Probabilistic Robotics Course

Finding Neighbors

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Neighbor Search

Given:

- a collection of vectors $\mathcal{P} = \{\mathbf{p}_n\}_{n=1:N}$ $\qquad \mathbf{p}_n \in \Re^k$
- a query vector $\mathbf{p}_q \in \Re^k$
- a distance metric $d(\mathbf{p}_n, \mathbf{p}_q) \in \Re^+$

Find either:

- the point in the collection that is the ***closest*** to the query, according to the metric

$$\mathbf{p}_i = \operatorname*{argmin}_{\mathbf{p}_n \in \mathcal{P}} d(\mathbf{p}_q, \mathbf{p}_n)$$

- the points in the collection whose distance from the query is smaller than a value ϵ

$$\mathcal{P}' = \{\mathbf{p}_i \in \mathcal{P}, d(\mathbf{p}_q, \mathbf{p}_i) < \epsilon\}$$

Distance Metrics

Examples:

- Squared Norm

$$\|\mathbf{p}_i - \mathbf{p}_j\|^2 = (\mathbf{p}_i - \mathbf{p}_j)^T (\mathbf{p}_i - \mathbf{p}_j)$$

- Omega Norm

this should look familiar

$$\|\mathbf{p}_i - \mathbf{p}_j\|^2_{\mathbf{\Omega}} = (\mathbf{p}_i - \mathbf{p}_j)^T \mathbf{\Omega} (\mathbf{p}_i - \mathbf{p}_j)$$

- Hamming distance (for binary descriptors)

Integer valued distance between two bit strings having the same dimension. Its value is the number of different bits.

example:

```
hamming(100100,101000)= 2
```

Trivial Approach

Brute Force:

compute the distance metric between the query point and *each* of the points in the collection and update the minimum.

Complexity: `O(N*cost_distance_metric)`

If we need to perform many queries, this results in unacceptable delays.

