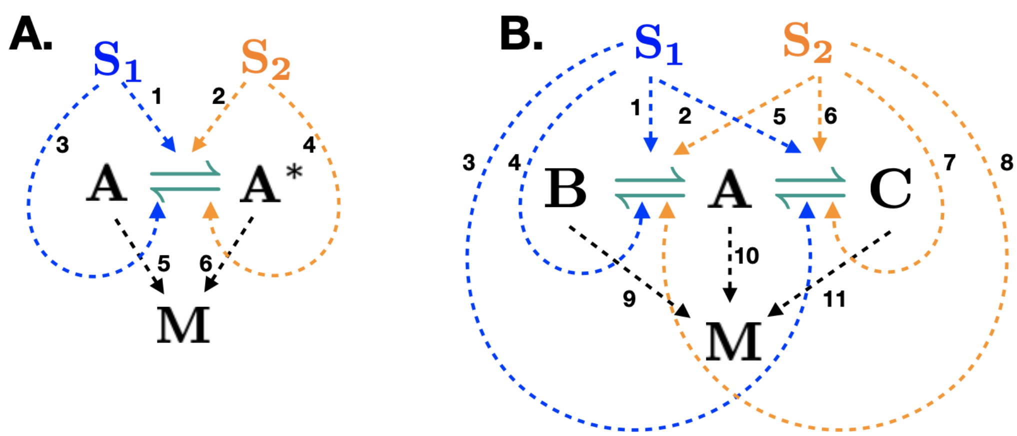
Minimal conversion network for value antagonism

Here we analyze the following general networks using conversion mechanism-



For the derivations we use the math –

$$A_1 \overset{S \downarrow}{\rightleftharpoons} A_2$$

$$\dot{a_2} = k_1 s a_1 - k_2 a_2$$
$$a_1 + a_2 = k_0$$

This analysis will be similar to analysis in Appendix C for binding networks where P_i determines the presence or absence of an edge-

Network A - five nodes

Dynamics-

In[∘]:=
```
dadt = (k_B + P_3 * k_3 * s_1 + P_4 * k_4 * s_2) * (a0 - a) - (k_F + P_1 * k_1 * s_1 + P_2 * k_2 * s_2) * a;
dmdt = k_m0 + P_5 * k_5 * a + P_6 * k_6 * (a0 - a) - k_dm * m;
Solve[{dadt == 0, dmdt == 0}, {a, m}]
```

Out[∘]=

$$\left\{\left\{a \to \frac{a0\,(k_B + k_3 P_3 s_1 + k_4 P_4 s_2)}{k_B + k_F + k_1 P_1 s_1 + k_3 P_3 s_1 + k_2 P_2 s_2 + k_4 P_4 s_2},\right.\right.$$

$$m \to -((-k_B k_{m0} - k_F k_{m0} - a0\,k_5 k_B P_5 - a0\,k_6 k_F P_6 - k_1 k_{m0} P_1 s_1 - k_3 k_{m0} P_3 s_1 - a0\,k_3 k_5 P_3 P_5 s_1 - a0\,k_1 k_6 P_1 P_6 s_1 - k_2 k_{m0} P_2 s_2 - k_4 k_{m0} P_4 s_2 - a0\,k_4 k_5 P_4 P_5 s_2 - a0\,k_2 k_6 P_2 P_6 s_2) / \left.\left.(k_{dm}\,(k_B + k_F + k_1 P_1 s_1 + k_3 P_3 s_1 + k_2 P_2 s_2 + k_4 P_4 s_2)))\right\}\right\}$$

Steady state expressions when both signals are present-

In[∘]:=
```
mP = FullSimplify[
  -((-k_B k_m0 - k_F k_m0 - a0 k_5 k_B P_5 - a0 k_6 k_F P_6 - k_1 k_m0 P_1 s_1 - k_3 k_m0 P_3 s_1 - a0 k_3 k_5 P_3 P_5 s_1 -
      a0 k_1 k_6 P_1 P_6 s_1 - k_2 k_m0 P_2 s_2 - k_4 k_m0 P_4 s_2 - a0 k_4 k_5 P_4 P_5 s_2 - a0 k_2 k_6 P_2 P_6 s_2) /
    (k_dm (k_B + k_F + k_1 P_1 s_1 + k_3 P_3 s_1 + k_2 P_2 s_2 + k_4 P_4 s_2)))]
```

Out[∘]=

$$(k_F k_{m0} + k_B (k_{m0} + a0\,k_5 P_5) + a0\,k_6 k_F P_6 + k_1 k_{m0} P_1 s_1 + k_3 k_{m0} P_3 s_1 + a0\,k_3 k_5 P_3 P_5 s_1 + a0\,k_1 k_6 P_1 P_6 s_1 + (k_4 P_4 (k_{m0} + a0\,k_5 P_5) + k_2 P_2 (k_{m0} + a0\,k_6 P_6))\,s_2) / (k_{dm}\,(k_B + k_F + (k_1 P_1 + k_3 P_3)\,s_1 + (k_2 P_2 + k_4 P_4)\,s_2))$$

Steady state expressions when only one signal is present -

In[∘]:=
```
mP1 = FullSimplify[mP /. s_2 → 0]
```

Out[∘]=

$$\frac{k_B (k_{m0} + a0\,k_5 P_5) + k_F (k_{m0} + a0\,k_6 P_6) + (k_3 P_3 (k_{m0} + a0\,k_5 P_5) + k_1 P_1 (k_{m0} + a0\,k_6 P_6))\,s_1}{k_{dm}\,(k_B + k_F + (k_1 P_1 + k_3 P_3)\,s_1)}$$

In[∘]:=
```
mP2 = FullSimplify[mP /. s_1 → 0]
```

Out[∘]=

$$\frac{k_B (k_{m0} + a0\,k_5 P_5) + k_F (k_{m0} + a0\,k_6 P_6) + (k_4 P_4 (k_{m0} + a0\,k_5 P_5) + k_2 P_2 (k_{m0} + a0\,k_6 P_6))\,s_2}{k_{dm}\,(k_B + k_F + (k_2 P_2 + k_4 P_4)\,s_2)}$$

This is the code to test the successful five node conversion network for value antagonism -

```
npts = 10 000; (*number of random {s1,s2} pairs
 over which the antagonism condition is verified*)
smax = 100; (*maximum value of s1 and s2*)
nedges = 6; (*maximum number of edges*)
Print["Successful five node conversion networks for value antagonism-\n"]
For[i = 0, i < 2^nedges, i++, P1 = PadLeft[IntegerDigits[i, 2], nedges][[1]];
 P2 = PadLeft[IntegerDigits[i, 2], nedges][[2]];
 P3 = PadLeft[IntegerDigits[i, 2], nedges][[3]];
 P4 = PadLeft[IntegerDigits[i, 2], nedges][[4]];
 P5 = PadLeft[IntegerDigits[i, 2], nedges][[5]];
 P6 = PadLeft[IntegerDigits[i, 2], nedges][[6]];
 For[j = 0, j < npts, j++, s1 = smax * RandomReal[];
  s2 = smax * RandomReal[];
  mP = (2 + (P1 + P3) s1 + (P2 + P4) s2 + P6 (1 + P1 s1 + P2 s2) + P5 (1 + P3 s1 + P4 s2)) / (2 + (P1 + P3) s1 + (P2 + P4) s2);
  m00 = 1/2 × (2 + P5 + P6);
  mP1 = (2 + P5 + P6 + (P3 (1 + P5) + P1 (1 + P6)) s1) / (2 + (P1 + P3) s1);
  mP2 = (2 + P5 + P6 + (P4 (1 + P5) + P2 (1 + P6)) s2) / (2 + (P2 + P4) s2);
  If[mP1 > m00 && mP2 > m00 && mP < Min[mP1, mP2],
   Print["P1=", P1, ", P2=", P2, ", P3=", P3, ", P4=", P4, ", P5=", P5, ", P6=",
     P6, ", Total edges=", Total[IntegerDigits[i, 2]]] && Break[], 0]]]
```

Successful five node conversion networks for value antagonism-

Plotting some surface maps-

In[ ]:=
```
dadt = (1 + P3 * 1 * s1 + P4 * 1 * s2) * (1 - a) - (1 + P1 * 1 * s1 + P2 * 1 * s2) * a;
dmdt = 1 + P5 * 1 * a + P6 * 1 * (1 - a) - 1 * m;
Solve[{dadt == 0, dmdt == 0}, {a, m}]
```

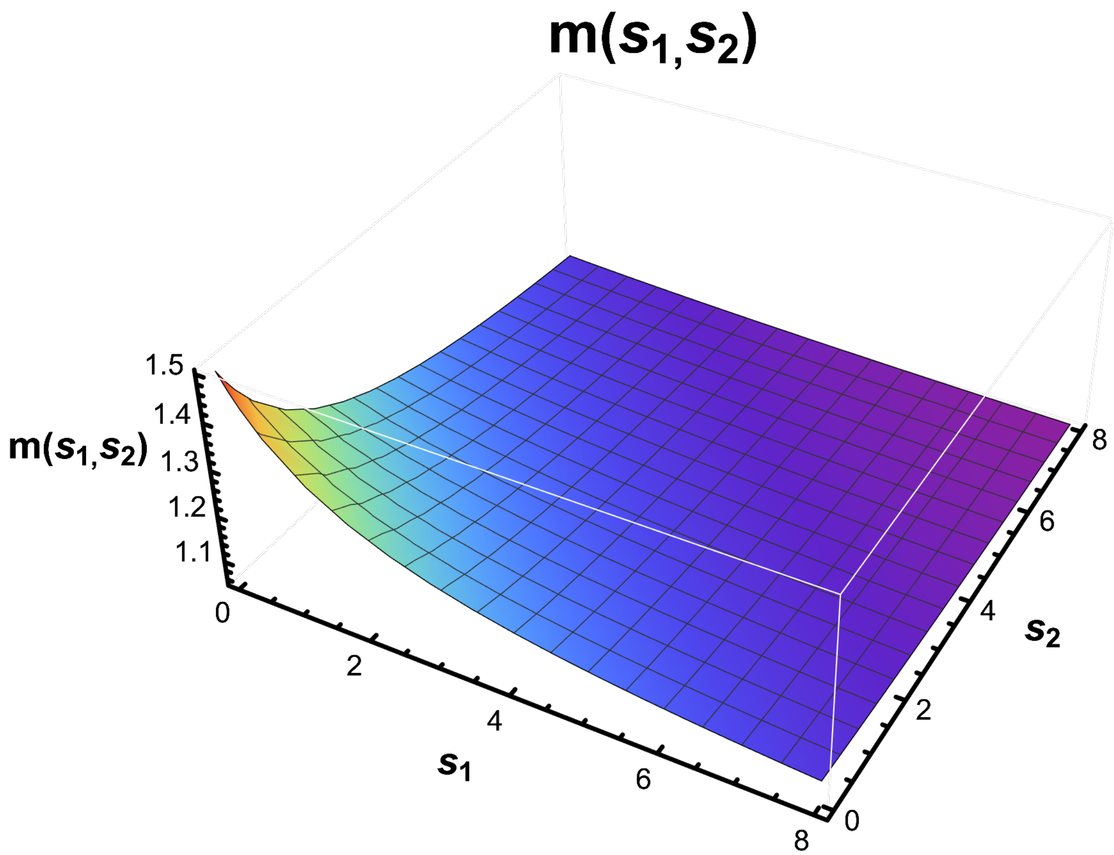
Out[ ]=

$$\left\{\left\{a \to -\frac{-1 - P_3\, s_1 - P_4\, s_2}{2 + P_1\, s_1 + P_3\, s_1 + P_2\, s_2 + P_4\, s_2},\; m \to -\frac{-2 - P_5 - P_6 - P_1\, s_1 - P_3\, s_1 - P_3\, P_5\, s_1 - P_1\, P_6\, s_1 - P_2\, s_2 - P_4\, s_2 - P_4\, P_5\, s_2 - P_2\, P_6\, s_2}{2 + P_1\, s_1 + P_3\, s_1 + P_2\, s_2 + P_4\, s_2}\right\}\right\}$$

