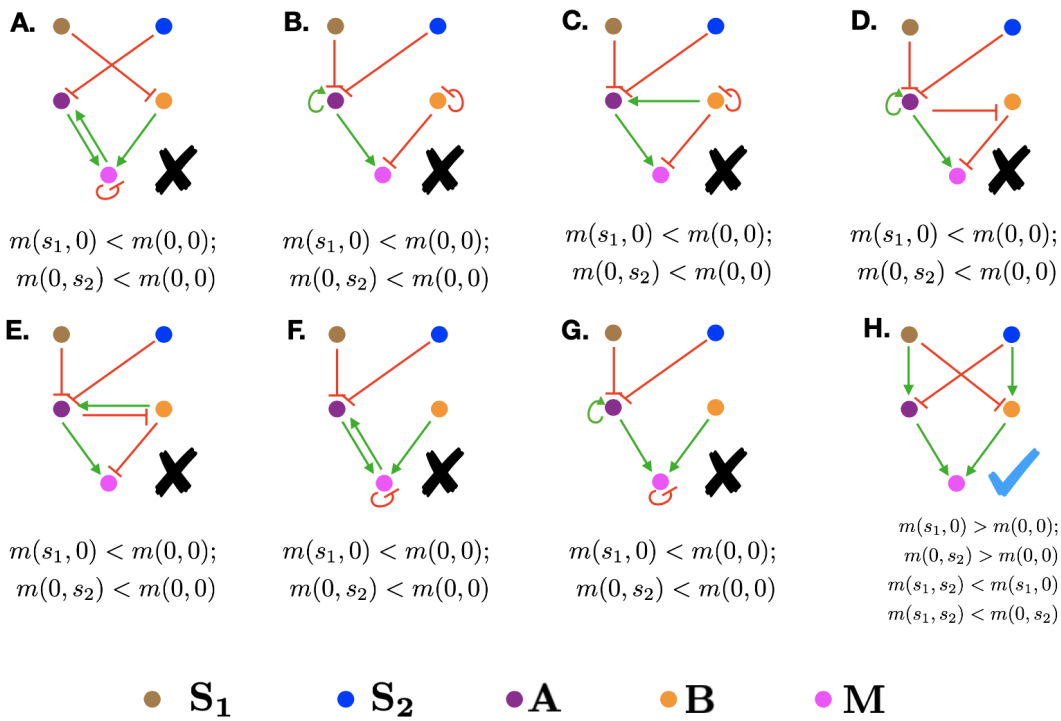
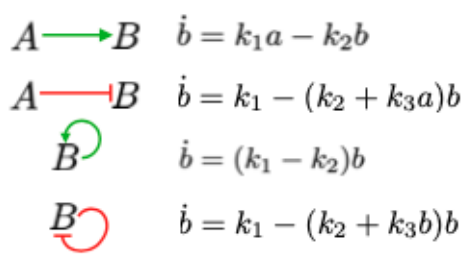
Minimal regulation network for value antagonism

In this section we analyse the following 8 networks to check which network can satisfy value antagonism-



The math is done using the following formulas-



Network A

The dynamics of A,B and M are-

In[526]:=
```
dadt = ka0 + kam m - (kda + kd2 * s2) * a;
dbdt = kb0 - (kdb + kd1 * s1) * b;
dmdt = km0 + kma * a + kmb * b - (kdm + krm * m) * m;
Solve[{dadt == 0, dbdt == 0, dmdt == 0}, {a, b, m}]
```

Out[529]=

$$\Bigg\{\Bigg\{a \to \frac{1}{-k_{da} - k_{d2}\, s_2}\Bigg(-k_{a0} - \frac{k_{am}\, k_{da}\, k_{db}\, k_{dm}}{2\,(-k_{da}\, k_{db}\, k_{rm} - k_{d1}\, k_{da}\, k_{rm}\, s_1 - k_{d2}\, k_{db}\, k_{rm}\, s_2 - k_{d1}\, k_{d2}\, k_{rm}\, s_1\, s_2)} + \frac{k_{am}^2\, k_{db}\, k_{ma}}{2\,(-k_{da}\, k_{db}\, k_{rm} - k_{d1}\, k_{da}\, k_{rm}\, s_1 - k_{d2}\, k_{db}\, k_{rm}\, s_2 - k_{d1}\, k_{d2}\, k_{rm}\, s_1\, s_2)} - \frac{k_{am}\, k_{d1}\, k_{da}\, k_{dm}\, s_1}{2\,(-k_{da}\, k_{db}\, k_{rm} - k_{d1}\, k_{da}\, k_{rm}\, s_1 - k_{d2}\, k_{db}\, k_{rm}\, s_2 - k_{d1}\, k_{d2}\, k_{rm}\, s_1\, s_2)} + \frac{k_{am}^2\, k_{d1}\, k_{ma}\, s_1}{2\,(-k_{da}\, k_{db}\, k_{rm} - k_{d1}\, k_{da}\, k_{rm}\, s_1 - k_{d2}\, k_{db}\, k_{rm}\, s_2 - k_{d1}\, k_{d2}\, k_{rm}\, s_1\, s_2)} - \frac{k_{am}\, k_{d2}\, k_{db}\, k_{dm}\, s_2}{2\,(-k_{da}\, k_{db}\, k_{rm} - k_{d1}\, k_{da}\, k_{rm}\, s_1 - k_{d2}\, k_{db}\, k_{rm}\, s_2 - k_{d1}\, k_{d2}\, k_{rm}\, s_1\, s_2)} - \frac{k_{am}\, k_{d1}\, k_{d2}\, k_{dm}\, s_1\, s_2}{2\,(-k_{da}\, k_{db}\, k_{rm} - k_{d1}\, k_{da}\, k_{rm}\, s_1 - k_{d2}\, k_{db}\, k_{rm}\, s_2 - k_{d1}\, k_{d2}\, k_{rm}\, s_1\, s_2)} + \Big(k_{am}\sqrt{\big((-k_{da}\, k_{db}\, k_{dm} + k_{am}\, k_{db}\, k_{ma} - k_{d1}\, k_{da}\, k_{dm}\, s_1 + k_{am}\, k_{d1}\, k_{ma}\, s_1 - k_{d2}\, k_{db}\, k_{dm}\, s_2 - k_{d1}\, k_{d2}\, k_{dm}\, s_1\, s_2)^2 - 4\,(k_{da}\, k_{db}\, k_{m0} + k_{a0}\, k_{db}\, k_{ma} + k_{b0}\, k_{da}\, k_{mb} + k_{d1}\, k_{da}\, k_{m0}\, s_1 + k_{a0}\, k_{d1}\, k_{ma}\, s_1 + k_{d2}\, k_{db}\, k_{m0}\, s_2 + k_{b0}\, k_{d2}\, k_{mb}\, s_2 + k_{d1}\, k_{d2}\, k_{m0}\, s_1\, s_2)\,(-k_{da}\, k_{db}\, k_{rm} - k_{d1}\, k_{da}\, k_{rm}\, s_1 - k_{d2}\, k_{db}\, k_{rm}\, s_2 - k_{d1}\, k_{d2}\, k_{rm}\, s_1\, s_2)\big)}\Big) \Big/ (2\,(-k_{da}\, k_{db}\, k_{rm} - k_{d1}\, k_{da}\, k_{rm}\, s_1 - k_{d2}\, k_{db}\, k_{rm}\, s_2 - k_{d1}\, k_{d2}\, k_{rm}\, s_1\, s_2))\Bigg),\ b \to \frac{k_{b0}}{k_{db} + k_{d1}\, s_1},$$

$$m \to \Big(k_{da}\, k_{db}\, k_{dm} - k_{am}\, k_{db}\, k_{ma} + k_{d1}\, k_{da}\, k_{dm}\, s_1 - k_{am}\, k_{d1}\, k_{ma}\, s_1 + k_{d2}\, k_{db}\, k_{dm}\, s_2 + k_{d1}\, k_{d2}\, k_{dm}\, s_1\, s_2 - \sqrt{\big((-k_{da}\, k_{db}\, k_{dm} + k_{am}\, k_{db}\, k_{ma} - k_{d1}\, k_{da}\, k_{dm}\, s_1 + k_{am}\, k_{d1}\, k_{ma}\, s_1 - k_{d2}\, k_{db}\, k_{dm}\, s_2 - k_{d1}\, k_{d2}\, k_{dm}\, s_1\, s_2)^2 - 4\,(k_{da}\, k_{db}\, k_{m0} + k_{a0}\, k_{db}\, k_{ma} + k_{b0}\, k_{da}\, k_{mb} + k_{d1}\, k_{da}\, k_{m0}\, s_1 + k_{a0}\, k_{d1}\, k_{ma}\, s_1 + k_{d2}\, k_{db}\, k_{m0}\, s_2 + k_{b0}\, k_{d2}\, k_{mb}\, s_2 + k_{d1}\, k_{d2}\, k_{m0}\, s_1\, s_2)\,(-k_{da}\, k_{db}\, k_{rm} - k_{d1}\, k_{da}\, k_{rm}\, s_1 - k_{d2}\, k_{db}\, k_{rm}\, s_2 - k_{d1}\, k_{d2}\, k_{rm}\, s_1\, s_2)\big)}\Big) \Big/ (2\,(-k_{da}\, k_{db}\, k_{rm} - k_{d1}\, k_{da}\, k_{rm}\, s_1 - k_{d2}\, k_{db}\, k_{rm}\, s_2 - k_{d1}\, k_{d2}\, k_{rm}\, s_1\, s_2))\Bigg\},$$

$$\Bigg\{a \to \frac{1}{-k_{da} - k_{d2}\, s_2}\Bigg(-k_{a0} - \frac{k_{am}\, k_{da}\, k_{db}\, k_{dm}}{2\,(-k_{da}\, k_{db}\, k_{rm} - k_{d1}\, k_{da}\, k_{rm}\, s_1 - k_{d2}\, k_{db}\, k_{rm}\, s_2 - k_{d1}\, k_{d2}\, k_{rm}\, s_1\, s_2)} + \frac{k_{am}^2\, k_{db}\, k_{ma}}{2\,(-k_{da}\, k_{db}\, k_{rm} - k_{d1}\, k_{da}\, k_{rm}\, s_1 - k_{d2}\, k_{db}\, k_{rm}\, s_2 - k_{d1}\, k_{d2}\, k_{rm}\, s_1\, s_2)} - \frac{k_{am}\, k_{d1}\, k_{da}\, k_{dm}\, s_1}{2\,(-k_{da}\, k_{db}\, k_{rm} - k_{d1}\, k_{da}\, k_{rm}\, s_1 - k_{d2}\, k_{db}\, k_{rm}\, s_2 - k_{d1}\, k_{d2}\, k_{rm}\, s_1\, s_2)} + \frac{k_{am}^2\, k_{d1}\, k_{ma}\, s_1}{2\,(-k_{da}\, k_{db}\, k_{rm} - k_{d1}\, k_{da}\, k_{rm}\, s_1 - k_{d2}\, k_{db}\, k_{rm}\, s_2 - k_{d1}\, k_{d2}\, k_{rm}\, s_1\, s_2)} - \frac{k_{am}\, k_{d2}\, k_{db}\, k_{dm}\, s_2}{2\,(-k_{da}\, k_{db}\, k_{rm} - k_{d1}\, k_{da}\, k_{rm}\, s_1 - k_{d2}\, k_{db}\, k_{rm}\, s_2 - k_{d1}\, k_{d2}\, k_{rm}\, s_1\, s_2)} -$$

$$\frac{k_{am}\, k_{d1}\, k_{d2}\, k_{dm}\, s_1\, s_2}{2\,(-k_{da}\, k_{db}\, k_{rm} - k_{d1}\, k_{da}\, k_{rm}\, s_1 - k_{d2}\, k_{db}\, k_{rm}\, s_2 - k_{d1}\, k_{d2}\, k_{rm}\, s_1\, s_2)} -$$

$$\Big(k_{am}\sqrt{\big((-k_{da}\, k_{db}\, k_{dm} + k_{am}\, k_{db}\, k_{ma} - k_{d1}\, k_{da}\, k_{dm}\, s_1 + k_{am}\, k_{d1}\, k_{ma}\, s_1 - k_{d2}\, k_{db}\, k_{dm}\, s_2 - k_{d1}\, k_{d2}\, k_{dm}\, s_1\, s_2)^2 - 4\,(k_{da}\, k_{db}\, k_{m0} + k_{a0}\, k_{db}\, k_{ma} + k_{b0}\, k_{da}\, k_{mb} + k_{d1}\, k_{da}\, k_{m0}\, s_1 + k_{a0}\, k_{d1}\, k_{ma}\, s_1 + k_{d2}\, k_{db}\, k_{m0}\, s_2 + k_{b0}\, k_{d2}\, k_{mb}\, s_2 + k_{d1}\, k_{d2}\, k_{m0}\, s_1\, s_2)\,(-k_{da}\, k_{db}\, k_{rm} - k_{d1}\, k_{da}\, k_{rm}\, s_1 - k_{d2}\, k_{db}\, k_{rm}\, s_2 - k_{d1}\, k_{d2}\, k_{rm}\, s_1\, s_2)\big)}\Big) \Big/ \big(2\,(-k_{da}\, k_{db}\, k_{rm} - k_{d1}\, k_{da}\, k_{rm}\, s_1 - k_{d2}\, k_{db}\, k_{rm}\, s_2 - k_{d1}\, k_{d2}\, k_{rm}\, s_1\, s_2)\big)\Big),\ b \to \frac{k_{b0}}{k_{db} + k_{d1}\, s_1},$$

$$m \to \Big(k_{da}\, k_{db}\, k_{dm} - k_{am}\, k_{db}\, k_{ma} + k_{d1}\, k_{da}\, k_{dm}\, s_1 - k_{am}\, k_{d1}\, k_{ma}\, s_1 + k_{d2}\, k_{db}\, k_{dm}\, s_2 + k_{d1}\, k_{d2}\, k_{dm}\, s_1\, s_2 + \sqrt{\big((-k_{da}\, k_{db}\, k_{dm} + k_{am}\, k_{db}\, k_{ma} - k_{d1}\, k_{da}\, k_{dm}\, s_1 + k_{am}\, k_{d1}\, k_{ma}\, s_1 - k_{d2}\, k_{db}\, k_{dm}\, s_2 - k_{d1}\, k_{d2}\, k_{dm}\, s_1\, s_2)^2 - 4\,(k_{da}\, k_{db}\, k_{m0} + k_{a0}\, k_{db}\, k_{ma} + k_{b0}\, k_{da}\, k_{mb} + k_{d1}\, k_{da}\, k_{m0}\, s_1 + k_{a0}\, k_{d1}\, k_{ma}\, s_1 + k_{d2}\, k_{db}\, k_{m0}\, s_2 + k_{b0}\, k_{d2}\, k_{mb}\, s_2 + k_{d1}\, k_{d2}\, k_{m0}\, s_1\, s_2)\,(-k_{da}\, k_{db}\, k_{rm} - k_{d1}\, k_{da}\, k_{rm}\, s_1 - k_{d2}\, k_{db}\, k_{rm}\, s_2 - k_{d1}\, k_{d2}\, k_{rm}\, s_1\, s_2)\big)}\Big) \Big/ \big(2\,(-k_{da}\, k_{db}\, k_{rm} - k_{d1}\, k_{da}\, k_{rm}\, s_1 - k_{d2}\, k_{db}\, k_{rm}\, s_2 - k_{d1}\, k_{d2}\, k_{rm}\, s_1\, s_2)\big)\Big\}\Big\}$$

For plotting we set all parameters to 1.

In[530]:=
```
dadt = 1 + m - (1 + s2) * a;
dbdt = 1 - (1 + s1) * b;
dmdt = 1 + a + b - (1 + m) * m;
Solve[{dadt == 0, dbdt == 0, dmdt == 0}, {a, b, m}]
```

Out[533]=

$$\Big\{\Big\{a \to \frac{-1 + \frac{s_2}{2\times(1+s_1+s_2+s_1\,s_2)} + \frac{s_1\,s_2}{2\times(1+s_1+s_2+s_1\,s_2)} + \frac{\sqrt{1+s_1}\,\sqrt{12+8\,s_1+20\,s_2+12\,s_1\,s_2+9\,s_2^2+5\,s_1\,s_2^2}}{2\times(1+s_1+s_2+s_1\,s_2)}}{-1 - s_2},$$

$$b \to \frac{1}{1+s_1},\ m \to \frac{-s_2 - s_1\,s_2 - \sqrt{1+s_1}\,\sqrt{12+8\,s_1+20\,s_2+12\,s_1\,s_2+9\,s_2^2+5\,s_1\,s_2^2}}{2\times(1+s_1+s_2+s_1\,s_2)}\Big\},$$

$$\Big\{a \to \frac{-1 + \frac{s_2}{2\times(1+s_1+s_2+s_1\,s_2)} + \frac{s_1\,s_2}{2\times(1+s_1+s_2+s_1\,s_2)} - \frac{\sqrt{1+s_1}\,\sqrt{12+8\,s_1+20\,s_2+12\,s_1\,s_2+9\,s_2^2+5\,s_1\,s_2^2}}{2\times(1+s_1+s_2+s_1\,s_2)}}{-1 - s_2},$$

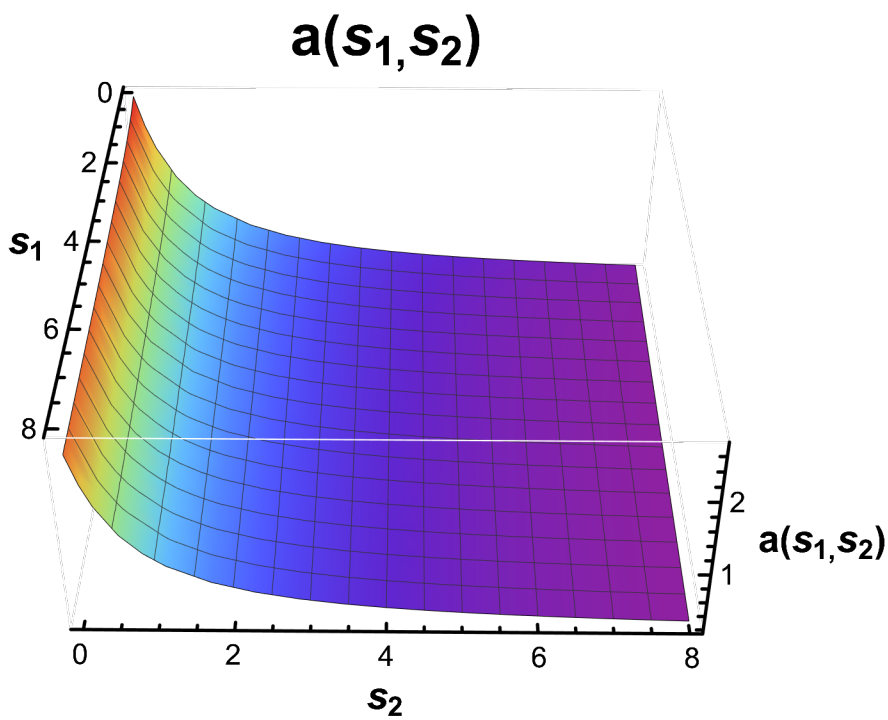
$$b \to \frac{1}{1+s_1},\ m \to \frac{-s_2 - s_1\,s_2 + \sqrt{1+s_1}\,\sqrt{12+8\,s_1+20\,s_2+12\,s_1\,s_2+9\,s_2^2+5\,s_1\,s_2^2}}{2\times(1+s_1+s_2+s_1\,s_2)}\Big\}\Big\}$$

Expressions and surface map in steady state-

In[534]:=
$$aP = \frac{-1 + \frac{s_2}{2\times(1+s_1+s_2+s_1 s_2)} + \frac{s_1 s_2}{2\times(1+s_1+s_2+s_1 s_2)} - \frac{\sqrt{1+s_1}\ \sqrt{12+8 s_1+20 s_2+12 s_1 s_2+9 s_2^2+5 s_1 s_2^2}}{2\times(1+s_1+s_2+s_1 s_2)}}{-1 - s_2};$$

