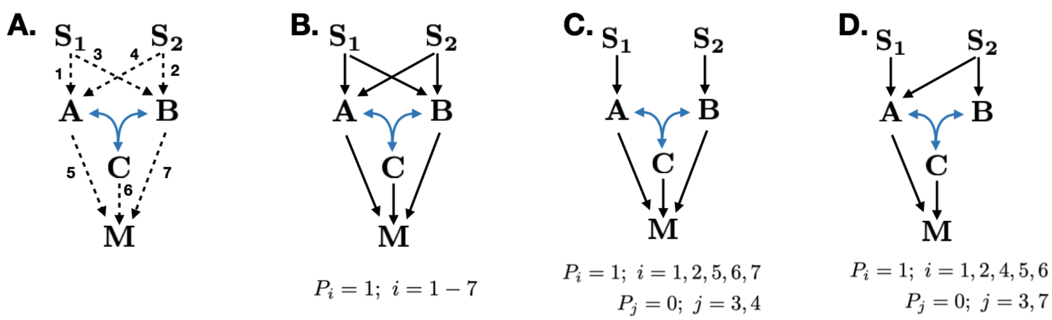
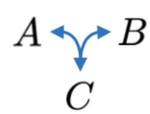
# Minimal binding network for slope antagonism

Here we analyze possible binding networks given in the following picture-




$$P_i = \{0, 1\} \equiv \{\textbf{edge absent, edge present}\}$$

We first perform calculation for the general network A using-




$$\dot{c} = k_1 ab - k_2 c$$

The dynamical equations are-

In[443]:=
$$\text{dadt} = k_{a0} + P_1 * k_1 * s_1 + P_4 * k_4 * s_2 - k_{da} * a - \gamma * a * b$$
$$\text{dbdt} = k_{b0} + P_3 * k_3 * s_1 + P_2 * k_2 * s_2 - k_{db} * b - \gamma * a * b$$
```
Solve[{dadt == 0, dbdt == 0}, {a, b}]
```

Out[443]=
$$-a\,b\,\gamma + k_{a0} - a\,k_{da} + k_1 P_1 s_1 + k_4 P_4 s_2$$

Out[444]=
$$-a\,b\,\gamma + k_{b0} - b\,k_{db} + k_3 P_3 s_1 + k_2 P_2 s_2$$

Out[445]=
$$\Bigg\{\Bigg\{a \to \frac{1}{2\gamma k_{da}}\Big(\gamma k_{a0} - \gamma k_{b0} - k_{da} k_{db} + \gamma k_1 P_1 s_1 - \gamma k_3 P_3 s_1 - \gamma k_2 P_2 s_2 + \gamma k_4 P_4 s_2 - \sqrt{\left((-\gamma k_{a0} + \gamma k_{b0} + k_{da} k_{db} - \gamma k_1 P_1 s_1 + \gamma k_3 P_3 s_1 + \gamma k_2 P_2 s_2 - \gamma k_4 P_4 s_2)^2 - 4\gamma k_{da}(-k_{a0} k_{db} - k_1 k_{db} P_1 s_1 - k_4 k_{db} P_4 s_2)\right)}\Big),$$
$$b \to \frac{1}{k_{db}}\Bigg(-\frac{k_{a0}}{2} + \frac{k_{b0}}{2} - \frac{k_{da} k_{db}}{2\gamma} - \frac{1}{2} k_1 P_1 s_1 + \frac{1}{2} k_3 P_3 s_1 + \frac{1}{2} k_2 P_2 s_2 - \frac{1}{2} k_4 P_4 s_2 - \frac{1}{2\gamma}\left(\sqrt{\left((-\gamma k_{a0} + \gamma k_{b0} + k_{da} k_{db} - \gamma k_1 P_1 s_1 + \gamma k_3 P_3 s_1 + \gamma k_2 P_2 s_2 - \gamma k_4 P_4 s_2)^2 - 4\gamma k_{da}(-k_{a0} k_{db} - k_1 k_{db} P_1 s_1 - k_4 k_{db} P_4 s_2)\right)}\right)\Bigg)\Bigg\},$$
$$\Bigg\{a \to \frac{1}{2\gamma k_{da}}\Big(\gamma k_{a0} - \gamma k_{b0} - k_{da} k_{db} + \gamma k_1 P_1 s_1 - \gamma k_3 P_3 s_1 - \gamma k_2 P_2 s_2 + \gamma k_4 P_4 s_2 + \sqrt{\left((-\gamma k_{a0} + \gamma k_{b0} + k_{da} k_{db} - \gamma k_1 P_1 s_1 + \gamma k_3 P_3 s_1 + \gamma k_2 P_2 s_2 - \gamma k_4 P_4 s_2)^2 - 4\gamma k_{da}(-k_{a0} k_{db} - k_1 k_{db} P_1 s_1 - k_4 k_{db} P_4 s_2)\right)}\Big),$$
$$b \to \frac{1}{k_{db}}\Bigg(-\frac{k_{a0}}{2} + \frac{k_{b0}}{2} - \frac{k_{da} k_{db}}{2\gamma} - \frac{1}{2} k_1 P_1 s_1 + \frac{1}{2} k_3 P_3 s_1 + \frac{1}{2} k_2 P_2 s_2 - \frac{1}{2} k_4 P_4 s_2 + \frac{1}{2\gamma}\left(\sqrt{\left((-\gamma k_{a0} + \gamma k_{b0} + k_{da} k_{db} - \gamma k_1 P_1 s_1 + \gamma k_3 P_3 s_1 + \gamma k_2 P_2 s_2 - \gamma k_4 P_4 s_2)^2 - 4\gamma k_{da}(-k_{a0} k_{db} - k_1 k_{db} P_1 s_1 - k_4 k_{db} P_4 s_2)\right)}\right)\Bigg)\Bigg\}\Bigg\}$$

