TP2 MPA

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Question 1

Chaque y_i est indépendant et $y_i \sim \mathcal{B}(\theta_2),\ \forall i$, donc :

$$\begin{aligned} p\left(y|c=1,\theta_1,\theta_2\right) &= \prod_{i=1}^{n} p(y_i|c=1,\theta_1,\theta_2) \\ &= \prod_{i=1}^{n} \theta_2^{y_i}\left(1-\theta_2\right)^{1-y_i} \\ &= \theta_2^{\sum\limits_{i=1}^{n} y_i}\left(1-\theta_2\right)^{n-\sum\limits_{i=1}^{n} y_i} \end{aligned}$$

Question 2

$\forall i \in [1, c-1],\ y_i \sim \mathcal{B}(\theta_1)$ et $\forall i \in [c, n],\ y_i \sim \mathcal{B}(\theta_2)$, donc pour $c = k,\ k \geq 2$:

$$\begin{aligned} p\left(y|c,\theta_1,\theta_2\right) &= \prod_{i=1}^{c-1} p(y_i|c,\theta_1,\theta_2) \prod_{i=c}^{n} p(y_i|c,\theta_1,\theta_2) \\ &= \prod_{i=1}^{c-1} \theta_1^{y_i}\left(1-\theta_1\right)^{1-y_i} \prod_{i=c}^{n} \theta_2^{y_i}\left(1-\theta_2\right)^{1-y_i} \\ &= \theta_1^{\sum\limits_{i=1}^{c-1} y_i}\left(1-\theta_1\right)^{c-1-\sum\limits_{i=1}^{c-1} y_i} \theta_2^{\sum\limits_{i=c}^{n} y_i}\left(1-\theta_2\right)^{n-c+1-\sum\limits_{i=c}^{n} y_i} \end{aligned}$$

Question 3

On utilise la formule de Bayes :

$$\begin{aligned} p(y|c=1,\theta_1,\theta_2) &= \frac{p(c=1|y,\theta_1,\theta_2)p(y,\theta_1,\theta_2)}{p(c=1,\theta_1,\theta_2)} \\ &\text{et} \\ p(y|c,\theta_1,\theta_2) &= \frac{p(c|y,\theta_1,\theta_2)p(y,\theta_1,\theta_2)}{p(c,\theta_1,\theta_2)} \end{aligned}$$

donc, en remarquant que $p(c=1|y,\theta_1,\theta_2) = p(c=1|y)$ et $p(c|y,\theta_1,\theta_2) = p(c|y)$ car c, θ_1 et θ_2 sont indépendants, on a:

$$\begin{aligned} \frac{p(y|c,\theta_1,\theta_2)}{p(y|c=1,\theta_1,\theta_2)} &= \frac{p(c|y)p(y,\theta_1,\theta_2)}{p(c,\theta_1,\theta_2)} \times \frac{p(c=1,\theta_1,\theta_2)}{p(c=1|y)p(y,\theta_1,\theta_2)} \\ &= \frac{p(c|y)}{\frac{1}{n}} \times \frac{\frac{1}{n}}{p(c=1|y)} = \frac{p(c|y)}{p(c=1|y)}. \end{aligned}$$

D'où

$$\frac{p(c|y)}{p(c=1|y)} = \frac{\prod_{i=1}^{c-1} \theta_1^{y_i} \left(1-\theta_1\right)^{1-y_i} \prod_{i=c}^{n} \theta_2^{y_i} \left(1-\theta_2\right)^{1-y_i}}{\prod_{i=1}^{n} \theta_2^{y_i} \left(1-\theta_2\right)^{1-y_i}} = \prod_{i=1}^{c-1} \frac{\theta_1^{y_i} \left(1-\theta_1\right)^{1-y_i}}{\theta_2^{y_i} \left(1-\theta_2\right)^{1-y_i}}$$