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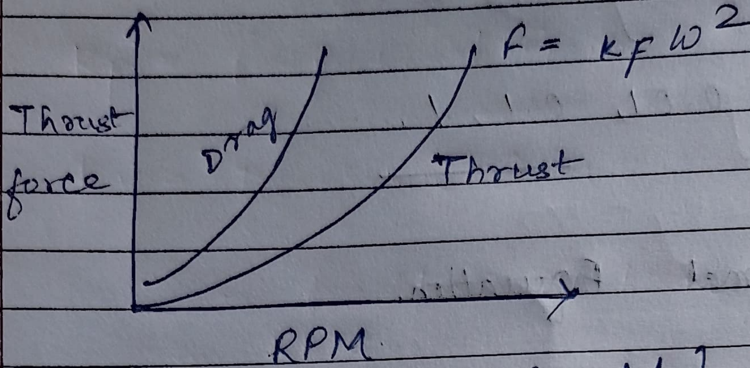
Upenn Aerial Robotics.

• Basic Mechanics →



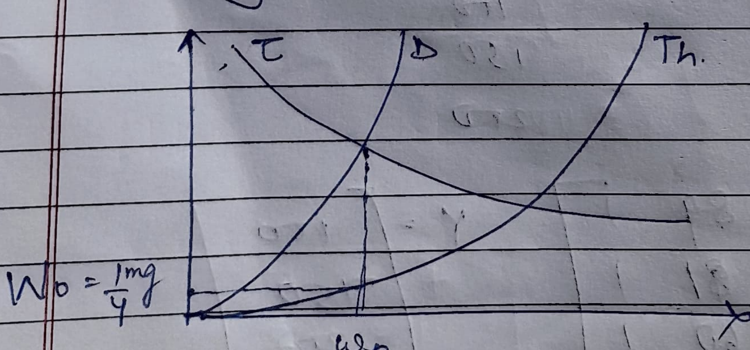
rotor speed (rad/s)

This relation is quadratic in nature.

Both Thrust and drag are quadratic in nature.

In the state of equilibrium - every rotor has to support 1/4th of the weight in equilibrium.

Sizing the Motor -



At ω_0, torque should balance out the associated drag to help the rotor stay afloat.

Power = Torque × rad/s
Nm × $\frac{1}{s}$

Motor speeds →

$$k_F \cdot \omega_i^2 = \frac{1}{4} mg$$

ω → rotor speed.

Motor Torque →

$$T_i \to k_M \cdot \omega_i^2$$

Dynamics and 1-D control. —

$$\sum_{i=1}^{4} k_F \omega_i^2 + mg = ma$$

Sum of forces.

$$a = \frac{d^2x}{dt^2} = \ddot{x}$$

$$u = \frac{\text{Sum of forces}}{\text{mass}}$$

$$u = \frac{1}{m}\left[\sum_{i=1}^{4} k_F \omega_i^2 + mg\right]$$

$$\therefore \quad u = a = \ddot{x}$$

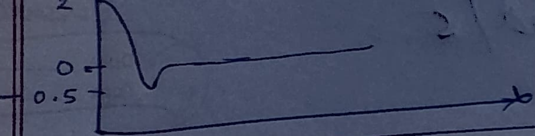
- Objective is to determine the function u so that we can determine x.

$$e(t) = x^{des}(t) - x(t)$$

e(t) = error function
we want e(t) to converge exponentially to zero.

$$\ddot{e} + K_v\dot{e} + K_p e = 0 \quad \text{—— (1)}$$

It is critical to ensure kp and kv must be zero such that e(t) will go down to 0.



$e(t)$

PD

$$\boxed{u(t) = \ddot{x}^{des}(t) + K_v \dot{e}(t) + K_p e(t)}$$

Control equation.

K_p = Proportional Gain

K_v = Derivative Gain

$\ddot{x}^{des}$ = Feedforward ~~term~~, to guide on how the trajectory should be.

- K_p acts like a spring (capacitance) response

$\uparrow K_p$, the more springy system response becomes.

$\therefore \uparrow K_p, \uparrow$ overshoot

- K_v acts like a viscous dashpot.

More K_v, makes the system over damped but the system converges slowly.