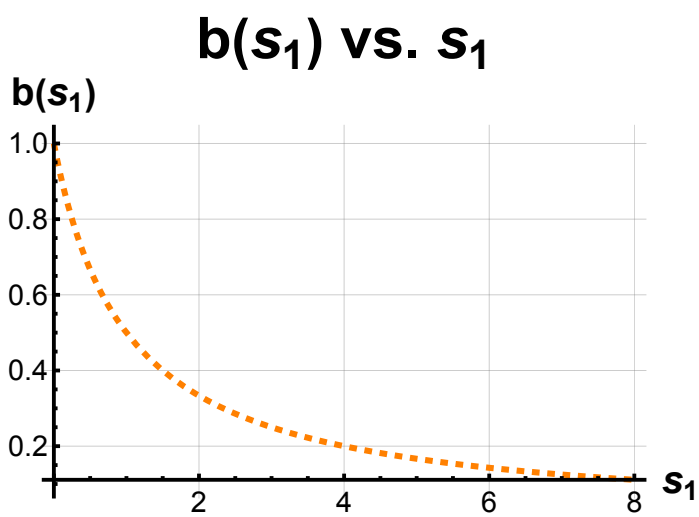
```
Plot3D[aP, {s1, 0, 8}, {s2, 0, 8}, AxesLabel →
  {Style["s1", Bold, 20, FontColor → Black], Style["s2", Bold, 20, FontColor → Black],
   Style["a(s1,s2)", Bold, 20, FontColor → Black]},
 PlotLabel → Style["a(s1,s2)", Bold, 30, FontColor → Black],
 ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
 TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

Out[535]=



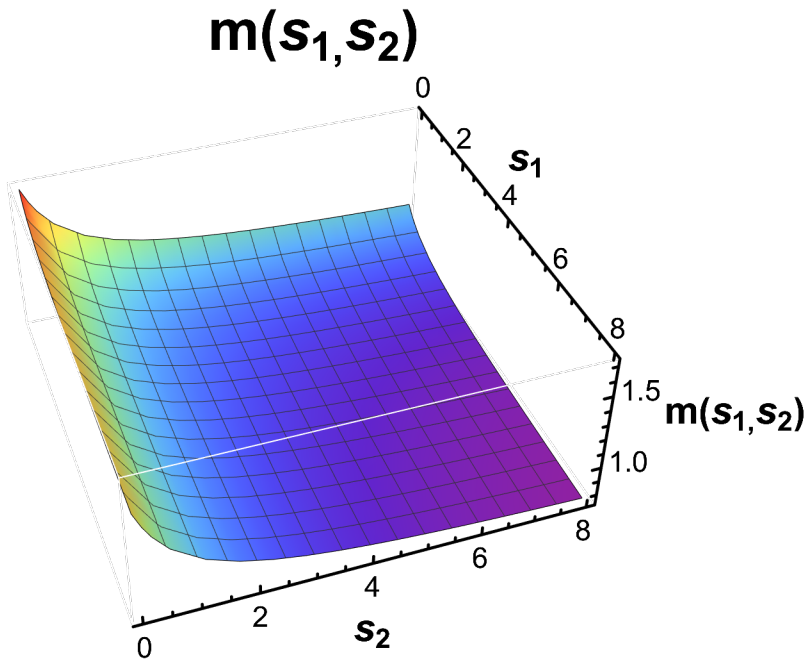
In[536]:=
```
bp = 1/(1 + s1);
plotset = {PlotStyle → Thick, AxesStyle → Thick,
   FrameStyle → Thick, TicksStyle → Directive[Black, 15]};
Plot[bp, {s1, 0, 8},
 AxesLabel → {Style["s1", Bold, 20, FontColor → Black],
   Style["b(s1)", Bold, 20, FontColor → Black]},
 PlotLabel → Style["b(s1) vs. s1", Bold, 30, FontColor → Black],
 GridLines → Automatic,
 PlotStyle → {Orange, Dashed, Thickness[0.01]}, Evaluate@plotset]
```

Out[538]=



In[539]:=
```
mP = (-s2 - s1 s2 + Sqrt[1 + s1] Sqrt[12 + 8 s1 + 20 s2 + 12 s1 s2 + 9 s2^2 + 5 s1 s2^2]) / (2 × (1 + s1 + s2 + s1 s2));
Plot3D[mP, {s1, 0, 8}, {s2, 0, 8}, AxesLabel →
  {Style["s1", Bold, 20, FontColor → Black], Style["s2", Bold, 20, FontColor → Black],
   Style["m(s1,s2)", Bold, 20, FontColor → Black]},
 PlotLabel → Style["m(s1,s2)", Bold, 30, FontColor → Black],
 ColorFunction → "Rainbow", AxesStyle → Thickness[0.005],
 BoxStyle → GrayLevel[2], TicksStyle → Directive[Black, 15], ImageSize → Large]
```

Out[540]=



So we will have $m(s_1, 0)$<m(0,0) & $m(0, s_2)$<m(0,0). So network A fails.

Network B

Dynamics-

In[541]:=
```
dadt = ka0 + kaa * a - (kda + kd1 * s1 + kd2 * s2) * a;
dbdt = kb0 - (kdb + kbb * b) * b;
dmdt = km0 + kma * a - (kdm + kmb * b) * m;
Solve[{dadt == 0, dbdt == 0, dmdt == 0}, {a, b, m}]
```

Out[544]=

$$\left\{\left\{a \to \frac{k_{a0}}{-k_{aa}+k_{da}+k_{d1}\,s_1+k_{d2}\,s_2},\ b \to -\frac{k_{db}-\sqrt{4\,k_{b0}\,k_{bb}+k_{db}^2}}{2\,k_{bb}},\right.\right.$$
$$m \to \left(-2\,k_{aa}\,k_{bb}\,k_{dm}\,k_{m0}+2\,k_{bb}\,k_{da}\,k_{dm}\,k_{m0}+2\,k_{a0}\,k_{bb}\,k_{dm}\,k_{ma}+k_{aa}\,k_{db}\,k_{m0}\,k_{mb}-k_{da}\,k_{db}\,k_{m0}\,k_{mb}+\right.$$
$$k_{aa}\,\sqrt{4\,k_{b0}\,k_{bb}+k_{db}^2}\,k_{m0}\,k_{mb}-k_{da}\,\sqrt{4\,k_{b0}\,k_{bb}+k_{db}^2}\,k_{m0}\,k_{mb}-k_{a0}\,k_{db}\,k_{ma}\,k_{mb}-$$
$$k_{a0}\,\sqrt{4\,k_{b0}\,k_{bb}+k_{db}^2}\,k_{ma}\,k_{mb}+2\,k_{bb}\,k_{d1}\,k_{dm}\,k_{m0}\,s_1-k_{d1}\,k_{db}\,k_{m0}\,k_{mb}\,s_1-k_{d1}\,\sqrt{4\,k_{b0}\,k_{bb}+k_{db}^2}$$
$$\left.k_{m0}\,k_{mb}\,s_1+2\,k_{bb}\,k_{d2}\,k_{dm}\,k_{m0}\,s_2-k_{d2}\,k_{db}\,k_{m0}\,k_{mb}\,s_2-k_{d2}\,\sqrt{4\,k_{b0}\,k_{bb}+k_{db}^2}\,k_{m0}\,k_{mb}\,s_2\right)\Big/$$
$$\left(2\,\left(-k_{aa}\,k_{bb}\,k_{dm}^2+k_{bb}\,k_{da}\,k_{dm}^2+k_{aa}\,k_{db}\,k_{dm}\,k_{mb}-k_{da}\,k_{db}\,k_{dm}\,k_{mb}+k_{aa}\,k_{b0}\,k_{mb}^2-\right.\right.$$
$$k_{b0}\,k_{da}\,k_{mb}^2+k_{bb}\,k_{d1}\,k_{dm}^2\,s_1-k_{d1}\,k_{db}\,k_{dm}\,k_{mb}\,s_1-k_{b0}\,k_{d1}\,k_{mb}^2\,s_1+$$
$$\left.\left.\left.k_{bb}\,k_{d2}\,k_{dm}^2\,s_2-k_{d2}\,k_{db}\,k_{dm}\,k_{mb}\,s_2-k_{b0}\,k_{d2}\,k_{mb}^2\,s_2\right)\right)\right\},$$

$$\left\{a \to \frac{k_{a0}}{-k_{aa}+k_{da}+k_{d1}\,s_1+k_{d2}\,s_2},\ b \to -\frac{k_{db}+\sqrt{4\,k_{b0}\,k_{bb}+k_{db}^2}}{2\,k_{bb}},\right.$$
$$m \to \left(-2\,k_{aa}\,k_{bb}\,k_{dm}\,k_{m0}+2\,k_{bb}\,k_{da}\,k_{dm}\,k_{m0}+2\,k_{a0}\,k_{bb}\,k_{dm}\,k_{ma}+k_{aa}\,k_{db}\,k_{m0}\,k_{mb}-k_{da}\,k_{db}\,k_{m0}\,k_{mb}-\right.$$
$$k_{aa}\,\sqrt{4\,k_{b0}\,k_{bb}+k_{db}^2}\,k_{m0}\,k_{mb}+k_{da}\,\sqrt{4\,k_{b0}\,k_{bb}+k_{db}^2}\,k_{m0}\,k_{mb}-k_{a0}\,k_{db}\,k_{ma}\,k_{mb}+$$
$$k_{a0}\,\sqrt{4\,k_{b0}\,k_{bb}+k_{db}^2}\,k_{ma}\,k_{mb}+2\,k_{bb}\,k_{d1}\,k_{dm}\,k_{m0}\,s_1-k_{d1}\,k_{db}\,k_{m0}\,k_{mb}\,s_1+k_{d1}\,\sqrt{4\,k_{b0}\,k_{bb}+k_{db}^2}$$
$$\left.k_{m0}\,k_{mb}\,s_1+2\,k_{bb}\,k_{d2}\,k_{dm}\,k_{m0}\,s_2-k_{d2}\,k_{db}\,k_{m0}\,k_{mb}\,s_2+k_{d2}\,\sqrt{4\,k_{b0}\,k_{bb}+k_{db}^2}\,k_{m0}\,k_{mb}\,s_2\right)\Big/$$
$$\left(2\,\left(-k_{aa}\,k_{bb}\,k_{dm}^2+k_{bb}\,k_{da}\,k_{dm}^2+k_{aa}\,k_{db}\,k_{dm}\,k_{mb}-k_{da}\,k_{db}\,k_{dm}\,k_{mb}+k_{aa}\,k_{b0}\,k_{mb}^2-\right.\right.$$
$$k_{b0}\,k_{da}\,k_{mb}^2+k_{bb}\,k_{d1}\,k_{dm}^2\,s_1-k_{d1}\,k_{db}\,k_{dm}\,k_{mb}\,s_1-k_{b0}\,k_{d1}\,k_{mb}^2\,s_1+$$
$$\left.\left.\left.k_{bb}\,k_{d2}\,k_{dm}^2\,s_2-k_{d2}\,k_{db}\,k_{dm}\,k_{mb}\,s_2-k_{b0}\,k_{d2}\,k_{mb}^2\,s_2\right)\right)\right\}\right\}$$

For plotting we set all parameters to 1.

In[545]:=
```
dadt = 1 + 1 * a - (2 + 1 * s1 + 1 * s2) * a;
dbdt = 1 - (1 + 1 * b) * b;
dmdt = 1 + 1 * a - (1 + 1 * b) * m;
Solve[{dadt == 0, dbdt == 0, dmdt == 0}, {a, b, m}]
```

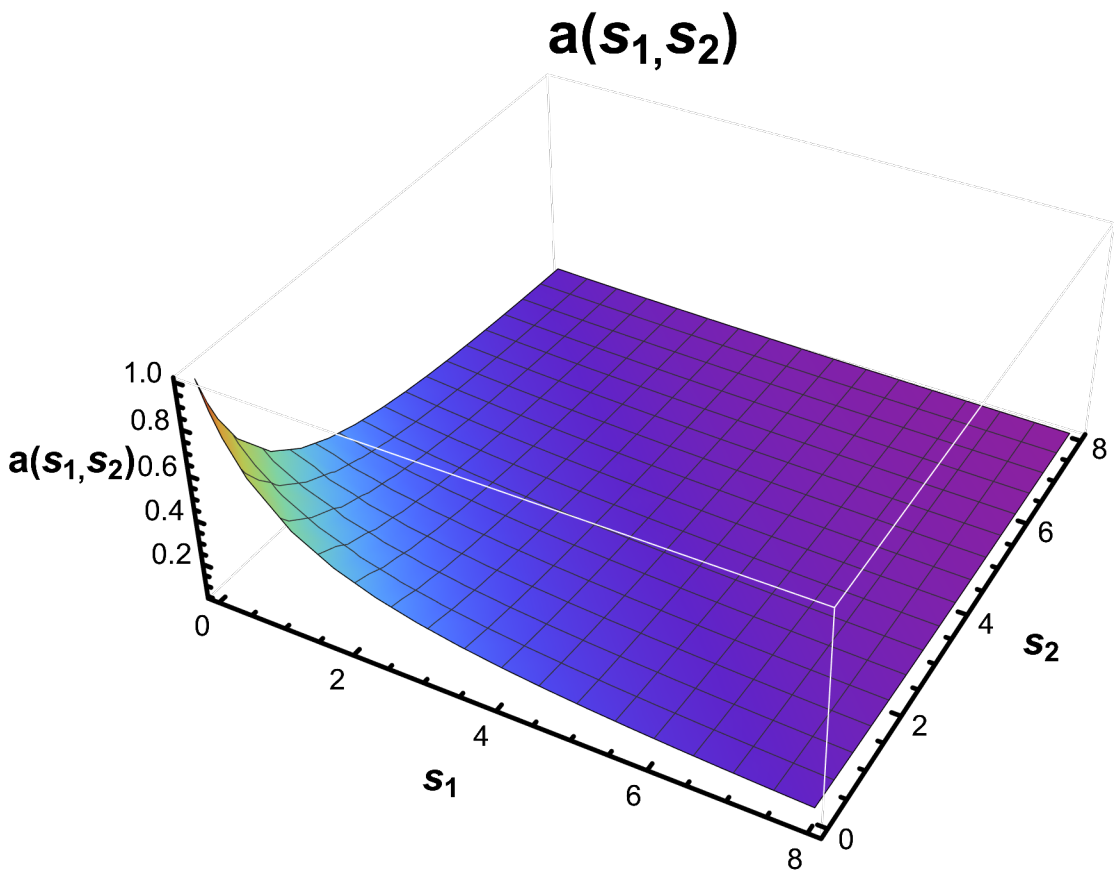
Out[548]=

$$\left\{\left\{a \to \frac{1}{1+s_1+s_2},\ b \to \frac{1}{2}\times\left(-1-\sqrt{5}\right),\ m \to \frac{-2-2\,\sqrt{5}-s_1-\sqrt{5}\,s_1-s_2-\sqrt{5}\,s_2}{2\times(1+s_1+s_2)}\right\},\right.$$
$$\left.\left\{a \to \frac{1}{1+s_1+s_2},\ b \to \frac{1}{2}\times\left(-1+\sqrt{5}\right),\ m \to \frac{-2+2\,\sqrt{5}-s_1+\sqrt{5}\,s_1-s_2+\sqrt{5}\,s_2}{2\times(1+s_1+s_2)}\right\}\right\}$$

Expressions in steady states and surface maps-

In[549]:=
```
aP = 1/(1 + s1 + s2);
Plot3D[aP, {s1, 0, 8}, {s2, 0, 8}, AxesLabel →
  {Style["s1", Bold, 20, FontColor → Black], Style["s2", Bold, 20, FontColor → Black],
   Style["a(s1,s2)", Bold, 20, FontColor → Black]},
 PlotLabel → Style["a(s1,s2)", Bold, 30, FontColor → Black],
 ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
 TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

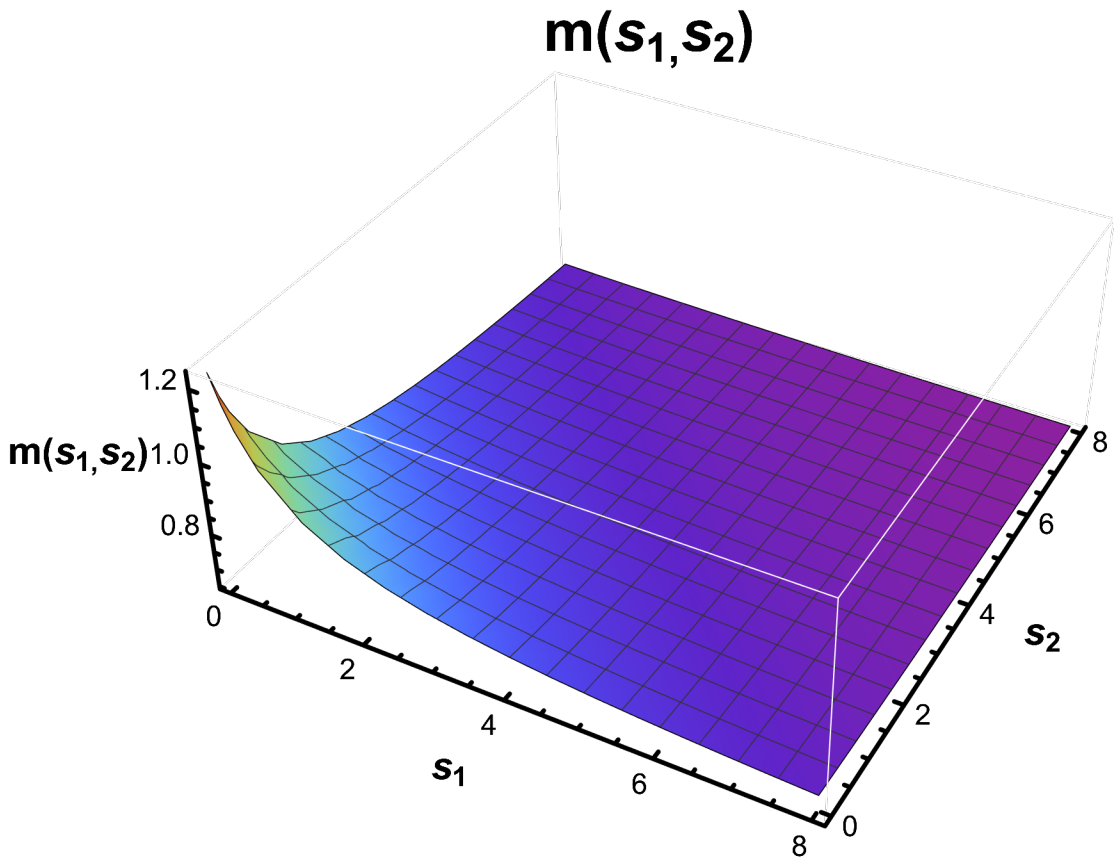
Out[550]=



b is constant.