Accounting for the positivity of a and b we can write the steady state expressions for a and b as,

In[446]:=
$$\text{aSS} = \text{FullSimplify}\Big[\frac{1}{2\gamma k_{da}}\Big(\gamma k_{a0} - \gamma k_{b0} - k_{da} k_{db} + \gamma k_1 P_1 s_1 - \gamma k_3 P_3 s_1 - \gamma k_2 P_2 s_2 + \gamma k_4 P_4 s_2 + \sqrt{\left((-\gamma k_{a0} + \gamma k_{b0} + k_{da} k_{db} - \gamma k_1 P_1 s_1 + \gamma k_3 P_3 s_1 + \gamma k_2 P_2 s_2 - \gamma k_4 P_4 s_2)^2 - 4\gamma k_{da}(-k_{a0} k_{db} - k_1 k_{db} P_1 s_1 - k_4 k_{db} P_4 s_2)\right)}\Big)\Big]$$

Out[446]=
$$\frac{1}{2\gamma k_{da}}\Big(-k_{da} k_{db} + \gamma\left(k_{a0} - k_{b0} + (k_1 P_1 - k_3 P_3)\, s_1 + (-k_2 P_2 + k_4 P_4)\, s_2\right) + \sqrt{\left(4\gamma k_{da} k_{db}(k_{a0} + k_1 P_1 s_1 + k_4 P_4 s_2) + \left(k_{da} k_{db} + \gamma(-k_{a0} + k_{b0} + (-k_1 P_1 + k_3 P_3)\, s_1 + (k_2 P_2 - k_4 P_4)\, s_2)\right)^2\right)}\Big)$$

In[447]:= **bSS =**

$$\mathtt{FullSimplify}\Big[\frac{1}{k_{db}}\Big(-\frac{k_{a0}}{2}+\frac{k_{b0}}{2}-\frac{k_{da}\,k_{db}}{2\,\gamma}-\frac{1}{2}\,k_1\,P_1\,s_1+\frac{1}{2}\,k_3\,P_3\,s_1+\frac{1}{2}\,k_2\,P_2\,s_2-\frac{1}{2}\,k_4\,P_4\,s_2+\frac{1}{2\,\gamma}\Big(\sqrt{\big((-\gamma\,k_{a0}+\gamma\,k_{b0}+k_{da}\,k_{db}-\gamma\,k_1\,P_1\,s_1+\gamma\,k_3\,P_3\,s_1+\gamma\,k_2\,P_2\,s_2-\gamma\,k_4\,P_4\,s_2)^2-4\,\gamma\,k_{da}\,(-k_{a0}\,k_{db}-k_1\,k_{db}\,P_1\,s_1-k_4\,k_{db}\,P_4\,s_2)\big)}\Big)\Big)\Big]$$

Out[447]=

$$\frac{1}{2\,\gamma\,k_{db}}\Big(-k_{da}\,k_{db}+\gamma\,(-k_{a0}+k_{b0}+(-k_1\,P_1+k_3\,P_3)\,s_1+(k_2\,P_2-k_4\,P_4)\,s_2)+\sqrt{\big(4\,\gamma\,k_{da}\,k_{db}\,(k_{a0}+k_1\,P_1\,s_1+k_4\,P_4\,s_2)+(k_{da}\,k_{db}+\gamma\,(-k_{a0}+k_{b0}+(-k_1\,P_1+k_3\,P_3)\,s_1+(k_2\,P_2-k_4\,P_4)\,s_2))^2\big)}\Big)$$

The expression for c in steady state is,

In[449]:= **cSS = FullSimplify[γ * aSS * bSS / $k_{dc}$]**

Out[449]=

$$\frac{1}{2\,\gamma\,k_{dc}}\Big(k_{da}\,k_{db}+\gamma\,(k_{a0}+k_{b0}+(k_1\,P_1+k_3\,P_3)\,s_1+(k_2\,P_2+k_4\,P_4)\,s_2)-\sqrt{\big(4\,\gamma\,k_{da}\,k_{db}\,(k_{a0}+k_1\,P_1\,s_1+k_4\,P_4\,s_2)+(k_{da}\,k_{db}+\gamma\,(-k_{a0}+k_{b0}+(-k_1\,P_1+k_3\,P_3)\,s_1+(k_2\,P_2-k_4\,P_4)\,s_2))^2\big)}\Big)$$