- PID Control –
  In case of disturbances such as wind it is desirable to use PID

Desirable to keep K_p and K_v +ve

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Design considerations. —

Effect of maximum thrust

We know that the motor thrust is limited due to its capacity and hence it cannot provide thrust greater than the threshold.

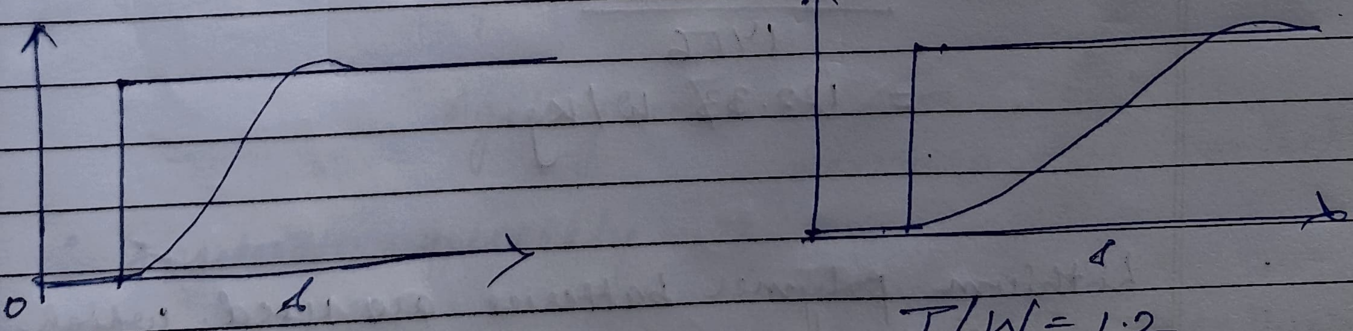
$$\sum_{i=1}^{4} k_f \omega_i^2 + mg = ma.$$

Maximum thrust is limited by peak motor torque.

Max Thrust $= T_{max}$

$$u_{max} = \frac{1}{m}\left[T_{max}\uparrow + mg\downarrow \right]$$ Take vector sum.

· Thrust / Weight Ratio →



$T/W = 2$

$T/W = 1.2$

More thrust per unit weight leads to a faster response.

Sikorsky UH-60 L. —

Total weight ⇒ 10,500 kg.

Total power ⇒ 1,410 kW, 2 GE engines

⇒ 2,8,20 kW

Power per unit mass ⇒ $\frac{2820 \times 1000}{10,500}$

⇒ 268.57 W/kg.

Cessna 182 skylane. —

weight ⇒ 1406 kg.

Power ⇒ 172 kW.

P/W ⇒ $\frac{172 \times 1000}{1406}$

= 122.33 W/kg.

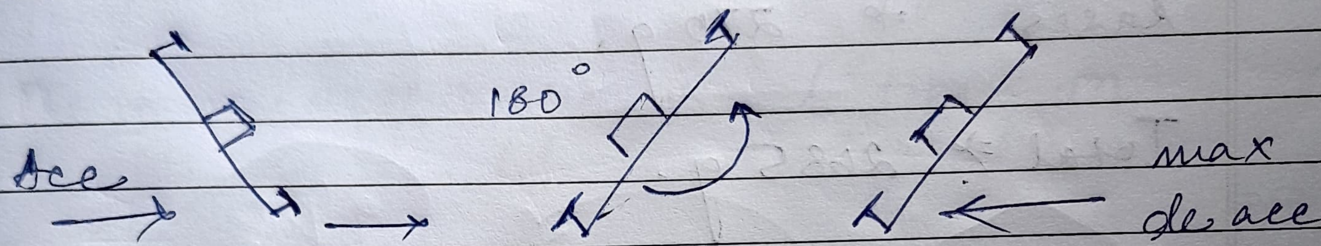
Lithium polymer batteries are used. which ~~provide~~ greater than 200 W/kg to support the power consumption

Motor Mass Distribution →

Batteries ~ 33%

Motor ~ 25%

Max Acc to Rest



$V^2 = 100$

$u^2 = 0$

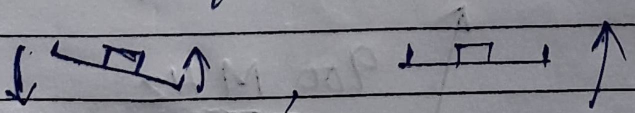
$V^2 - u^2 = 2as$

$\frac{100 - 0}{2 \times 5} = s$

$s = 10m.$

Agility. —

- Accelerate quickly. → maximize $\frac{u_1}{W}$ (Max Thr)
- Roll/Pitch quick. maximize $\frac{u_2}{I_{xx}}$ (max Turning moment



Moment of inertia along ~~Ixx~~ x-axis.