Idea: *use auxiliary search structures.*

Distance Map

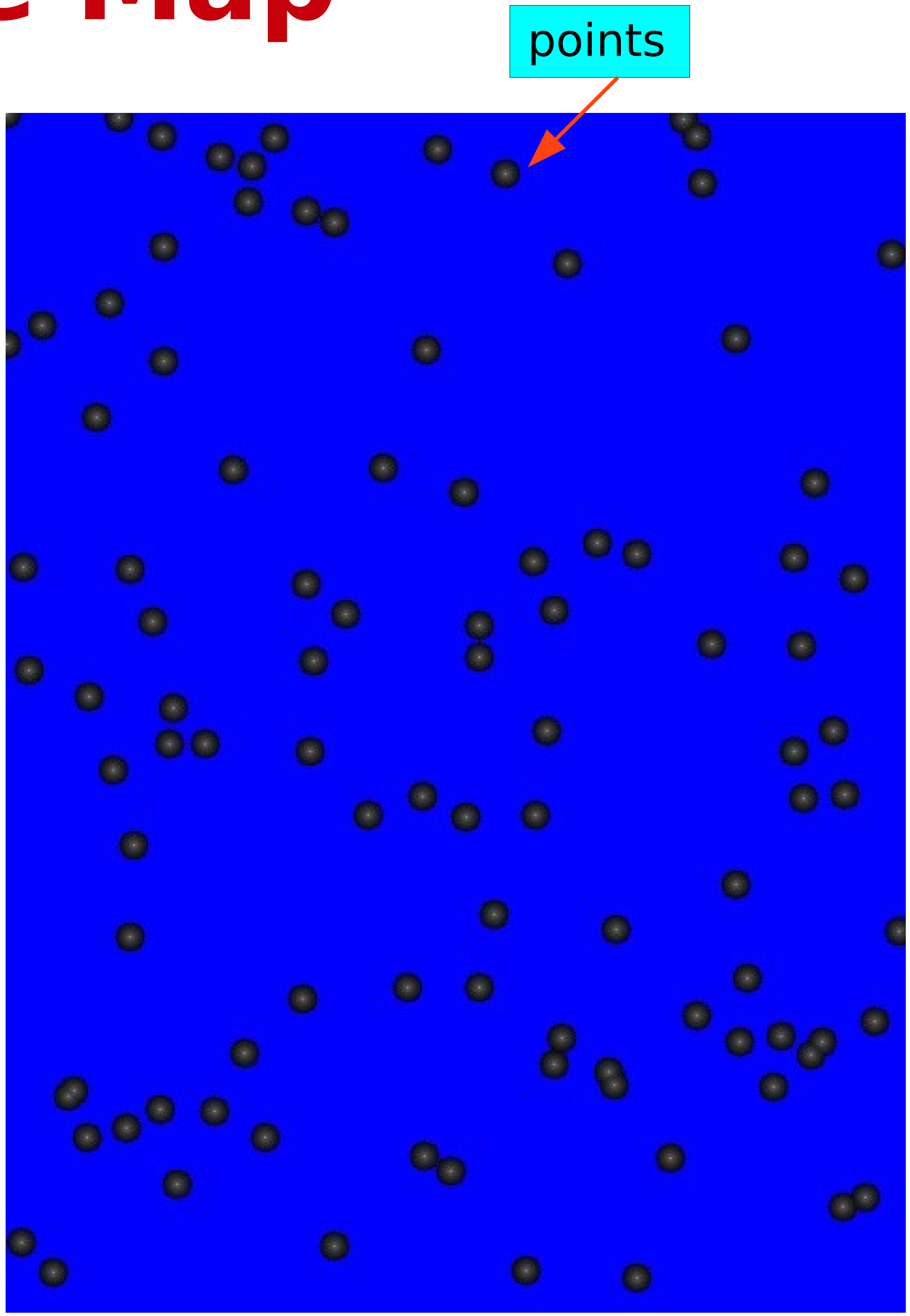
If

- the dimension of the vectors is small (< 3)
- they are spread in a relatively small region of the space

we can *pre-compute* a **grid lookup table**.

Each cell of the grid contains:

- the distance from the closest point.
- the identity of the closest point.