And finally the steady state expressions for m is-

In[450]:= **mSS = FullSimplify[($k_{m0}$ + $P_5$ * $k_5$ * aSS + $P_6$ * $k_6$ * cSS + $P_7$ * $k_7$ * bSS) / $k_{dm}$]**

Out[450]=

$$\frac{1}{2\,k_{dm}}\Bigg(2\,k_{m0}+\frac{1}{\gamma\,k_{dc}}k_6\,P_6\,\Big(k_{da}\,k_{db}+\gamma\,(k_{a0}+k_{b0}+(k_1\,P_1+k_3\,P_3)\,s_1+(k_2\,P_2+k_4\,P_4)\,s_2)-\sqrt{\big(4\,\gamma\,k_{da}\,k_{db}\,(k_{a0}+k_1\,P_1\,s_1+k_4\,P_4\,s_2)+(k_{da}\,k_{db}+\gamma\,(-k_{a0}+k_{b0}+(-k_1\,P_1+k_3\,P_3)\,s_1+(k_2\,P_2-k_4\,P_4)\,s_2))^2\big)}\Big)+\frac{1}{\gamma\,k_{db}}k_7\,P_7\,\Big(-k_{da}\,k_{db}+\gamma\,(-k_{a0}+k_{b0}+(-k_1\,P_1+k_3\,P_3)\,s_1+(k_2\,P_2-k_4\,P_4)\,s_2)+\sqrt{\big(4\,\gamma\,k_{da}\,k_{db}\,(k_{a0}+k_1\,P_1\,s_1+k_4\,P_4\,s_2)+(k_{da}\,k_{db}+\gamma\,(-k_{a0}+k_{b0}+(-k_1\,P_1+k_3\,P_3)\,s_1+(k_2\,P_2-k_4\,P_4)\,s_2))^2\big)}\Big)+\frac{1}{\gamma\,k_{da}}k_5\,P_5\,\Big(-k_{da}\,k_{db}+\gamma\,(k_{a0}-k_{b0}+(k_1\,P_1-k_3\,P_3)\,s_1+(-k_2\,P_2+k_4\,P_4)\,s_2)+\sqrt{\big(4\,\gamma\,k_{da}\,k_{db}\,(k_{a0}+k_1\,P_1\,s_1+k_4\,P_4\,s_2)+(k_{da}\,k_{db}+\gamma\,(-k_{a0}+k_{b0}+(-k_1\,P_1+k_3\,P_3)\,s_1+(k_2\,P_2-k_4\,P_4)\,s_2))^2\big)}\Big)\Bigg)$$

## Now we set all reaction rates to 1 and determine the expressions for a,b,c and m in steady state for further analysis-

In[451]:=
```
dadt = 1 + P1 * 1 * s1 + P4 * 1 * s2 - 1 * a - 1 * a * b
dbdt = 1 + P3 * 1 * s1 + P2 * 1 * s2 - 1 * b - 1 * a * b
Solve[{dadt == 0, dbdt == 0}, {a, b}]
```

Out[451]= $1 - a - a\,b + P_1\,s_1 + P_4\,s_2$

Out[452]= $1 - b - a\,b + P_3\,s_1 + P_2\,s_2$

Out[453]=
$$\left\{\left\{a \to \frac{1}{2}\times\left(-1 + P_1\,s_1 - P_3\,s_1 - P_2\,s_2 + P_4\,s_2 - \sqrt{-4\times(-1 - P_1\,s_1 - P_4\,s_2) + (1 - P_1\,s_1 + P_3\,s_1 + P_2\,s_2 - P_4\,s_2)^2}\right),\; b \to \frac{1}{2}\times\left(-1 - P_1\,s_1 + P_3\,s_1 + P_2\,s_2 - P_4\,s_2 - \sqrt{-4\times(-1 - P_1\,s_1 - P_4\,s_2) + (1 - P_1\,s_1 + P_3\,s_1 + P_2\,s_2 - P_4\,s_2)^2}\right)\right\},\right.$$
$$\left.\left\{a \to \frac{1}{2}\times\left(-1 + P_1\,s_1 - P_3\,s_1 - P_2\,s_2 + P_4\,s_2 + \sqrt{-4\times(-1 - P_1\,s_1 - P_4\,s_2) + (1 - P_1\,s_1 + P_3\,s_1 + P_2\,s_2 - P_4\,s_2)^2}\right),\; b \to \frac{1}{2}\times\left(-1 - P_1\,s_1 + P_3\,s_1 + P_2\,s_2 - P_4\,s_2 + \sqrt{-4\times(-1 - P_1\,s_1 - P_4\,s_2) + (1 - P_1\,s_1 + P_3\,s_1 + P_2\,s_2 - P_4\,s_2)^2}\right)\right\}\right\}$$