Component Selection -

- DJI F550 + E600 →

Frame + Pixhawk + propulsion ⇒ 1494 g.

Battery ⇒ 721 g.

Computer ⇒ 200 g.

Laser ⇒ 270 g.

Total ⇒ 2685 g.

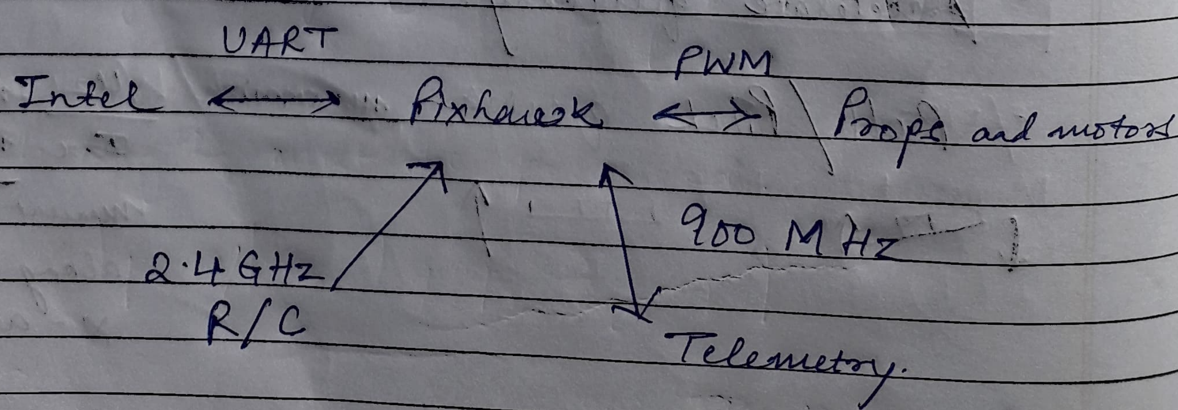
Max Thrust = 9600 N.

$$T/W = \frac{9600}{2685} \doteq 3.57$$

Basic Hardware -

1) Pixhawk. Ardupilot

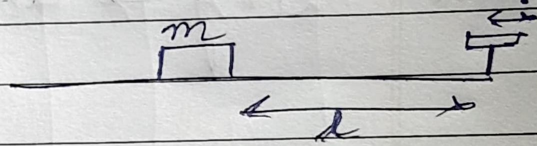
2) Intel NUC i7, High level processor.



Effect of size →

a) Mass and Inertia →

$m \sim l^3$, $I \sim t^5$



mass of the system is almost proportional to the cube of length l.

The moment of inertia is l^2 times m. since m is l^3 ∴ $I \sim l^5$.

b) Thrust →

r → rotor radius.

$F \sim \pi r^2 \times (\omega r^2)$

ω = angular velocity of rotor

ωr = blade tip speed.

c) Moment →

v = blade tip speed.

$M \sim Fl$

$$a \sim \frac{v^2}{l}, \quad \alpha \sim \frac{v^2}{l^2}$$

The maximum angular acceleration tends to be higher in small vehicles making them more agile.

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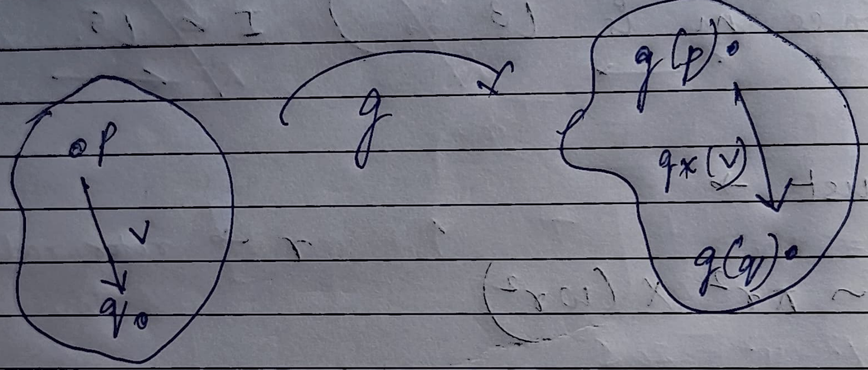
1 kg → 200 W.