Surface plot 1

In[∘]:=
```
P1 = 0;
P2 = 0;
P3 = 1;
P4 = 1;
P5 = 0;
P6 = 1;
```
$$mP = -\frac{-2 - P_5 - P_6 - P_1 s_1 - P_3 s_1 - P_3 P_5 s_1 - P_1 P_6 s_1 - P_2 s_2 - P_4 s_2 - P_4 P_5 s_2 - P_2 P_6 s_2}{2 + P_1 s_1 + P_3 s_1 + P_2 s_2 + P_4 s_2};$$
```
Plot3D[mP, {s1, 0, 8}, {s2, 0, 8}, AxesLabel →
  {Style["s1", Bold, 20, FontColor → Black], Style["s2", Bold, 20, FontColor → Black],
   Style["m(s1,s2)", Bold, 20, FontColor → Black]},
 PlotLabel → Style["m(s1,s2)", Bold, 30, FontColor → Black],
 ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
 TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

Out[∘]=



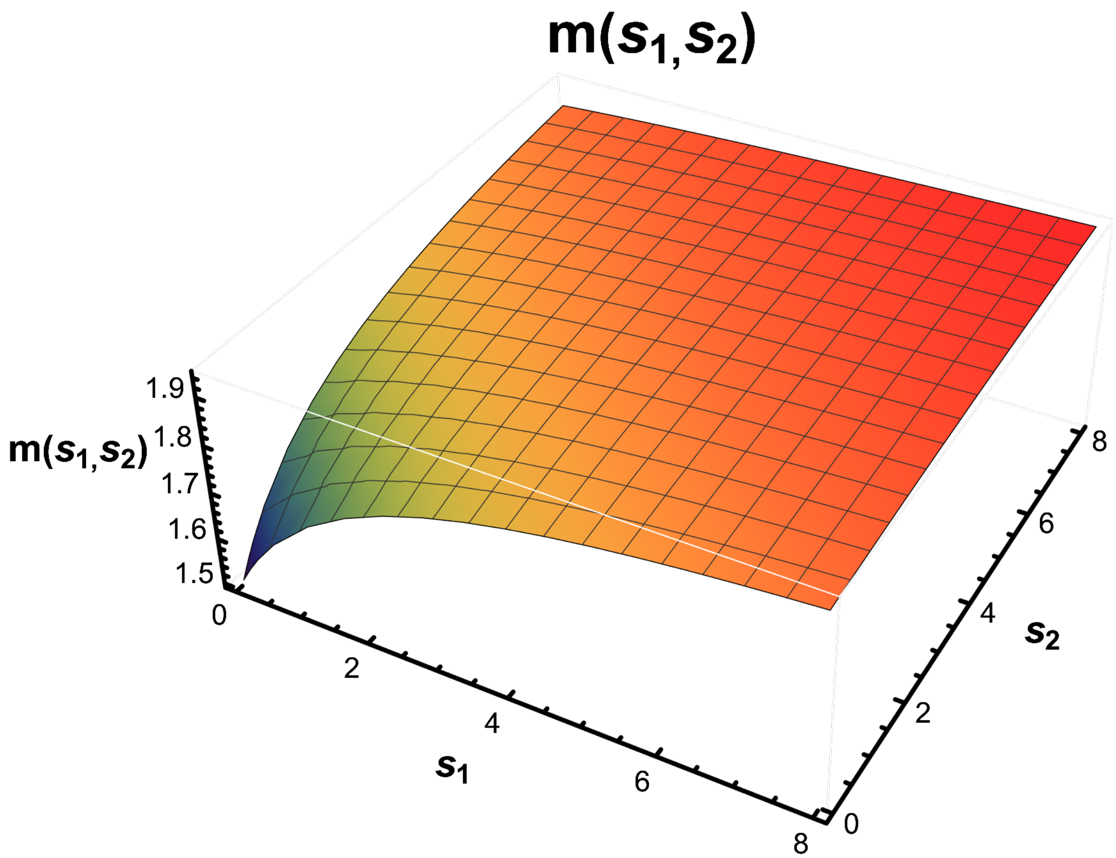
Surface plot 2

In[∘]:=
```
P₁ = 0;
P₂ = 0;
P₃ = 1;
P₄ = 1;
P₅ = 1;
P₆ = 0;
```

$$mP = -\frac{-2 - P_5 - P_6 - P_1 s_1 - P_3 s_1 - P_3 P_5 s_1 - P_1 P_6 s_1 - P_2 s_2 - P_4 s_2 - P_4 P_5 s_2 - P_2 P_6 s_2}{2 + P_1 s_1 + P_3 s_1 + P_2 s_2 + P_4 s_2};$$

```
Plot3D[mP, {s₁, 0, 8}, {s₂, 0, 8}, AxesLabel →
   {Style["s₁", Bold, 20, FontColor → Black], Style["s₂", Bold, 20, FontColor → Black],
    Style["m(s₁,s₂)", Bold, 20, FontColor → Black]},
  PlotLabel → Style["m(s₁,s₂)", Bold, 30, FontColor → Black],
  ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
  TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