The steady state expressions are,

In[454]:=
```
aP = FullSimplify[1/2 × (-1 + P1 s1 - P3 s1 - P2 s2 +
    P4 s2 + Sqrt[-4 × (-1 - P1 s1 - P4 s2) + (1 - P1 s1 + P3 s1 + P2 s2 - P4 s2)^2])]
```

Out[454]=
$$\frac{1}{2}\times\left(-1 + P_1\,s_1 - P_3\,s_1 - P_2\,s_2 + P_4\,s_2 + \sqrt{(1 + (-P_1 + P_3)\,s_1 + (P_2 - P_4)\,s_2)^2 + 4\times(1 + P_1\,s_1 + P_4\,s_2)}\right)$$

In[455]:=
```
bP = FullSimplify[1/2 × (-1 - P1 s1 + P3 s1 + P2 s2 -
    P4 s2 + Sqrt[-4 × (-1 - P1 s1 - P4 s2) + (1 - P1 s1 + P3 s1 + P2 s2 - P4 s2)^2])]
```

Out[455]=
$$\frac{1}{2}\times\left(-1 - P_1\,s_1 + P_3\,s_1 + P_2\,s_2 - P_4\,s_2 + \sqrt{(1 + (-P_1 + P_3)\,s_1 + (P_2 - P_4)\,s_2)^2 + 4\times(1 + P_1\,s_1 + P_4\,s_2)}\right)$$

The steady state expressions for c is,

In[456]:=
```
cP = FullSimplify[aP * bP]
```

Out[456]=
$$\frac{1}{2}\times\left(3 + P_1\,s_1 + P_3\,s_1 + P_2\,s_2 + P_4\,s_2 - \sqrt{(1 + (-P_1 + P_3)\,s_1 + (P_2 - P_4)\,s_2)^2 + 4\times(1 + P_1\,s_1 + P_4\,s_2)}\right)$$

And, finally the expression for the output m-

In[457]:= **mP = FullSimplify[1 + P5 * 1 * aP + P6 * 1 * cP + P7 * 1 * bP]**

Out[457]=
$$\frac{1}{2}\times\Bigg(2+P_6\left(3+P_1\,s_1+P_3\,s_1+P_2\,s_2+P_4\,s_2-\sqrt{(1+(-P_1+P_3)\,s_1+(P_2-P_4)\,s_2)^2+4\times(1+P_1\,s_1+P_4\,s_2)}\right)+P_7\left(-1-P_1\,s_1+P_3\,s_1+P_2\,s_2-P_4\,s_2+\sqrt{(1+(-P_1+P_3)\,s_1+(P_2-P_4)\,s_2)^2+4\times(1+P_1\,s_1+P_4\,s_2)}\right)+P_5\left(-1+P_1\,s_1-P_3\,s_1-P_2\,s_2+P_4\,s_2+\sqrt{(1+(-P_1+P_3)\,s_1+(P_2-P_4)\,s_2)^2+4\times(1+P_1\,s_1+P_4\,s_2)}\right)\Bigg)$$

Expression for m only when one input is present,

In[466]:= **mP1 = FullSimplify[mP /. s1 → 0]**

Out[466]=
$$\frac{1}{2}\times\Bigg(2+P_6\left(3+P_1\,s_1+P_3\,s_1-\sqrt{4+4\,P_1\,s_1+(1+(-P_1+P_3)\,s_1)^2}\right)+P_5\left(-1+P_1\,s_1-P_3\,s_1+\sqrt{4+4\,P_1\,s_1+(1+(-P_1+P_3)\,s_1)^2}\right)+P_7\left(-1-P_1\,s_1+P_3\,s_1+\sqrt{4+4\,P_1\,s_1+(1+(-P_1+P_3)\,s_1)^2}\right)\Bigg)$$

In[467]:= **mP2 = FullSimplify[mP /. s1 → 0]**

Out[467]=
$$\frac{1}{2}\times\Bigg(2+3\,P_6-P_7+P_2\,(P_6+P_7)\,s_2-(P_6-P_7)\left(-P_4\,s_2+\sqrt{5+2\,(P_2+P_4)\,s_2+(P_2-P_4)^2\,s_2^2}\right)+P_5\left(-1+(-P_2+P_4)\,s_2+\sqrt{5+2\,(P_2+P_4)\,s_2+(P_2-P_4)^2\,s_2^2}\right)\Bigg)$$

Derivatives

In[468]:= **m1 = FullSimplify[D[mP1, s1]]**

Out[468]=
$$\frac{1}{2}\Bigg(P_6\left(P_1+P_3+\frac{-P_1-P_3-(P_1-P_3)^2\,s_1}{\sqrt{4+4\,P_1\,s_1+(1+(-P_1+P_3)\,s_1)^2}}\right)+P_5\left(P_1-P_3+\frac{P_1+P_3+(P_1-P_3)^2\,s_1}{\sqrt{4+4\,P_1\,s_1+(1+(-P_1+P_3)\,s_1)^2}}\right)+P_7\left(-P_1+P_3+\frac{P_1+P_3+(P_1-P_3)^2\,s_1}{\sqrt{4+4\,P_1\,s_1+(1+(-P_1+P_3)\,s_1)^2}}\right)\Bigg)$$