Comp. 100 g → 20 W. + 30 W → 50 W

Lacea 200 g → 40 W. + 20 W → 60 W

Week - 2

Rigid Body Transformations.



Ans 1 →

a) g(P) is the correct expression of point P'

b) P' - Q' is represented by -

⇒ $g(P) - g(Q)$ and $g*(P-Q)$

1. Length must be preserved.

$$|p-q| = |g(p) - g(q)|$$

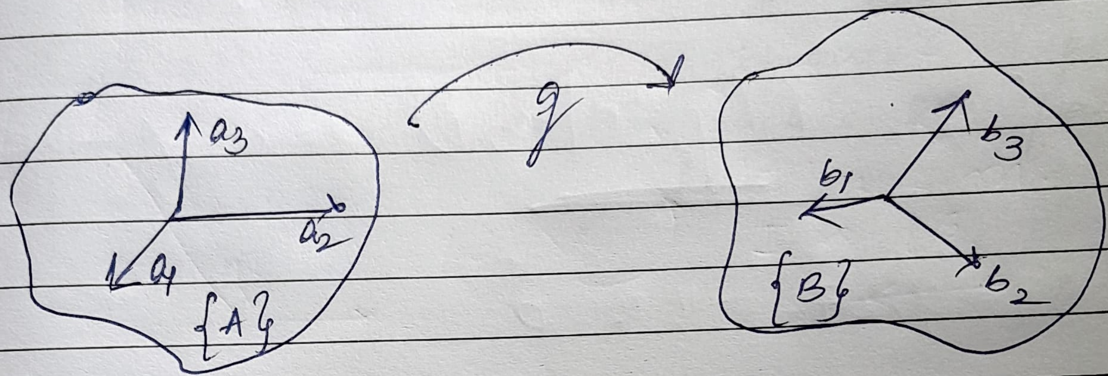
2. Cross products of vectors ~~are~~ preserved.

$$g*(v) \times g*(w) = g*(v \times w)$$

$p = [0, 0.5, 0.25]$

$q = [0.5, 0, 0.5]^T$

$$p \cdot q = 0 + 0 + 0.25 \times 0.5$$
$$= 0.125$$



$$b_1 = R_{11} a_1 + R_{12} a_2 + R_{13} a_3$$

$$b_2 = R_{21} a_1 + R_{22} a_2 + R_{23} a_3$$

$$b_3 = R_{31} a_1 + R_{32} a_2 + R_{33} a_3$$

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$$R = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$$ R = Rotation Matrix

Q

$$R = \begin{bmatrix} 0.9129 & 0.1695 & 0.3714 \\ -0.3651 & 0.7459 & 0.5571 \\ 0.1826 & 0.6442 & -0.7428 \end{bmatrix}$$

