Two-Stage Stochastic Capacity Expansion Problem

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Problem Description

A company must decide how much production capacity to build at several facilities before knowing the future demand. After the demand is realized (under a given scenario), it must allocate production to satisfy demand at minimum cost, including transportation costs and potential penalties for unmet demand.

Sets and Indices

- I: Set of facilities
- J: Set of customer zones
- Ω: Set of demand scenarios

Parameters

- c_i: Cost to build capacity at facility $i \in I$
- t_{ij}: Transportation cost per unit from facility i to customer j under scenario $\omega \in \Omega$
- d_j^ω: Demand at customer j under scenario $\omega \in \Omega$
- p_j: Penalty cost per unit of unmet demand at customer j
- q_ω: Probability of scenario $\omega \in \Omega$

Decision Variables

First-stage (before knowing demand):

- x_i: Capacity to build at facility i

Second-stage (after demand is realized):

- y_{ij}^ω: Units shipped from facility i to customer j under scenario ω
- s_j^ω: Unmet demand at customer j under scenario ω

Stochastic Optimization Model

First-stage problem:

$$\begin{aligned} \min \quad & \sum_{i \in I} c_i x_i + \sum_{\omega \in \Omega} q_\omega Q(x, \omega) \\ \text{s.t.} \quad & x_i \geq 0 \quad \forall i \in I \end{aligned}$$

Second-stage problem (for each scenario $\omega \in \Omega$):

$$\begin{aligned} Q(x, \omega) = \quad \min \quad & \sum_{i \in I} \sum_{j \in J} t_{ij}^\omega y_{ij}^\omega + \sum_{j \in J} p_j s_j^\omega \\ \text{s.t.} \quad & \sum_{j \in J} y_{ij}^\omega \leq x_i \quad \forall i \in I \\ & \sum_{i \in I} y_{ij}^\omega + s_j^\omega \geq d_j^\omega \quad \forall j \in J \\ & y_{ij}^\omega \geq 0, \quad s_j^\omega \geq 0 \end{aligned}$$

Scenario Generation

We consider that each consumer have 3 levels of demand. For each customer j, $d_j \in \{80, 100, 120\}$, with probabilities $\{0.25, 0.5, 0.25\}$.