In[470]:= **m2 = FullSimplify[D[mP2, s2]]**

Out[470]=
$$\frac{1}{2}\Bigg(P_2\,(P_6+P_7)-(P_6-P_7)\left(-P_4+\frac{P_2+P_4+(P_2-P_4)^2\,s_2}{\sqrt{5+2\,(P_2+P_4)\,s_2+(P_2-P_4)^2\,s_2^2}}\right)+P_5\left(-P_2+P_4+\frac{P_2+P_4+(P_2-P_4)^2\,s_2}{\sqrt{5+2\,(P_2+P_4)\,s_2+(P_2-P_4)^2\,s_2^2}}\right)\Bigg)$$

In[471]:= $m_{12}$ = FullSimplify[D[mP, $s_1$] + D[mP, $s_2$]]

Out[471]=

$$\frac{1}{2}\left(P_6\left(P_1+P_3-\frac{4P_1+2(-P_1+P_3)(1+(-P_1+P_3)s_1+(P_2-P_4)s_2)}{2\sqrt{(1+(-P_1+P_3)s_1+(P_2-P_4)s_2)^2+4\times(1+P_1 s_1+P_4 s_2)}}\right)+\right.$$
$$P_5\left(P_1-P_3+\frac{4P_1+2(-P_1+P_3)(1+(-P_1+P_3)s_1+(P_2-P_4)s_2)}{2\sqrt{(1+(-P_1+P_3)s_1+(P_2-P_4)s_2)^2+4\times(1+P_1 s_1+P_4 s_2)}}\right)+$$
$$P_7\left(-P_1+P_3+\frac{4P_1+2(-P_1+P_3)(1+(-P_1+P_3)s_1+(P_2-P_4)s_2)}{2\sqrt{(1+(-P_1+P_3)s_1+(P_2-P_4)s_2)^2+4\times(1+P_1 s_1+P_4 s_2)}}\right)+$$
$$P_6\left(P_2+P_4-\frac{4P_4+2(P_2-P_4)(1+(-P_1+P_3)s_1+(P_2-P_4)s_2)}{2\sqrt{(1+(-P_1+P_3)s_1+(P_2-P_4)s_2)^2+4\times(1+P_1 s_1+P_4 s_2)}}\right)+$$
$$P_7\left(P_2-P_4+\frac{4P_4+2(P_2-P_4)(1+(-P_1+P_3)s_1+(P_2-P_4)s_2)}{2\sqrt{(1+(-P_1+P_3)s_1+(P_2-P_4)s_2)^2+4\times(1+P_1 s_1+P_4 s_2)}}\right)+$$
$$\left.P_5\left(-P_2+P_4+\frac{4P_4+2(P_2-P_4)(1+(-P_1+P_3)s_1+(P_2-P_4)s_2)}{2\sqrt{(1+(-P_1+P_3)s_1+(P_2-P_4)s_2)^2+4\times(1+P_1 s_1+P_4 s_2)}}\right)\right)$$