$|R|$

$\rightarrow -0.8451 + 0.0287 + -0.0367 \neq 0$

$\therefore$ NO

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Rotation Matrices →

Special Orthogonal Matrices

$R \in R^{3\times 3} \mid R^T R = R R^T \mid \det R = 1$

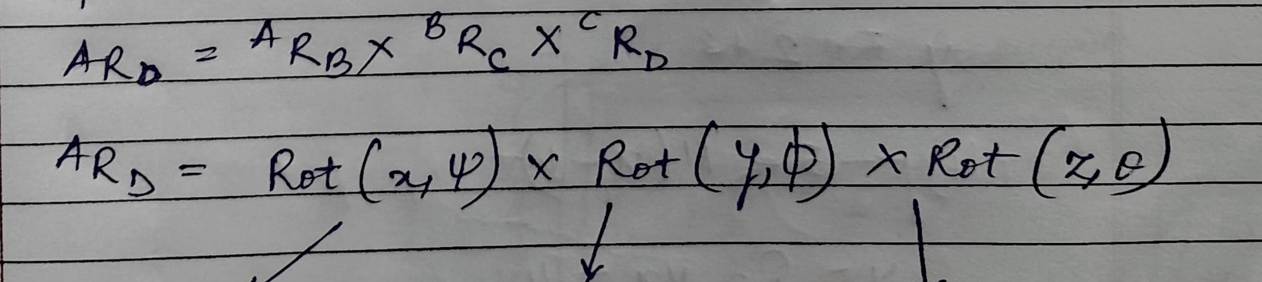
- SO(3) special orthogonal group.

Minimum number of charts needed to cover the earth's surface — Two.

Euler Angles

Euler showed that 3 coordinates are necessary to describe a general rotation

These coordinates are called as Euler angles



$${}^A R_D = {}^A R_B \times {}^B R_C \times {}^C R_D$$

$${}^A R_D = Rot(x, \psi) \times Rot(y, \phi) \times Rot(z, \theta)$$

Roll on axis a_1

Pitch on axis b_2

Yaw on axis c_3

$x + y = 100$

$25x + 40y = 2800$

$25x + 40(100 - x) = 2800$

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X-Y-Z Euler Angles

$^{A}R_{D} = {}^{A}R_{B} \times {}^{B}R_{C} \times {}^{C}R_{D}$

Z-X-Y Euler Angles [Used more commonly].

Rot (z, ψ)
Rot (x, ϕ)
Rot (y, θ)

$a + y + z = 1$

$ax - ay + 3z = 5$

$5x - 3y + az = 6$

$y = 2x^4 + 3x$ $\qquad a = \log x$

$\Rightarrow 4 \cdot 2 \cdot x^3 + 3$

$\Rightarrow 8x^3 + 3 \Rightarrow 8(1) + 3$

$\Rightarrow$

$y = 2 + 3$

$y - 5 = 11(x - 1)$

$y - 5 = 11x - 11$

$y = 11x -$

$y = x^x$

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Dynamic equation of motion of the quadrotor in

z direction is $\Rightarrow \ddot{z} = \frac{u}{m} - g$

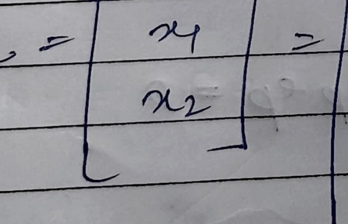
u = motor thrust

mg = weight

$u = m(\ddot{z}_{des} + K_p e + K_v \dot{e} + g)$

24/6/22 Planar Quadrotor Model.

$$\begin{bmatrix} \ddot{y} \\ \ddot{z} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} 0 \\ -g \\ 0 \end{bmatrix} + \begin{bmatrix} -\frac{1}{m}\sin\phi & 0 \\ \frac{1}{m}\cos\phi & 0 \\ 0 & \frac{1}{I_{xx}} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$



The equations of motion of the planar model are—

i) Linear in Input

2) Non linear in output.

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} y \\ z \\ \phi \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} \dot{y} \\ \dot{z} \\ \dot{\phi} \\ 0 \\ -g \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\frac{1}{m}\sin\phi & 0 \\ \frac{1}{m}\cos\phi & 0 \\ 0 & \frac{1}{I_{xx}} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\ddot{y} = -\frac{u_1}{m} \sin(\phi)$$

$$\ddot{z} = -g + \frac{u_1}{m} \cos(\phi)$$

$$\ddot{\phi} = \frac{u_2}{I_{xx}}$$

Equilibrium hover configuration -

$y_0, z_0, \phi_0 = 0$

$u_{1,0} = mg$

$u_{2,0} = 0.$

Trajectory Tracking

$e_p = r_T(t) - r$

$e_v = \dot{r}_T(t) - \dot{r}$

Want →

$$(\ddot{r}_T(t) - \ddot{r}_c) + K_d e_v + k_p e_p = 0$$

Trajectory tracking in 3 Dimensions

$$r_T(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \\ \psi(t) \end{bmatrix}$$