In[551]:=
```
mP = (-2 + 2 √5 - s1 + √5 s1 - s2 + √5 s2)/(2 × (1 + s1 + s2));
Plot3D[mP, {s1, 0, 8}, {s2, 0, 8}, AxesLabel →
  {Style["s1", Bold, 20, FontColor → Black], Style["s2", Bold, 20, FontColor → Black],
   Style["m(s1,s2)", Bold, 20, FontColor → Black]},
 PlotLabel → Style["m(s1,s2)", Bold, 30, FontColor → Black],
 ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
 TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

Out[552]=



So we will have $m(s_1, 0) < m(0, 0)$ & $m(0, s_2) < m(0, 0)$. So network B fails.

Network C

Dynamics-

In[553]:=
```
dadt = ka0 + kab * b - (kda + kd1 * s1 + kd2 * s2) * a;
dbdt = kb0 - (kdb + krb * b) * b;
dmdt = km0 + kma * a - (kdm + kmb * b) * m;
Solve[{dadt == 0, dbdt == 0, dmdt == 0}, {a, b, m}]
```

Out[556]=

$$\left\{\left\{a \to \frac{2\,k_{a0} - \frac{k_{ab}\,k_{db}}{k_{rb}} + \frac{k_{ab}\,\sqrt{k_{db}^2 + 4\,k_{b0}\,k_{rb}}}{k_{rb}}}{2\,(k_{da} + k_{d1}\,s_1 + k_{d2}\,s_2)},\ b \to -\frac{k_{db} - \sqrt{k_{db}^2 + 4\,k_{b0}\,k_{rb}}}{2\,k_{rb}},\right.\right.$$

$$m \to \Big(k_{ab}\,k_{db}\,k_{dm}\,k_{ma} + k_{da}\,k_{db}\,k_{m0}\,k_{mb} + 2\,k_{ab}\,k_{b0}\,k_{ma}\,k_{mb} + k_{a0}\,k_{db}\,k_{ma}\,k_{mb} - 2\,k_{da}\,k_{dm}\,k_{m0}\,k_{rb} - 2\,k_{a0}\,k_{dm}\,k_{ma}\,k_{rb} - k_{ab}\,k_{dm}\,k_{ma}\,\sqrt{k_{db}^2 + 4\,k_{b0}\,k_{rb}} + k_{da}\,k_{m0}\,k_{mb}\,\sqrt{k_{db}^2 + 4\,k_{b0}\,k_{rb}} + k_{a0}\,k_{ma}\,k_{mb}\,\sqrt{k_{db}^2 + 4\,k_{b0}\,k_{rb}} + k_{d1}\,k_{db}\,k_{m0}\,k_{mb}\,s_1 - 2\,k_{d1}\,k_{dm}\,k_{m0}\,k_{rb}\,s_1 + k_{d1}\,k_{m0}\,k_{mb}\,\sqrt{k_{db}^2 + 4\,k_{b0}\,k_{rb}}\,s_1 + k_{d2}\,k_{db}\,k_{m0}\,k_{mb}\,s_2 - 2\,k_{d2}\,k_{dm}\,k_{m0}\,k_{rb}\,s_2 + k_{d2}\,k_{m0}\,k_{mb}\,\sqrt{k_{db}^2 + 4\,k_{b0}\,k_{rb}}\,s_2\Big) \Big/ \Big(2\,\big(k_{da}\,k_{db}\,k_{dm}\,k_{mb} + k_{b0}\,k_{da}\,k_{mb}^2 - k_{da}\,k_{dm}^2\,k_{rb} + k_{d1}\,k_{db}\,k_{dm}\,k_{mb}\,s_1 + k_{b0}\,k_{d1}\,k_{mb}^2\,s_1 - k_{d1}\,k_{dm}^2\,k_{rb}\,s_1 + k_{d2}\,k_{db}\,k_{dm}\,k_{mb}\,s_2 + k_{b0}\,k_{d2}\,k_{mb}^2\,s_2 - k_{d2}\,k_{dm}^2\,k_{rb}\,s_2\big)\Big)\Big\},$$

$$\left\{a \to \frac{2\,k_{a0} - \frac{k_{ab}\,k_{db}}{k_{rb}} - \frac{k_{ab}\,\sqrt{k_{db}^2 + 4\,k_{b0}\,k_{rb}}}{k_{rb}}}{2\,(k_{da} + k_{d1}\,s_1 + k_{d2}\,s_2)},\ b \to -\frac{k_{db} + \sqrt{k_{db}^2 + 4\,k_{b0}\,k_{rb}}}{2\,k_{rb}},\right.$$

$$m \to \Big(k_{ab}\,k_{db}\,k_{dm}\,k_{ma} + k_{da}\,k_{db}\,k_{m0}\,k_{mb} + 2\,k_{ab}\,k_{b0}\,k_{ma}\,k_{mb} + k_{a0}\,k_{db}\,k_{ma}\,k_{mb} - 2\,k_{da}\,k_{dm}\,k_{m0}\,k_{rb} - 2\,k_{a0}\,k_{dm}\,k_{ma}\,k_{rb} + k_{ab}\,k_{dm}\,k_{ma}\,\sqrt{k_{db}^2 + 4\,k_{b0}\,k_{rb}} - k_{da}\,k_{m0}\,k_{mb}\,\sqrt{k_{db}^2 + 4\,k_{b0}\,k_{rb}} - k_{a0}\,k_{ma}\,k_{mb}\,\sqrt{k_{db}^2 + 4\,k_{b0}\,k_{rb}} + k_{d1}\,k_{db}\,k_{m0}\,k_{mb}\,s_1 - 2\,k_{d1}\,k_{dm}\,k_{m0}\,k_{rb}\,s_1 - k_{d1}\,k_{m0}\,k_{mb}\,\sqrt{k_{db}^2 + 4\,k_{b0}\,k_{rb}}\,s_1 + k_{d2}\,k_{db}\,k_{m0}\,k_{mb}\,s_2 - 2\,k_{d2}\,k_{dm}\,k_{m0}\,k_{rb}\,s_2 - k_{d2}\,k_{m0}\,k_{mb}\,\sqrt{k_{db}^2 + 4\,k_{b0}\,k_{rb}}\,s_2\Big) \Big/ \Big(2\,\big(k_{da}\,k_{db}\,k_{dm}\,k_{mb} + k_{b0}\,k_{da}\,k_{mb}^2 - k_{da}\,k_{dm}^2\,k_{rb} + k_{d1}\,k_{db}\,k_{dm}\,k_{mb}\,s_1 + k_{b0}\,k_{d1}\,k_{mb}^2\,s_1 - k_{d1}\,k_{dm}^2\,k_{rb}\,s_1 + k_{d2}\,k_{db}\,k_{dm}\,k_{mb}\,s_2 + k_{b0}\,k_{d2}\,k_{mb}^2\,s_2 - k_{d2}\,k_{dm}^2\,k_{rb}\,s_2\big)\Big)\Big\}\Big\}$$

For plotting we set all parameters to 1.

In[557]:=
```
dadt = 1 + 1 * b - (1 + 1 * s1 + 1 * s2) * a;
dbdt = 1 - (1 + 1 * b) * b;
dmdt = 1 + 1 * a - (1 + 1 * b) * m;
Solve[{dadt == 0, dbdt == 0, dmdt == 0}, {a, b, m}]
```

Out[560]=

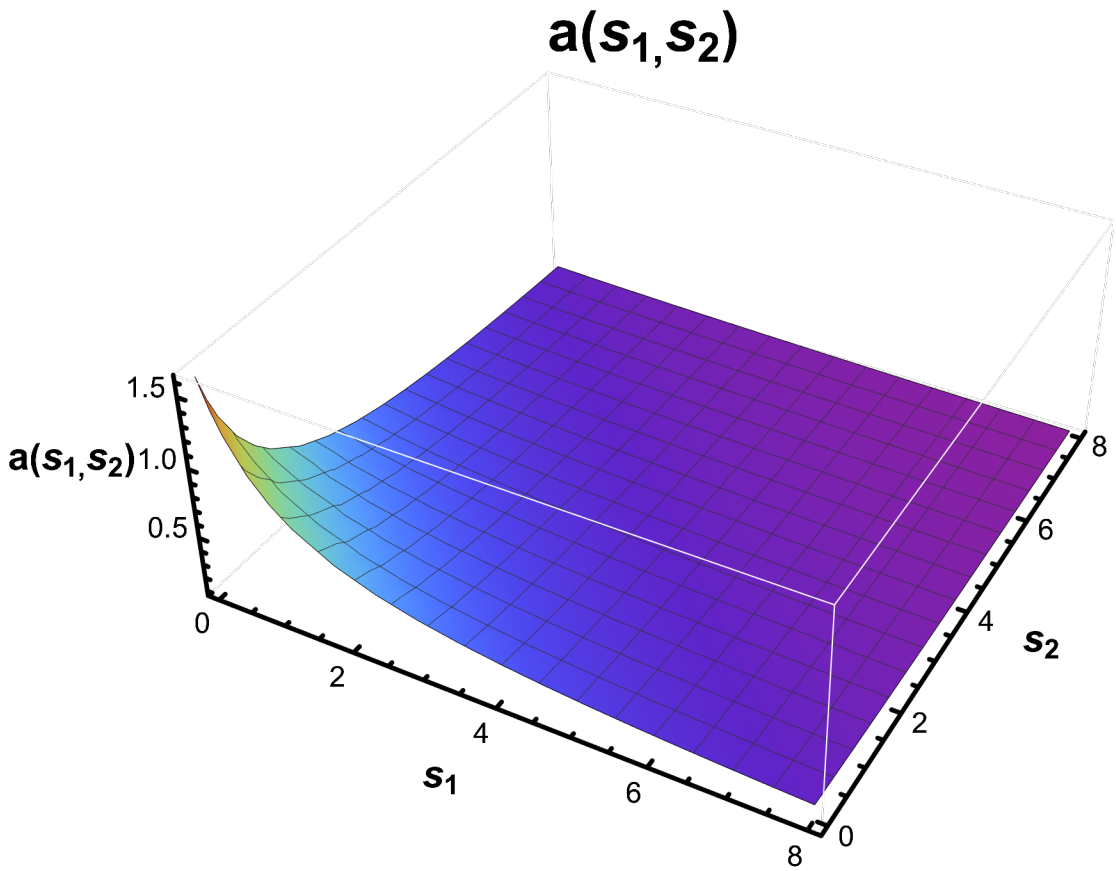
$$\left\{\left\{a \to \frac{1 - \sqrt{5}}{2 \times (1 + s_1 + s_2)},\ b \to \frac{1}{2} \times \left(-1 - \sqrt{5}\right),\ m \to \frac{1 - \sqrt{5} - s_1 - \sqrt{5}\,s_1 - s_2 - \sqrt{5}\,s_2}{2 \times (1 + s_1 + s_2)}\right\},\right.$$

$$\left.\left\{a \to \frac{1 + \sqrt{5}}{2 \times (1 + s_1 + s_2)},\ b \to \frac{1}{2} \times \left(-1 + \sqrt{5}\right),\ m \to \frac{1 + \sqrt{5} - s_1 + \sqrt{5}\,s_1 - s_2 + \sqrt{5}\,s_2}{2 \times (1 + s_1 + s_2)}\right\}\right\}$$

Steady state values and surface plots-

In[561]:=
```
aP = (1 + Sqrt[5])/(2 × (1 + s1 + s2));
Plot3D[aPC, {s1, 0, 8}, {s2, 0, 8}, AxesLabel →
  {Style["s1", Bold, 20, FontColor → Black], Style["s2", Bold, 20, FontColor → Black],
   Style["a(s1,s2)", Bold, 20, FontColor → Black]},
 PlotLabel → Style["a(s1,s2)", Bold, 30, FontColor → Black],
 ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
 TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

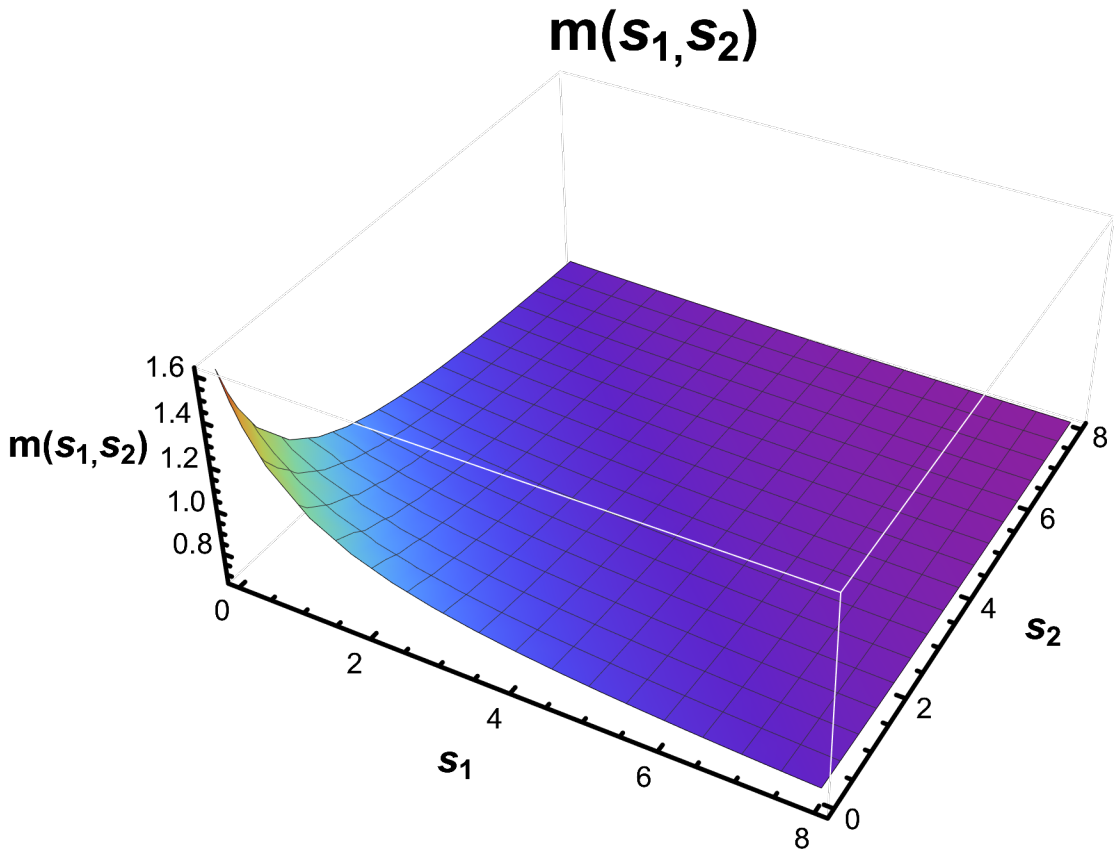
Out[562]=



b is a constant.

In[563]:=
```
mP = (1 + √5 - s1 + √5 s1 - s2 + √5 s2)/(2 × (1 + s1 + s2));
Plot3D[mP, {s1, 0, 8}, {s2, 0, 8}, AxesLabel →
  {Style["s1", Bold, 20, FontColor → Black], Style["s2", Bold, 20, FontColor → Black],
   Style["m(s1,s2)", Bold, 20, FontColor → Black]},
 PlotLabel → Style["m(s1,s2)", Bold, 30, FontColor → Black],
 ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
 TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

Out[564]=



So we will have m(s1, 0) < m(0, 0) & m(0, s2) < m(0, 0). So network C fails.

Network D

Dynamics-

In[565]:=
```
dadt = ka0 + kaa * a - (kda + kd1 * s1 + kd2 * s2) * a;
dbdt = kb0 - (kdb + krb * a) * b;
dmdt = km0 + kma * a - (kdm + kmb * b) * m;
Solve[{dadt == 0, dbdt == 0, dmdt == 0}, {a, b, m}]
```

Out[568]=
$$\left\{\left\{a \to \frac{k_{a0}}{-k_{aa}+k_{da}+k_{d1}\,s_1+k_{d2}\,s_2},\ b \to \frac{k_{b0}\,(k_{aa}-k_{da}-k_{d1}\,s_1-k_{d2}\,s_2)}{k_{aa}\,k_{db}-k_{da}\,k_{db}-k_{a0}\,k_{rb}-k_{d1}\,k_{db}\,s_1-k_{d2}\,k_{db}\,s_2},\right.\right.$$
$$m \to ((k_{aa}\,k_{db}-k_{da}\,k_{db}-k_{a0}\,k_{rb}-k_{d1}\,k_{db}\,s_1-k_{d2}\,k_{db}\,s_2)$$
$$(k_{aa}\,k_{m0}-k_{da}\,k_{m0}-k_{a0}\,k_{ma}-k_{d1}\,k_{m0}\,s_1-k_{d2}\,k_{m0}\,s_2))\ /$$
$$((k_{aa}-k_{da}-k_{d1}\,s_1-k_{d2}\,s_2)\ (k_{aa}\,k_{db}\,k_{dm}-k_{da}\,k_{db}\,k_{dm}+k_{aa}\,k_{b0}\,k_{mb}-k_{b0}\,k_{da}\,k_{mb}-$$
$$\left.\left.k_{a0}\,k_{dm}\,k_{rb}-k_{d1}\,k_{db}\,k_{dm}\,s_1-k_{b0}\,k_{d1}\,k_{mb}\,s_1-k_{d2}\,k_{db}\,k_{dm}\,s_2-k_{b0}\,k_{d2}\,k_{mb}\,s_2))\right\}\right\}$$

For plotting we set all parameters to 1.

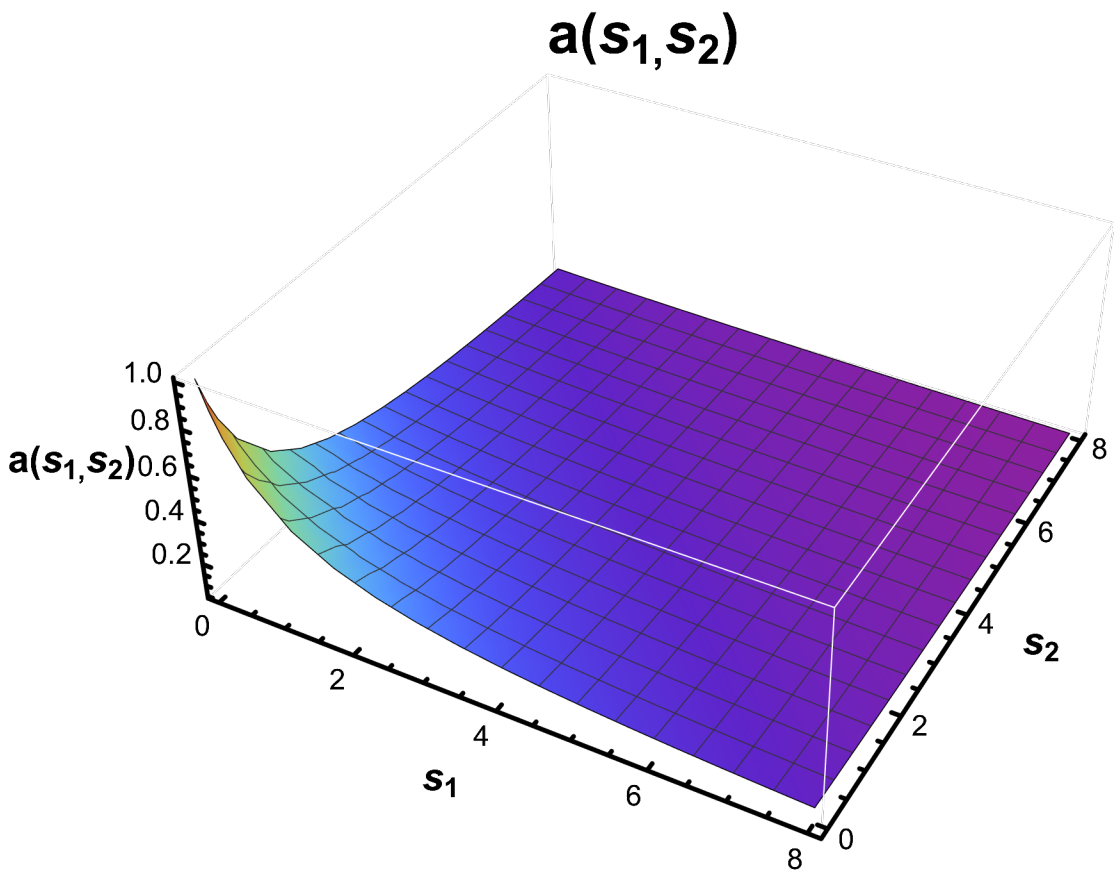
In[569]:=
```
dadt = 1 + 1 * a - (2 + 1 * s1 + 1 * s2) * a;
dbdt = 1 - (1 + 1 * a) * b;
dmdt = 1 + 1 * a - (1 + 1 * b) * m;
Solve[{dadt == 0, dbdt == 0, dmdt == 0}, {a, b, m}]
```

Out[572]=
$$\left\{\left\{a \to \frac{1}{1+s_1+s_2},\ b \to \frac{1+s_1+s_2}{2+s_1+s_2},\ m \to \frac{(2+s_1+s_2)^2}{3+5\,s_1+2\,s_1^2+5\,s_2+4\,s_1\,s_2+2\,s_2^2}\right\}\right\}$$

Steady state values and surface plots-

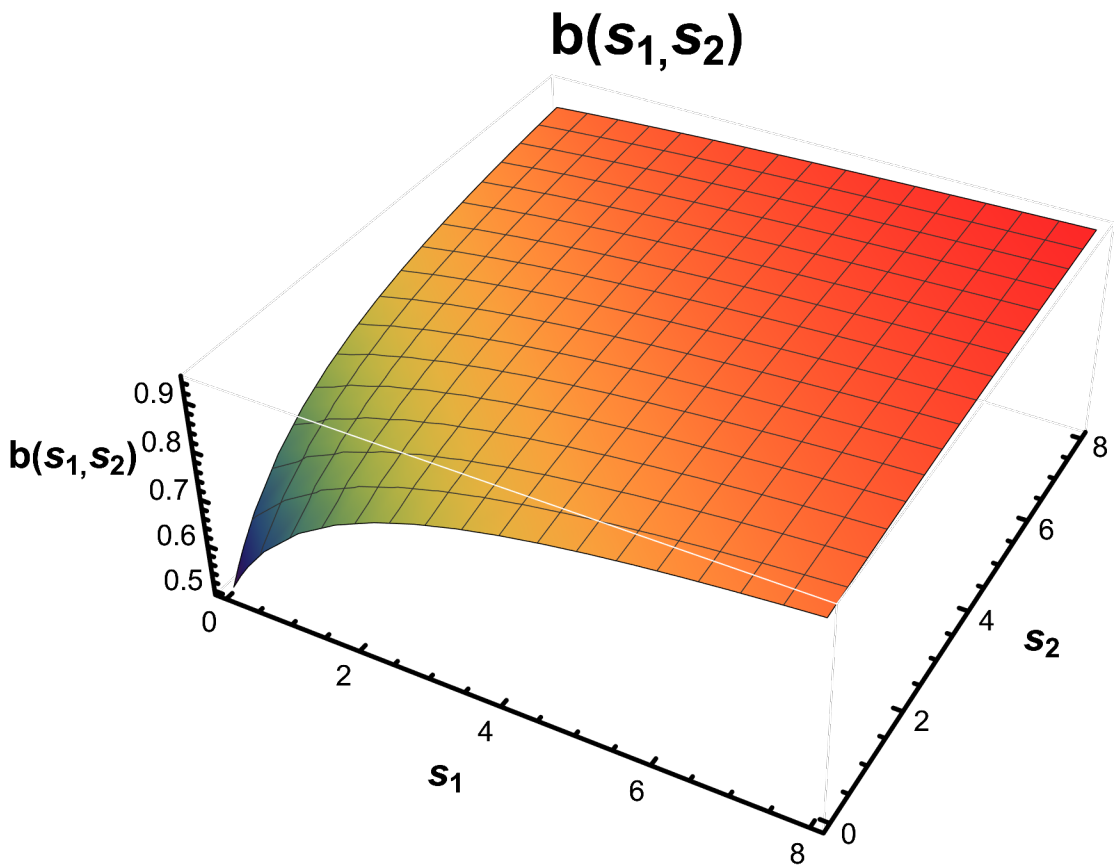
In[573]:=
```
aP = 1/(1 + s1 + s2);
Plot3D[aP, {s1, 0, 8}, {s2, 0, 8}, AxesLabel →
  {Style["s1", Bold, 20, FontColor → Black], Style["s2", Bold, 20, FontColor → Black],
   Style["a(s1,s2)", Bold, 20, FontColor → Black]},
 PlotLabel → Style["a(s1,s2)", Bold, 30, FontColor → Black],
 ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
 TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