Distance Map

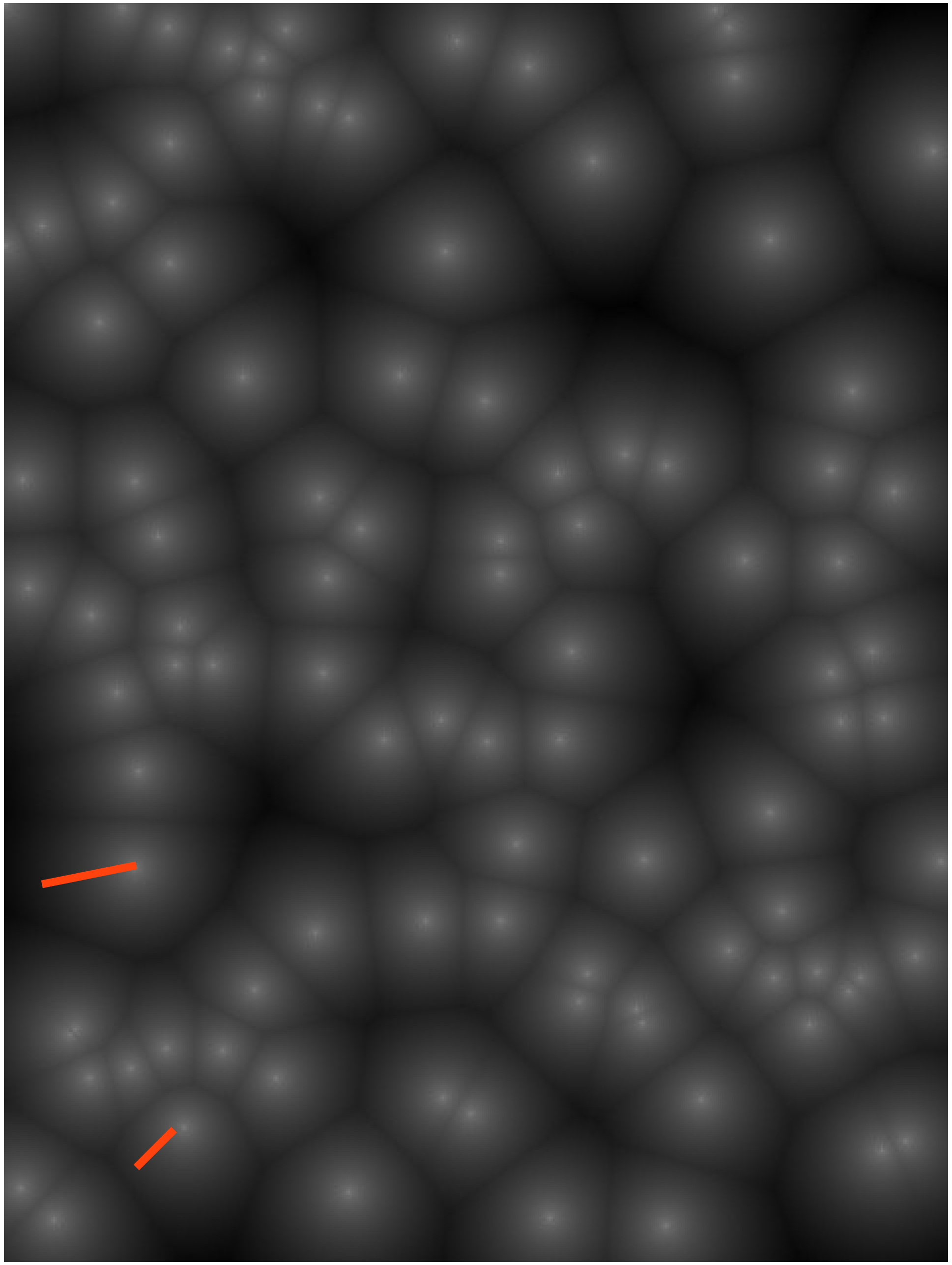
If

- the dimension of the vectors is small (`< 3`)
- they are spread in a relatively small region of the space

we can *pre-compute* a **grid lookup table**.

Each cell of the grid contains:

- the distance from the closest point `(gray value)`
- the identity of the closest point `(orange line)`