Out[∘]=



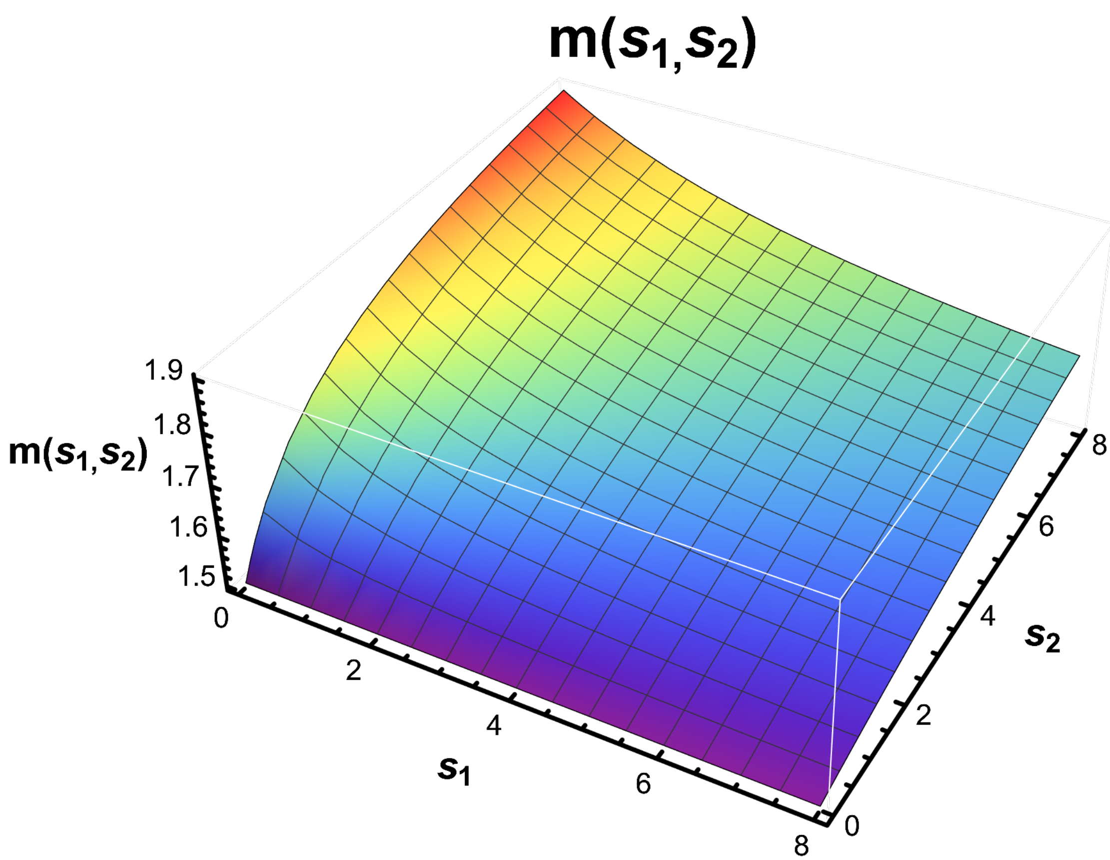
Surface plot 3

In[∘]:=
```
P1 = 1;
P2 = 0;
P3 = 1;
P4 = 1;
P5 = 1;
P6 = 0;
```

$$mP = -\frac{-2 - P_5 - P_6 - P_1 s_1 - P_3 s_1 - P_3 P_5 s_1 - P_1 P_6 s_1 - P_2 s_2 - P_4 s_2 - P_4 P_5 s_2 - P_2 P_6 s_2}{2 + P_1 s_1 + P_3 s_1 + P_2 s_2 + P_4 s_2};$$

```
Plot3D[mP, {s1, 0, 8}, {s2, 0, 8}, AxesLabel →
   {Style["s1", Bold, 20, FontColor → Black], Style["s2", Bold, 20, FontColor → Black],
    Style["m(s1,s2)", Bold, 20, FontColor → Black]},
  PlotLabel → Style["m(s1,s2)", Bold, 30, FontColor → Black],
  ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
  TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

Out[∘]=



Network B - six nodes

Dynamics-

In[∘]:=

$$\mathbf{dcdt = (k_{CA} + P_2 * k_2 * s_1 + P_6 * k_6 * s_2) * (a0 - c - b) - (k_{FC} + P_3 * k_3 * s_1 + P_7 * k_7 * s_2) * c;}$$
$$\mathbf{dbdt = (k_{BA} + P_4 * k_4 * s_1 + P_8 * k_8 * s_2) * (a0 - c - b) - (k_{FB} + P_1 * k_1 * s_1 + P_5 * k_5 * s_2) * b;}$$
$$\mathbf{dmdt = k_{m0} + P_{10} * k_{10} * (a0 - c - b) + P_9 * k_9 * b + P_{11} * k_{11} * c - k_{dm} * m;}$$
$$\mathbf{Solve[\{dcdt == 0, dbdt == 0, dmdt == 0\}, \{b, c, m\}]}$$

Out[∘]=

$$\Big\{\Big\{b \to -\big((a0\,(k_{CA} + k_2 P_2 s_1 + k_6 P_6 s_2)\,(-k_{BA} - k_4 P_4 s_1 - k_8 P_8 s_2) - a0\,(-k_{CA} - k_{FC} - k_2 P_2 s_1 - k_3 P_3 s_1 - k_6 P_6 s_2 - k_7 P_7 s_2)\,(k_{BA} + k_4 P_4 s_1 + k_8 P_8 s_2))\,/\,((-k_{CA} - k_2 P_2 s_1 - k_6 P_6 s_2)\,(-k_{BA} - k_4 P_4 s_1 - k_8 P_8 s_2) - (-k_{CA} - k_{FC} - k_2 P_2 s_1 - k_3 P_3 s_1 - k_6 P_6 s_2 - k_7 P_7 s_2)\,(-k_{BA} - k_{FB} - k_1 P_1 s_1 - k_4 P_4 s_1 - k_5 P_5 s_2 - k_8 P_8 s_2))\big),$$

$$c \to -\Big(\big(-a0\,k_{CA}\,k_{FB} - a0\,k_1\,k_{CA}\,P_1\,s_1 - a0\,k_2\,k_{FB}\,P_2\,s_1 - a0\,k_1\,k_2\,P_1\,P_2\,s_1^2 - a0\,k_5\,k_{CA}\,P_5\,s_2 - a0\,k_6\,k_{FB}\,P_6\,s_2 - a0\,k_2\,k_5\,P_2\,P_5\,s_1\,s_2 - a0\,k_1\,k_6\,P_1\,P_6\,s_1\,s_2 - a0\,k_5\,k_6\,P_5\,P_6\,s_2^2\big)\,/\,\big(k_{CA}\,k_{FB} + k_{BA}\,k_{FC} + k_{FB}\,k_{FC} + k_1\,k_{CA}\,P_1\,s_1 + k_1\,k_{FC}\,P_1\,s_1 + k_2\,k_{FB}\,P_2\,s_1 + k_3\,k_{BA}\,P_3\,s_1 + k_3\,k_{FB}\,P_3\,s_1 + k_4\,k_{FC}\,P_4\,s_1 + k_1\,k_2\,P_1\,P_2\,s_1^2 + k_1\,k_3\,P_1\,P_3\,s_1^2 + k_3\,k_4\,P_3\,P_4\,s_1^2 + k_5\,k_{CA}\,P_5\,s_2 + k_5\,k_{FC}\,P_5\,s_2 + k_6\,k_{FB}\,P_6\,s_2 + k_7\,k_{BA}\,P_7\,s_2 + k_7\,k_{FB}\,P_7\,s_2 + k_8\,k_{FC}\,P_8\,s_2 + k_2\,k_5\,P_2\,P_5\,s_1\,s_2 + k_3\,k_5\,P_3\,P_5\,s_1\,s_2 + k_1\,k_6\,P_1\,P_6\,s_1\,s_2 + k_1\,k_7\,P_1\,P_7\,s_1\,s_2 + k_4\,k_7\,P_4\,P_7\,s_1\,s_2 + k_3\,k_8\,P_3\,P_8\,s_1\,s_2 + k_5\,k_6\,P_5\,P_6\,s_2^2 + k_5\,k_7\,P_5\,P_7\,s_2^2 + k_7\,k_8\,P_7\,P_8\,s_2^2\big)\Big),$$

$$m \to -\Big(\big(-k_{CA}\,k_{FB}\,k_{m0} - k_{BA}\,k_{FC}\,k_{m0} - k_{FB}\,k_{FC}\,k_{m0} - a0\,k_9\,k_{BA}\,k_{FC}\,P_9 - a0\,k_{10}\,k_{FB}\,k_{FC}\,P_{10} - a0\,k_{11}\,k_{CA}\,k_{FB}\,P_{11} - k_1\,k_{CA}\,k_{m0}\,P_1\,s_1 - k_1\,k_{FC}\,k_{m0}\,P_1\,s_1 - k_2\,k_{FB}\,k_{m0}\,P_2\,s_1 - k_3\,k_{BA}\,k_{m0}\,P_3\,s_1 - k_3\,k_{FB}\,k_{m0}\,P_3\,s_1 - k_4\,k_{FC}\,k_{m0}\,P_4\,s_1 - a0\,k_3\,k_9\,k_{BA}\,P_3\,P_9\,s_1 - a0\,k_4\,k_9\,k_{FC}\,P_4\,P_9\,s_1 - a0\,k_1\,k_{10}\,k_{FC}\,P_1\,P_{10}\,s_1 - a0\,k_3\,k_{10}\,k_{FB}\,P_3\,P_{10}\,s_1 - a0\,k_1\,k_{11}\,k_{CA}\,P_1\,P_{11}\,s_1 - a0\,k_2\,k_{11}\,k_{FB}\,P_2\,P_{11}\,s_1 - k_1\,k_2\,k_{m0}\,P_1\,P_2\,s_1^2 - k_1\,k_3\,k_{m0}\,P_1\,P_3\,s_1^2 - k_3\,k_4\,k_{m0}\,P_3\,P_4\,s_1^2 - a0\,k_3\,k_4\,k_9\,P_3\,P_4\,P_9\,s_1^2 - a0\,k_1\,k_3\,k_{10}\,P_1\,P_3\,P_{10}\,s_1^2 - a0\,k_1\,k_2\,k_{11}\,P_1\,P_2\,P_{11}\,s_1^2 - k_5\,k_{CA}\,k_{m0}\,P_5\,s_2 - k_5\,k_{FC}\,k_{m0}\,P_5\,s_2 - k_6\,k_{FB}\,k_{m0}\,P_6\,s_2 - k_7\,k_{BA}\,k_{m0}\,P_7\,s_2 - k_7\,k_{FB}\,k_{m0}\,P_7\,s_2 - k_8\,k_{FC}\,k_{m0}\,P_8\,s_2 - a0\,k_7\,k_9\,k_{BA}\,P_7\,P_9\,s_2 - a0\,k_8\,k_9\,k_{FC}\,P_8\,P_9\,s_2 - a0\,k_5\,k_{10}\,k_{FC}\,P_5\,P_{10}\,s_2 - a0\,k_7\,k_{10}\,k_{FB}\,P_7\,P_{10}\,s_2 - a0\,k_5\,k_{11}\,k_{CA}\,P_5\,P_{11}\,s_2 - a0\,k_6\,k_{11}\,k_{FB}\,P_6\,P_{11}\,s_2 - k_2\,k_5\,k_{m0}\,P_2\,P_5\,s_1\,s_2 - k_3\,k_5\,k_{m0}\,P_3\,P_5\,s_1\,s_2 - k_1\,k_6\,k_{m0}\,P_1\,P_6\,s_1\,s_2 - k_1\,k_7\,k_{m0}\,P_1\,P_7\,s_1\,s_2 - k_4\,k_7\,k_{m0}\,P_4\,P_7\,s_1\,s_2 - k_3\,k_8\,k_{m0}\,P_3\,P_8\,s_1\,s_2 - a0\,k_4\,k_7\,k_9\,P_4\,P_7\,P_9\,s_1\,s_2 - a0\,k_3\,k_8\,k_9\,P_3\,P_8\,P_9\,s_1\,s_2 - a0\,k_3\,k_5\,k_{10}\,P_3\,P_5\,P_{10}\,s_1\,s_2 - a0\,k_1\,k_7\,k_{10}\,P_1\,P_7\,P_{10}\,s_1\,s_2 - a0\,k_2\,k_5\,k_{11}\,P_2\,P_5\,P_{11}\,s_1\,s_2 - a0\,k_1\,k_6\,k_{11}\,P_1\,P_6\,P_{11}\,s_1\,s_2 - k_5\,k_6\,k_{m0}\,P_5\,P_6\,s_2^2 - k_5\,k_7\,k_{m0}\,P_5\,P_7\,s_2^2 - k_7\,k_8\,k_{m0}\,P_7\,P_8\,s_2^2 - a0\,k_7\,k_8\,k_9\,P_7\,P_8\,P_9\,s_2^2 - a0\,k_5\,k_7\,k_{10}\,P_5\,P_7\,P_{10}\,s_2^2 - a0\,k_5\,k_6\,k_{11}\,P_5\,P_6\,P_{11}\,s_2^2\big)\,/\,\big(k_{dm}\,(k_{CA}\,k_{FB} + k_{BA}\,k_{FC} + k_{FB}\,k_{FC} + k_1\,k_{CA}\,P_1\,s_1 + k_1\,k_{FC}\,P_1\,s_1 + k_2\,k_{FB}\,P_2\,s_1 + k_3\,k_{BA}\,P_3\,s_1 + k_3\,k_{FB}\,P_3\,s_1 + k_4\,k_{FC}\,P_4\,s_1 + k_1\,k_2\,P_1\,P_2\,s_1^2 + k_1\,k_3\,P_1\,P_3\,s_1^2 + k_3\,k_4\,P_3\,P_4\,s_1^2 + k_5\,k_{CA}\,P_5\,s_2 + k_5\,k_{FC}\,P_5\,s_2 + k_6\,k_{FB}\,P_6\,s_2 + k_7\,k_{BA}\,P_7\,s_2 + k_7\,k_{FB}\,P_7\,s_2 + k_8\,k_{FC}\,P_8\,s_2 + k_2\,k_5\,P_2\,P_5\,s_1\,s_2 + k_3\,k_5\,P_3\,P_5\,s_1\,s_2 + k_1\,k_6\,P_1\,P_6\,s_1\,s_2 + k_1\,k_7\,P_1\,P_7\,s_1\,s_2 + k_4\,k_7\,P_4\,P_7\,s_1\,s_2 + k_3\,k_8\,P_3\,P_8\,s_1\,s_2 + k_5\,k_6\,P_5\,P_6\,s_2^2 + k_5\,k_7\,P_5\,P_7\,s_2^2 + k_7\,k_8\,P_7\,P_8\,s_2^2)\big)\Big)\Big\}\Big\}$$

Steady state expressions when both signals are present-

In[ ]:= mP = FullSimplify[

$$-\left(\left(-k_{CA}\,k_{FB}\,k_{m0}-k_{BA}\,k_{FC}\,k_{m0}-k_{FB}\,k_{FC}\,k_{m0}-a0\,k_9\,k_{BA}\,k_{FC}\,P_9-a0\,k_{10}\,k_{FB}\,k_{FC}\,P_{10}-a0\,k_{11}\,k_{CA}\,k_{FB}\,P_{11}-k_1\,k_{CA}\,k_{m0}\,P_1\,s_1-k_1\,k_{FC}\,k_{m0}\,P_1\,s_1-k_2\,k_{FB}\,k_{m0}\,P_2\,s_1-k_3\,k_{BA}\,k_{m0}\,P_3\,s_1-k_3\,k_{FB}\,k_{m0}\,P_3\,s_1-k_4\,k_{FC}\,k_{m0}\,P_4\,s_1-a0\,k_3\,k_9\,k_{BA}\,P_3\,P_9\,s_1-a0\,k_4\,k_9\,k_{FC}\,P_4\,P_9\,s_1-a0\,k_1\,k_{10}\,k_{FC}\,P_1\,P_{10}\,s_1-a0\,k_3\,k_{10}\,k_{FB}\,P_3\,P_{10}\,s_1-a0\,k_1\,k_{11}\,k_{CA}\,P_1\,P_{11}\,s_1-a0\,k_2\,k_{11}\,k_{FB}\,P_2\,P_{11}\,s_1-k_1\,k_2\,k_{m0}\,P_1\,P_2\,s_1^2-k_1\,k_3\,k_{m0}\,P_1\,P_3\,s_1^2-k_3\,k_4\,k_{m0}\,P_3\,P_4\,s_1^2-a0\,k_3\,k_4\,k_9\,P_3\,P_4\,P_9\,s_1^2-a0\,k_1\,k_3\,k_{10}\,P_1\,P_3\,P_{10}\,s_1^2-a0\,k_1\,k_2\,k_{11}\,P_1\,P_2\,P_{11}\,s_1^2-k_5\,k_{CA}\,k_{m0}\,P_5\,s_2-k_5\,k_{FC}\,k_{m0}\,P_5\,s_2-k_6\,k_{FB}\,k_{m0}\,P_6\,s_2-k_7\,k_{BA}\,k_{m0}\,P_7\,s_2-k_7\,k_{FB}\,k_{m0}\,P_7\,s_2-k_8\,k_{FC}\,k_{m0}\,P_8\,s_2-a0\,k_7\,k_9\,k_{BA}\,P_7\,P_9\,s_2-a0\,k_8\,k_9\,k_{FC}\,P_8\,P_9\,s_2-a0\,k_5\,k_{10}\,k_{FC}\,P_5\,P_{10}\,s_2-a0\,k_7\,k_{10}\,k_{FB}\,P_7\,P_{10}\,s_2-a0\,k_5\,k_{11}\,k_{CA}\,P_5\,P_{11}\,s_2-a0\,k_6\,k_{11}\,k_{FB}\,P_6\,P_{11}\,s_2-k_2\,k_5\,k_{m0}\,P_2\,P_5\,s_1\,s_2-k_3\,k_5\,k_{m0}\,P_3\,P_5\,s_1\,s_2-k_1\,k_6\,k_{m0}\,P_1\,P_6\,s_1\,s_2-k_1\,k_7\,k_{m0}\,P_1\,P_7\,s_1\,s_2-k_4\,k_7\,k_{m0}\,P_4\,P_7\,s_1\,s_2-k_3\,k_8\,k_{m0}\,P_3\,P_8\,s_1\,s_2-a0\,k_4\,k_7\,k_9\,P_4\,P_7\,P_9\,s_1\,s_2-a0\,k_3\,k_8\,k_9\,P_3\,P_8\,P_9\,s_1\,s_2-a0\,k_3\,k_5\,k_{10}\,P_3\,P_5\,P_{10}\,s_1\,s_2-a0\,k_1\,k_7\,k_{10}\,P_1\,P_7\,P_{10}\,s_1\,s_2-a0\,k_2\,k_5\,k_{11}\,P_2\,P_5\,P_{11}\,s_1\,s_2-a0\,k_1\,k_6\,k_{11}\,P_1\,P_6\,P_{11}\,s_1\,s_2-k_5\,k_6\,k_{m0}\,P_5\,P_6\,s_2^2-k_5\,k_7\,k_{m0}\,P_5\,P_7\,s_2^2-k_7\,k_8\,k_{m0}\,P_7\,P_8\,s_2^2-a0\,k_7\,k_8\,k_9\,P_7\,P_8\,P_9\,s_2^2-a0\,k_5\,k_7\,k_{10}\,P_5\,P_7\,P_{10}\,s_2^2-a0\,k_5\,k_6\,k_{11}\,P_5\,P_6\,P_{11}\,s_2^2\right)\Big/\left(k_{dm}\left(k_{CA}\,k_{FB}+k_{BA}\,k_{FC}+k_{FB}\,k_{FC}+k_1\,k_{CA}\,P_1\,s_1+k_1\,k_{FC}\,P_1\,s_1+k_2\,k_{FB}\,P_2\,s_1+k_3\,k_{BA}\,P_3\,s_1+k_3\,k_{FB}\,P_3\,s_1+k_4\,k_{FC}\,P_4\,s_1+k_1\,k_2\,P_1\,P_2\,s_1^2+k_1\,k_3\,P_1\,P_3\,s_1^2+k_3\,k_4\,P_3\,P_4\,s_1^2+k_5\,k_{CA}\,P_5\,s_2+k_5\,k_{FC}\,P_5\,s_2+k_6\,k_{FB}\,P_6\,s_2+k_7\,k_{BA}\,P_7\,s_2+k_7\,k_{FB}\,P_7\,s_2+k_8\,k_{FC}\,P_8\,s_2+k_2\,k_5\,P_2\,P_5\,s_1\,s_2+k_3\,k_5\,P_3\,P_5\,s_1\,s_2+k_1\,k_6\,P_1\,P_6\,s_1\,s_2+k_1\,k_7\,P_1\,P_7\,s_1\,s_2+k_4\,k_7\,P_4\,P_7\,s_1\,s_2+k_3\,k_8\,P_3\,P_8\,s_1\,s_2+k_5\,k_6\,P_5\,P_6\,s_2^2+k_5\,k_7\,P_5\,P_7\,s_2^2+k_7\,k_8\,P_7\,P_8\,s_2^2\right)\right)\right)\Big]$$