ψ - yaw angle

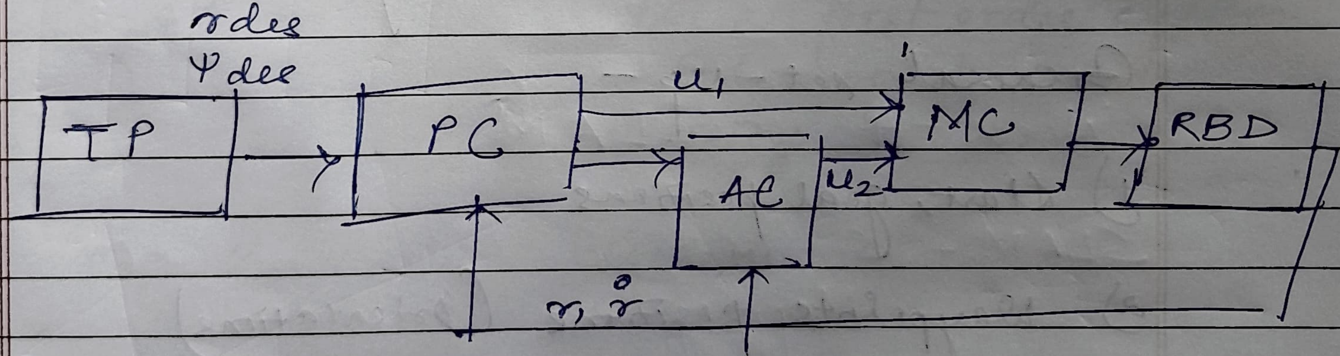
$x, y, z \rightarrow$ position vector.

$r_T(t)$ desired trajectory (pos, yaw).

$$e_p = r_T(t) - r$$

$$e_v = \dot{r}_T(t) - \dot{r}$$

$$(\ddot{r}_T(t) - \ddot{r}_c) + K_d e_v + K_p e_p = 0.$$



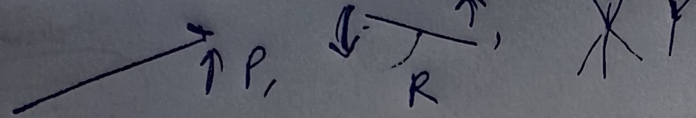
TP = Trajectory planner.

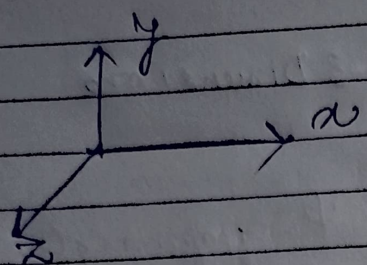
PC = Position Controller

AC = Attitude Controller.

MC = Motor controller.

RBD = Rigid Body Dynamics.





Q While hovering the following must be zero →

1) Roll angle

2) Linear velocity along z

3) Pitch

25/6/22 Week - 3

Time motion and trajectories

General set-up -

1) Start, goal positions

2) Waypoints positions (Orientations)

3) Smoothness criterion

4) Order of the system (n)

First Order systems - You can specify velocity. [Kinematic Model]

Second order system - " " acceleration.

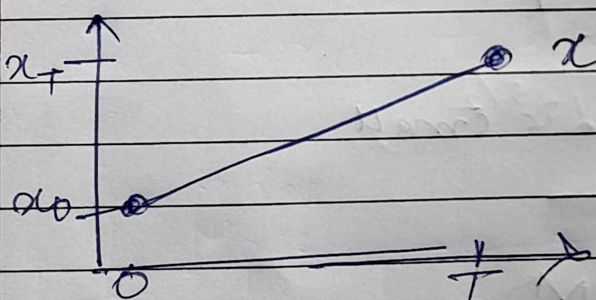
Third order system - third order derivative or Jerk forth derivative - snap

Order of the system determines the input with boundary conditions on $(n-1)^{th}$ order

Euler - Lagrange Equation →

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

[Necessary condition satisfied by the 'optimal' function $x(t)$



$x = c_1 t + c_0$

For a first order system with input as velocity.

$$m = \frac{x_T - x_0}{T} = \text{velocity}$$

$n = 1$ shortest distance

$n = 2$ min acc

$n = 3$ min jerk

$n = 4$ min snap

$$x^*(t) = \underset{x(t)}{\operatorname{argmin}} \int_0^T \left(x^n\right)^2 dt$$

The position control system is a 4th order system.