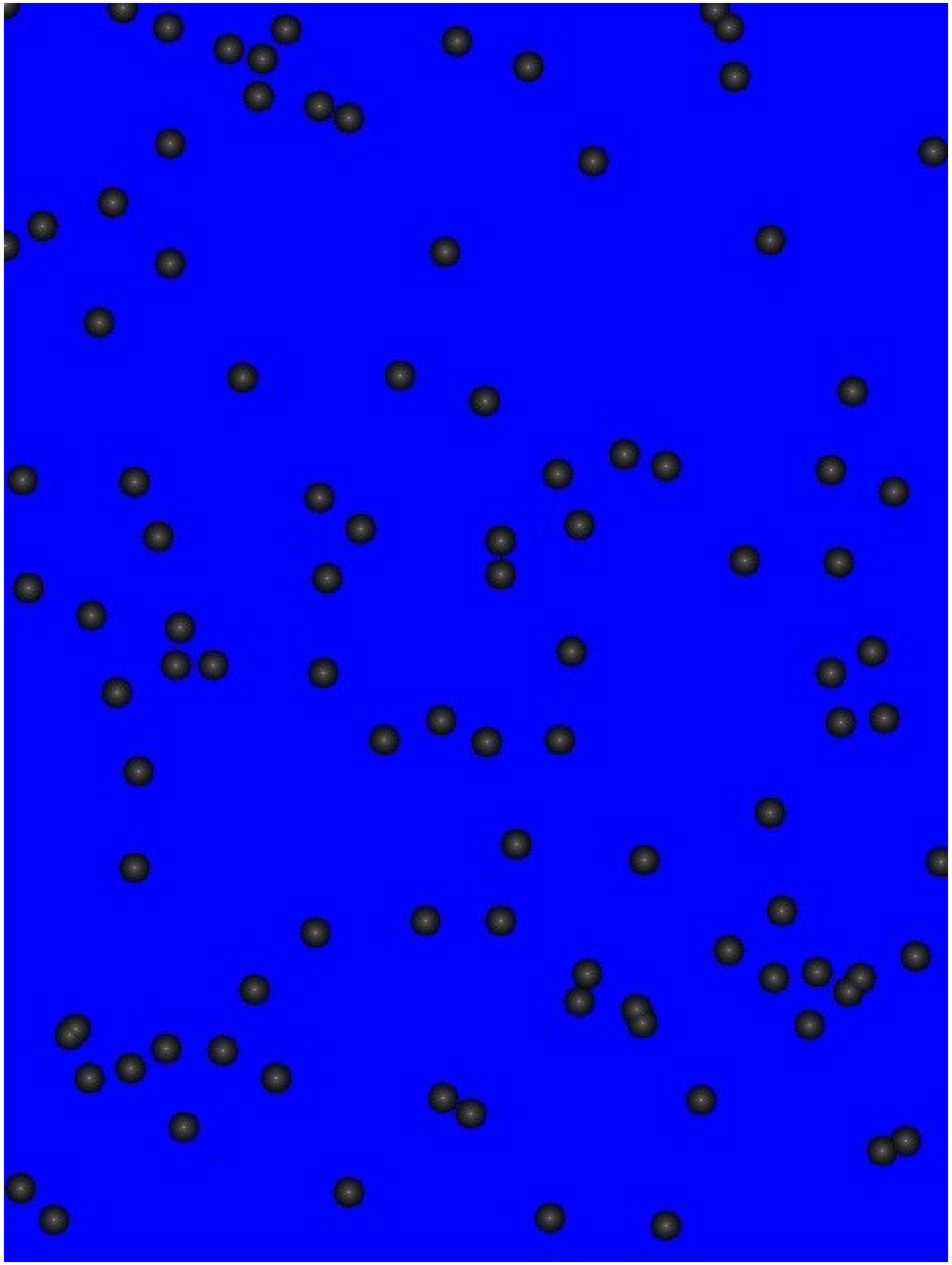
Computing the Distance Map

Modified Version of *Dijkstra algorithm.*

Each grid cell `c` stores:

- d: distance from nearest point
- `parent`: pointer to the nearest point

- We initialize the grid cells which corresponds to the points in $\mathcal{P}$ as:
  - `p.d = 0`
  - `p.parent = p`