Out[ ]=

$$\Big(k_{BA}\,k_{FC}\,k_{m0}+k_{FB}\,k_{FC}\,k_{m0}+a0\,k_9\,k_{BA}\,k_{FC}\,P_9+a0\,k_{10}\,k_{FB}\,k_{FC}\,P_{10}+k_1\,k_{FC}\,k_{m0}\,P_1\,s_1+k_2\,k_{FB}\,k_{m0}\,P_2\,s_1+k_3\,k_{BA}\,k_{m0}\,P_3\,s_1+k_3\,k_{FB}\,k_{m0}\,P_3\,s_1+k_4\,k_{FC}\,k_{m0}\,P_4\,s_1+a0\,k_3\,k_9\,k_{BA}\,P_3\,P_9\,s_1+a0\,k_4\,k_9\,k_{FC}\,P_4\,P_9\,s_1+a0\,k_1\,k_{10}\,k_{FC}\,P_1\,P_{10}\,s_1+a0\,k_3\,k_{10}\,k_{FB}\,P_3\,P_{10}\,s_1+a0\,k_2\,k_{11}\,k_{FB}\,P_2\,P_{11}\,s_1+k_1\,k_2\,k_{m0}\,P_1\,P_2\,s_1^2+k_1\,k_3\,k_{m0}\,P_1\,P_3\,s_1^2+k_3\,k_4\,k_{m0}\,P_3\,P_4\,s_1^2+a0\,k_3\,k_4\,k_9\,P_3\,P_4\,P_9\,s_1^2+a0\,k_1\,k_3\,k_{10}\,P_1\,P_3\,P_{10}\,s_1^2+a0\,k_1\,k_2\,k_{11}\,P_1\,P_2\,P_{11}\,s_1^2+\big(k_6\,k_{FB}\,k_{m0}\,P_6+k_7\,k_{BA}\,k_{m0}\,P_7+k_7\,k_{FB}\,k_{m0}\,P_7+k_8\,k_{FC}\,k_{m0}\,P_8+a0\,k_7\,k_9\,k_{BA}\,P_7\,P_9+a0\,k_8\,k_9\,k_{FC}\,P_8\,P_9+a0\,k_7\,k_{10}\,k_{FB}\,P_7\,P_{10}+a0\,k_6\,k_{11}\,k_{FB}\,P_6\,P_{11}+\big((k_4\,k_7\,P_4\,P_7+k_3\,k_8\,P_3\,P_8)\,(k_{m0}+a0\,k_9\,P_9)+k_1\,P_1\,(k_7\,P_7\,(k_{m0}+a0\,k_{10}\,P_{10})+k_6\,P_6\,(k_{m0}+a0\,k_{11}\,P_{11}))\big)\,s_1+k_5\,P_5\,(k_{FC}\,(k_{m0}+a0\,k_{10}\,P_{10})+(k_3\,P_3\,(k_{m0}+a0\,k_{10}\,P_{10})+k_2\,P_2\,(k_{m0}+a0\,k_{11}\,P_{11}))\,s_1)\big)\,s_2+(k_7\,k_8\,P_7\,P_8\,(k_{m0}+a0\,k_9\,P_9)+k_5\,P_5\,(k_7\,P_7\,(k_{m0}+a0\,k_{10}\,P_{10})+k_6\,P_6\,(k_{m0}+a0\,k_{11}\,P_{11})))\,s_2^2+k_{CA}\,(k_{m0}+a0\,k_{11}\,P_{11})\,(k_{FB}+k_1\,P_1\,s_1+k_5\,P_5\,s_2)\Big)\Big/\Big(k_{dm}\,\big(k_{BA}\,k_{FC}+s_1\,(k_4\,k_{FC}\,P_4+k_1\,P_1\,(k_{FC}+(k_2\,P_2+k_3\,P_3)\,s_1)+k_3\,P_3\,(k_{BA}+k_4\,P_4\,s_1))+(k_1\,k_6\,P_1\,P_6\,s_1+k_8\,P_8\,(k_{FC}+k_3\,P_3\,s_1)+k_5\,P_5\,(k_{FC}+(k_2\,P_2+k_3\,P_3)\,s_1)+k_7\,P_7\,(k_{BA}+(k_1\,P_1+k_4\,P_4)\,s_1))\,s_2+(k_5\,P_5\,(k_6\,P_6+k_7\,P_7)+k_7\,k_8\,P_7\,P_8)\,s_2^2+k_{CA}\,(k_{FB}+k_1\,P_1\,s_1+k_5\,P_5\,s_2)+k_{FB}\,(k_{FC}+(k_2\,P_2+k_3\,P_3)\,s_1+(k_6\,P_6+k_7\,P_7)\,s_2)\big)\Big)$$

Steady state expressions when only one signal is present -

In[∘]:= **mP1 = FullSimplify[mP /. s_2 → 0]**

Out[∘]=
$$\Big(k_{FC}\,(k_{BA}\,(k_{m0}+a0\,k_9\,P_9)+k_{FB}\,(k_{m0}+a0\,k_{10}\,P_{10}))+(k_3\,k_{BA}\,k_{m0}\,P_3+k_3\,k_{FB}\,k_{m0}\,P_3+k_4\,k_{FC}\,k_{m0}\,P_4+a0\,k_3\,k_9\,k_{BA}\,P_3\,P_9+a0\,k_4\,k_9\,k_{FC}\,P_4\,P_9+a0\,k_3\,k_{10}\,k_{FB}\,P_3\,P_{10}+k_1\,k_{FC}\,P_1\,(k_{m0}+a0\,k_{10}\,P_{10})+k_2\,k_{FB}\,P_2\,(k_{m0}+a0\,k_{11}\,P_{11}))\,s_1+(k_3\,k_4\,P_3\,P_4\,(k_{m0}+a0\,k_9\,P_9)+k_1\,P_1\,(k_3\,P_3\,(k_{m0}+a0\,k_{10}\,P_{10})+k_2\,P_2\,(k_{m0}+a0\,k_{11}\,P_{11})))\,s_1^2+k_{CA}\,(k_{m0}+a0\,k_{11}\,P_{11})\,(k_{FB}+k_1\,P_1\,s_1)\Big)\Big/\Big(k_{dm}\,\Big(k_{BA}\,k_{FC}+k_{FB}\,(k_{CA}+k_{FC})+(k_1\,(k_{CA}+k_{FC})\,P_1+k_2\,k_{FB}\,P_2+k_3\,(k_{BA}+k_{FB})\,P_3+k_4\,k_{FC}\,P_4)\,s_1+(k_1\,P_1\,(k_2\,P_2+k_3\,P_3)+k_3\,k_4\,P_3\,P_4)\,s_1^2\Big)\Big)$$

In[∘]:= **mP2 = FullSimplify[mP /. s_1 → 0]**

Out[∘]=
$$\Big(k_{FC}\,(k_{BA}\,(k_{m0}+a0\,k_9\,P_9)+k_{FB}\,(k_{m0}+a0\,k_{10}\,P_{10}))+(k_7\,k_{BA}\,k_{m0}\,P_7+k_7\,k_{FB}\,k_{m0}\,P_7+k_8\,k_{FC}\,k_{m0}\,P_8+a0\,k_7\,k_9\,k_{BA}\,P_7\,P_9+a0\,k_8\,k_9\,k_{FC}\,P_8\,P_9+a0\,k_7\,k_{10}\,k_{FB}\,P_7\,P_{10}+k_5\,k_{FC}\,P_5\,(k_{m0}+a0\,k_{10}\,P_{10})+k_6\,k_{FB}\,P_6\,(k_{m0}+a0\,k_{11}\,P_{11}))\,s_2+(k_7\,k_8\,P_7\,P_8\,(k_{m0}+a0\,k_9\,P_9)+k_5\,P_5\,(k_7\,P_7\,(k_{m0}+a0\,k_{10}\,P_{10})+k_6\,P_6\,(k_{m0}+a0\,k_{11}\,P_{11})))\,s_2^2+k_{CA}\,(k_{m0}+a0\,k_{11}\,P_{11})\,(k_{FB}+k_5\,P_5\,s_2)\Big)\Big/\Big(k_{dm}\,\Big(k_{BA}\,k_{FC}+k_{FB}\,(k_{CA}+k_{FC})+(k_5\,(k_{CA}+k_{FC})\,P_5+k_6\,k_{FB}\,P_6+k_7\,(k_{BA}+k_{FB})\,P_7+k_8\,k_{FC}\,P_8)\,s_2+(k_5\,P_5\,(k_6\,P_6+k_7\,P_7)+k_7\,k_8\,P_7\,P_8)\,s_2^2\Big)\Big)$$

This is the code to test the successful six node conversion network for value antagonism

