# 1 Coursework 3

#### 1.0.1 Problem 2

$$f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{if } x \neq 2 \\ 4 & \text{if } x = 2 \end{cases}, \quad g(x) = \begin{cases} x & \text{if } x < 4 \\ x^2 - 3x & \text{if } x \geq 4 \end{cases}$$

Show that $g(f(x))$ is continuous at $x = 0$.

Two approaches to the problem:

1. Find some theorem saying that if $f(x)$ is continuous somewhere and $g(x)$ is continuous somewhere, then $g(f(x))$ is continuous.
2. Start with scratch: simplify $g(f(x))$.

## 1.1 Problem 4

Given that

$$f(x+y) = f(x)f(y)$$

, show $f'(x)$ exists if $f'(0)$ exists.

Difference quotient
$= \frac{f(x+h)-f(x)}{h}(h \neq 0)$
$= \frac{f(x)f(h)-f(x)}{h}$
$= f(x) \cdot \frac{f(h)-1}{h}$

Given that $f'(0)$ exists, we know that $lim_{h \to 0} \frac{f(h)-f(0)}{h}$ is defined.

We notice that $\frac{f(h)-f(0)}{h}$ does not look the same as $\frac{f(h)-1}{h}$. Hence we want to prove that $f(0) = 1$.