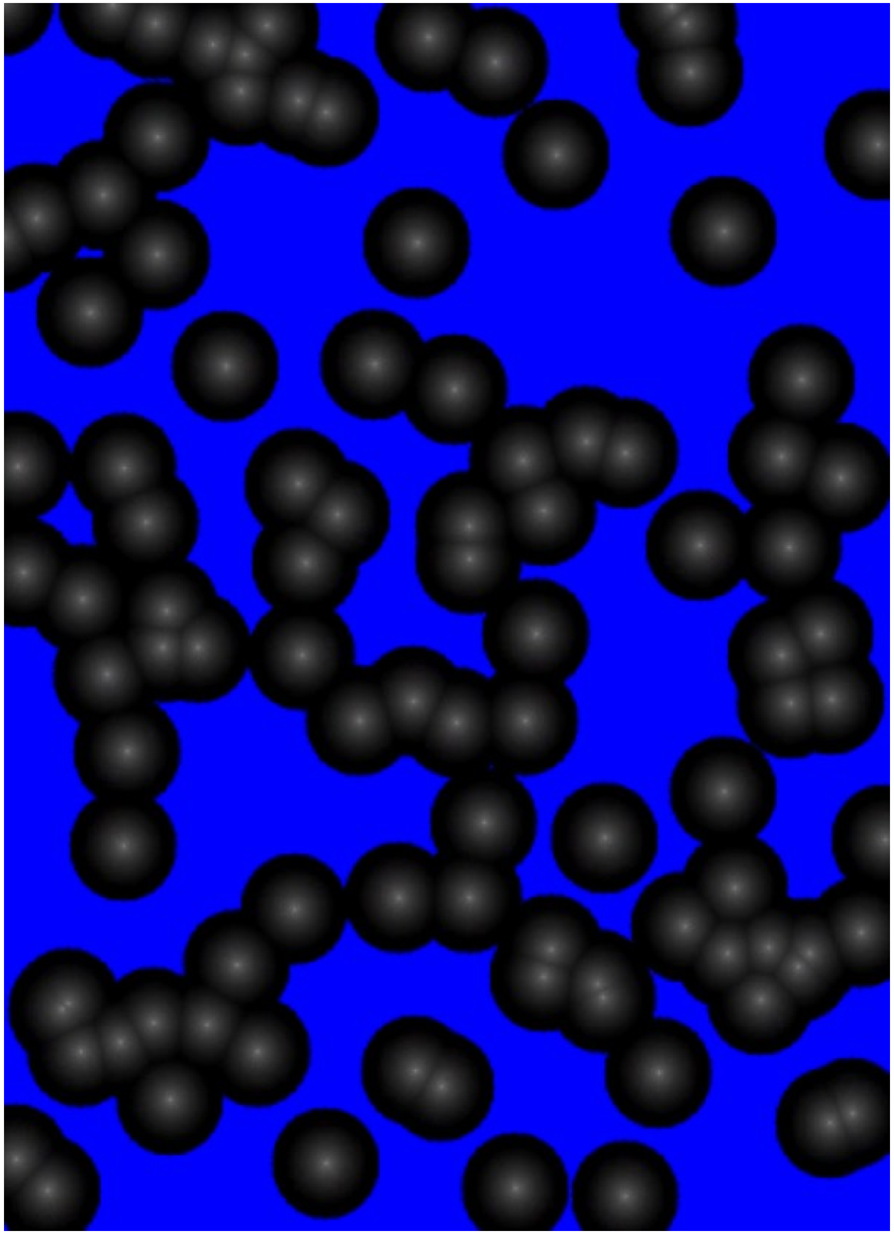


Computing the Distance Map

- We expand each initialized grid cell p, *i.e.* we take the 8 neighbors.
- For each of these expanded cells c_i

```
d_{p,c} = distance(p,c_i)
if (c_i.isEmpty()) {
    c_i.d = d_{p,c}
    c_i.parent = p.parent
} else if (c_i.d > d_{p,c}) {
        c_i.d = d_{p,c}
        c_i.parent = p.parent
}
```

- Continue iteratively for all the expanded cells.



Distance Map: Complexity

Computing the distance map requires a time

`O(grid_size*log(grid_size))`

One query on the distance map requires

`O(1)`

Good when

- we have many low dimensional points in the collection
- the grid is small
- we can tolerate small association errors

Distance Map: Heuristics

Gating:

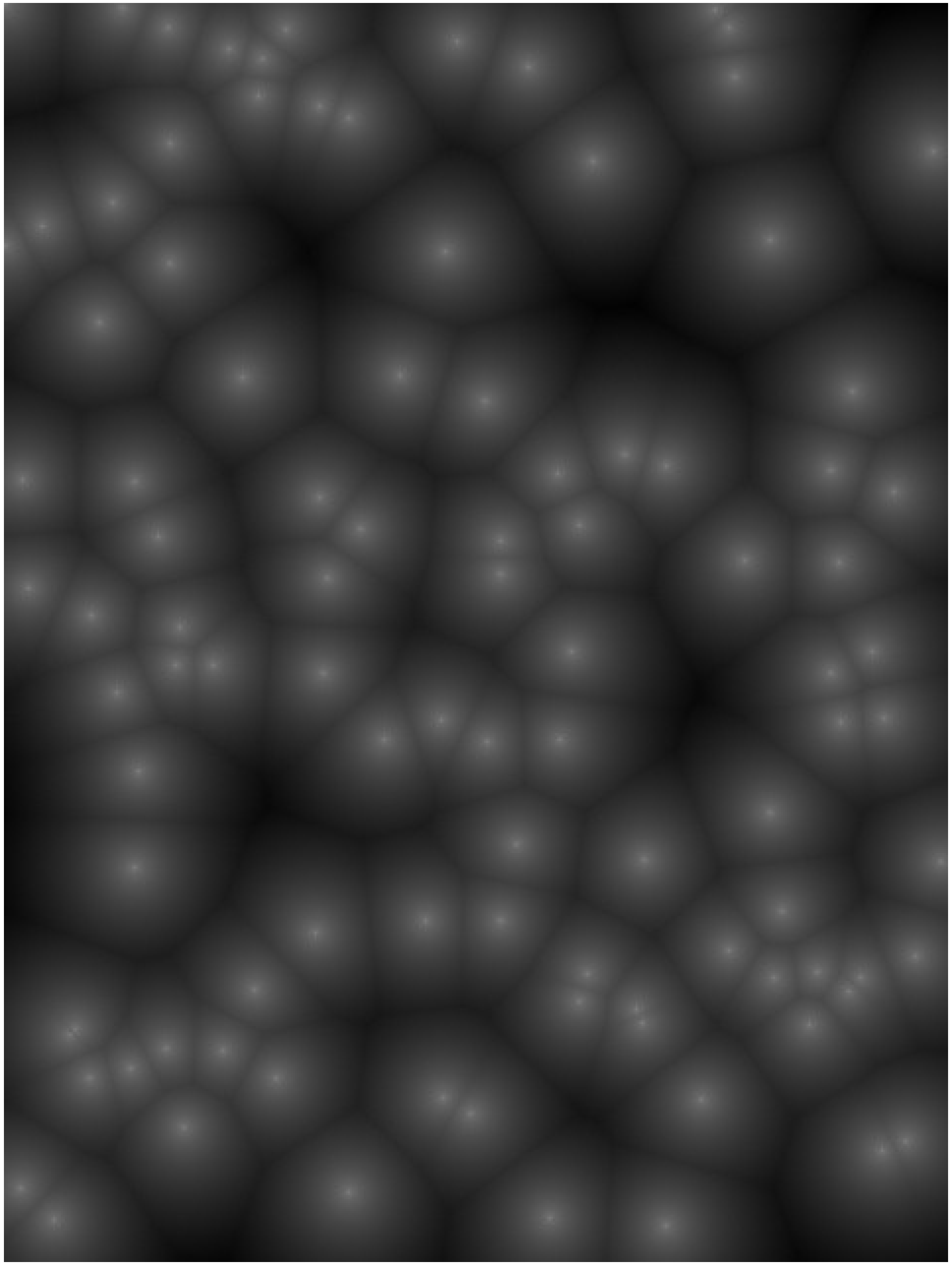
- expand up to a maximum distance

Best friends:

- construct a distance map also for the measurements and cross check

Lonely Best Friends:

- looking up a region of the distance map instead of a single point



KD-Trees

What if the points are K-dimensional, with `K` large?

- The distance map does not scale well:

  memory grows with `K`*, and so the time to construct it*

KD-Tree:

- search structure that partitions the database according to their spatial distribution
- If the tree is balanced, a search takes `log(N)` with `N` the number of points

KD-Trees

Constructing KD-trees can be done with a trivial recursion.

At each time, the set is split in two parts until the number of points in a leaf is smaller than a threshold.