Out[574]=



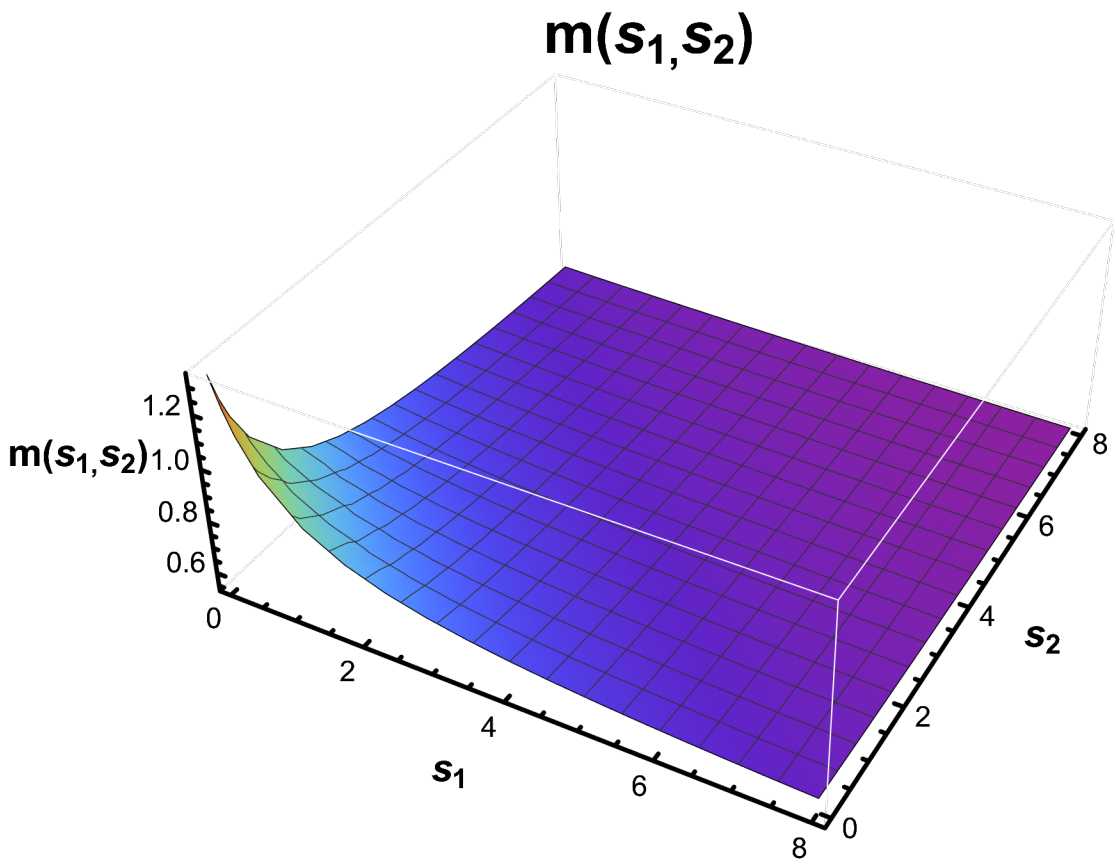
In[575]:=
```
bP = (1 + s1 + s2)/(2 + s1 + s2);
Plot3D[bP, {s1, 0, 8}, {s2, 0, 8}, AxesLabel →
  {Style["s1", Bold, 20, FontColor → Black], Style["s2", Bold, 20, FontColor → Black],
   Style["b(s1,s2)", Bold, 20, FontColor → Black]},
 PlotLabel → Style["b(s1,s2)", Bold, 30, FontColor → Black],
 ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
 TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

Out[576]=



In[577]:=
```
mP = (2 + s1 + s2)^2 / (3 + 5 s1 + 2 s1^2 + 5 s2 + 4 s1 s2 + 2 s2^2);
Plot3D[mP, {s1, 0, 8}, {s2, 0, 8}, AxesLabel →
  {Style["s1", Bold, 20, FontColor → Black], Style["s2", Bold, 20, FontColor → Black],
   Style["m(s1,s2)", Bold, 20, FontColor → Black]},
 PlotLabel → Style["m(s1,s2)", Bold, 30, FontColor → Black],
 ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
 TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

Out[578]=



So we will have m(s1, 0) < m(0, 0) & m(0, s2) < m(0, 0). So network D fails.

Network E

Dynamics-

In[579]:=
```
dadt = ka0 + kab * b - (kda + kd1 * s1 + kd2 * s2) * a;
dbdt = kb0 - (kdb + krb * a) * b;
dmdt = km0 + kma * a - (kdm + kmb * b) * m;
Solve[{dadt == 0, dbdt == 0, dmdt == 0}, {a, b, m}]
```

Out[582]=

$$\Bigg\{\Bigg\{a \to \Big(k_{da}\,k_{db} - k_{a0}\,k_{rb} + k_{d1}\,k_{db}\,s_1 + k_{d2}\,k_{db}\,s_2 - \sqrt{\big((-k_{da}\,k_{db} + k_{a0}\,k_{rb} - k_{d1}\,k_{db}\,s_1 - k_{d2}\,k_{db}\,s_2)^2 - 4\,(k_{ab}\,k_{b0} + k_{a0}\,k_{db})\,(-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2)}\Big) \Big/ \big(2\,(-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2)\big),$$

$$b \to \frac{1}{k_{ab}}\Bigg(-k_{a0} + \frac{k_{da}^2\,k_{db}}{2\,(-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2)} - \frac{k_{a0}\,k_{da}\,k_{rb}}{2\,(-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2)} + \frac{k_{d1}\,k_{da}\,k_{db}\,s_1}{-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2} - \frac{k_{a0}\,k_{d1}\,k_{rb}\,s_1}{2\,(-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2)} + \frac{k_{d1}^2\,k_{db}\,s_1^2}{2\,(-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2)} + \frac{k_{d2}\,k_{da}\,k_{db}\,s_2}{-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2} - \frac{k_{a0}\,k_{d2}\,k_{rb}\,s_2}{2\,(-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2)} + \frac{k_{d1}\,k_{d2}\,k_{db}\,s_1\,s_2}{-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2} + \frac{k_{d2}^2\,k_{db}\,s_2^2}{2\,(-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2)} - \Big(k_{da}\sqrt{\big((-k_{da}\,k_{db} + k_{a0}\,k_{rb} - k_{d1}\,k_{db}\,s_1 - k_{d2}\,k_{db}\,s_2)^2 - 4\,(k_{ab}\,k_{b0} + k_{a0}\,k_{db})\,(-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2)}\Big) \Big/ \big(2\,(-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2)\big) - \Big(k_{d1}\,s_1\sqrt{\big((-k_{da}\,k_{db} + k_{a0}\,k_{rb} - k_{d1}\,k_{db}\,s_1 - k_{d2}\,k_{db}\,s_2)^2 - 4\,(k_{ab}\,k_{b0} + k_{a0}\,k_{db})\,(-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2)}\Big) \Big/ \big(2\,(-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2)\big) - \Big(k_{d2}\,s_2\sqrt{\big((-k_{da}\,k_{db} + k_{a0}\,k_{rb} - k_{d1}\,k_{db}\,s_1 - k_{d2}\,k_{db}\,s_2)^2 - 4\,(k_{ab}\,k_{b0} + k_{a0}\,k_{db})\,(-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2)}\Big) \Big/ \big(2\,(-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2)\big)\Bigg),$$

$$m \to \Bigg(-k_{da}\,k_{db}\,k_{m0}\,k_{mb} - k_{ab}\,k_{b0}\,k_{ma}\,k_{mb} - k_{a0}\,k_{db}\,k_{ma}\,k_{mb} + k_{ab}\,k_{dm}\,k_{m0}\,k_{rb} - k_{d1}\,k_{db}\,k_{m0}\,k_{mb}\,s_1 - k_{d2}\,k_{db}\,k_{m0}\,k_{mb}\,s_2 + \frac{k_{ab}\,k_{da}\,k_{db}\,k_{dm}\,k_{ma}\,k_{rb}}{2\,(-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2)} - \frac{k_{da}^2\,k_{db}\,k_{m0}\,k_{mb}\,k_{rb}}{2\,(-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2)} - \frac{k_{a0}\,k_{da}\,k_{db}\,k_{ma}\,k_{mb}\,k_{rb}}{2\,(-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2)} - \frac{k_{a0}\,k_{ab}\,k_{dm}\,k_{ma}\,k_{rb}^2}{2\,(-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2)} + \frac{k_{a0}\,k_{da}\,k_{m0}\,k_{mb}\,k_{rb}^2}{2\,(-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2)} + \frac{k_{a0}^2\,k_{ma}\,k_{mb}\,k_{rb}^2}{2\,(-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2)} + \frac{k_{ab}\,k_{d1}\,k_{db}\,k_{dm}\,k_{ma}\,k_{rb}\,s_1}{2\,(-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2)} - \frac{k_{d1}\,k_{da}\,k_{db}\,k_{m0}\,k_{mb}\,k_{rb}\,s_1}{-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2} - \frac{k_{a0}\,k_{d1}\,k_{db}\,k_{ma}\,k_{mb}\,k_{rb}\,s_1}{2\,(-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2)} + \frac{k_{a0}\,k_{d1}\,k_{m0}\,k_{mb}\,k_{rb}^2\,s_1}{2\,(-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2)} - \frac{k_{d1}^2\,k_{db}\,k_{m0}\,k_{mb}\,k_{rb}\,s_1^2}{2\,(-k_{da}\,k_{rb} - k_{d1}\,k_{rb}\,s_1 - k_{d2}\,k_{rb}\,s_2)} +$$

$$\frac{k_{ab}\,k_{d2}\,k_{db}\,k_{dm}\,k_{ma}\,k_{rb}\,s_2}{2\,(-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2)}-\frac{k_{d2}\,k_{da}\,k_{db}\,k_{m0}\,k_{mb}\,k_{rb}\,s_2}{-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2}-$$
$$\frac{k_{a0}\,k_{d2}\,k_{db}\,k_{ma}\,k_{mb}\,k_{rb}\,s_2}{2\,(-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2)}+\frac{k_{a0}\,k_{d2}\,k_{m0}\,k_{mb}\,k_{rb}^2\,s_2}{2\,(-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2)}-$$
$$\frac{k_{d1}\,k_{d2}\,k_{db}\,k_{m0}\,k_{mb}\,k_{rb}\,s_1\,s_2}{-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2}-\frac{k_{d2}^2\,k_{db}\,k_{m0}\,k_{mb}\,k_{rb}\,s_2^2}{2\,(-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2)}-$$
$$\Big(k_{ab}\,k_{dm}\,k_{ma}\,k_{rb}\sqrt{\big((-k_{da}\,k_{db}+k_{a0}\,k_{rb}-k_{d1}\,k_{db}\,s_1-k_{d2}\,k_{db}\,s_2)^2-4\,(k_{ab}\,k_{b0}+k_{a0}\,k_{db})}$$
$$(-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2)\big)\Big)\Big/(2\,(-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2))+$$
$$\Big(k_{da}\,k_{m0}\,k_{mb}\,k_{rb}\sqrt{\big((-k_{da}\,k_{db}+k_{a0}\,k_{rb}-k_{d1}\,k_{db}\,s_1-k_{d2}\,k_{db}\,s_2)^2-4\,(k_{ab}\,k_{b0}+k_{a0}\,k_{db})}$$
$$(-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2)\big)\Big)\Big/(2\,(-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2))+$$
$$\Big(k_{a0}\,k_{ma}\,k_{mb}\,k_{rb}\sqrt{\big((-k_{da}\,k_{db}+k_{a0}\,k_{rb}-k_{d1}\,k_{db}\,s_1-k_{d2}\,k_{db}\,s_2)^2-4\,(k_{ab}\,k_{b0}+k_{a0}\,k_{db})}$$
$$(-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2)\big)\Big)\Big/(2\,(-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2))+$$
$$\Big(k_{d1}\,k_{m0}\,k_{mb}\,k_{rb}\,s_1\sqrt{\big((-k_{da}\,k_{db}+k_{a0}\,k_{rb}-k_{d1}\,k_{db}\,s_1-k_{d2}\,k_{db}\,s_2)^2-4\,(k_{ab}\,k_{b0}+k_{a0}\,k_{db})}$$
$$(-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2)\big)\Big)\Big/(2\,(-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2))+$$
$$\Big(k_{d2}\,k_{m0}\,k_{mb}\,k_{rb}\,s_2\sqrt{\big((-k_{da}\,k_{db}+k_{a0}\,k_{rb}-k_{d1}\,k_{db}\,s_1-k_{d2}\,k_{db}\,s_2)^2-4\,(k_{ab}\,k_{b0}+k_{a0}\,k_{db})}$$
$$(-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2)\big)\Big)\Big/(2\,(-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2))\Big)\Big/$$
$$\big(-k_{da}\,k_{db}\,k_{dm}\,k_{mb}-k_{b0}\,k_{da}\,k_{mb}^2+k_{ab}\,k_{dm}^2\,k_{rb}-k_{a0}\,k_{dm}\,k_{mb}\,k_{rb}-k_{d1}\,k_{db}\,k_{dm}\,k_{mb}\,s_1-$$
$$k_{b0}\,k_{d1}\,k_{mb}^2\,s_1-k_{d2}\,k_{db}\,k_{dm}\,k_{mb}\,s_2-k_{b0}\,k_{d2}\,k_{mb}^2\,s_2\big)\Big\},$$
$$\Big\{a\to\Big(k_{da}\,k_{db}-k_{a0}\,k_{rb}+k_{d1}\,k_{db}\,s_1+k_{d2}\,k_{db}\,s_2+\sqrt{\big((-k_{da}\,k_{db}+k_{a0}\,k_{rb}-k_{d1}\,k_{db}\,s_1-k_{d2}\,k_{db}\,s_2)^2-}$$
$$4\,(k_{ab}\,k_{b0}+k_{a0}\,k_{db})\,(-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2)\big)\Big)\Big/$$
$$(2\,(-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2)),\ b\to\frac{1}{k_{ab}}$$
$$\Big(-k_{a0}+\frac{k_{da}^2\,k_{db}}{2\,(-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2)}-\frac{k_{a0}\,k_{da}\,k_{rb}}{2\,(-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2)}+$$
$$\frac{k_{d1}\,k_{da}\,k_{db}\,s_1}{-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2}-\frac{k_{a0}\,k_{d1}\,k_{rb}\,s_1}{2\,(-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2)}+$$
$$\frac{k_{d1}^2\,k_{db}\,s_1^2}{2\,(-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2)}+$$
$$\frac{k_{d2}\,k_{da}\,k_{db}\,s_2}{-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2}-\frac{k_{a0}\,k_{d2}\,k_{rb}\,s_2}{2\,(-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2)}+$$
$$\frac{k_{d1}\,k_{d2}\,k_{db}\,s_1\,s_2}{-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2}+\frac{k_{d2}^2\,k_{db}\,s_2^2}{2\,(-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2)}+$$
$$\Big(k_{da}\sqrt{\big((-k_{da}\,k_{db}+k_{a0}\,k_{rb}-k_{d1}\,k_{db}\,s_1-k_{d2}\,k_{db}\,s_2)^2-4\,(k_{ab}\,k_{b0}+k_{a0}\,k_{db})}$$
$$(-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2)\big)\Big)\Big/(2\,(-k_{da}\,k_{rb}-k_{d1}\,k_{rb}\,s_1-k_{d2}\,k_{rb}\,s_2))+$$

$$\left(k_{d1}\, s_1 \sqrt{\left((-k_{da}\, k_{db} + k_{a0}\, k_{rb} - k_{d1}\, k_{db}\, s_1 - k_{d2}\, k_{db}\, s_2)^2 - 4\,(k_{ab}\, k_{b0} + k_{a0}\, k_{db})\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)\right)} \Big/ (2\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)) + \right.$$

$$\left. k_{d2}\, s_2 \sqrt{\left((-k_{da}\, k_{db} + k_{a0}\, k_{rb} - k_{d1}\, k_{db}\, s_1 - k_{d2}\, k_{db}\, s_2)^2 - 4\,(k_{ab}\, k_{b0} + k_{a0}\, k_{db})\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)\right)} \Big/ (2\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)) \right),$$

$$m \to \Bigg(-k_{da}\, k_{db}\, k_{m0}\, k_{mb} - k_{ab}\, k_{b0}\, k_{ma}\, k_{mb} - k_{a0}\, k_{db}\, k_{ma}\, k_{mb} + k_{ab}\, k_{dm}\, k_{m0}\, k_{rb} - k_{d1}\, k_{db}\, k_{m0}\, k_{mb}\, s_1 -$$

$$k_{d2}\, k_{db}\, k_{m0}\, k_{mb}\, s_2 + \frac{k_{ab}\, k_{da}\, k_{db}\, k_{dm}\, k_{ma}\, k_{rb}}{2\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)} -$$

$$\frac{k_{da}^2\, k_{db}\, k_{m0}\, k_{mb}\, k_{rb}}{2\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)} - \frac{k_{a0}\, k_{da}\, k_{db}\, k_{ma}\, k_{mb}\, k_{rb}}{2\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)} -$$

