EKF SLAM

Map → not known

Landmark → we are still able to recognize specific ID for the landmark
→ but the position of the landmark not known! → STATE GROWS OVER TIME

Scenario

POINT SENSOR ← velocity, etc. → KF
landmark measurement → KF

Odometry → 2D plane
→ $\Delta\rho$, $\Delta\theta$ (translational ⊕ rotational velocities)

1. Predict: incorporate new control
2. Correct: incorporate new measurement $\left(z_t - h(\mu_{t|t-1}) \right) \equiv$ INNOVATION
3. SLAM: add new landmarks to state

Domains

State Space:

Robot $\longrightarrow X_t^{[r]} = [R_t | t_t] \in SE(2) \Longrightarrow x_t = \begin{pmatrix} x_t \\ y_t \\ \theta_t \end{pmatrix} \in \mathbb{R}^3$

Landmark $\rightarrow X_t^{[n]} = \begin{pmatrix} x_t^{[n]} \\ y_t^{[n]} \end{pmatrix} \in \mathbb{R}^2$, $n = 1, \dots, N$

$\Downarrow$

Full State Vector $— X_t = \begin{pmatrix} X_t^{[r]} \\ x_t^{[1]} \\ \vdots \\ X_t^{[N]} \end{pmatrix} \in \mathbb{R}^{(3+2\times N)}$

Controls + Measurements:

No changes $\Leftarrow$ $u_t = \begin{pmatrix} u_t^{[1]} \\ u_t^{[2]} \end{pmatrix} \in \mathbb{R}^2$; $z_t^{[m]} = \begin{pmatrix} x_t^{[m]} \\ y_t^{[m]} \end{pmatrix} \in \mathbb{R}^2$

translational rotational

$m = 1, \dots, M$

Transition Function

$$x_t = f(x_{t-1}, u_{t-1}) = \begin{pmatrix} x_{t-1} + u_{t-1}^{[1]} \cdot \cos(\theta_{t-1}) \\ y_{t-1} + u_{t-1}^{[1]} \cdot \sin(\theta_{t-1}) \\ \theta_{t-1} + u_{t-1}^{[2]} \\ x_{t-1}^{[1]} \\ x_{t-1}^{[2]} \\ \vdots \\ x_{t-1}^{[N]} \end{pmatrix}$$

landmarks do not move!

Measurement Function

1 measurement function for each landmark $x_t^{[n]}$, $n = 1, \dots, N$

$$z_t^{[n]} = h^{[n]}(x_t) = h(x_t^{[1]}, x_t^{[n]}) =$$

$$= R_t^T (x_t^{[n]} - t_t) =$$

$$= \begin{pmatrix} c_{\theta_t} \cdot (x_t^{[n]} - x_t) + s_{\theta_t} \cdot (y_t^{[n]} - y_t) \\ -s_{\theta_t} \cdot (x_t^{[n]} - x_t) + c_{\theta_t} \cdot (y_t^{[n]} - y_t) \end{pmatrix}$$

what about the update of the landmark pose?...

$\Downarrow$

$h(\cdot)$ relates state → measurement

Control Noise

$$n_{u,t} \sim N\Big(n_{u,t}; \emptyset, \underbrace{\begin{pmatrix} \sigma_u^2 + \sigma_T^2 & \emptyset \\ \emptyset & \sigma_u^2 + \sigma_R^2 \end{pmatrix}}_{\Sigma_u}\Big)$$

Measurement Noise

$$n_z \sim N\left(n_z; \emptyset, \begin{pmatrix} \sigma_z^2 & \emptyset \\ \emptyset & \sigma_z^2 \end{pmatrix}\right)$$