```
In[1]:= npts = 10 000; (*number of random {s1,s2} pairs
         over which the antagonism condition is verified*)
        smax = 100; (*maximum value of s1 and s2*)
        nedges = 11; (*maximum number of edges*)
        Print["Successful six node conversion networks for value antagonism-\n"]
        For[i = 0, i < 2^nedges, i++, P1 = PadLeft[IntegerDigits[i, 2], nedges][[1]];
         P2 = PadLeft[IntegerDigits[i, 2], nedges][[2]];
         P3 = PadLeft[IntegerDigits[i, 2], nedges][[3]];
         P4 = PadLeft[IntegerDigits[i, 2], nedges][[4]];
         P5 = PadLeft[IntegerDigits[i, 2], nedges][[5]];
         P6 = PadLeft[IntegerDigits[i, 2], nedges][[6]];
         P7 = PadLeft[IntegerDigits[i, 2], nedges][[7]];
         P8 = PadLeft[IntegerDigits[i, 2], nedges][[8]];
         P9 = PadLeft[IntegerDigits[i, 2], nedges][[9]];
         P10 = PadLeft[IntegerDigits[i, 2], nedges][[10]];
         P11 = PadLeft[IntegerDigits[i, 2], nedges][[11]];
         For[j = 0, j < npts, j++, s1 = smax * RandomReal[];
          s2 = smax * RandomReal[];
          mP = (3 + P11 + (2 P1 + P2 + 2 P3 + P4) s1 +
               s1 ((P1 + P2) P11 + (P3 P4 + P1 (P3 + P2 (1 + P11))) s1) + (2 P5 + P6 + 2 P7 + P8) s2 +
               ((P5 + P6) P11 + (P4 P7 + P3 (P5 + P8) + P2 P5 (1 + P11) + P1 (P7 + P6 (1 + P11))) s1) s2 +
               (P7 P8 + P5 (P7 + P6 (1 + P11))) s2^2 + P10 (1 + P1 s1 + P5 s2) × (1 + P3 s1 + P7 s2) +
               P9 (1 + P3 s1 + P7 s2) × (1 + P4 s1 + P8 s2)) /
             (3 + s1 (P2 + P4 + P1 (2 + (P2 + P3) s1) + P3 (2 + P4 s1)) + (2 P5 + P6 + 2 P7 + P8) s2 +
               (P2 P5 + P1 P6 + (P1 + P4) P7 + P3 (P5 + P8)) s1 s2 + (P5 (P6 + P7) + P7 P8) s2^2);
          m00 = 1/3 × (3 + P9 + P10 + P11);
          mP1 = (3 + P9 + P10 + P11 + (P4 (1 + P9) + P3 (2 + P9 + P10) + P2 (1 + P11) + P1 (2 + P10 + P11)) s1 +
               (P3 P4 (1 + P9) + P1 (P3 (1 + P10) + P2 (1 + P11))) s1^2) /
             (3 + s1 (P2 + P4 + P1 (2 + (P2 + P3) s1) + P3 (2 + P4 s1)));
          mP2 = (3 + P9 + P10 + P11 + (P8 (1 + P9) + P7 (2 + P9 + P10) + P6 (1 + P11) + P5 (2 + P10 + P11)) s2 +
               (P7 P8 (1 + P9) + P5 (P7 (1 + P10) + P6 (1 + P11))) s2^2) /
             (3 + s2 (P6 + P8 + P5 (2 + (P6 + P7) s2) + P7 (2 + P8 s2)));
          If[mP1 > m00 && mP2 > m00 && mP < Min[mP1, mP2],
           Print["P1=", P1, ", P2=", P2, ", P3=", P3, ", P4=", P4, ", P5=", P5, ", P6=",
             P6, ", P7=", P7, ", P8=", P8, ", P9=", P9, ", P10=", P10, ", P11=",
             P11, ", Total edges=", Total[IntegerDigits[i, 2]]] && Break[], 0]]]
```

Successful six node conversion networks for value antagonism-

P_1=0, P_2=0, P_3=1, P_4=1, P_5=1, P_6=1, P_7=0, P_8=0, P_9=1, P_{10}=0, P_{11}=1, Total edges=6

P_1=0, P_2=0, P_3=1, P_4=1, P_5=1, P_6=1, P_7=0, P_8=1, P_9=1, P_{10}=0, P_{11}=1, Total edges=7

P_1=0, P_2=1, P_3=1, P_4=1, P_5=1, P_6=0, P_7=1, P_8=0, P_9=1, P_{10}=1, P_{11}=0, Total edges=7

P_1=0, P_2=1, P_3=1, P_4=1, P_5=1, P_6=0, P_7=1, P_8=1, P_9=1, P_{10}=1, P_{11}=0, Total edges=8

P_1=0, P_2=1, P_3=1, P_4=1, P_5=1, P_6=1, P_7=0, P_8=0, P_9=1, P_{10}=0, P_{11}=1, Total edges=7

P_1=0, P_2=1, P_3=1, P_4=1, P_5=1, P_6=1, P_7=0, P_8=1, P_9=1, P_{10}=0, P_{11}=1, Total edges=8

P_1=1, P_2=0, P_3=1, P_4=0, P_5=0, P_6=1, P_7=1, P_8=1, P_9=1, P_{10}=1, P_{11}=0, Total edges=7

P_1=1, P_2=0, P_3=1, P_4=0, P_5=1, P_6=1, P_7=0, P_8=1, P_9=0, P_{10}=1, P_{11}=1, Total edges=7

P_1=1, P_2=0, P_3=1, P_4=1, P_5=0, P_6=1, P_7=1, P_8=1, P_9=1, P_{10}=1, P_{11}=0, Total edges=8

P_1=1, P_2=1, P_3=0, P_4=0, P_5=0, P_6=0, P_7=1, P_8=1, P_9=1, P_{10}=0, P_{11}=1, Total edges=6

P_1=1, P_2=1, P_3=0, P_4=0, P_5=0, P_6=1, P_7=1, P_8=1, P_9=1, P_{10}=0, P_{11}=1, Total edges=7

P_1=1, P_2=1, P_3=0, P_4=1, P_5=0, P_6=0, P_7=1, P_8=1, P_9=1, P_{10}=0, P_{11}=1, Total edges=7

P_1=1, P_2=1, P_3=0, P_4=1, P_5=0, P_6=1, P_7=1, P_8=1, P_9=1, P_{10}=0, P_{11}=1, Total edges=8

P_1=1, P_2=1, P_3=0, P_4=1, P_5=1, P_6=0, P_7=1, P_8=0, P_9=0, P_{10}=1, P_{11}=1, Total edges=7

P_1=1, P_2=1, P_3=0, P_4=1, P_5=1, P_6=1, P_7=1, P_8=0, P_9=0, P_{10}=1, P_{11}=1, Total edges=8

P_1=1, P_2=1, P_3=1, P_4=0, P_5=1, P_6=1, P_7=0, P_8=1, P_9=0, P_{10}=1, P_{11}=1, Total edges=8

Some surface plots for network B

In[ ]:=

$$\mathrm{dcdt} = (1 + P_2 * 1 * s_1 + P_6 * 1 * s_2) * (1 - c - b) - (1 + P_3 * 1 * s_1 + P_7 * 1 * s_2) * c;$$
$$\mathrm{dbdt} = (1 + P_4 * 1 * s_1 + P_8 * 1 * s_2) * (1 - c - b) - (1 + P_1 * 1 * s_1 + P_5 * 1 * s_2) * b;$$
$$\mathrm{dmdt} = 1 + P_{10} * 1 * (1 - c - b) + P_9 * 1 * b + P_{11} * 1 * c - 1 * m;$$
$$\mathrm{Solve}[\{\mathrm{dcdt} == 0, \mathrm{dbdt} == 0, \mathrm{dmdt} == 0\}, \{b, c, m\}]$$

Out[ ]=

$$\{\{b \to -(((1 + P_2 s_1 + P_6 s_2) \times (-1 - P_4 s_1 - P_8 s_2) - (-2 - P_2 s_1 - P_3 s_1 - P_6 s_2 - P_7 s_2) \times (1 + P_4 s_1 + P_8 s_2)) / ((-1 - P_2 s_1 - P_6 s_2) \times (-1 - P_4 s_1 - P_8 s_2) - (-2 - P_2 s_1 - P_3 s_1 - P_6 s_2 - P_7 s_2) \times (-2 - P_1 s_1 - P_4 s_1 - P_5 s_2 - P_8 s_2))),$$