Question:

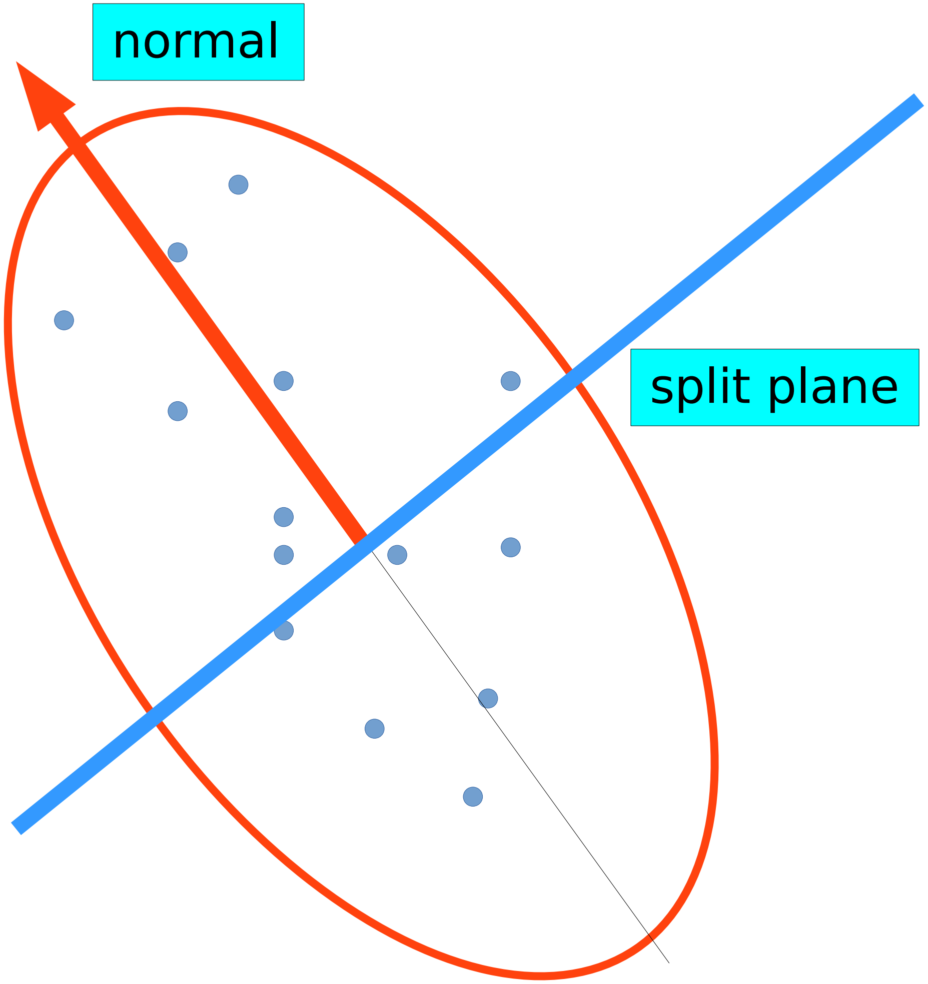
how to split?

KD-Trees: simple splitting

Consider the K-dimensional points as if they were normally distributed.

Compute the **covariance** matrix of the distribution.

Chose a splitting (hyper) plane that passes through the mean and has a normal aligned with the longest axis of the **covariance** matrix.



KD-Trees

A split plane is characterized by

- a *normal* $\mathbf{n}_i$
- a *mean* μ_i

We can check if a point lies on the one side of the plane by evaluating

$$\mathbf{n}_i^T(\mathbf{p}_q - \mu_i) > 0$$

KD-Trees

Each intermediate node of the tree contains

- The normal of the splitting plane
- The mean
- Pointers to the children nodes

A query requires traversing the tree from the top to the bottom and at each time going left or right.

The query result is an approximation, *i.e.* the tree is a heuristic that might return not the real minimum, depending on how the tree was built.

Efficient randomized variants (see ANN C++ library)

Other Tools

Projection to lower dimension

- Reduce the dimension of the query points, by projecting them for instance in 2D
- Use some easy heuristic to perform the association in the lower dimensional space

Bag of words

- Used to determine the appearance similarity of multiple points.
- An item of the search collection contains many points.

How to speed up the Localizer?

- What would you use?
- Why?
- How?
- What do you expect?

What about SLAM?

- What would you use?
- Why?
- How?
- What do you expect?