## Surface maps for networks B,C,D

# Network B

In[491]:=

$P_1 = 1;$
$P_2 = 1;$
$P_3 = 1;$
$P_4 = 1;$
$P_5 = 1;$
$P_6 = 1;$
$P_7 = 1;$

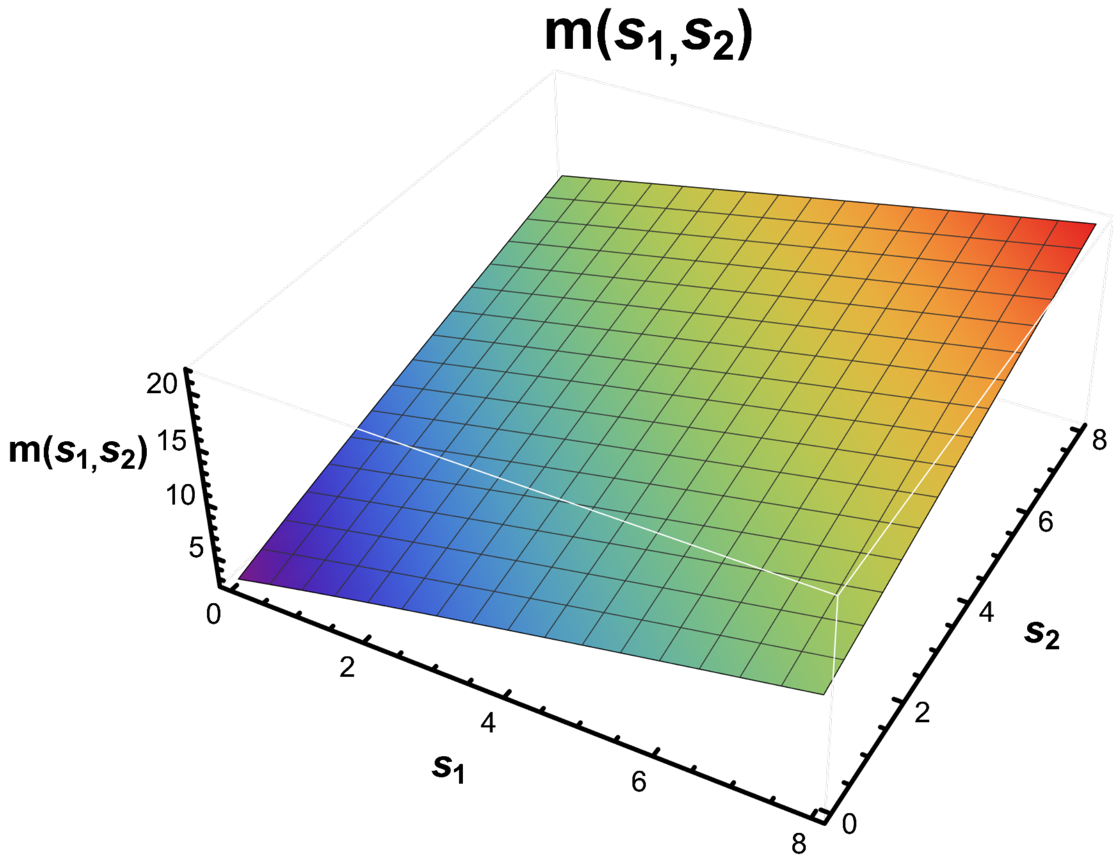
$$\text{mPB} = \frac{1}{2}\times\left(2+P_6\left(3+P_1 s_1+P_3 s_1+P_2 s_2+P_4 s_2-\sqrt{(1+(-P_1+P_3)s_1+(P_2-P_4)s_2)^2+4\times(1+P_1 s_1+P_4 s_2)}\right)+P_7\left(-1-P_1 s_1+P_3 s_1+P_2 s_2-P_4 s_2+\sqrt{(1+(-P_1+P_3)s_1+(P_2-P_4)s_2)^2+4\times(1+P_1 s_1+P_4 s_2)}\right)+P_5\left(-1+P_1 s_1-P_3 s_1-P_2 s_2+P_4 s_2+\sqrt{(1+(-P_1+P_3)s_1+(P_2-P_4)s_2)^2+4\times(1+P_1 s_1+P_4 s_2)}\right)\right)$$

