

MatheMagic

Backpropagation

Step 0: Read input and output

```
import numpy as np
x = np.array([[1,0,1,0],[1,0,1,1],[0,1,0,1]])
y = np.array([1,1,0]).reshape(3,-1)
```

x			
1	0	1	0
1	0	1	1
0	1	0	1

y
1
1
0

Step 1: Initialize weights and biases with random values

```
wh = np.random.rand(4,3)
bh = np.random.rand(3)
```

```
wout = np.random.rand(3,1)
```

```
bout = np.random.rand(1)
```

wh		
0.51010119	0.08058941	0.81539472
0.94568682	0.24722592	0.1687749
0.20810438	0.88061006	0.01211583
0.19951766	0.16610815	0.15118213

bh		
0.05253469	0.23126176	0.64622242

wout
0.48488218
0.65993833
0.84282406

bout
0.83612087

Step 2: Calculate hidden layer input:

```
hidden_layer_input = x.dot(wh) + bh
```

hidden_layer_input		
0.77074025	1.19246122	1.47373298
0.97025791	1.35856938	1.6249151
1.19773916	0.64459583	0.96617946

Step 3: Sigmoid activation on hidden layer

```
def sigmoid(x):  
    return (1/(1+np.exp(-x)))  
hidden_layer_activation = sigmoid(hidden_layer_input)
```

hidden_layer_activation		
0.683681	0.76718096	0.81362412
0.7251709	0.79552709	0.835471877
0.76812235	0.65579161	0.72435733

Step 4: Perform linear and non-linear transformation of hidden layer activation at output layer

```
output_layer = hidden_layer_activation.dot(wout) + bout  
output = sigmoid(output_layer)
```

output

0.91369898
0.91810681
0.904810649

Step 5: Calculate Error E

$$E = y - \text{output}$$

E
0.08630102
0.0818931
-0.90481064

Step 6: Compute slope at output and hidden layer

```
def derivative_sigmoid(x):
    sigmoid_X = sigmoid(x)
    return (sigmoid_X*(1-sigmoid_X))
slope_output_layer = derivative_sigmoid(output)
slope_hidden_layer = derivative_sigmoid(hidden_layer_activation)
```

slope_output_layer
0.20430816
0.20392272

0.20508267

slope_hidden_layer		
0.22292008	0.21654365	0.21279683
0.21981279	0.21427262	0.21098953
0.21646905	0.22493655	0.21987491

Step 7: Compute delta at output layer

```
lr = 1
d_output = E * slope_output_layer * lr
```

d_output
0.017632
0.01669988
-0.18556098

Step 8: Calculate Error at hidden layer

```
error_at_hidden_layer = d_output * np.transpose(wout)
```

error_at_hidden_layer		
0.00854944	0.01163604	0.01486068

0.00809747	0.01102089	0.01407506
-0.08997521	-0.12245881	-0.15639526

Step 9: Compute delta at hidden layer

```
d_hidden_layer = error_at_hidden_layer * slope_hidden_layer
```

d_hidden_layer		
0.00190584	0.00251971	0.0031623
0.00177993	0.00236148	0.00296969
-0.01947685	-0.02754546	-0.03438739

Step 10: Update weight at both output and hidden layer

```
#Updating weights
```

```
wout = wout + (np.transpose(hidden_layer_activation).dot(
d_output))*lr
```

```
wh = wh + (np.transpose(x).dot(d_hidden_layer))*lr
```

wh		
0.51378696	0.08547059	0.82152672
0.92620997	0.21968046	0.13438751
0.21179015	0.88549124	0.01824783

0.18182074	0.14092417	0.11976443
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wout
0.12977637
0.37530676],
0.524481

Step 11: Update biases at both output and hidden layer

```
#Updating bias
bh = bh + np.sum(d_hidden_layer,axis=0)*lr
bout = bout + np.sum(d_output,axis=0)*lr
```

bout
0.68489177

bh		
0.03674361	0.20859748	0.61796702