Jacobian

$$\Downarrow$$

$$f(x_{t-1}, u_{t-1}) = \begin{pmatrix} x_{t-1} + u_{t-1}^{[1]} \cdot c_{\theta_{t-1}} \\ y_{t-1} + u_{t-1}^{[1]} \cdot s_{\theta_{t-1}} \\ \theta_{t-1} + u_{t-1}^{[2]} \\ x_{t-1}^{[1]} \\ \vdots \\ x_{t-1}^{[N]} \end{pmatrix}$$

$$A_t = \frac{\partial f(\cdot)}{\partial x}\bigg|_{x=\mu_{t-1|t-1}} = \begin{pmatrix} \frac{\partial f(\cdot)}{\partial x^{[1]}} & \frac{\partial f(\cdot)}{\partial x^{[2]}} & \cdots & \frac{\partial f(\cdot)}{\partial x^{[N]}} \end{pmatrix}$$

(State)

$$\frac{\partial f(\cdot)}{\partial x^{[1]}} = \begin{pmatrix} 1 & \emptyset & -s_{\theta_{t-1}} \cdot u_{t-1}^{[1]} \\ \emptyset & 1 & c_{\theta_{t-1}} \cdot u_{t-1}^{[1]} \\ \emptyset & \emptyset & 1 \\ & & \\ \emptyset & & \emptyset \end{pmatrix} \; ; \quad \frac{\partial f(\cdot)}{\partial x^{[m]}} = \begin{pmatrix} \emptyset & \emptyset \\ \emptyset & \emptyset \\ \emptyset & \emptyset \\ \vdots & \\ 1 & \emptyset \\ \emptyset & 1 \\ \vdots & \end{pmatrix}$$

$$B_t = \frac{\partial f(\cdot)}{\partial u}\bigg|_{x=\mu_{t-1|t-1}} = \begin{pmatrix} c_{\theta_{t-1}} & \emptyset \\ s_{\theta_{t-1}} & \emptyset \\ \emptyset & 1 \\ & \\ \emptyset & \end{pmatrix}$$

→ pose block

→ Landmarks block $(N \times 2)$

(Controls)

$$h^{[n]}(x_t) = R_t^T \left(x_t^{[m]} - t_t \right)$$

↓

$$C_t^{[m]} = \frac{\partial h^{[m]}(\cdot)}{\partial x}\bigg|_{\mu_{t|t-1}} = \left(\frac{\partial h^{[m]}(\cdot)}{\partial x^{[1]}} \cdots \emptyset \cdots \frac{\partial h^{[m]}(\cdot)}{\partial x^{[m]}} \cdots \emptyset \cdots \right)$$

↳ $$\frac{\partial h^{[n]}}{\partial x^{[1]}} = \frac{\partial R_t^T}{\partial x^{[1]}} \cdot \left(x_t^{[m]} - t_t \right) - R_t^T$$

↳ $$\frac{\partial h^{[m]}}{\partial x^{[m]}} = R_t^T$$

DATA ASSOCIATION → determine the association between observations and state

something I've seen before // or something new?

$j(m) \in [1, 2, \dots, N]$ ≡ determine which measurement is generated by which landmark in the state!

↳ e.g. $j(3) = 5$

≡

measurement 3 ⇐ landmark 5

↓

for now, we assume to know the assignment $j(m)$ //

POPULATING THE MAP = REQUIRED EVERYTIME WE SEE A NEW LANDMARK

id_to_state_map = $(-1 \;\dots\; -1)$

state_to_id_map = $(-1, \dots\; -1)$

$$\mu_{t+3} = \begin{pmatrix} x_{t+3}^{[1]} \\ x_{t+3}^{[7]} \\ x_{t+3}^{[9]} \\ x_{t+3}^{[1]} \end{pmatrix}$$

landmark 1 (index in id_to_state_map) is represented in state as index 3 .

id_to_state_map = $(3, 1, -1, -1, -2, -1, 1, -1, 2, \dots, -1)$

state_to_id_map = $(7, 9, 1, -1, -1, -1, -1, \dots, \dots, -1)$

≡

element 1 in state vector represents landmark w/ ID = 7

UPDATING THE MAP | already known landmarks → use them to perform an EKF correction new landmarks → add them to state AFTER EKF correction

↓

covariance initialized with covariance of the measurement

$$\begin{pmatrix} \sigma_\theta^2 & \\ & \sigma_z^2 \end{pmatrix}$$

that originated the landmark.