Out[498]=

$$\frac{1}{2}\times\left(3+2s_1+2s_2+\sqrt{1+4\times(1+s_1+s_2)}\right)$$

In[490]:=
```
Plot3D[mPB, {s1, 0, 8}, {s2, 0, 8}, AxesLabel →
  {Style["s1", Bold, 20, FontColor → Black], Style["s2", Bold, 20, FontColor → Black],
   Style["m(s1,s2)", Bold, 20, FontColor → Black]},
 PlotLabel → Style["m(s1,s2)", Bold, 30, FontColor → Black],
 ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
 TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

Out[490]=



# Network C

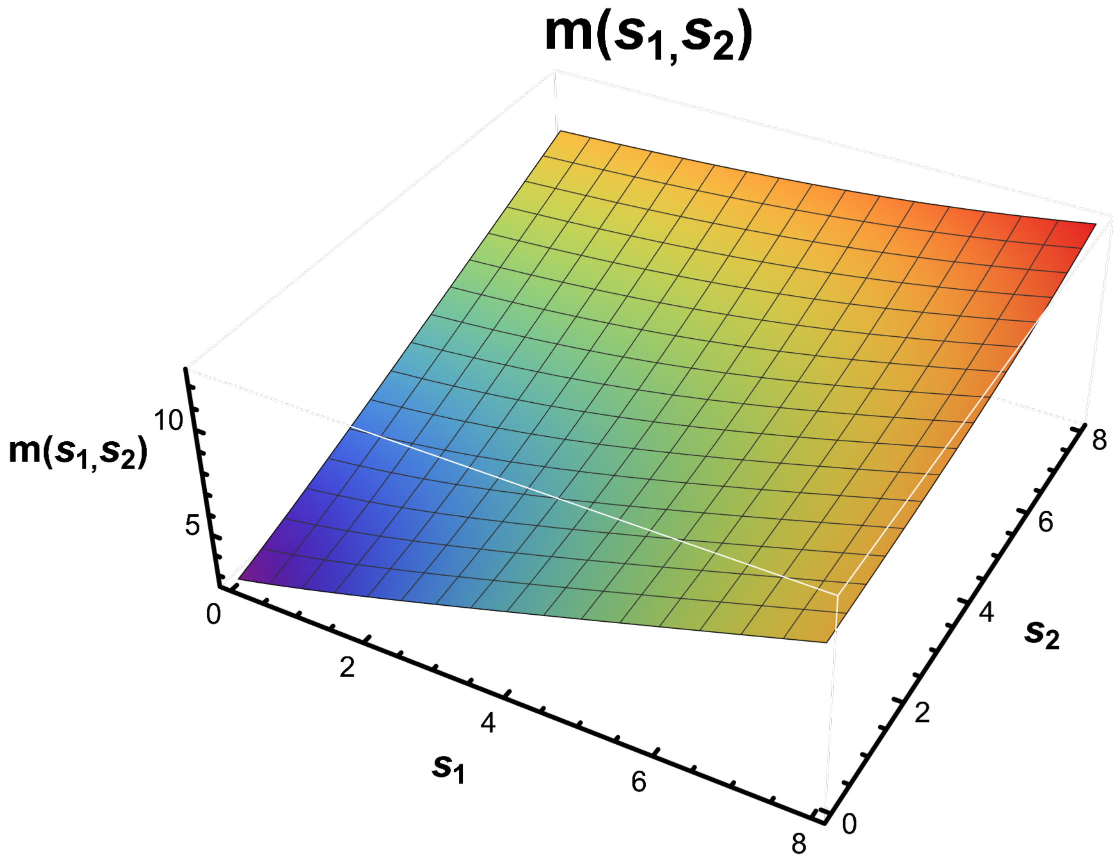
In[499]:=
```
P1 = 1;
P2 = 1;
P3 = 0;
P4 = 0;
P5 = 1;
P6 = 1;
P7 = 1;
```

$$mPC = \frac{1}{2} \times \left(2 + P_6 \left(3 + P_1 s_1 + P_3 s_1 + P_2 s_2 + P_4 s_2 - \sqrt{(1 + (-P_1 + P_3) s_1 + (P_2 - P_4) s_2)^2 + 4 \times (1 + P_1 s_1 + P_4 s_2)}\right) + P_7 \left(-1 - P_1 s_1 + P_3 s_1 + P_2 s_2 - P_4 s_2 + \sqrt{(1 + (-P_1 + P_3) s_1 + (P_2 - P_4) s_2)^2 + 4 \times (1 + P_1 s_1 + P_4 s_2)}\right) + P_5 \left(-1 + P_1 s_1 - P_3 s_1 - P_2 s_2 + P_4 s_2 + \sqrt{(1 + (-P_1 + P_3) s_1 + (P_2 - P_4) s_2)^2 + 4 \times (1 + P_1 s_1 + P_4 s_2)}\right)\right)$$

Out[506]=
$$\frac{1}{2} \times \left(3 + s_1 + s_2 + \sqrt{4 \times (1 + s_1) + (1 - s_1 + s_2)^2}\right)$$

In[507]:=
```
Plot3D[mPC, {s₁, 0, 8}, {s₂, 0, 8}, AxesLabel →
  {Style["s₁", Bold, 20, FontColor → Black], Style["s₂", Bold, 20, FontColor → Black],
   Style["m(s₁,s₂)", Bold, 20, FontColor → Black]},
 PlotLabel → Style["m(s₁,s₂)", Bold, 30, FontColor → Black],
 ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
 TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