$$c \to -\Big(\big(-1 - P_1 s_1 - P_2 s_1 - P_1 P_2 s_1^2 - P_5 s_2 - P_6 s_2 - P_2 P_5 s_1 s_2 - P_1 P_6 s_1 s_2 - P_5 P_6 s_2^2\big) \Big/ \big(3 + 2 P_1 s_1 + P_2 s_1 + 2 P_3 s_1 + P_4 s_1 + P_1 P_2 s_1^2 + P_1 P_3 s_1^2 + P_3 P_4 s_1^2 + 2 P_5 s_2 + P_6 s_2 + 2 P_7 s_2 + P_8 s_2 + P_2 P_5 s_1 s_2 + P_3 P_5 s_1 s_2 + P_1 P_6 s_1 s_2 + P_1 P_7 s_1 s_2 + P_4 P_7 s_1 s_2 + P_3 P_8 s_1 s_2 + P_5 P_6 s_2^2 + P_5 P_7 s_2^2 + P_7 P_8 s_2^2\big)\Big),$$
$$m \to -\Big(\big(-3 - P_9 - P_{10} - P_{11} - 2 P_1 s_1 - P_2 s_1 - 2 P_3 s_1 - P_4 s_1 - P_3 P_9 s_1 - P_4 P_9 s_1 - P_1 P_{10} s_1 - P_3 P_{10} s_1 - P_1 P_{11} s_1 - P_2 P_{11} s_1 - P_1 P_2 s_1^2 - P_1 P_3 s_1^2 - P_3 P_4 s_1^2 - P_3 P_4 P_9 s_1^2 - P_1 P_3 P_{10} s_1^2 - P_1 P_2 P_{11} s_1^2 - 2 P_5 s_2 - P_6 s_2 - 2 P_7 s_2 - P_8 s_2 - P_7 P_9 s_2 - P_8 P_9 s_2 - P_5 P_{10} s_2 - P_7 P_{10} s_2 - P_5 P_{11} s_2 - P_6 P_{11} s_2 - P_2 P_5 s_1 s_2 - P_3 P_5 s_1 s_2 - P_1 P_6 s_1 s_2 - P_1 P_7 s_1 s_2 - P_4 P_7 s_1 s_2 - P_3 P_8 s_1 s_2 - P_4 P_7 P_9 s_1 s_2 - P_3 P_8 P_9 s_1 s_2 - P_3 P_5 P_{10} s_1 s_2 - P_1 P_7 P_{10} s_1 s_2 - P_2 P_5 P_{11} s_1 s_2 - P_1 P_6 P_{11} s_1 s_2 - P_5 P_6 s_2^2 - P_5 P_7 s_2^2 - P_7 P_8 s_2^2 - P_7 P_8 P_9 s_2^2 - P_5 P_7 P_{10} s_2^2 - P_5 P_6 P_{11} s_2^2\big) \Big/ \big(3 + 2 P_1 s_1 + P_2 s_1 + 2 P_3 s_1 + P_4 s_1 + P_1 P_2 s_1^2 + P_1 P_3 s_1^2 + P_3 P_4 s_1^2 + 2 P_5 s_2 + P_6 s_2 + 2 P_7 s_2 + P_8 s_2 + P_2 P_5 s_1 s_2 + P_3 P_5 s_1 s_2 + P_1 P_6 s_1 s_2 + P_1 P_7 s_1 s_2 + P_4 P_7 s_1 s_2 + P_3 P_8 s_1 s_2 + P_5 P_6 s_2^2 + P_5 P_7 s_2^2 + P_7 P_8 s_2^2\big)\Big)\}\}$$

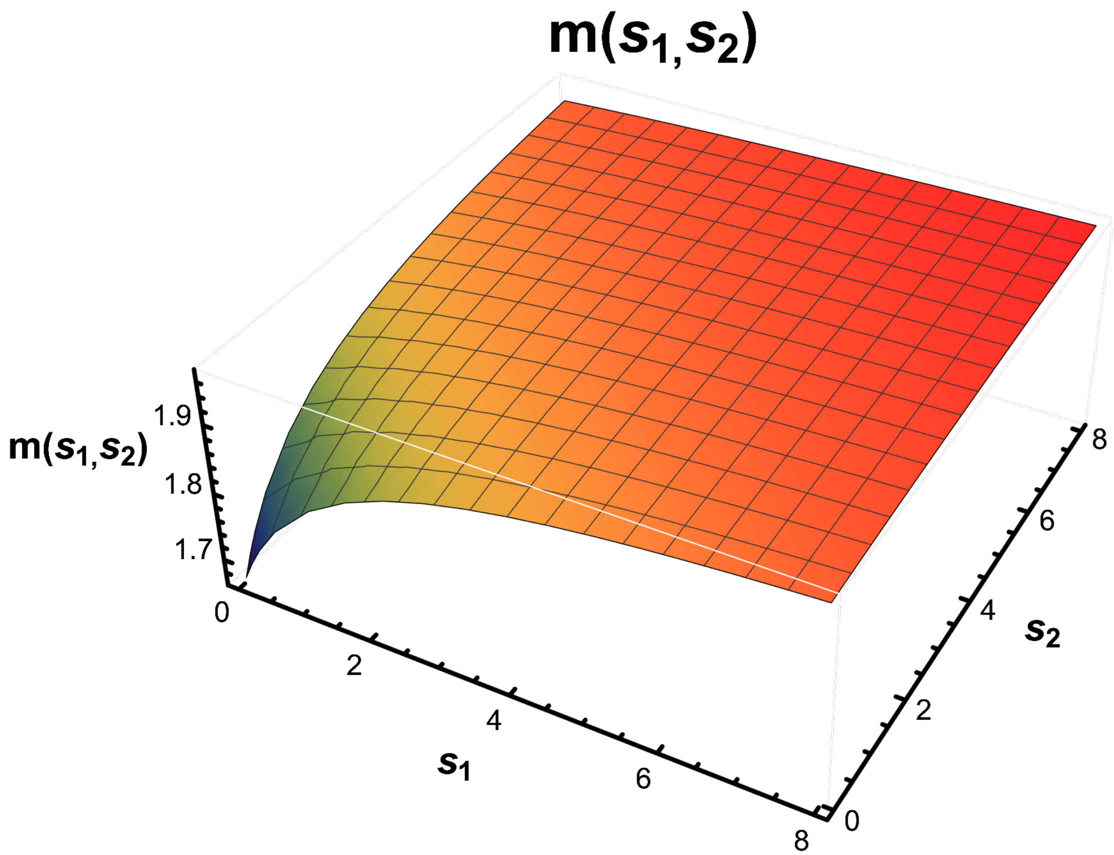
Out[ ]=

$$\{\{b \to -(((1 + P_2 s_1 + P_6 s_2) \times (-1 - P_4 s_1 - P_8 s_2) - (-2 - P_2 s_1 - P_3 s_1 - P_6 s_2 - P_7 s_2) \times (1 + P_4 s_1 + P_8 s_2)) / ((-1 - P_2 s_1 - P_6 s_2) \times (-1 - P_4 s_1 - P_8 s_2) - (-2 - P_2 s_1 - P_3 s_1 - P_6 s_2 - P_7 s_2) \times (-2 - P_1 s_1 - P_4 s_1 - P_5 s_2 - P_8 s_2))),$$
$$c \to -\Big(\big(-1 - P_1 s_1 - P_2 s_1 - P_1 P_2 s_1^2 - P_5 s_2 - P_6 s_2 - P_2 P_5 s_1 s_2 - P_1 P_6 s_1 s_2 - P_5 P_6 s_2^2\big) \Big/ \big(3 + 2 P_1 s_1 + P_2 s_1 + 2 P_3 s_1 + P_4 s_1 + P_1 P_2 s_1^2 + P_1 P_3 s_1^2 + P_3 P_4 s_1^2 + 2 P_5 s_2 + P_6 s_2 + 2 P_7 s_2 + P_8 s_2 + P_2 P_5 s_1 s_2 + P_3 P_5 s_1 s_2 + P_1 P_6 s_1 s_2 + P_1 P_7 s_1 s_2 + P_4 P_7 s_1 s_2 + P_3 P_8 s_1 s_2 + P_5 P_6 s_2^2 + P_5 P_7 s_2^2 + P_7 P_8 s_2^2\big)\Big),$$
$$m \to -\Big(\big(-3 - P_9 - P_{10} - P_{11} - 2 P_1 s_1 - P_2 s_1 - 2 P_3 s_1 - P_4 s_1 - P_3 P_9 s_1 - P_4 P_9 s_1 - P_1 P_{10} s_1 - P_3 P_{10} s_1 - P_1 P_{11} s_1 - P_2 P_{11} s_1 - P_1 P_2 s_1^2 - P_1 P_3 s_1^2 - P_3 P_4 s_1^2 - P_3 P_4 P_9 s_1^2 - P_1 P_3 P_{10} s_1^2 - P_1 P_2 P_{11} s_1^2 - 2 P_5 s_2 - P_6 s_2 - 2 P_7 s_2 - P_8 s_2 - P_7 P_9 s_2 - P_8 P_9 s_2 - P_5 P_{10} s_2 - P_7 P_{10} s_2 - P_5 P_{11} s_2 - P_6 P_{11} s_2 - P_2 P_5 s_1 s_2 - P_3 P_5 s_1 s_2 - P_1 P_6 s_1 s_2 - P_1 P_7 s_1 s_2 - P_4 P_7 s_1 s_2 - P_3 P_8 s_1 s_2 - P_4 P_7 P_9 s_1 s_2 - P_3 P_8 P_9 s_1 s_2 - P_3 P_5 P_{10} s_1 s_2 - P_1 P_7 P_{10} s_1 s_2 - P_2 P_5 P_{11} s_1 s_2 - P_1 P_6 P_{11} s_1 s_2 - P_5 P_6 s_2^2 - P_5 P_7 s_2^2 - P_7 P_8 s_2^2 - P_7 P_8 P_9 s_2^2 - P_5 P_7 P_{10} s_2^2 - P_5 P_6 P_{11} s_2^2\big) \Big/ \big(3 + 2 P_1 s_1 + P_2 s_1 + 2 P_3 s_1 + P_4 s_1 + P_1 P_2 s_1^2 + P_1 P_3 s_1^2 + P_3 P_4 s_1^2 + 2 P_5 s_2 + P_6 s_2 + 2 P_7 s_2 + P_8 s_2 + P_2 P_5 s_1 s_2 + P_3 P_5 s_1 s_2 + P_1 P_6 s_1 s_2 + P_1 P_7 s_1 s_2 + P_4 P_7 s_1 s_2 + P_3 P_8 s_1 s_2 + P_5 P_6 s_2^2 + P_5 P_7 s_2^2 + P_7 P_8 s_2^2\big)\Big)\}\}$$

Surface plot 1

In[∘]:=
```
P1 = 0;
P2 = 1;
P3 = 0;
P4 = 1;
P5 = 0;
P6 = 1;
P7 = 0;
P8 = 1;
P9 = 1;
P10 = 0;
P11 = 1;
mP =
  -((-3 - P9 - P10 - P11 - 2 P1 s1 - P2 s1 - 2 P3 s1 - P4 s1 - P3 P9 s1 - P4 P9 s1 - P1 P10 s1 - P3 P10 s1 -
       P1 P11 s1 - P2 P11 s1 - P1 P2 s1^2 - P1 P3 s1^2 - P3 P4 s1^2 - P3 P4 P9 s1^2 - P1 P3 P10 s1^2 - P1 P2 P11 s1^2 -
       2 P5 s2 - P6 s2 - 2 P7 s2 - P8 s2 - P7 P9 s2 - P8 P9 s2 - P5 P10 s2 - P7 P10 s2 - P5 P11 s2 -
       P6 P11 s2 - P2 P5 s1 s2 - P3 P5 s1 s2 - P1 P6 s1 s2 - P1 P7 s1 s2 - P4 P7 s1 s2 - P3 P8 s1 s2 -
       P4 P7 P9 s1 s2 - P3 P8 P9 s1 s2 - P3 P5 P10 s1 s2 - P1 P7 P10 s1 s2 - P2 P5 P11 s1 s2 -
       P1 P6 P11 s1 s2 - P5 P6 s2^2 - P5 P7 s2^2 - P7 P8 s2^2 - P7 P8 P9 s2^2 - P5 P7 P10 s2^2 - P5 P6 P11 s2^2) /
     (3 + 2 P1 s1 + P2 s1 + 2 P3 s1 + P4 s1 + P1 P2 s1^2 + P1 P3 s1^2 + P3 P4 s1^2 + 2 P5 s2 +
       P6 s2 + 2 P7 s2 + P8 s2 + P2 P5 s1 s2 + P3 P5 s1 s2 + P1 P6 s1 s2 +
       P1 P7 s1 s2 + P4 P7 s1 s2 + P3 P8 s1 s2 + P5 P6 s2^2 + P5 P7 s2^2 + P7 P8 s2^2));
Plot3D[mP, {s1, 0, 8}, {s2, 0, 8}, AxesLabel → {Style["s1", Bold, 20, FontColor → Black],
   Style["s2", Bold, 20, FontColor → Black],
   Style["m(s1,s2)", Bold, 20, FontColor → Black]},
 PlotLabel → Style["m(s1,s2)", Bold, 30, FontColor → Black],
 ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
 TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

