

13.6.5 Random walk

Random walk (RW) is a good model for *Brownian motion*, *diffusion* of molecules or gas particles, behaviour of long chain *polymers in solution* and in many other areas. To build up a model, we can assume that a walker moves randomly, one step at a time, in any direction from some origin. We can ask the question: How far does the walker move after certain steps (end-to-end distance)?

Random walk in 1D:

Let us consider a random walk in one dimension. A walker moves randomly either right (+1) or left (-1), at each time step, on a straight line. If we keep track of the position of the walker from the beginning, we can then calculate the cumulative distance (distance from origin) and check how it varies with time. For example, if the walker goes 1 step right, then 1 step left and then again 1 step right, the steps will be +1, -1, +1, respectively. The cumulative position at the end of three steps = $+1 - 1 + 1 = 1$ (one step right). In order to implement in computer, we can map this problem into a coin tossing problem with two options (Head or Tail). For this, we assign Head = +1 (one step right) and Tail = -1 (one step left). In the following, we generate a random walk of many steps at one go.

Random walk in 2D:

Consider a random walk on a two-dimensional square grid. Starting from the origin, the walker moves in each step to any of the 4 nearest neighbour nodes on four directions: right $(1, 0)$, left $(-1, 0)$, up $(0, 1)$ and down $(0, -1)$.