Out[507]=



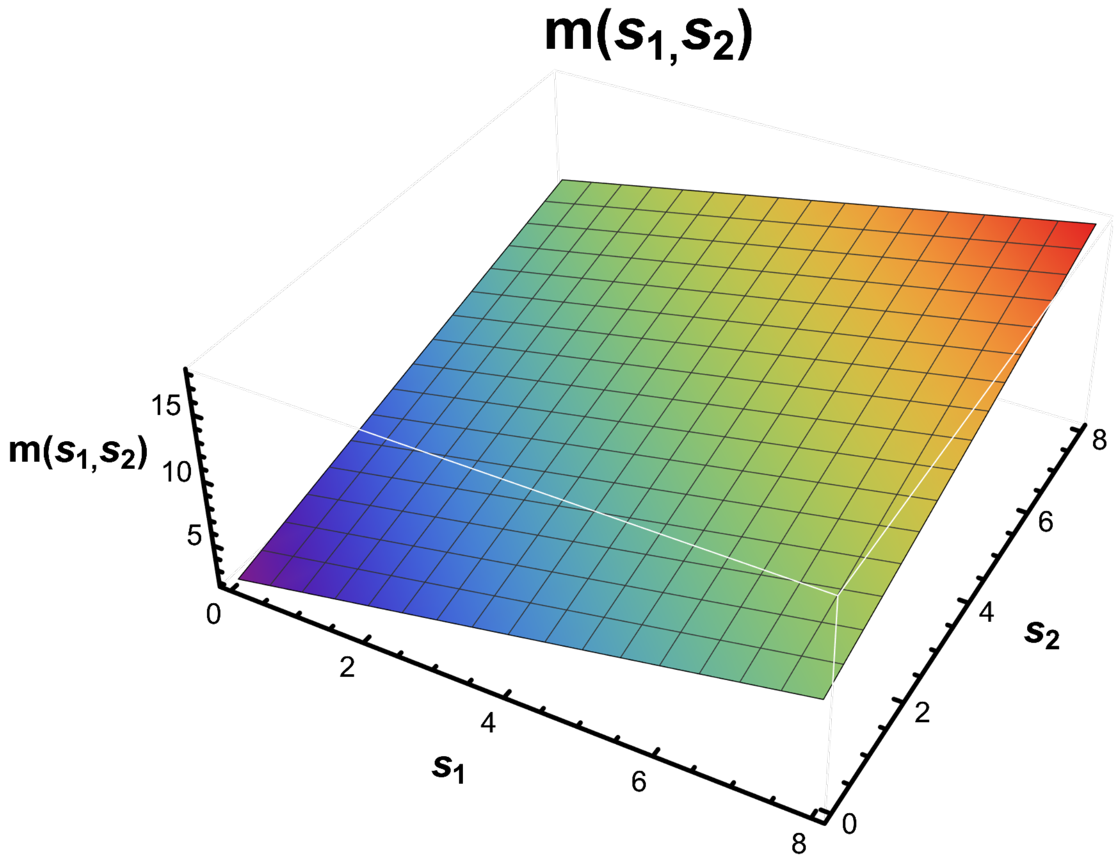
# Network D

In[508]:=
$$P_1 = 1;$$
$$P_2 = 1;$$
$$P_3 = 0;$$
$$P_4 = 1;$$
$$P_5 = 1;$$
$$P_6 = 1;$$
$$P_7 = 0;$$
$$\text{mPD} = \frac{1}{2} \times \Big(2 + P_6 \Big(3 + P_1\, s_1 + P_3\, s_1 + P_2\, s_2 + P_4\, s_2 - \sqrt{(1 + (-P_1 + P_3)\, s_1 + (P_2 - P_4)\, s_2)^2 + 4 \times (1 + P_1\, s_1 + P_4\, s_2)}\Big) + P_7 \Big(-1 - P_1\, s_1 + P_3\, s_1 + P_2\, s_2 - P_4\, s_2 + \sqrt{(1 + (-P_1 + P_3)\, s_1 + (P_2 - P_4)\, s_2)^2 + 4 \times (1 + P_1\, s_1 + P_4\, s_2)}\Big) + P_5 \Big(-1 + P_1\, s_1 - P_3\, s_1 - P_2\, s_2 + P_4\, s_2 + \sqrt{(1 + (-P_1 + P_3)\, s_1 + (P_2 - P_4)\, s_2)^2 + 4 \times (1 + P_1\, s_1 + P_4\, s_2)}\Big)\Big)$$

Out[515]=
$$\frac{1}{2} \times (4 + 2\, s_1 + 2\, s_2)$$

In[516]:=
```
Plot3D[mPD, {s₁, 0, 8}, {s₂, 0, 8}, AxesLabel →
  {Style["s₁", Bold, 20, FontColor → Black], Style["s₂", Bold, 20, FontColor → Black],
   Style["m(s₁,s₂)", Bold, 20, FontColor → Black]},
 PlotLabel → Style["m(s₁,s₂)", Bold, 30, FontColor → Black],
 ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
 TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

Out[516]=




# Successful network

This is the surface map for the successful network-

In[517]:=
```
P₁ = 1;
P₂ = 1;
P₃ = 0;
P₄ = 0;
P₅ = 1;
P₆ = 0;
P₇ = 1;
```

$$mPS = \frac{1}{2} \times \left(2 + P_6 \left(3 + P_1 s_1 + P_3 s_1 + P_2 s_2 + P_4 s_2 - \sqrt{(1 + (-P_1 + P_3) s_1 + (P_2 - P_4) s_2)^2 + 4 \times (1 + P_1 s_1 + P_4 s_2)}\right) + P_7 \left(-1 - P_1 s_1 + P_3 s_1 + P_2 s_2 - P_4 s_2 + \sqrt{(1 + (-P_1 + P_3) s_1 + (P_2 - P_4) s_2)^2 + 4 \times (1 + P_1 s_1 + P_4 s_2)}\right) + P_5 \left(-1 + P_1 s_1 - P_3 s_1 - P_2 s_2 + P_4 s_2 + \sqrt{(1 + (-P_1 + P_3) s_1 + (P_2 - P_4) s_2)^2 + 4 \times (1 + P_1 s_1 + P_4 s_2)}\right)\right)$$

Out[524]= $\sqrt{4 \times (1 + s_1) + (1 - s_1 + s_2)^2}$

In[525]:=
```
Plot3D[mPS, {s₁, 0, 8}, {s₂, 0, 8}, AxesLabel →
  {Style["s₁", Bold, 20, FontColor → Black], Style["s₂", Bold, 20, FontColor → Black],
   Style["m(s₁,s₂)", Bold, 20, FontColor → Black]},
 PlotLabel → Style["m(s₁,s₂)", Bold, 30, FontColor → Black],
 ColorFunction → "Rainbow", AxesStyle → Thickness[0.005], BoxStyle → GrayLevel[2],
 TicksStyle → Directive[Black, 15], ImageSize → Large, PlotRange → Full]
```

Out[525]=