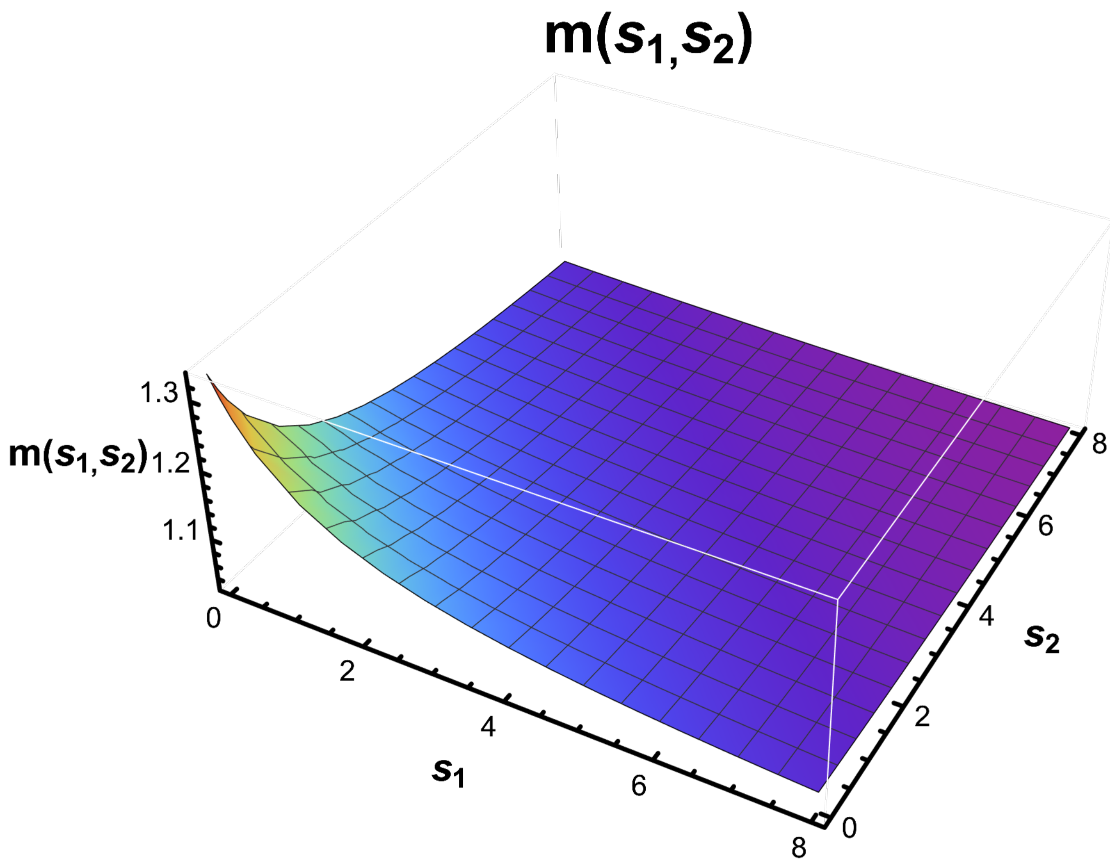
Out[ ]=



Surface plot 2

```
In[∘]:= P₁ = 0;
P₂ = 1;
P₃ = 0;
P₄ = 1;
P₅ = 0;
P₆ = 1;
P₇ = 0;
P₈ = 1;
P₉ = 0;
P₁₀ = 1;
P₁₁ = 0;
mP =
  -((-3 - P₉ - P₁₀ - P₁₁ - 2 P₁ s₁ - P₂ s₁ - 2 P₃ s₁ - P₄ s₁ - P₃ P₉ s₁ - P₄ P₉ s₁ - P₁ P₁₀ s₁ - P₃ P₁₀ s₁ -
        P₁ P₁₁ s₁ - P₂ P₁₁ s₁ - P₁ P₂ s₁² - P₁ P₃ s₁² - P₃ P₄ s₁² - P₃ P₄ P₉ s₁² - P₁ P₃ P₁₀ s₁² - P₁ P₂ P₁₁ s₁² -
        2 P₅ s₂ - P₆ s₂ - 2 P₇ s₂ - P₈ s₂ - P₇ P₉ s₂ - P₈ P₉ s₂ - P₅ P₁₀ s₂ - P₇ P₁₀ s₂ - P₅ P₁₁ s₂ -
        P₆ P₁₁ s₂ - P₂ P₅ s₁ s₂ - P₃ P₅ s₁ s₂ - P₁ P₆ s₁ s₂ - P₁ P₇ s₁ s₂ - P₄ P₇ s₁ s₂ - P₃ P₈ s₁ s₂ -
        P₄ P₇ P₉ s₁ s₂ - P₃ P₈ P₉ s₁ s₂ - P₃ P₅ P₁₀ s₁ s₂ - P₁ P₇ P₁₀ s₁ s₂ - P₂ P₅ P₁₁ s₁ s₂ -
        P₁ P₆ P₁₁ s₁ s₂ - P₅ P₆ s₂² - P₅ P₇ s₂² - P₇ P₈ s₂² - P₇ P₈ P₉ s₂² - P₅ P₇ P₁₀ s₂² - P₅ P₆ P₁₁ s₂²) /
     (3 + 2 P₁ s₁ + P₂ s₁ + 2 P₃ s₁ + P₄ s₁ + P₁ P₂ s₁² + P₁ P₃ s₁² + P₃ P₄ s₁² + 2 P₅ s₂ +
        P₆ s₂ + 2 P₇ s₂ + P₈ s₂ + P₂ P₅ s₁ s₂ + P₃ P₅ s₁ s₂ + P₁ P₆ s₁ s₂ +
        P₁ P₇ s₁ s₂ + P₄ P₇ s₁ s₂ + P₃ P₈ s₁ s₂ + P₅ P₆ s₂² + P₅ P₇ s₂² + P₇ P₈ s₂²));
Plot3D[mP, {s₁, 0, 8}, {s₂, 0, 8}, AxesLabel → {Style["s₁", Bold, 20, FontColor → Black],
   Style["s₂", Bold, 20, FontColor → Black],
   Style["m(s₁,s₂)", Bold, 20, FontColor → Black]},
 PlotLabel → Style["m(s₁,s₂)", Bold, 30, FontColor → Black],
 ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
 TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

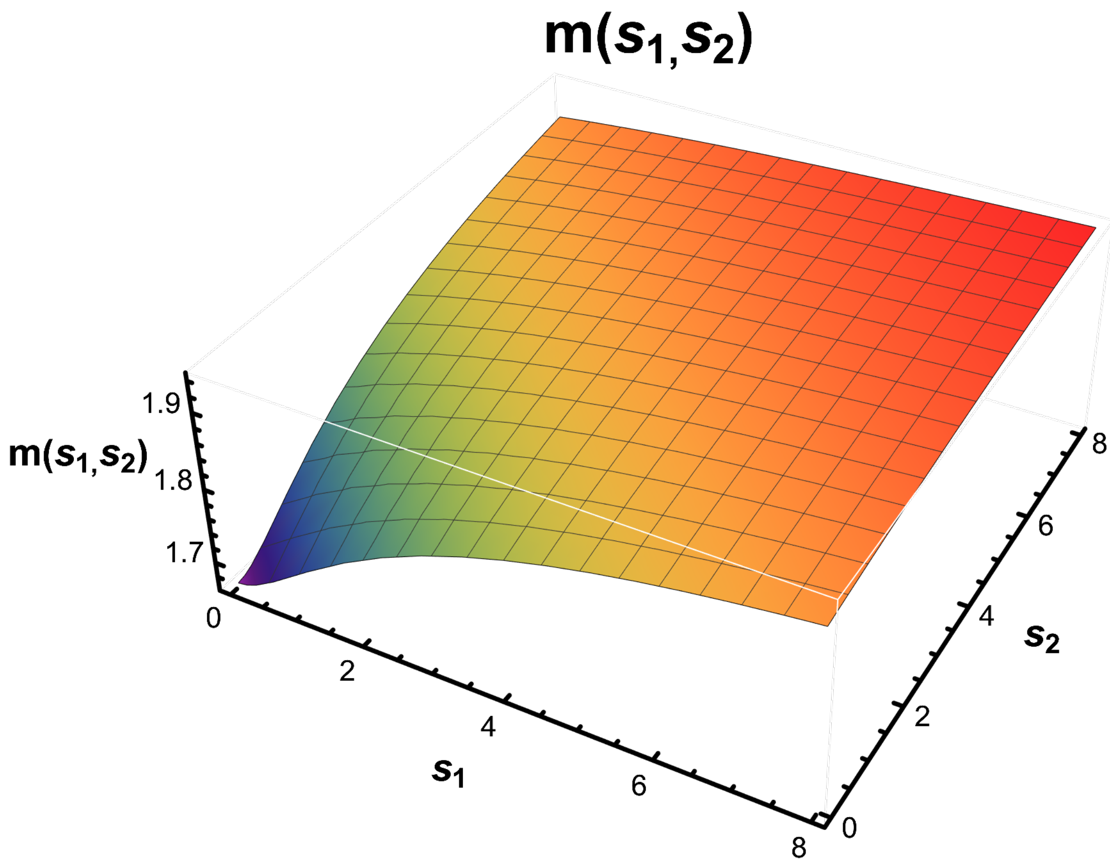
Out[ ]=



Surface plot 3

In[ ]:=
```
P₁ = 1;
P₂ = 1;
P₃ = 0;
P₄ = 0;
P₅ = 1;
P₆ = 1;
P₇ = 0;
P₈ = 0;
P₉ = 1;
P₁₀ = 0;
P₁₁ = 1;
mP =
  -((-3 - P₉ - P₁₀ - P₁₁ - 2 P₁ s₁ - P₂ s₁ - 2 P₃ s₁ - P₄ s₁ - P₃ P₉ s₁ - P₄ P₉ s₁ - P₁ P₁₀ s₁ - P₃ P₁₀ s₁ -
        P₁ P₁₁ s₁ - P₂ P₁₁ s₁ - P₁ P₂ s₁² - P₁ P₃ s₁² - P₃ P₄ s₁² - P₃ P₄ P₉ s₁² - P₁ P₃ P₁₀ s₁² - P₁ P₂ P₁₁ s₁² -
        2 P₅ s₂ - P₆ s₂ - 2 P₇ s₂ - P₈ s₂ - P₇ P₉ s₂ - P₈ P₉ s₂ - P₅ P₁₀ s₂ - P₇ P₁₀ s₂ - P₅ P₁₁ s₂ -
        P₆ P₁₁ s₂ - P₂ P₅ s₁ s₂ - P₃ P₅ s₁ s₂ - P₁ P₆ s₁ s₂ - P₁ P₇ s₁ s₂ - P₄ P₇ s₁ s₂ - P₃ P₈ s₁ s₂ -
        P₄ P₇ P₉ s₁ s₂ - P₃ P₈ P₉ s₁ s₂ - P₃ P₅ P₁₀ s₁ s₂ - P₁ P₇ P₁₀ s₁ s₂ - P₂ P₅ P₁₁ s₁ s₂ -
        P₁ P₆ P₁₁ s₁ s₂ - P₅ P₆ s₂² - P₅ P₇ s₂² - P₇ P₈ s₂² - P₇ P₈ P₉ s₂² - P₅ P₇ P₁₀ s₂² - P₅ P₆ P₁₁ s₂²) /
     (3 + 2 P₁ s₁ + P₂ s₁ + 2 P₃ s₁ + P₄ s₁ + P₁ P₂ s₁² + P₁ P₃ s₁² + P₃ P₄ s₁² + 2 P₅ s₂ +
        P₆ s₂ + 2 P₇ s₂ + P₈ s₂ + P₂ P₅ s₁ s₂ + P₃ P₅ s₁ s₂ + P₁ P₆ s₁ s₂ +
        P₁ P₇ s₁ s₂ + P₄ P₇ s₁ s₂ + P₃ P₈ s₁ s₂ + P₅ P₆ s₂² + P₅ P₇ s₂² + P₇ P₈ s₂²));
Plot3D[mP, {s₁, 0, 8}, {s₂, 0, 8}, AxesLabel → {Style["s₁", Bold, 20, FontColor → Black],
    Style["s₂", Bold, 20, FontColor → Black],
    Style["m(s₁,s₂)", Bold, 20, FontColor → Black]},
  PlotLabel → Style["m(s₁,s₂)", Bold, 30, FontColor → Black],
  ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
  TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

Out[•]=



Surface plot 4 - One of the successful networks

In[◦]:=
```
P₁ = 1;
P₂ = 1;
P₃ = 0;
P₄ = 0;
P₅ = 0;
P₆ = 0;
P₇ = 1;
P₈ = 1;
P₉ = 1;
P₁₀ = 0;
P₁₁ = 1;
mP =
  -((-3 - P₉ - P₁₀ - P₁₁ - 2 P₁ s₁ - P₂ s₁ - 2 P₃ s₁ - P₄ s₁ - P₃ P₉ s₁ - P₄ P₉ s₁ - P₁ P₁₀ s₁ - P₃ P₁₀ s₁ -
        P₁ P₁₁ s₁ - P₂ P₁₁ s₁ - P₁ P₂ s₁^2 - P₁ P₃ s₁^2 - P₃ P₄ s₁^2 - P₃ P₄ P₉ s₁^2 - P₁ P₃ P₁₀ s₁^2 - P₁ P₂ P₁₁ s₁^2 -
        2 P₅ s₂ - P₆ s₂ - 2 P₇ s₂ - P₈ s₂ - P₇ P₉ s₂ - P₈ P₉ s₂ - P₅ P₁₀ s₂ - P₇ P₁₀ s₂ - P₅ P₁₁ s₂ -
        P₆ P₁₁ s₂ - P₂ P₅ s₁ s₂ - P₃ P₅ s₁ s₂ - P₁ P₆ s₁ s₂ - P₁ P₇ s₁ s₂ - P₄ P₇ s₁ s₂ - P₃ P₈ s₁ s₂ -
        P₄ P₇ P₉ s₁ s₂ - P₃ P₈ P₉ s₁ s₂ - P₃ P₅ P₁₀ s₁ s₂ - P₁ P₇ P₁₀ s₁ s₂ - P₂ P₅ P₁₁ s₁ s₂ -
        P₁ P₆ P₁₁ s₁ s₂ - P₅ P₆ s₂^2 - P₅ P₇ s₂^2 - P₇ P₈ s₂^2 - P₇ P₈ P₉ s₂^2 - P₅ P₇ P₁₀ s₂^2 - P₅ P₆ P₁₁ s₂^2) /
     (3 + 2 P₁ s₁ + P₂ s₁ + 2 P₃ s₁ + P₄ s₁ + P₁ P₂ s₁^2 + P₁ P₃ s₁^2 + P₃ P₄ s₁^2 + 2 P₅ s₂ +
        P₆ s₂ + 2 P₇ s₂ + P₈ s₂ + P₂ P₅ s₁ s₂ + P₃ P₅ s₁ s₂ + P₁ P₆ s₁ s₂ +
        P₁ P₇ s₁ s₂ + P₄ P₇ s₁ s₂ + P₃ P₈ s₁ s₂ + P₅ P₆ s₂^2 + P₅ P₇ s₂^2 + P₇ P₈ s₂^2));
Plot3D[mP, {s₁, 0, 8}, {s₂, 0, 8}, AxesLabel → {Style["s₁", Bold, 20, FontColor → Black],
    Style["s₂", Bold, 20, FontColor → Black],
    Style["m(s₁,s₂)", Bold, 20, FontColor → Black]},
  PlotLabel → Style["m(s₁,s₂)", Bold, 30, FontColor → Black],
  ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
  TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

Out[ ]=