$$\frac{k_{a0}\, k_{ab}\, k_{dm}\, k_{ma}\, k_{rb}^2}{2\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)} + \frac{k_{a0}\, k_{da}\, k_{m0}\, k_{mb}\, k_{rb}^2}{2\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)} +$$

$$\frac{k_{a0}^2\, k_{ma}\, k_{mb}\, k_{rb}^2}{2\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)} + \frac{k_{ab}\, k_{d1}\, k_{db}\, k_{dm}\, k_{ma}\, k_{rb}\, s_1}{2\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)} -$$

$$\frac{k_{d1}\, k_{da}\, k_{db}\, k_{m0}\, k_{mb}\, k_{rb}\, s_1}{-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2} - \frac{k_{a0}\, k_{d1}\, k_{db}\, k_{ma}\, k_{mb}\, k_{rb}\, s_1}{2\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)} +$$

$$\frac{k_{a0}\, k_{d1}\, k_{m0}\, k_{mb}\, k_{rb}^2\, s_1}{2\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)} - \frac{k_{d1}^2\, k_{db}\, k_{m0}\, k_{mb}\, k_{rb}\, s_1^2}{2\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)} +$$

$$\frac{k_{ab}\, k_{d2}\, k_{db}\, k_{dm}\, k_{ma}\, k_{rb}\, s_2}{2\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)} - \frac{k_{d2}\, k_{da}\, k_{db}\, k_{m0}\, k_{mb}\, k_{rb}\, s_2}{-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2} -$$

$$\frac{k_{a0}\, k_{d2}\, k_{db}\, k_{ma}\, k_{mb}\, k_{rb}\, s_2}{2\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)} + \frac{k_{a0}\, k_{d2}\, k_{m0}\, k_{mb}\, k_{rb}^2\, s_2}{2\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)} -$$

$$\frac{k_{d1}\, k_{d2}\, k_{db}\, k_{m0}\, k_{mb}\, k_{rb}\, s_1\, s_2}{-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2} - \frac{k_{d2}^2\, k_{db}\, k_{m0}\, k_{mb}\, k_{rb}\, s_2^2}{2\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)} +$$

$$\left(k_{ab}\, k_{dm}\, k_{ma}\, k_{rb} \sqrt{\left((-k_{da}\, k_{db} + k_{a0}\, k_{rb} - k_{d1}\, k_{db}\, s_1 - k_{d2}\, k_{db}\, s_2)^2 - 4\,(k_{ab}\, k_{b0} + k_{a0}\, k_{db})\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)\right)} \Big/ (2\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)) - \right.$$

$$\left(k_{da}\, k_{m0}\, k_{mb}\, k_{rb} \sqrt{\left((-k_{da}\, k_{db} + k_{a0}\, k_{rb} - k_{d1}\, k_{db}\, s_1 - k_{d2}\, k_{db}\, s_2)^2 - 4\,(k_{ab}\, k_{b0} + k_{a0}\, k_{db})\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)\right)} \Big/ (2\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)) - \right.$$

$$\left(k_{a0}\, k_{ma}\, k_{mb}\, k_{rb} \sqrt{\left((-k_{da}\, k_{db} + k_{a0}\, k_{rb} - k_{d1}\, k_{db}\, s_1 - k_{d2}\, k_{db}\, s_2)^2 - 4\,(k_{ab}\, k_{b0} + k_{a0}\, k_{db})\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)\right)} \Big/ (2\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)) - \right.$$

$$\left(k_{d1}\, k_{m0}\, k_{mb}\, k_{rb}\, s_1 \sqrt{\left((-k_{da}\, k_{db} + k_{a0}\, k_{rb} - k_{d1}\, k_{db}\, s_1 - k_{d2}\, k_{db}\, s_2)^2 - 4\,(k_{ab}\, k_{b0} + k_{a0}\, k_{db})\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)\right)} \Big/ (2\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)) - \right.$$

$$\left(k_{d2}\, k_{m0}\, k_{mb}\, k_{rb}\, s_2 \sqrt{\left((-k_{da}\, k_{db} + k_{a0}\, k_{rb} - k_{d1}\, k_{db}\, s_1 - k_{d2}\, k_{db}\, s_2)^2 - 4\,(k_{ab}\, k_{b0} + k_{a0}\, k_{db})\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)\right)} \Big/ (2\,(-k_{da}\, k_{rb} - k_{d1}\, k_{rb}\, s_1 - k_{d2}\, k_{rb}\, s_2)) \right) \Big/$$

$$\left(-k_{da}\,k_{db}\,k_{dm}\,k_{mb}-k_{b0}\,k_{da}\,k_{mb}^2+k_{ab}\,k_{dm}^2\,k_{rb}-k_{a0}\,k_{dm}\,k_{mb}\,k_{rb}-k_{d1}\,k_{db}\,k_{dm}\,k_{mb}\,s_1-k_{b0}\,k_{d1}\,k_{mb}^2\,s_1-k_{d2}\,k_{db}\,k_{dm}\,k_{mb}\,s_2-k_{b0}\,k_{d2}\,k_{mb}^2\,s_2\right)\Big\}\Big\}$$

For plotting-

In[583]:=
```
dadt = 1 + 1 * b - (1 + 1 * s1 + 1 * s2) * a;
dbdt = 1 - (1 + 1 * a) * b;
dmdt = 1 + 1 * a - (1 + 1 * b) * m;
Solve[{dadt == 0, dbdt == 0, dmdt == 0}, {a, b, m}]
```

Out[586]=

$$\left\{\left\{a\to\frac{-s_1-s_2-\sqrt{(s_1+s_2)^2+8\times(1+s_1+s_2)}}{2\times(1+s_1+s_2)},\right.\right.$$

$$b\to-1-\frac{s_1}{2\times(1+s_1+s_2)}-\frac{s_1^2}{2\times(1+s_1+s_2)}-\frac{s_2}{2\times(1+s_1+s_2)}-\frac{s_1\,s_2}{1+s_1+s_2}-\frac{s_2^2}{2\times(1+s_1+s_2)}-\frac{\sqrt{(s_1+s_2)^2+8\times(1+s_1+s_2)}}{2\times(1+s_1+s_2)}-\frac{s_1\sqrt{(s_1+s_2)^2+8\times(1+s_1+s_2)}}{2\times(1+s_1+s_2)}-\frac{s_2\sqrt{(s_1+s_2)^2+8\times(1+s_1+s_2)}}{2\times(1+s_1+s_2)},$$

$$m\to\frac{1}{2+2\,s_1+2\,s_2}\left(2+s_1+s_2-\frac{s_1}{2\times(1+s_1+s_2)}-\frac{s_1^2}{2\times(1+s_1+s_2)}-\frac{s_2}{2\times(1+s_1+s_2)}-\frac{s_1\,s_2}{1+s_1+s_2}-\frac{s_2^2}{2\times(1+s_1+s_2)}-\frac{\sqrt{(s_1+s_2)^2+8\times(1+s_1+s_2)}}{2\times(1+s_1+s_2)}-\frac{s_1\sqrt{(s_1+s_2)^2+8\times(1+s_1+s_2)}}{2\times(1+s_1+s_2)}-\frac{s_2\sqrt{(s_1+s_2)^2+8\times(1+s_1+s_2)}}{2\times(1+s_1+s_2)}\right)\Big\},$$

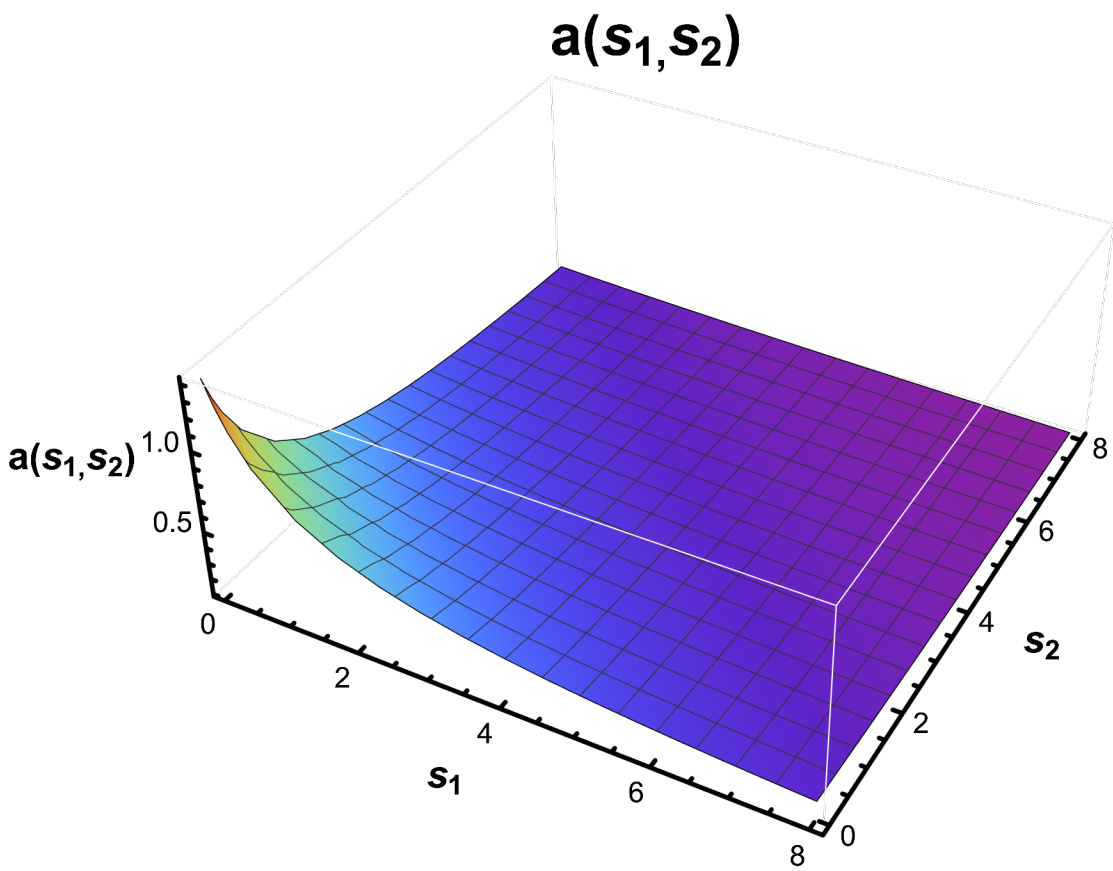
$$\left\{a\to\frac{-s_1-s_2+\sqrt{(s_1+s_2)^2+8\times(1+s_1+s_2)}}{2\times(1+s_1+s_2)},\ b\to-1-\frac{s_1}{2\times(1+s_1+s_2)}-\frac{s_1^2}{2\times(1+s_1+s_2)}-\frac{s_2}{2\times(1+s_1+s_2)}-\frac{s_1\,s_2}{1+s_1+s_2}-\frac{s_2^2}{2\times(1+s_1+s_2)}+\frac{\sqrt{(s_1+s_2)^2+8\times(1+s_1+s_2)}}{2\times(1+s_1+s_2)}+\frac{s_1\sqrt{(s_1+s_2)^2+8\times(1+s_1+s_2)}}{2\times(1+s_1+s_2)}+\frac{s_2\sqrt{(s_1+s_2)^2+8\times(1+s_1+s_2)}}{2\times(1+s_1+s_2)},\right.$$

$$m\to\frac{1}{2+2\,s_1+2\,s_2}\left(2+s_1+s_2-\frac{s_1}{2\times(1+s_1+s_2)}-\frac{s_1^2}{2\times(1+s_1+s_2)}-\frac{s_2}{2\times(1+s_1+s_2)}-\frac{s_1\,s_2}{1+s_1+s_2}-\frac{s_2^2}{2\times(1+s_1+s_2)}+\frac{\sqrt{(s_1+s_2)^2+8\times(1+s_1+s_2)}}{2\times(1+s_1+s_2)}+\frac{s_1\sqrt{(s_1+s_2)^2+8\times(1+s_1+s_2)}}{2\times(1+s_1+s_2)}+\frac{s_2\sqrt{(s_1+s_2)^2+8\times(1+s_1+s_2)}}{2\times(1+s_1+s_2)}\right)\Big\}\Big\}$$

Surface plots-

In[587]:=
```
aP = (-s1 - s2 + Sqrt[(s1 + s2)^2 + 8 × (1 + s1 + s2)]) / (2 × (1 + s1 + s2));
Plot3D[aP, {s1, 0, 8}, {s2, 0, 8}, AxesLabel →
  {Style["s1", Bold, 20, FontColor → Black], Style["s2", Bold, 20, FontColor → Black],
   Style["a(s1,s2)", Bold, 20, FontColor → Black]},
 PlotLabel → Style["a(s1,s2)", Bold, 30, FontColor → Black],
 ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
 TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

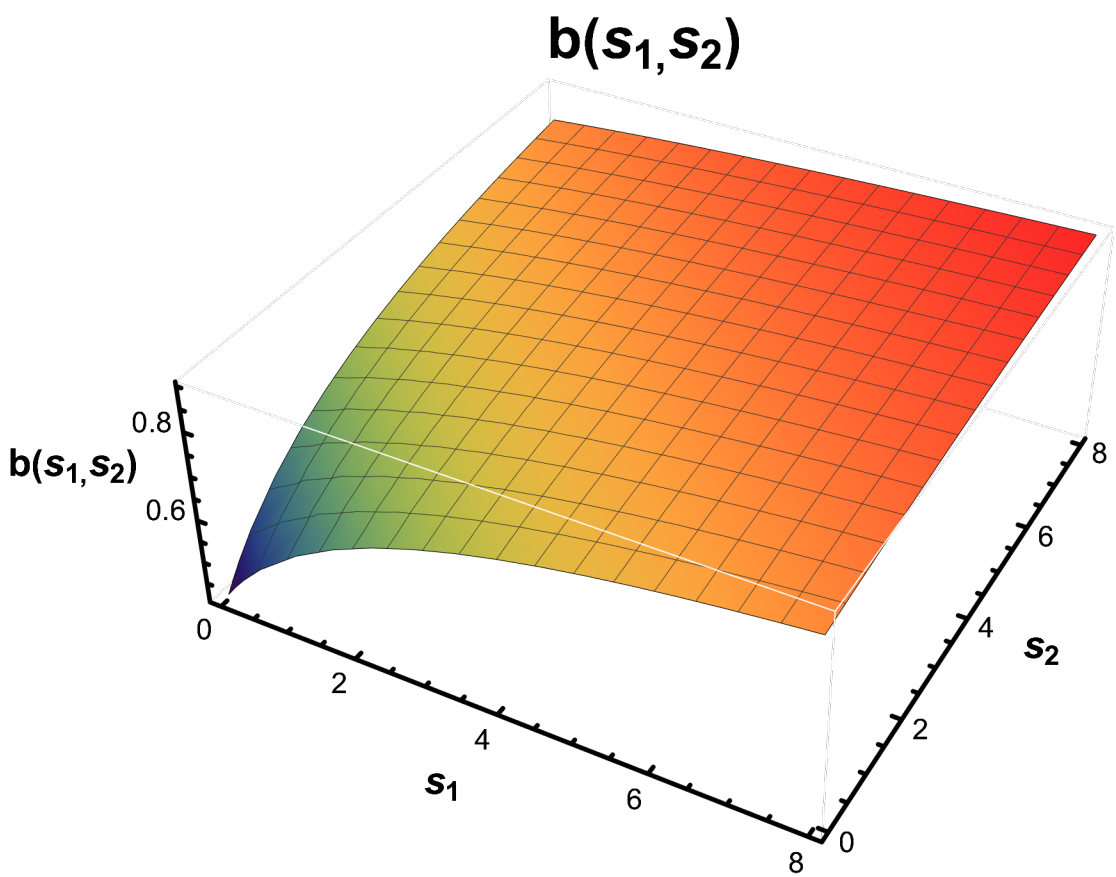
Out[588]=



In[589]:=
$$bP = -1 - \frac{s_1}{2 \times (1 + s_1 + s_2)} - \frac{s_1^2}{2 \times (1 + s_1 + s_2)} - \frac{s_2}{2 \times (1 + s_1 + s_2)} - \frac{s_1 s_2}{1 + s_1 + s_2} - \frac{s_2^2}{2 \times (1 + s_1 + s_2)} + \frac{\sqrt{(s_1 + s_2)^2 + 8 \times (1 + s_1 + s_2)}}{2 \times (1 + s_1 + s_2)} + \frac{s_1 \sqrt{(s_1 + s_2)^2 + 8 \times (1 + s_1 + s_2)}}{2 \times (1 + s_1 + s_2)} + \frac{s_2 \sqrt{(s_1 + s_2)^2 + 8 \times (1 + s_1 + s_2)}}{2 \times (1 + s_1 + s_2)};$$

```
Plot3D[bP, {s1, 0, 8}, {s2, 0, 8}, AxesLabel →
  {Style["s1", Bold, 20, FontColor → Black], Style["s2", Bold, 20, FontColor → Black],
   Style["b(s1,s2)", Bold, 20, FontColor → Black]},
 PlotLabel → Style["b(s1,s2)", Bold, 30, FontColor → Black],
 ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
 TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

