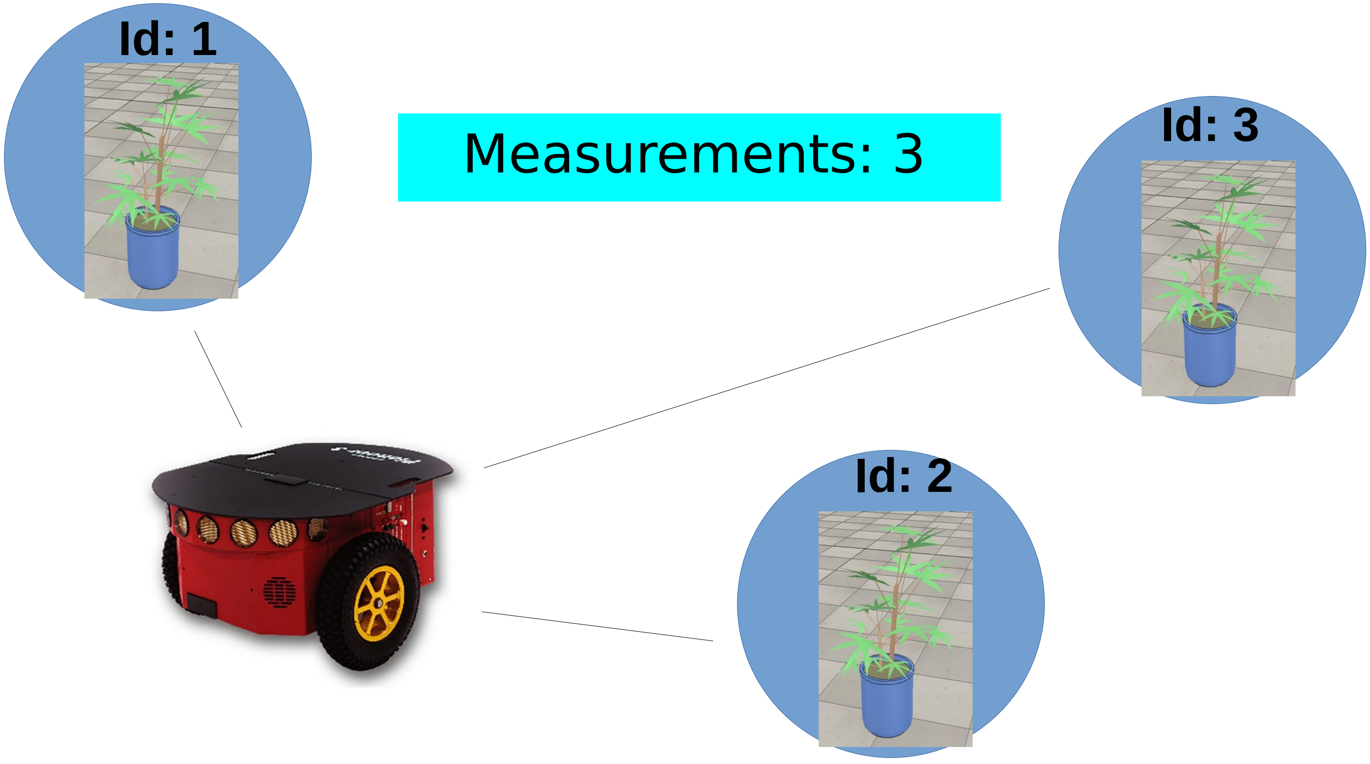
Time: t

current state:

$$\mu_t = \begin{pmatrix} \mathbf{x}_t^{[r]} \\ \mathbf{x}_t^{[1]} \\ \mathbf{x}_t^{[2]} \\ \mathbf{x}_t^{[3]} \end{pmatrix}$$

The first time an unmatched landmark is seen, results in the creation and assignment of a new id