There we want trajectories that can be differentiated four times.

∴ Minimum snap trajectory.

Q Week-3 26|6|22

1) Outer - Position, Inner - Orientation

2) 9 $(5 \times 2 - 1) = k$

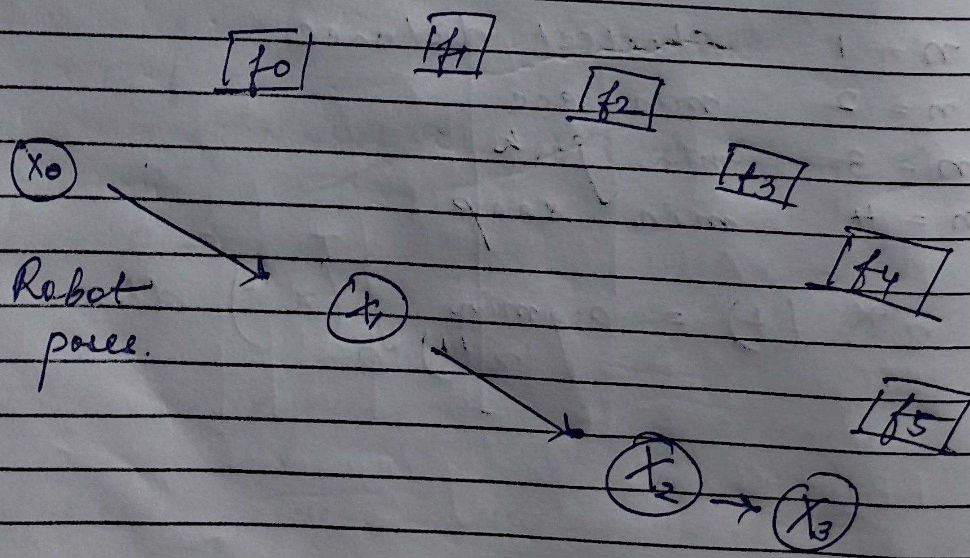
$\Rightarrow 10 - 1 = k$

$k = 9$

3) All except, quadcopter is small

Week-4 SLAM

SLAM - Simultaneous Localization and Mapping.



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The robot takes the IMU measurements at each point from X_0 to X_3 and integrates them

Q What type of info does the IMU measurement provide

a) Info about absolute attitude (orientation) of the bot

b) Info (via integration) about the relative motion from X_{i-1} to X_i

Q What does the laser scanner provide -

a) Info about bearing of features relative to current robot pos.

b) Info about depth of features. " " "

Q Which sensors are most likely to work well in lift-buildings →

a) Camera
b) IMU
c) Laser Scanner

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We want Non-Linear control to perform the complex maneuvers.

$$r_T(t) = \begin{bmatrix} x_{des}(t) \\ y_{des}(t) \\ z_{des}(t) \\ \psi_{des}(t) \end{bmatrix}$$

$$u_1 = (\ddot{r}_{des} + k_v e_{\dot{r}} + k_p e_r + mga_3)$$

$$u_2 = \omega \times I\omega + I(-k_R e_R - k_\omega e_\omega)$$

Basin of attraction $\rightarrow \frac{1}{L^{5/2}}$

Q Why are we incorporating the 2nd derivative of the yaw angle to the cost functional and not the 4th derivative

Ans We only need the 2nd derivative of the yaw.
While we need the 4th derivative of position

Checkout → Transportation and Construction. [2:54 – 3:45]

classmate

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28/6/22 Swarm Robotics →

- Each individual acts independently [ie no master]
- Actions are based on local information
- Anonymity in coordination.

Complexity.

n – robots, m – obstacles

1) Dimensionality of the state space increases linearly with 'n' $O(n)$

2) No of potential interactions with neighbours ↑ as n^2

3) Potential obstacles ⇒ $m \times n$

$\{O(mn + n^2)\}$

4) No of assignments of robots to goal position ⇒ $O(n!)$

Hungarian Algorithm. – $O(n^3)$