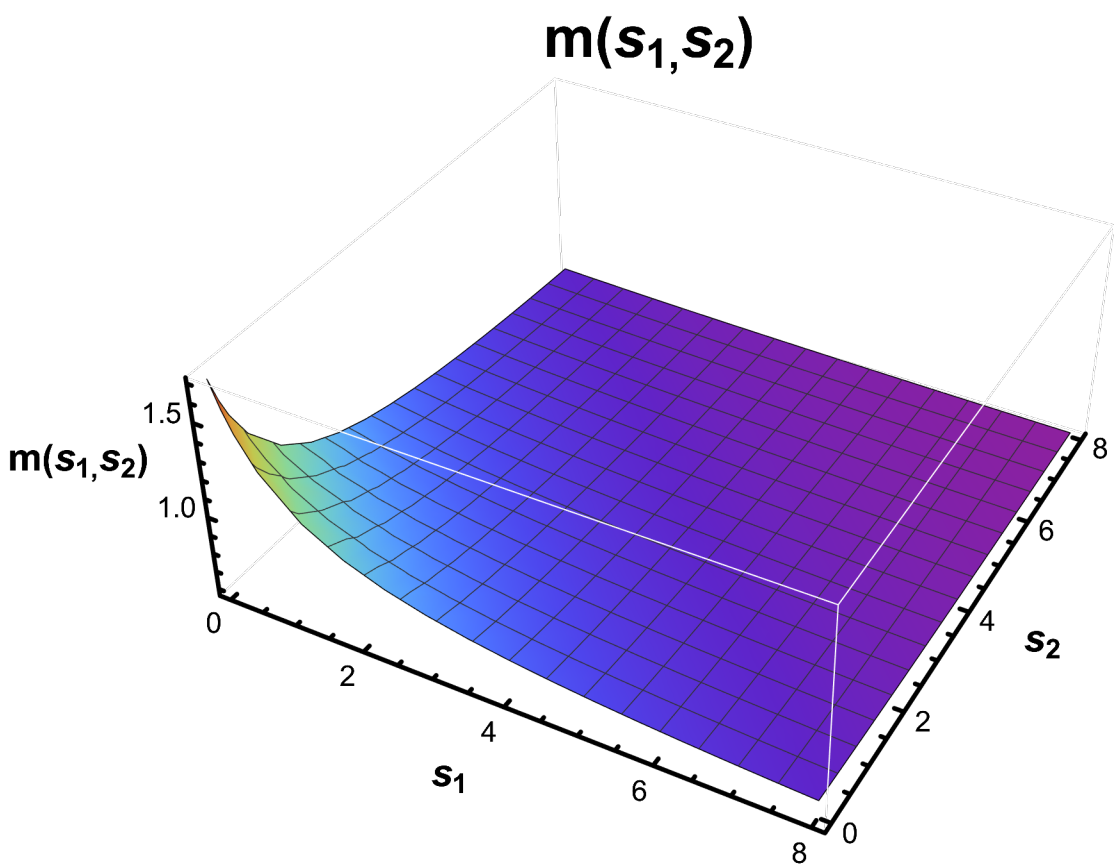
Out[590]=



In[591]:=
$$mP = \frac{1}{2+2s_1+2s_2}\left(2+s_1+s_2-\frac{s_1}{2\times(1+s_1+s_2)}-\frac{s_1^2}{2\times(1+s_1+s_2)}-\frac{s_2}{2\times(1+s_1+s_2)}-\frac{s_1 s_2}{1+s_1+s_2}-\frac{s_2^2}{2\times(1+s_1+s_2)}+\frac{\sqrt{(s_1+s_2)^2+8\times(1+s_1+s_2)}}{2\times(1+s_1+s_2)}+\frac{s_1\sqrt{(s_1+s_2)^2+8\times(1+s_1+s_2)}}{2\times(1+s_1+s_2)}+\frac{s_2\sqrt{(s_1+s_2)^2+8\times(1+s_1+s_2)}}{2\times(1+s_1+s_2)}\right);$$

```
Plot3D[mP, {s1, 0, 8}, {s2, 0, 8}, AxesLabel →
  {Style["s1", Bold, 20, FontColor → Black], Style["s2", Bold, 20, FontColor → Black],
   Style["m(s1,s2)", Bold, 20, FontColor → Black]},
 PlotLabel → Style["m(s1,s2)", Bold, 30, FontColor → Black],
 ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
 TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

Out[592]=



So we will have m(s1, 0) < m(0, 0) & m(0, s2) < m(0, 0). So network E fails.

Network F

Dynamics-

In[593]:=
```
dadt = ka0 + kam * m - (kda + kd1 * s1 + kd2 * s2) * a;
dbdt = kb0 - kdb * b;
dmdt = km0 + kma * a + kmb * b - (kdm + kmm * m) * m;
Solve[{dadt == 0, dbdt == 0, dmdt == 0}, {a, b, m}]
```

Out[596]=

$$\Big\{\Big\{a \to \Big(-k_{am}\,k_{da}\,k_{db}\,k_{dm} + k_{am}^2\,k_{db}\,k_{ma} + 2\,k_{a0}\,k_{da}\,k_{db}\,k_{mm} - k_{am}\,k_{d1}\,k_{db}\,k_{dm}\,s_1 + 2\,k_{a0}\,k_{d1}\,k_{db}\,k_{mm}\,s_1 - k_{am}\,k_{d2}\,k_{db}\,k_{dm}\,s_2 + 2\,k_{a0}\,k_{d2}\,k_{db}\,k_{mm}\,s_2 - \sqrt{\Big(\big(k_{am}\,k_{da}\,k_{db}\,k_{dm} - k_{am}^2\,k_{db}\,k_{ma} - 2\,k_{a0}\,k_{da}\,k_{db}\,k_{mm} + k_{am}\,k_{d1}\,k_{db}\,k_{dm}\,s_1 - 2\,k_{a0}\,k_{d1}\,k_{db}\,k_{mm}\,s_1 + k_{am}\,k_{d2}\,k_{db}\,k_{dm}\,s_2 - 2\,k_{a0}\,k_{d2}\,k_{db}\,k_{mm}\,s_2\big)^2 - 4\,\big(-k_{a0}\,k_{am}\,k_{db}\,k_{dm} - k_{am}^2\,k_{db}\,k_{m0} - k_{am}^2\,k_{b0}\,k_{mb} + k_{a0}^2\,k_{db}\,k_{mm}\big)\,\big(k_{da}^2\,k_{db}\,k_{mm} + 2\,k_{d1}\,k_{da}\,k_{db}\,k_{mm}\,s_1 + k_{d1}^2\,k_{db}\,k_{mm}\,s_1^2 + 2\,k_{d2}\,k_{da}\,k_{db}\,k_{mm}\,s_2 + 2\,k_{d1}\,k_{d2}\,k_{db}\,k_{mm}\,s_1\,s_2 + k_{d2}^2\,k_{db}\,k_{mm}\,s_2^2\big)\Big)}\Big) \Big/ \Big(2\,\big(k_{da}^2\,k_{db}\,k_{mm} + 2\,k_{d1}\,k_{da}\,k_{db}\,k_{mm}\,s_1 + k_{d1}^2\,k_{db}\,k_{mm}\,s_1^2 + 2\,k_{d2}\,k_{da}\,k_{db}\,k_{mm}\,s_2 + 2\,k_{d1}\,k_{d2}\,k_{db}\,k_{mm}\,s_1\,s_2 + k_{d2}^2\,k_{db}\,k_{mm}\,s_2^2\big)\Big),\ b \to \frac{k_{b0}}{k_{db}},$$

$$m \to \frac{1}{k_{am}}\Big(-k_{a0} - \big(k_{am}\,k_{da}^2\,k_{db}\,k_{dm}\big) \Big/ \big(2\,\big(k_{da}^2\,k_{db}\,k_{mm} + 2\,k_{d1}\,k_{da}\,k_{db}\,k_{mm}\,s_1 + k_{d1}^2\,k_{db}\,k_{mm}\,s_1^2 + 2\,k_{d2}\,k_{da}\,k_{db}\,k_{mm}\,s_2 + 2\,k_{d1}\,k_{d2}\,k_{db}\,k_{mm}\,s_1\,s_2 + k_{d2}^2\,k_{db}\,k_{mm}\,s_2^2\big)\big) + \big(k_{am}^2\,k_{da}\,k_{db}\,k_{ma}\big) \Big/ \big(2\,\big(k_{da}^2\,k_{db}\,k_{mm} + 2\,k_{d1}\,k_{da}\,k_{db}\,k_{mm}\,s_1 + k_{d1}^2\,k_{db}\,k_{mm}\,s_1^2 + 2\,k_{d2}\,k_{da}\,k_{db}\,k_{mm}\,s_2 + 2\,k_{d1}\,k_{d2}\,k_{db}\,k_{mm}\,s_1\,s_2 + k_{d2}^2\,k_{db}\,k_{mm}\,s_2^2\big)\big) + \big(k_{a0}\,k_{da}^2\,k_{db}\,k_{mm}\big) \Big/ \big(k_{da}^2\,k_{db}\,k_{mm} + 2\,k_{d1}\,k_{da}\,k_{db}\,k_{mm}\,s_1 + k_{d1}^2\,k_{db}\,k_{mm}\,s_1^2 + 2\,k_{d2}\,k_{da}\,k_{db}\,k_{mm}\,s_2 + 2\,k_{d1}\,k_{d2}\,k_{db}\,k_{mm}\,s_1\,s_2 + k_{d2}^2\,k_{db}\,k_{mm}\,s_2^2\big) - \big(k_{am}\,k_{d1}\,k_{da}\,k_{db}\,k_{dm}\,s_1\big) \Big/ \big(k_{da}^2\,k_{db}\,k_{mm} + 2\,k_{d1}\,k_{da}\,k_{db}\,k_{mm}\,s_1 + k_{d1}^2\,k_{db}\,k_{mm}\,s_1^2 + 2\,k_{d2}\,k_{da}\,k_{db}\,k_{mm}\,s_2 + 2\,k_{d1}\,k_{d2}\,k_{db}\,k_{mm}\,s_1\,s_2 + k_{d2}^2\,k_{db}\,k_{mm}\,s_2^2\big) + \big(k_{am}^2\,k_{d1}\,k_{db}\,k_{ma}\,s_1\big) \Big/ \big(2\,\big(k_{da}^2\,k_{db}\,k_{mm} + 2\,k_{d1}\,k_{da}\,k_{db}\,k_{mm}\,s_1 + k_{d1}^2\,k_{db}\,k_{mm}\,s_1^2 + 2\,k_{d2}\,k_{da}\,k_{db}\,k_{mm}\,s_2 + 2\,k_{d1}\,k_{d2}\,k_{db}\,k_{mm}\,s_1\,s_2 + k_{d2}^2\,k_{db}\,k_{mm}\,s_2^2\big)\big) + \big(2\,k_{a0}\,k_{d1}\,k_{da}\,k_{db}\,k_{mm}\,s_1\big) \Big/ \big(k_{da}^2\,k_{db}\,k_{mm} + 2\,k_{d1}\,k_{da}\,k_{db}\,k_{mm}\,s_1 + k_{d1}^2\,k_{db}\,k_{mm}\,s_1^2 + 2\,k_{d2}\,k_{da}\,k_{db}\,k_{mm}\,s_2 + 2\,k_{d1}\,k_{d2}\,k_{db}\,k_{mm}\,s_1\,s_2 + k_{d2}^2\,k_{db}\,k_{mm}\,s_2^2\big) - \big(k_{am}\,k_{d1}^2\,k_{db}\,k_{dm}\,s_1^2\big) \Big/ \big(2\,\big(k_{da}^2\,k_{db}\,k_{mm} + 2\,k_{d1}\,k_{da}\,k_{db}\,k_{mm}\,s_1 + k_{d1}^2\,k_{db}\,k_{mm}\,s_1^2 + 2\,k_{d2}\,k_{da}\,k_{db}\,k_{mm}\,s_2 + 2\,k_{d1}\,k_{d2}\,k_{db}\,k_{mm}\,s_1\,s_2 + k_{d2}^2\,k_{db}\,k_{mm}\,s_2^2\big)\big) + \big(k_{a0}\,k_{d1}^2\,k_{db}\,k_{mm}\,s_1^2\big) \Big/ \big(k_{da}^2\,k_{db}\,k_{mm} + 2\,k_{d1}\,k_{da}\,k_{db}\,k_{mm}\,s_1 + k_{d1}^2\,k_{db}\,k_{mm}\,s_1^2 + 2\,k_{d2}\,k_{da}\,k_{db}\,k_{mm}\,s_2 + 2\,k_{d1}\,k_{d2}\,k_{db}\,k_{mm}\,s_1\,s_2 + k_{d2}^2\,k_{db}\,k_{mm}\,s_2^2\big) - \big(k_{am}\,k_{d2}\,k_{da}\,k_{db}\,k_{dm}\,s_2\big) \Big/ \big(k_{da}^2\,k_{db}\,k_{mm} + 2\,k_{d1}\,k_{da}\,k_{db}\,k_{mm}\,s_1 + k_{d1}^2\,k_{db}\,k_{mm}\,s_1^2 + 2\,k_{d2}\,k_{da}\,k_{db}\,k_{mm}\,s_2 + 2\,k_{d1}\,k_{d2}\,k_{db}\,k_{mm}\,s_1\,s_2 + k_{d2}^2\,k_{db}\,k_{mm}\,s_2^2\big) + \big(k_{am}^2\,k_{d2}\,k_{db}\,k_{ma}\,s_2\big) \Big/ \big(2\,\big(k_{da}^2\,k_{db}\,k_{mm} + 2\,k_{d1}\,k_{da}\,k_{db}\,k_{mm}\,s_1 + k_{d1}^2\,k_{db}\,k_{mm}\,s_1^2 + 2\,k_{d2}\,k_{da}\,k_{db}\,k_{mm}\,s_2 + 2\,k_{d1}\,k_{d2}\,k_{db}\,k_{mm}\,s_1\,s_2 + k_{d2}^2\,k_{db}\,k_{mm}\,s_2^2\big)\big) +$$

$$
\begin{aligned}
&(2\, k_{a0}\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2) \Big/ \Big(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + \\
&\quad 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\Big) - \\
&(k_{am}\, k_{d1}\, k_{d2}\, k_{db}\, k_{dm}\, s_1\, s_2) \Big/ \Big(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + \\
&\quad 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\Big) + \\
&(2\, k_{a0}\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2) \Big/ \Big(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + \\
&\quad 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\Big) - \\
&\Big(k_{am}\, k_{d2}^2\, k_{db}\, k_{dm}\, s_2^2\Big) \Big/ \Big(2\, \Big(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + \\
&\qquad 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\Big)\Big) + \\
&\Big(k_{a0}\, k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\Big) \Big/ \Big(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + \\
&\quad 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\Big) - \\
&\Big(k_{da} \sqrt{\Big(\Big(k_{am}\, k_{da}\, k_{db}\, k_{dm} - k_{am}^2\, k_{db}\, k_{ma} - 2\, k_{a0}\, k_{da}\, k_{db}\, k_{mm} + k_{am}\, k_{d1}\, k_{db}\, k_{dm}\, s_1 - 2\, k_{a0}\, k_{d1} \\
&\qquad k_{db}\, k_{mm}\, s_1 + k_{am}\, k_{d2}\, k_{db}\, k_{dm}\, s_2 - 2\, k_{a0}\, k_{d2}\, k_{db}\, k_{mm}\, s_2\Big)^2 - 4\, \Big(-k_{a0}\, k_{am}\, k_{db}\, k_{dm} - \\
&\qquad k_{am}^2\, k_{db}\, k_{m0} - k_{am}^2\, k_{b0}\, k_{mb} + k_{a0}^2\, k_{db}\, k_{mm}\Big)\, \Big(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + \\
&\qquad k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\Big)\Big)\Big) \Big/ \\
&\quad \Big(2\, \Big(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + \\
&\qquad 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\Big)\Big) - \\
&\Big(k_{d1}\, s_1 \sqrt{\Big(\Big(k_{am}\, k_{da}\, k_{db}\, k_{dm} - k_{am}^2\, k_{db}\, k_{ma} - 2\, k_{a0}\, k_{da}\, k_{db}\, k_{mm} + k_{am}\, k_{d1}\, k_{db}\, k_{dm}\, s_1 - 2\, k_{a0} \\
&\qquad k_{d1}\, k_{db}\, k_{mm}\, s_1 + k_{am}\, k_{d2}\, k_{db}\, k_{dm}\, s_2 - 2\, k_{a0}\, k_{d2}\, k_{db}\, k_{mm}\, s_2\Big)^2 - 4\, \Big(-k_{a0}\, k_{am}\, k_{db}\, k_{dm} - \\
&\qquad k_{am}^2\, k_{db}\, k_{m0} - k_{am}^2\, k_{b0}\, k_{mb} + k_{a0}^2\, k_{db}\, k_{mm}\Big)\, \Big(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + \\
&\qquad k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\Big)\Big)\Big) \Big/ \\
&\quad \Big(2\, \Big(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + \\
&\qquad 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\Big)\Big) - \\
&\Big(k_{d2}\, s_2 \sqrt{\Big(\Big(k_{am}\, k_{da}\, k_{db}\, k_{dm} - k_{am}^2\, k_{db}\, k_{ma} - 2\, k_{a0}\, k_{da}\, k_{db}\, k_{mm} + k_{am}\, k_{d1}\, k_{db}\, k_{dm}\, s_1 - 2\, k_{a0} \\
&\qquad k_{d1}\, k_{db}\, k_{mm}\, s_1 + k_{am}\, k_{d2}\, k_{db}\, k_{dm}\, s_2 - 2\, k_{a0}\, k_{d2}\, k_{db}\, k_{mm}\, s_2\Big)^2 - 4\, \Big(-k_{a0}\, k_{am}\, k_{db}\, k_{dm} - \\
&\qquad k_{am}^2\, k_{db}\, k_{m0} - k_{am}^2\, k_{b0}\, k_{mb} + k_{a0}^2\, k_{db}\, k_{mm}\Big)\, \Big(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + \\
&\qquad k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\Big)\Big)\Big) \Big/ \\
&\quad \Big(2\, \Big(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + \\
&\qquad 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\Big)\Big)\Big\},
\end{aligned}
$$

$$
\begin{aligned}
&\Big\{a \to \Big(-k_{am}\, k_{da}\, k_{db}\, k_{dm} + k_{am}^2\, k_{db}\, k_{ma} + 2\, k_{a0}\, k_{da}\, k_{db}\, k_{mm} - k_{am}\, k_{d1}\, k_{db}\, k_{dm}\, s_1 + \\
&\quad 2\, k_{a0}\, k_{d1}\, k_{db}\, k_{mm}\, s_1 - k_{am}\, k_{d2}\, k_{db}\, k_{dm}\, s_2 + 2\, k_{a0}\, k_{d2}\, k_{db}\, k_{mm}\, s_2 + \\
&\quad \sqrt{\Big(\Big(k_{am}\, k_{da}\, k_{db}\, k_{dm} - k_{am}^2\, k_{db}\, k_{ma} - 2\, k_{a0}\, k_{da}\, k_{db}\, k_{mm} + k_{am}\, k_{d1}\, k_{db}\, k_{dm}\, s_1 - \\
&\qquad 2\, k_{a0}\, k_{d1}\, k_{db}\, k_{mm}\, s_1 + k_{am}\, k_{d2}\, k_{db}\, k_{dm}\, s_2 - 2\, k_{a0}\, k_{d2}\, k_{db}\, k_{mm}\, s_2\Big)^2 - \\
&\qquad 4\, \Big(-k_{a0}\, k_{am}\, k_{db}\, k_{dm} - k_{am}^2\, k_{db}\, k_{m0} - k_{am}^2\, k_{b0}\, k_{mb} + k_{a0}^2\, k_{db}\, k_{mm}\Big)\, \Big(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm} \\
&\qquad s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\Big)\Big)\Big) \Big/ \\
&\quad \Big(2\, \Big(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + \\
&\qquad 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\Big)\Big),\ b \to \frac{k_{b0}}{k_{db}},
\end{aligned}
$$

$$
\begin{aligned}
m \to \frac{1}{k_{am}} \Big(& -k_{a0} - \left(k_{am}\, k_{da}^2\, k_{db}\, k_{dm}\right) \Big/ \Big(2 \left(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + \right. \\
& \left. 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\right)\Big) + \\
& \left(k_{am}^2\, k_{da}\, k_{db}\, k_{ma}\right) \Big/ \Big(2 \left(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + \right. \\
& \left. 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\right)\Big) + \\
& \left(k_{a0}\, k_{da}^2\, k_{db}\, k_{mm}\right) \Big/ \Big(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + \\
& 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\Big) - \\
& \left(k_{am}\, k_{d1}\, k_{da}\, k_{db}\, k_{dm}\, s_1\right) \Big/ \Big(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + \\
& 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\Big) + \\
& \left(k_{am}^2\, k_{d1}\, k_{db}\, k_{ma}\, s_1\right) \Big/ \Big(2 \left(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + \right. \\
& \left. 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\right)\Big) + \\
& \left(2\, k_{a0}\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1\right) \Big/ \Big(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + \\
& 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\Big) - \\
& \left(k_{am}\, k_{d1}^2\, k_{db}\, k_{dm}\, s_1^2\right) \Big/ \Big(2 \left(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + \right. \\
& \left. 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\right)\Big) + \\
& \left(k_{a0}\, k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2\right) \Big/ \Big(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + \\
& 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\Big) - \\
& \left(k_{am}\, k_{d2}\, k_{da}\, k_{db}\, k_{dm}\, s_2\right) \Big/ \Big(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + \\
& 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\Big) + \\
& \left(k_{am}^2\, k_{d2}\, k_{db}\, k_{ma}\, s_2\right) \Big/ \Big(2 \left(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + \right. \\
& \left. 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\right)\Big) + \\
& \left(2\, k_{a0}\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2\right) \Big/ \Big(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + \\
& 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\Big) - \\
& \left(k_{am}\, k_{d1}\, k_{d2}\, k_{db}\, k_{dm}\, s_1\, s_2\right) \Big/ \Big(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + \\
& 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\Big) + \\
& \left(2\, k_{a0}\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2\right) \Big/ \Big(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + \\
& 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\Big) - \\
& \left(k_{am}\, k_{d2}^2\, k_{db}\, k_{dm}\, s_2^2\right) \Big/ \Big(2 \left(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + \right. \\
& \left. 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\right)\Big) + \\
& \left(k_{a0}\, k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\right) \Big/ \Big(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + \\
& 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\Big) + \\
& \Big(k_{da} \sqrt{\Big(\left(k_{am}\, k_{da}\, k_{db}\, k_{dm} - k_{am}^2\, k_{db}\, k_{ma} - 2\, k_{a0}\, k_{da}\, k_{db}\, k_{mm} + k_{am}\, k_{d1}\, k_{db}\, k_{dm}\, s_1 - 2\, k_{a0}\, k_{d1}\, \right.} \\
& \left. k_{db}\, k_{mm}\, s_1 + k_{am}\, k_{d2}\, k_{db}\, k_{dm}\, s_2 - 2\, k_{a0}\, k_{d2}\, k_{db}\, k_{mm}\, s_2\right)^2 - 4 \left(-k_{a0}\, k_{am}\, k_{db}\, k_{dm} - \right. \\
& \left. k_{am}^2\, k_{db}\, k_{m0} - k_{am}^2\, k_{b0}\, k_{mb} + k_{a0}^2\, k_{db}\, k_{mm}\right) \left(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + \right. \\
& \left. k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\right)\Big)\Big) \Big/ \\
& \Big(2 \left(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + \right. \\
& \left. 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\right)\Big) + \\
& \Big(k_{d1}\, s_1 \sqrt{\Big(\left(k_{am}\, k_{da}\, k_{db}\, k_{dm} - k_{am}^2\, k_{db}\, k_{ma} - 2\, k_{a0}\, k_{da}\, k_{db}\, k_{mm} + k_{am}\, k_{d1}\, k_{db}\, k_{dm}\, s_1 - 2\, k_{a0}\, \right.}
\end{aligned}
$$

$$k_{d1}\, k_{db}\, k_{mm}\, s_1 + k_{am}\, k_{d2}\, k_{db}\, k_{dm}\, s_2 - 2\, k_{a0}\, k_{d2}\, k_{db}\, k_{mm}\, s_2\Big)^2 - 4\, \Big(-k_{a0}\, k_{am}\, k_{db}\, k_{dm} - k_{am}^2\, k_{db}\, k_{m0} - k_{am}^2\, k_{b0}\, k_{mb} + k_{a0}^2\, k_{db}\, k_{mm}\Big)\, \Big(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\Big)\Big)\Big) \Big/ \Big(2\, \Big(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\Big)\Big) + \Big(k_{d2}\, s_2 \sqrt{\Big(\Big(k_{am}\, k_{da}\, k_{db}\, k_{dm} - k_{am}^2\, k_{db}\, k_{ma} - 2\, k_{a0}\, k_{da}\, k_{db}\, k_{mm} + k_{am}\, k_{d1}\, k_{db}\, k_{dm}\, s_1 - 2\, k_{a0}\, k_{d1}\, k_{db}\, k_{mm}\, s_1 + k_{am}\, k_{d2}\, k_{db}\, k_{dm}\, s_2 - 2\, k_{a0}\, k_{d2}\, k_{db}\, k_{mm}\, s_2\Big)^2 - 4\, \Big(-k_{a0}\, k_{am}\, k_{db}\, k_{dm} - k_{am}^2\, k_{db}\, k_{m0} - k_{am}^2\, k_{b0}\, k_{mb} + k_{a0}^2\, k_{db}\, k_{mm}\Big)\, \Big(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\Big)\Big)\Big) \Big/ \Big(2\, \Big(k_{da}^2\, k_{db}\, k_{mm} + 2\, k_{d1}\, k_{da}\, k_{db}\, k_{mm}\, s_1 + k_{d1}^2\, k_{db}\, k_{mm}\, s_1^2 + 2\, k_{d2}\, k_{da}\, k_{db}\, k_{mm}\, s_2 + 2\, k_{d1}\, k_{d2}\, k_{db}\, k_{mm}\, s_1\, s_2 + k_{d2}^2\, k_{db}\, k_{mm}\, s_2^2\Big)\Big)\Big)\Big\}\Big\}$$

For plotting-

In[597]:=
```
dadt = 1 + 1 * m - (1 + 1 * s1 + 1 * s2) * a;
dbdt = 1 - 1 * b;
dmdt = 1 + 1 * a + 1 * b - (1 + 1 * m) * m;
Solve[{dadt == 0, dbdt == 0, dmdt == 0}, {a, b, m}]
```

Out[600]=

$$\left\{\left\{a \to \frac{2+s_1+s_2-\sqrt{12+20\,s_1+9\,s_1^2+20\,s_2+18\,s_1\,s_2+9\,s_2^2}}{2\times\left(1+2\,s_1+s_1^2+2\,s_2+2\,s_1\,s_2+s_2^2\right)},\right.\right.$$

$$b \to 1,\ m \to -1+\frac{1}{1+2\,s_1+s_1^2+2\,s_2+2\,s_1\,s_2+s_2^2}+$$

$$\frac{3\,s_1}{2\times\left(1+2\,s_1+s_1^2+2\,s_2+2\,s_1\,s_2+s_2^2\right)}+\frac{s_1^2}{2\times\left(1+2\,s_1+s_1^2+2\,s_2+2\,s_1\,s_2+s_2^2\right)}+$$

$$\frac{3\,s_2}{2\times\left(1+2\,s_1+s_1^2+2\,s_2+2\,s_1\,s_2+s_2^2\right)}+\frac{s_1\,s_2}{1+2\,s_1+s_1^2+2\,s_2+2\,s_1\,s_2+s_2^2}+$$

$$\frac{s_2^2}{2\times\left(1+2\,s_1+s_1^2+2\,s_2+2\,s_1\,s_2+s_2^2\right)}-\frac{\sqrt{12+20\,s_1+9\,s_1^2+20\,s_2+18\,s_1\,s_2+9\,s_2^2}}{2\times\left(1+2\,s_1+s_1^2+2\,s_2+2\,s_1\,s_2+s_2^2\right)}-$$

$$\left.\frac{s_1\,\sqrt{12+20\,s_1+9\,s_1^2+20\,s_2+18\,s_1\,s_2+9\,s_2^2}}{2\times\left(1+2\,s_1+s_1^2+2\,s_2+2\,s_1\,s_2+s_2^2\right)}-\frac{s_2\,\sqrt{12+20\,s_1+9\,s_1^2+20\,s_2+18\,s_1\,s_2+9\,s_2^2}}{2\times\left(1+2\,s_1+s_1^2+2\,s_2+2\,s_1\,s_2+s_2^2\right)}\right\},$$

$$\left\{a \to \frac{2+s_1+s_2+\sqrt{12+20\,s_1+9\,s_1^2+20\,s_2+18\,s_1\,s_2+9\,s_2^2}}{2\times\left(1+2\,s_1+s_1^2+2\,s_2+2\,s_1\,s_2+s_2^2\right)},\ b \to 1,\right.$$

$$m \to -1+\frac{1}{1+2\,s_1+s_1^2+2\,s_2+2\,s_1\,s_2+s_2^2}+\frac{3\,s_1}{2\times\left(1+2\,s_1+s_1^2+2\,s_2+2\,s_1\,s_2+s_2^2\right)}+$$

$$\frac{s_1^2}{2\times\left(1+2\,s_1+s_1^2+2\,s_2+2\,s_1\,s_2+s_2^2\right)}+\frac{3\,s_2}{2\times\left(1+2\,s_1+s_1^2+2\,s_2+2\,s_1\,s_2+s_2^2\right)}+$$

$$\frac{s_1\,s_2}{1+2\,s_1+s_1^2+2\,s_2+2\,s_1\,s_2+s_2^2}+\frac{s_2^2}{2\times\left(1+2\,s_1+s_1^2+2\,s_2+2\,s_1\,s_2+s_2^2\right)}+$$

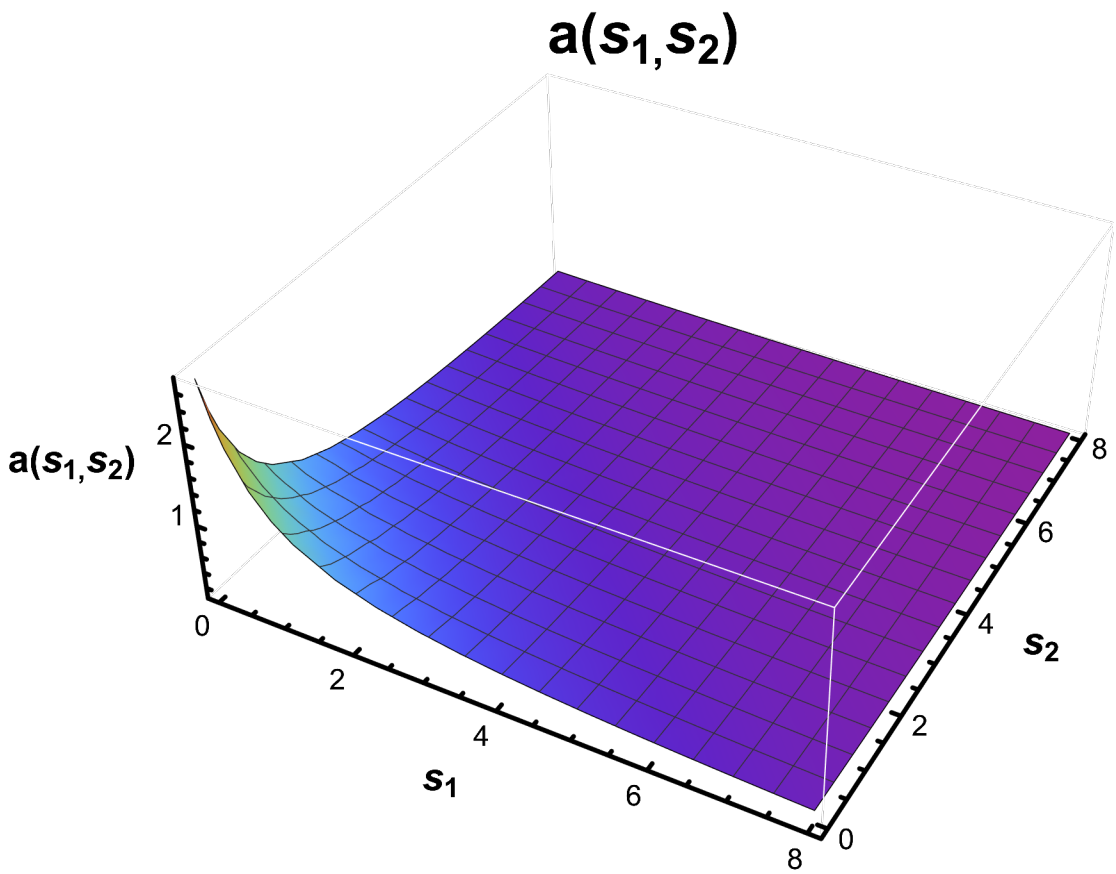
$$\frac{\sqrt{12+20\,s_1+9\,s_1^2+20\,s_2+18\,s_1\,s_2+9\,s_2^2}}{2\times\left(1+2\,s_1+s_1^2+2\,s_2+2\,s_1\,s_2+s_2^2\right)}+\frac{s_1\,\sqrt{12+20\,s_1+9\,s_1^2+20\,s_2+18\,s_1\,s_2+9\,s_2^2}}{2\times\left(1+2\,s_1+s_1^2+2\,s_2+2\,s_1\,s_2+s_2^2\right)}+$$

$$\left.\left.\frac{s_2\,\sqrt{12+20\,s_1+9\,s_1^2+20\,s_2+18\,s_1\,s_2+9\,s_2^2}}{2\times\left(1+2\,s_1+s_1^2+2\,s_2+2\,s_1\,s_2+s_2^2\right)}\right\}\right\}$$

Surface maps-

In[601]:=
```
aP = (2 + s1 + s2 + Sqrt[12 + 20 s1 + 9 s1^2 + 20 s2 + 18 s1 s2 + 9 s2^2]) / (2 × (1 + 2 s1 + s1^2 + 2 s2 + 2 s1 s2 + s2^2));
Plot3D[aP, {s1, 0, 8}, {s2, 0, 8}, AxesLabel →
  {Style["s1", Bold, 20, FontColor → Black], Style["s2", Bold, 20, FontColor → Black],
   Style["a(s1,s2)", Bold, 20, FontColor → Black]},
 PlotLabel → Style["a(s1,s2)", Bold, 30, FontColor → Black],
 ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
 TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