Measurements: 3

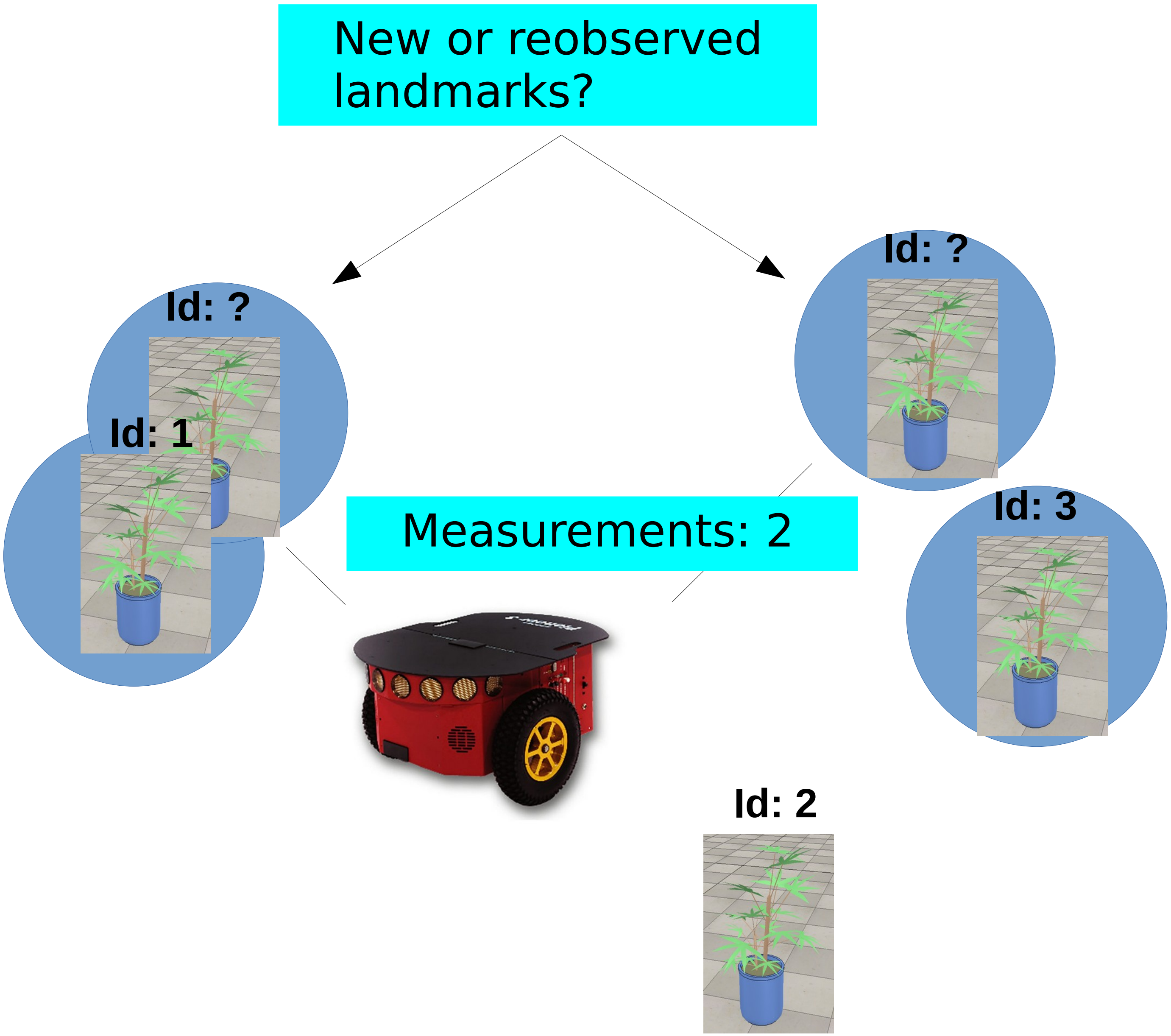


Data Association

Time: t+1

current state:

$$\mu_{t+1} = \begin{pmatrix} \mathbf{x}^{[r]} \\ \mathbf{x}_{t+1}^{[1]} \\ \mathbf{x}_{t+1}^{[2]} \\ \mathbf{x}_{t+1}^{[3]} \\ ?? \end{pmatrix}$$



Data Association

At each time step, compute the **likelihood** for each landmark/measurement pair:

$$a_{mn} = (\mathbf{z}^{[m]} - \mathbf{h}^{[n]}(\mathbf{x}_t))^\top \boldsymbol{\Omega}_{n,n} (\mathbf{z}^{[m]} - \mathbf{h}^{[n]}(\mathbf{x}_t))$$

with information matrix (canonical):

$$\Omega_{n,n} = {\Sigma_{n,n}}^{-1},\ \Sigma_{n,n} = C_t^{[n]} \Sigma_x \left(C_t^{[n]}\right)^T + \Sigma_{z|x}$$

and assemble them in a **cost matrix**:

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots a_{1N} \\ a_{21} & a_{22} & a_{23} & \cdots a_{2N} \\ \vdots & & & \\ a_{M1} & a_{M2} & a_{M3} & \cdots a_{MN} \end{pmatrix}$$

1. Gating

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots a_{1N} \\ a_{21} & a_{22} & a_{23} & \cdots a_{2N} \\ \vdots & & & \\ a_{M1} & a_{M2} & a_{M3} & \cdots a_{MN} \end{pmatrix}$$

- Choose a threshold τ_{accept}
- Extract the minimum for each row a_{mn}
- *If* $a_{mn} < \tau_{accept}$
  - *then* observation m is associated with landmark n
  - *otherwise*, m is a new landmark.

Multiple measurements can be assigned to the same landmark

2. Best Friends

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots a_{1N} \\ a_{21} & a_{22} & a_{23} & \cdots a_{2N} \\ \vdots & & & \\ a_{M1} & a_{M2} & a_{M3} & \cdots a_{MN} \end{pmatrix}$$

- Take all the accepted associations from gating and check
- *If* $\left[a_{mn} = \min_m a_{mn}\right] \mathrel{!=} \left[a_{mn} = \min_n a_{mn}\right]$
  - *then* discard it
  - o*therwise* keep the association

Only one measurement can be assigned to the same landmark

3. Lonely Best Friends

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots a_{1N} \\ a_{21} & a_{22} & a_{23} & \cdots a_{2N} \\ \vdots & & & \\ a_{M1} & a_{M2} & a_{M3} & \cdots a_{MN} \end{pmatrix}$$

- Define a small threshold $\gamma > 0$ and, for all surviving associations, extract the second best association for measurements $\hat{a}_m$ and landmarks $\hat{a}_n$ and check
- *If* $[\hat{a}_m - a_{mn} < \gamma]\ \mathrm{OR}\ [\hat{a}_n - a_{mn} < \gamma]$
  - *then* discard it
  - *otherwise* keep it

Hands On!