

# TP2 MPA

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## Question 1

Chaque  $y_i$  est indépendant et  $y_i \sim \mathcal{B}(\theta_2)$ ,  $\forall i$ , donc :

$$\begin{aligned} p(y|c=1, \theta_1, \theta_2) &= \prod_{i=1}^n p(y_i|c=1, \theta_1, \theta_2) \\ &= \prod_{i=1}^n \theta_2^{y_i} (1 - \theta_2)^{1-y_i} \\ &= \theta_2^{\sum_{i=1}^n y_i} (1 - \theta_2)^{n - \sum_{i=1}^n y_i} \end{aligned}$$

## Question 2

$\forall i \in [1, c-1]$ ,  $y_i \sim \mathcal{B}(\theta_1)$  et  $\forall i \in [c, n]$ ,  $y_i \sim \mathcal{B}(\theta_2)$ , donc pour  $c = k$ ,  $k \geq 2$  :

$$\begin{aligned} p(y|c, \theta_1, \theta_2) &= \prod_{i=1}^{c-1} p(y_i|c, \theta_1, \theta_2) \prod_{i=c}^n p(y_i|c, \theta_1, \theta_2) \\ &= \prod_{i=1}^{c-1} \theta_1^{y_i} (1 - \theta_1)^{1-y_i} \prod_{i=c}^n \theta_2^{y_i} (1 - \theta_2)^{1-y_i} \\ &= \theta_1^{\sum_{i=1}^{c-1} y_i} (1 - \theta_1)^{c-1 - \sum_{i=1}^{c-1} y_i} \theta_2^{\sum_{i=c}^n y_i} (1 - \theta_2)^{n-c+1 - \sum_{i=c}^n y_i} \end{aligned}$$

## Question 3

On utilise la formule de Bayes :

$$\begin{aligned} p(y|c=1, \theta_1, \theta_2) &= \frac{p(c=1|y, \theta_1, \theta_2)p(y, \theta_1, \theta_2)}{p(c=1, \theta_1, \theta_2)} \\ \text{et} \\ p(y|c, \theta_1, \theta_2) &= \frac{p(c|y, \theta_1, \theta_2)p(y, \theta_1, \theta_2)}{p(c, \theta_1, \theta_2)} \end{aligned}$$

donc, en remarquant que  $p(c=1|y, \theta_1, \theta_2) = p(c=1|y)$  et  $p(c|y, \theta_1, \theta_2) = p(c|y)$  car  $c$ ,  $\theta_1$  et  $\theta_2$  sont indépendants, on a :

$$\begin{aligned} \frac{p(y|c, \theta_1, \theta_2)}{p(y|c=1, \theta_1, \theta_2)} &= \frac{p(c|y)p(y, \theta_1, \theta_2)}{p(c, \theta_1, \theta_2)} \times \frac{p(c=1, \theta_1, \theta_2)}{p(c=1|y)p(y, \theta_1, \theta_2)} \\ &= \frac{p(c|y)}{\frac{1}{n}} \times \frac{\frac{1}{n}}{p(c=1|y)} = \frac{p(c|y)}{p(c=1|y)}. \end{aligned}$$

D'où

$$\frac{p(c|y)}{p(c=1|y)} = \frac{\prod_{i=1}^{c-1} \theta_1^{y_i} (1-\theta_1)^{1-y_i} \prod_{i=c}^n \theta_2^{y_i} (1-\theta_2)^{1-y_i}}{\prod_{i=1}^n \theta_2^{y_i} (1-\theta_2)^{1-y_i}} = \prod_{i=1}^{c-1} \frac{\theta_1^{y_i} (1-\theta_1)^{1-y_i}}{\theta_2^{y_i} (1-\theta_2)^{1-y_i}}$$

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n<-1000  
y<-rnorm(n, 0, 1)  
hist(y)
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