Out[602]=

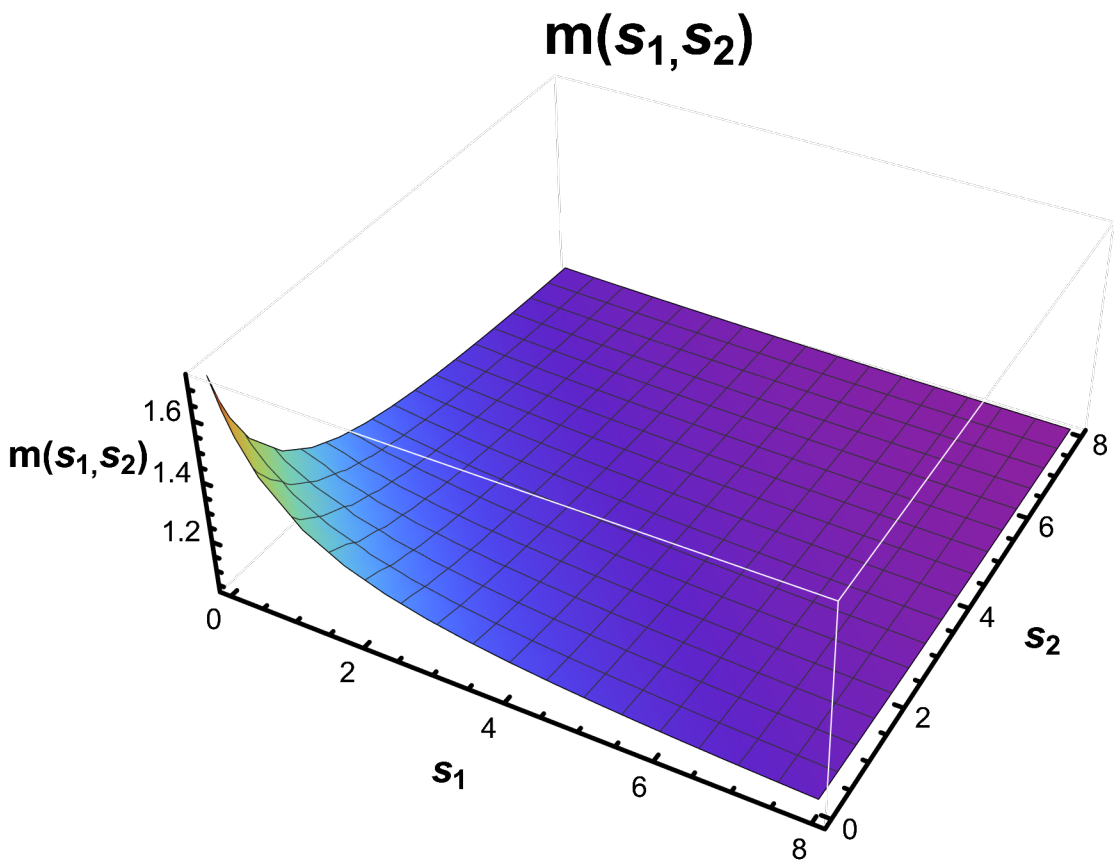


b is constant.

In[603]:=
$$mP = -1 + \frac{1}{1 + 2 s_1 + s_1^2 + 2 s_2 + 2 s_1 s_2 + s_2^2} + \frac{3 s_1}{2 \times (1 + 2 s_1 + s_1^2 + 2 s_2 + 2 s_1 s_2 + s_2^2)} + \frac{s_1^2}{2 \times (1 + 2 s_1 + s_1^2 + 2 s_2 + 2 s_1 s_2 + s_2^2)} + \frac{3 s_2}{2 \times (1 + 2 s_1 + s_1^2 + 2 s_2 + 2 s_1 s_2 + s_2^2)} + \frac{s_1 s_2}{1 + 2 s_1 + s_1^2 + 2 s_2 + 2 s_1 s_2 + s_2^2} + \frac{s_2^2}{2 \times (1 + 2 s_1 + s_1^2 + 2 s_2 + 2 s_1 s_2 + s_2^2)} + \frac{\sqrt{12 + 20 s_1 + 9 s_1^2 + 20 s_2 + 18 s_1 s_2 + 9 s_2^2}}{2 \times (1 + 2 s_1 + s_1^2 + 2 s_2 + 2 s_1 s_2 + s_2^2)} + \frac{s_1 \sqrt{12 + 20 s_1 + 9 s_1^2 + 20 s_2 + 18 s_1 s_2 + 9 s_2^2}}{2 \times (1 + 2 s_1 + s_1^2 + 2 s_2 + 2 s_1 s_2 + s_2^2)} + \frac{s_2 \sqrt{12 + 20 s_1 + 9 s_1^2 + 20 s_2 + 18 s_1 s_2 + 9 s_2^2}}{2 \times (1 + 2 s_1 + s_1^2 + 2 s_2 + 2 s_1 s_2 + s_2^2)};$$

```
Plot3D[mP, {s1, 0, 8}, {s2, 0, 8}, AxesLabel →
  {Style["s1", Bold, 20, FontColor → Black], Style["s2", Bold, 20, FontColor → Black],
   Style["m(s1,s2)", Bold, 20, FontColor → Black]},
 PlotLabel → Style["m(s1,s2)", Bold, 30, FontColor → Black],
 ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
 TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

Out[604]=



So we will have $m(s_1, 0) < m(0, 0)$ & $m(0, s_2) < m(0, 0)$. So network F fails.

Network G

Dynamics-

In[605]:=
```
dadt = ka0 + kaa * a - (kda + kd1 * s1 + kd2 * s2) * a;
dbdt = kb0 - kdb * b;
dmdt = km0 + kma * a + kmb * b - (kdm + kmm * m) * m;
Solve[{dadt == 0, dbdt == 0, dmdt == 0}, {a, b, m}]
```

Out[608]=

$$\left\{\left\{a \to \frac{k_{a0}}{-k_{aa} + k_{da} + k_{d1} s_1 + k_{d2} s_2},\ b \to \frac{k_{b0}}{k_{db}},\ m \to \left(k_{aa} k_{db} k_{dm} - k_{da} k_{db} k_{dm} - k_{d1} k_{db} k_{dm} s_1 - k_{d2} k_{db} k_{dm} s_2 - \sqrt{\left((-k_{aa} k_{db} k_{dm} + k_{da} k_{db} k_{dm} + k_{d1} k_{db} k_{dm} s_1 + k_{d2} k_{db} k_{dm} s_2)^2 - 4 (k_{aa} k_{db} k_{m0} - k_{da} k_{db} k_{m0} - k_{a0} k_{db} k_{ma} + k_{aa} k_{b0} k_{mb} - k_{b0} k_{da} k_{mb} - k_{d1} k_{db} k_{m0} s_1 - k_{b0} k_{d1} k_{mb} s_1 - k_{d2} k_{db} k_{m0} s_2 - k_{b0} k_{d2} k_{mb} s_2) (-k_{aa} k_{db} k_{mm} + k_{da} k_{db} k_{mm} + k_{d1} k_{db} k_{mm} s_1 + k_{d2} k_{db} k_{mm} s_2)\right)}\right) \Big/ (2 (-k_{aa} k_{db} k_{mm} + k_{da} k_{db} k_{mm} + k_{d1} k_{db} k_{mm} s_1 + k_{d2} k_{db} k_{mm} s_2))\right\},$$

$$\left\{a \to \frac{k_{a0}}{-k_{aa} + k_{da} + k_{d1} s_1 + k_{d2} s_2},\ b \to \frac{k_{b0}}{k_{db}},\ m \to \left(k_{aa} k_{db} k_{dm} - k_{da} k_{db} k_{dm} - k_{d1} k_{db} k_{dm} s_1 - k_{d2} k_{db} k_{dm} s_2 + \sqrt{\left((-k_{aa} k_{db} k_{dm} + k_{da} k_{db} k_{dm} + k_{d1} k_{db} k_{dm} s_1 + k_{d2} k_{db} k_{dm} s_2)^2 - 4 (k_{aa} k_{db} k_{m0} - k_{da} k_{db} k_{m0} - k_{a0} k_{db} k_{ma} + k_{aa} k_{b0} k_{mb} - k_{b0} k_{da} k_{mb} - k_{d1} k_{db} k_{m0} s_1 - k_{b0} k_{d1} k_{mb} s_1 - k_{d2} k_{db} k_{m0} s_2 - k_{b0} k_{d2} k_{mb} s_2) (-k_{aa} k_{db} k_{mm} + k_{da} k_{db} k_{mm} + k_{d1} k_{db} k_{mm} s_1 + k_{d2} k_{db} k_{mm} s_2)\right)}\right) \Big/ (2 (-k_{aa} k_{db} k_{mm} + k_{da} k_{db} k_{mm} + k_{d1} k_{db} k_{mm} s_1 + k_{d2} k_{db} k_{mm} s_2))\right\}\right\}$$

For plotting-

In[613]:=
```
dadt = 1 + 1 * a - (2 + 1 * s1 + 1 * s2) * a;
dbdt = 1 - 1 * b;
dmdt = 1 + 1 * a + 1 * b - (1 + 1 * m) * m;
Solve[{dadt == 0, dbdt == 0, dmdt == 0}, {a, b, m}]
```

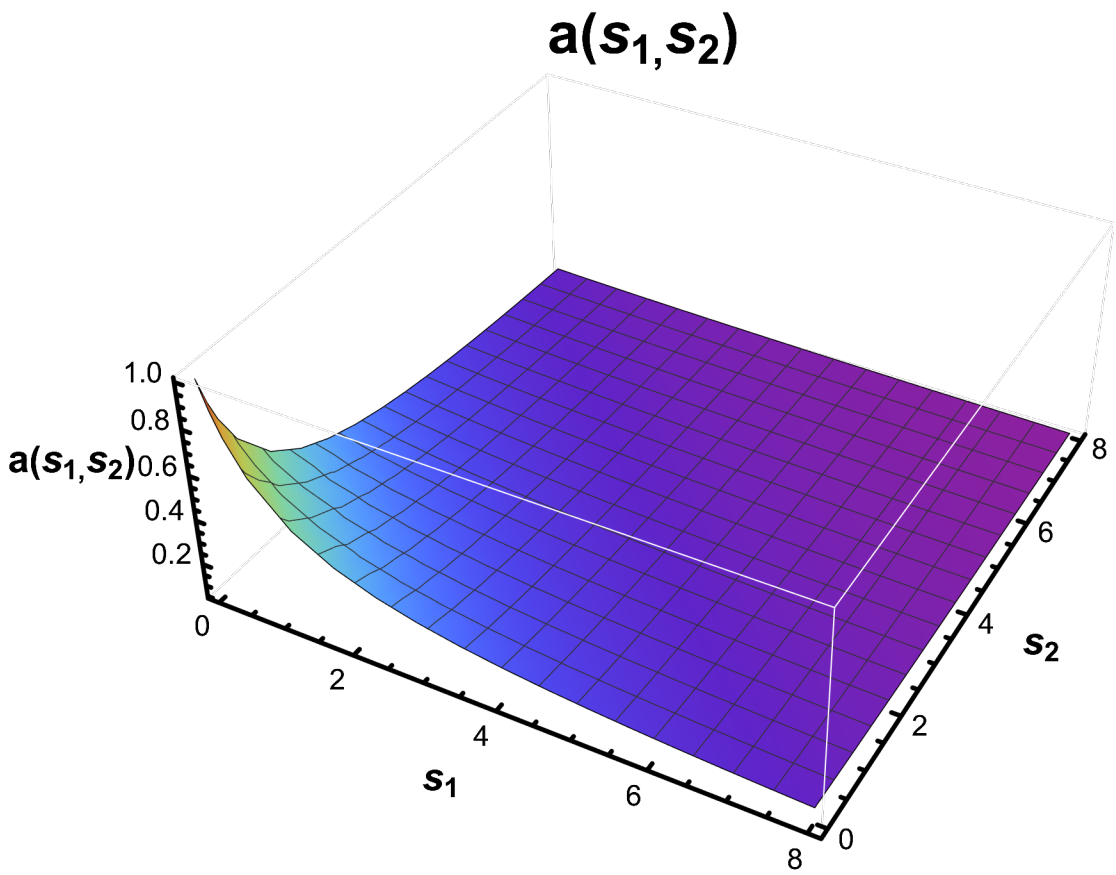
Out[616]=

$$\left\{\left\{a \to \frac{1}{1+s_1+s_2},\ b \to 1,\ m \to \frac{-1-s_1-s_2-\sqrt{13+22\,s_1+9\,s_1^2+22\,s_2+18\,s_1\,s_2+9\,s_2^2}}{2\times(1+s_1+s_2)}\right\},\right.$$
$$\left.\left\{a \to \frac{1}{1+s_1+s_2},\ b \to 1,\ m \to \frac{-1-s_1-s_2+\sqrt{13+22\,s_1+9\,s_1^2+22\,s_2+18\,s_1\,s_2+9\,s_2^2}}{2\times(1+s_1+s_2)}\right\}\right\}$$

Surface maps-

In[617]:=
```
aP = 1/(1 + s1 + s2);
Plot3D[aP, {s1, 0, 8}, {s2, 0, 8}, AxesLabel →
  {Style["s1", Bold, 20, FontColor → Black], Style["s2", Bold, 20, FontColor → Black],
   Style["a(s1,s2)", Bold, 20, FontColor → Black]},
 PlotLabel → Style["a(s1,s2)", Bold, 30, FontColor → Black],
 ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
 TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

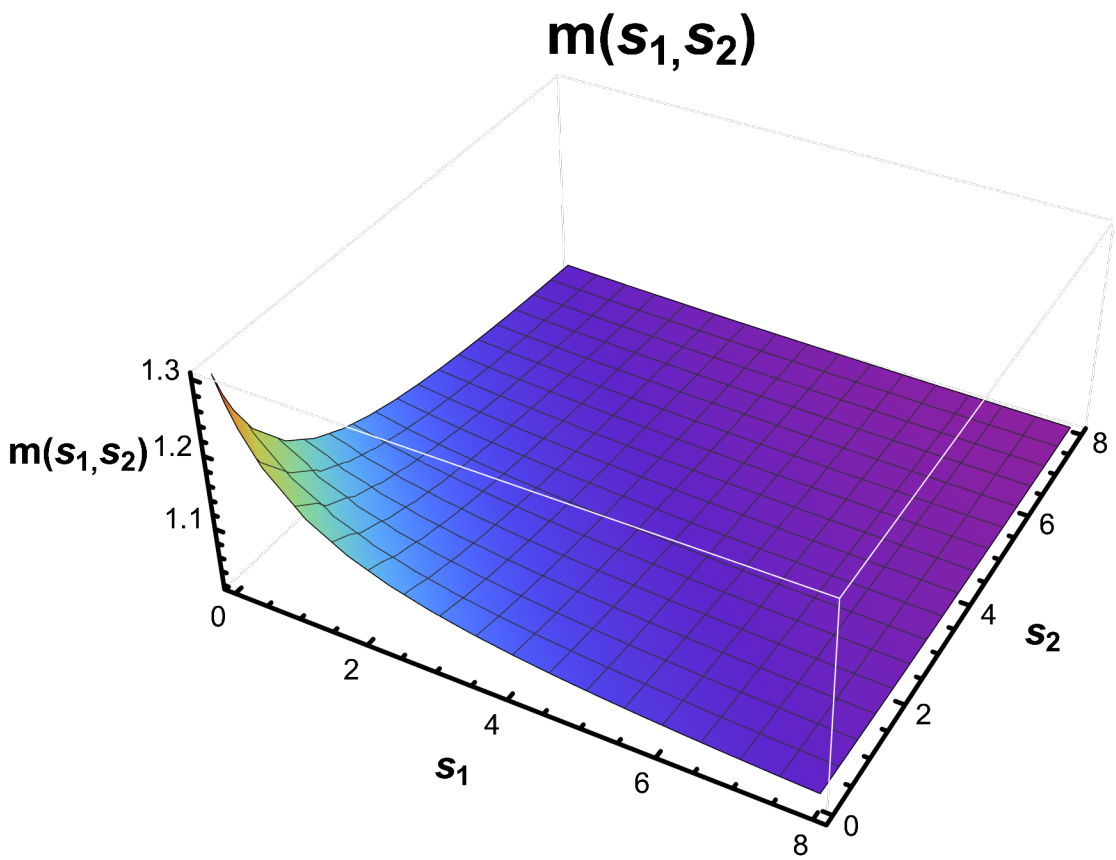
Out[618]=



b is constant.

In[619]:=
```
mP = (-1 - s1 - s2 + Sqrt[13 + 22 s1 + 9 s1^2 + 22 s2 + 18 s1 s2 + 9 s2^2]) / (2 × (1 + s1 + s2));
Plot3D[mP, {s1, 0, 8}, {s2, 0, 8}, AxesLabel →
  {Style["s1", Bold, 20, FontColor → Black], Style["s2", Bold, 20, FontColor → Black],
   Style["m(s1,s2)", Bold, 20, FontColor → Black]},
 PlotLabel → Style["m(s1,s2)", Bold, 30, FontColor → Black],
 ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
 TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

Out[620]=



So we will have m(s1, 0) < m(0, 0) & m(0, s2) < m(0, 0). So network G fails.

Network H

Dynamics-

In[625]:=
```
dadt = ka0 + k1 * s1 - (kda + kd2 * s2) * a;
dbdt = kb0 + k2 * s2 - (kdb + kd1 * s1) * b;
dmdt = km0 + kma * a + kmb * b - kdm * m;
Solve[{dadt == 0, dbdt == 0, dmdt == 0}, {a, b, m}]
```

Out[628]=
$$\left\{\left\{a \to -\frac{-k_{a0} - k_1 s_1}{k_{da} + k_{d2} s_2},\ b \to -\frac{-k_{b0} - k_2 s_2}{k_{db} + k_{d1} s_1},\ m \to -\frac{1}{k_{dm} (k_{db} + k_{d1} s_1) (k_{da} + k_{d2} s_2)} \left(-k_{da} k_{db} k_{m0} - k_{a0} k_{db} k_{ma} - k_{b0} k_{da} k_{mb} - k_{d1} k_{da} k_{m0} s_1 - k_{a0} k_{d1} k_{ma} s_1 - k_1 k_{db} k_{ma} s_1 - k_1 k_{d1} k_{ma} s_1^2 - k_{d2} k_{db} k_{m0} s_2 - k_{b0} k_{d2} k_{mb} s_2 - k_2 k_{da} k_{mb} s_2 - k_{d1} k_{d2} k_{m0} s_1 s_2 - k_2 k_{d2} k_{mb} s_2^2\right)\right\}\right\}$$

For plotting-

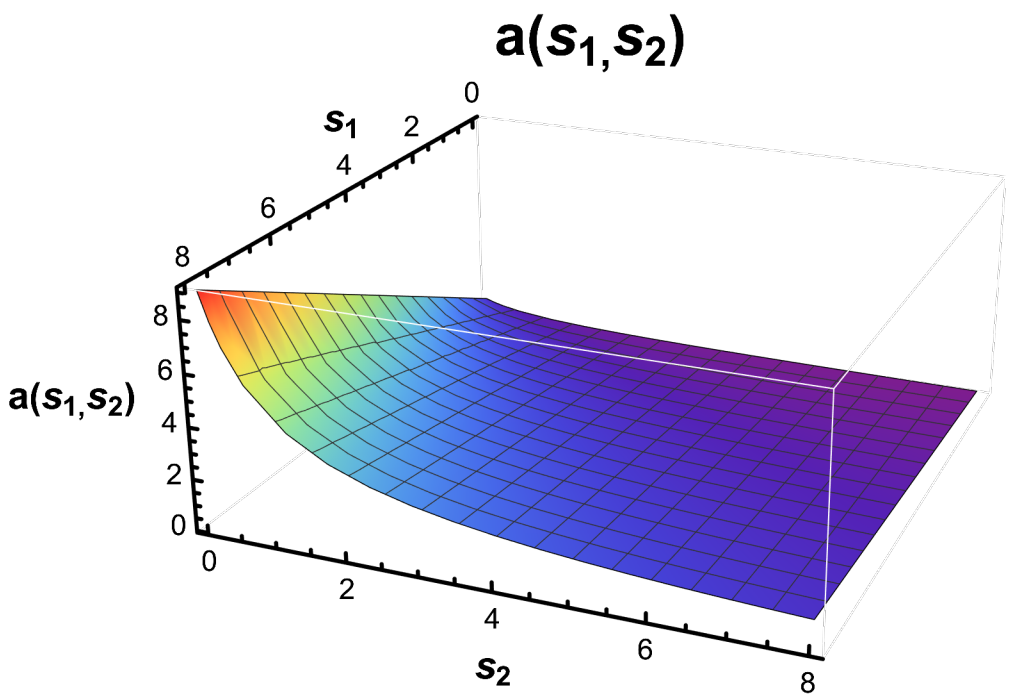
In[629]:=
```
dadt = 1 + s1 - (1 + s2) * a;
dbdt = 1 + s2 - (1 + s1) * b;
dmdt = 1 + a + b - m;
Solve[{dadt == 0, dbdt == 0, dmdt == 0}, {a, b, m}]
```

Out[632]=
$$\left\{\left\{a \to -\frac{-1 - s_1}{1 + s_2},\ b \to -\frac{-1 - s_2}{1 + s_1},\ m \to -\frac{-3 - 3 s_1 - s_1^2 - 3 s_2 - s_1 s_2 - s_2^2}{(1 + s_1) \times (1 + s_2)}\right\}\right\}$$

Surface maps-

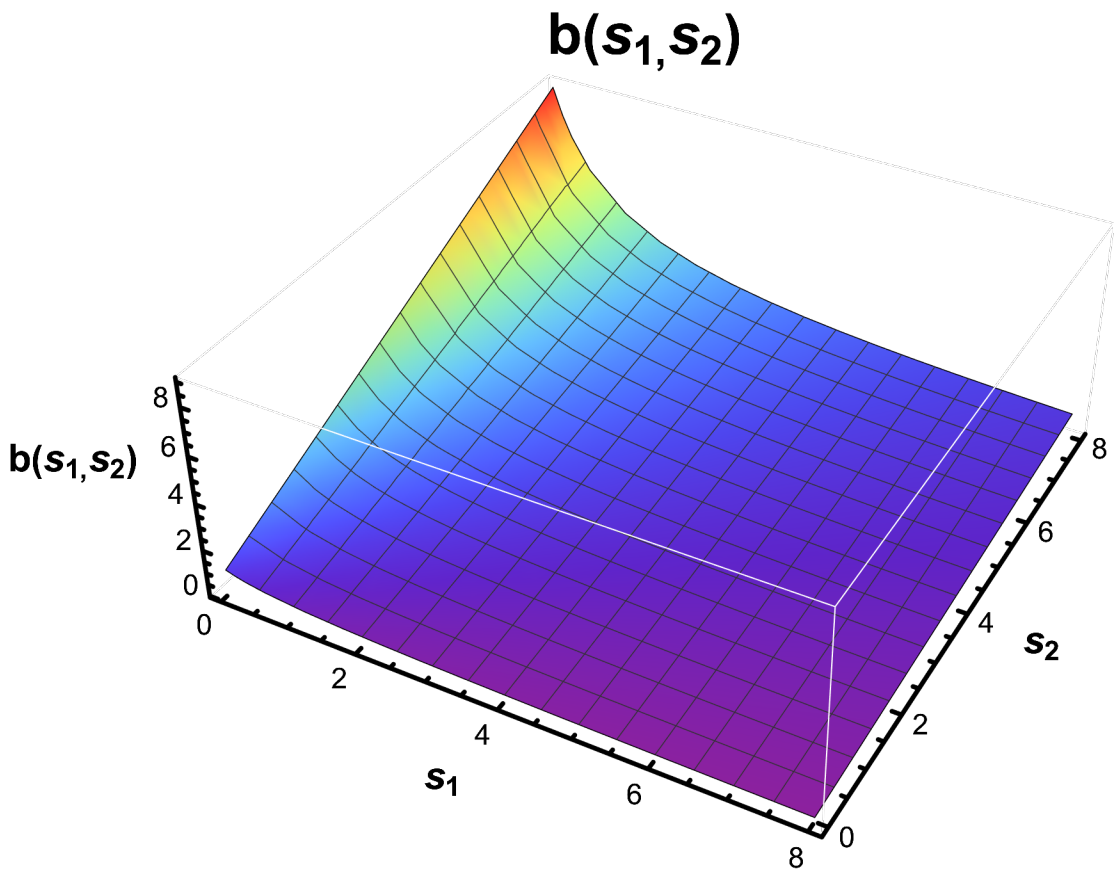
In[634]:=
```
aP = -(-1 - s1)/(1 + s2);
Plot3D[aP, {s1, 0, 8}, {s2, 0, 8}, AxesLabel →
  {Style["s1", Bold, 20, FontColor → Black], Style["s2", Bold, 20, FontColor → Black],
   Style["a(s1,s2)", Bold, 20, FontColor → Black]},
 PlotLabel → Style["a(s1,s2)", Bold, 30, FontColor → Black],
 ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
 TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

Out[635]=



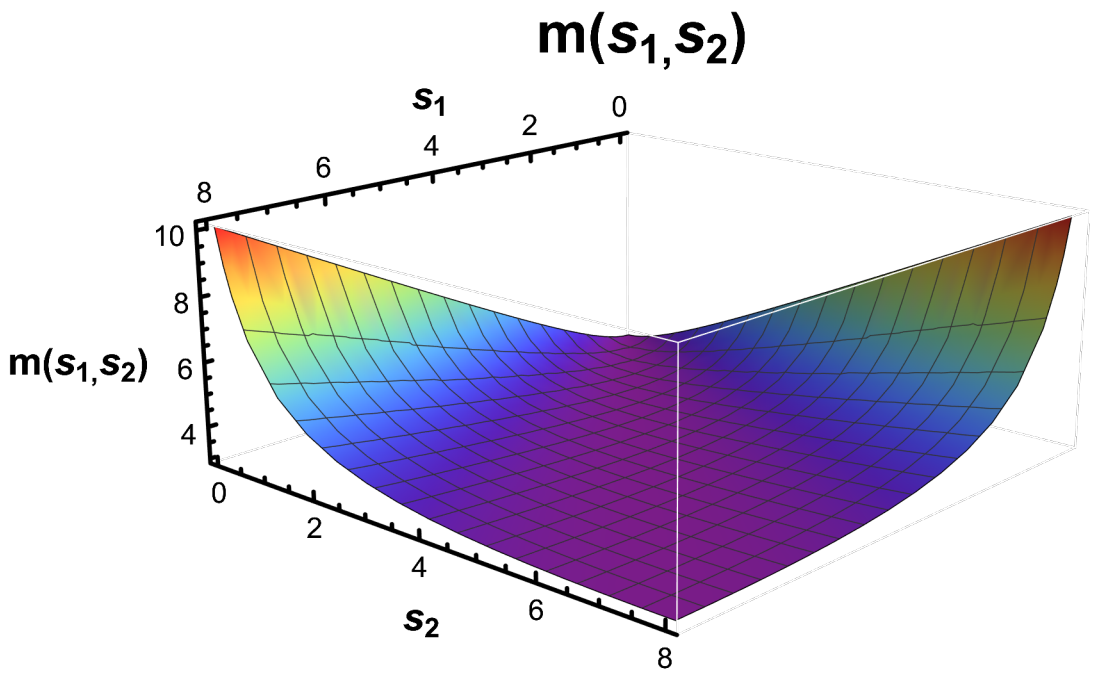
In[636]:=
```
bP = -(-1 - s2)/(1 + s1);
Plot3D[bP, {s1, 0, 8}, {s2, 0, 8}, AxesLabel →
  {Style["s1", Bold, 20, FontColor → Black], Style["s2", Bold, 20, FontColor → Black],
   Style["b(s1,s2)", Bold, 20, FontColor → Black]},
 PlotLabel → Style["b(s1,s2)", Bold, 30, FontColor → Black],
 ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
 TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

Out[637]=



In[639]:=
```
mP = -(-3 - 3 s1 - s1^2 - 3 s2 - s1 s2 - s2^2)/((1 + s1) × (1 + s2));
Plot3D[mP, {s1, 0, 8}, {s2, 0, 8}, AxesLabel →
  {Style["s1", Bold, 20, FontColor → Black], Style["s2", Bold, 20, FontColor → Black],
   Style["m(s1,s2)", Bold, 20, FontColor → Black]},
 PlotLabel → Style["m(s1,s2)", Bold, 30, FontColor → Black],
 ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
 TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

Out[640]=



This network satisfies all conditions. So, this is the chosen minimal network.