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# DIRECT NUMERICAL SIMULATIONS OF BUBBLES IN VERTICAL CHANNELS

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## ABSTRACT

Recent DNS studies of bubbly flows in channels are discussed. Simulations of nearly spherical bubbly flows in vertical channels show that the bubbles move towards the wall for upflow and away from the wall for downflow in such a way that the core is in hydrostatic equilibrium. For down flow the wall layer is free of bubbles but for upflow there is an excess of bubbles in the wall layer. The liquid velocity in the core is uniform. For laminar downflow the velocity in the wall layer can be computed analytically but for upflow the velocity is strongly influenced by the presence of the bubbles. Results for turbulent flow show similar behavior and for downflow the velocity is given (almost) by the law of the wall. Several simulations are used to examine the effect of void fraction and bubble size for turbulent downflow.

## INTRODUCTION

Bubbly flows in vertical pipes and channels are encountered in a wide variety of industrial systems. The best-known early study of such flows is by Serizawa, Kataoka and Michiyoshi (1975) who examined experimentally the void fraction distribution and the velocity profile in turbulent air-water bubbly flows. Other experiments have been done by by Wang, Lee, Jones and Lahey (1987), Liu and Bankoff (1993), Nakoryakov and Kashinsky (1981, 1996), Liu (1997), Kashinsky, Randin, and Timkin (1999), So, Morikita, Takagi, and Matsumoto, (2002), Guet, Ooms, Oliemans, and Mudde (2004), and Matos, Rosa and Franca (2004). The results show that for nearly spherical bubbles the void fraction distribution and the velocity profile in the core of the channel are relatively uniform and that a void fraction peak is generally found near the wall for upflow but not for downflow. Sufficiently deformable bubbles, on the other hand, show exactly the opposite behavior and migrate to the center of the channel in upflow and toward the walls in downflow. A number of authors have also developed two-fluid models of bubbly flows in vertical channels. General descriptions of the two-fluid model can be found in Delhay (1982), Kataoka and Serizawa (1989), Zhang and Prosperetti (1994) and Drew and Passman (1999), for example. Numerical studies, using the two-fluid model can be found in Lopez De Bertodano, Lahey, and Jones (1987, 1994), Kuo, Pan, and Chieng (1997), and Guet, Ooms and Oliemans (2005) and others. The model results generally reproduce the experimental results reasonably well.

While the flow is likely to be turbulent in most cases of practical interest, laminar flow is an important limiting case that can be used to explore aspects of multiphase flow modeling that do not depend on the specifics of the turbulence. This was recognized by Antal, Lahey and Flaherty (1991) who developed a two-fluid model for such flows and compared the model results with experimental predictions. The agreement between the model and the experiments was good, although for upflow there is a need to introduce a wall repulsion force to keep the center of the bubbles at least a radius away from the walls and the authors observed some dependency on the exact value of the lift coefficient used. Other studies of laminar flow include the experimental investigation by Song, Luo, Yang, and Wang (2001) who studied flows with both uniform and nonuniform distribution of bubble sizes and Lou, Pan, and Yang (2003) who examined the motion of light particles. Both studies were done for upflow and both found wall peaking.

The model of Antal, Lahey and Flaherty (1991) was studied analytically by Azpitarte and Buscaglia (2003) who made the very important observation that the fluid in the center region of the channel is always in hydrostatic equilibrium. Biswas, Esmaeeli and Tryggvason (2005) conducted direct numerical simulations of two-dimensional bubbly flows and compared their results with the model of Antal et al. Since the simulations assumed two-dimensional flow, Biswas et al. had to adjust the model parameters, but they found that once the parameters had been adjusted for one case, the model predicted other situations reasonably well.

 

## FORMULATION AND NUMERICAL METHOD

We examine the flow in a vertical channel between two parallel walls, where the streamwise, wall-normal and spanwise directions are denoted by x, y and z, respectively. The flow is driven upward or downward by a constant pressure gradient, and gravity acts in the negative x-direction. Periodic boundary conditions are imposed in both the streamwise and the spanwise directions and no-slip boundary conditions are enforced at the walls.

Here we only consider the statistically steady state where the macroscopic momentum balance for the computational domain at a statistically steady state is given by

$$2\tau_w A - \frac{dp}{dx}V + \rho_{av} g V = 0. \quad (1)$$

In equation (1), $\tau_w$ is the average wall shear stress, dp/dx is the pressure gradient, $\rho_{av}$ is the average density of the bubbly flow, A is the area of each wall and V is the volume of the whole channel. The fluid and the bubbles are taken to be incompressible, so $\rho_{av}$ is constant throughout the simulations. Since the mixture is driven by a constant pressure gradient, $\beta = -dp/dx + \rho_{av} g$ is also constant. The direction of the flow depends on the sign of β. Since $V = A \cdot 2H$, where H=1 is the half width of the channel, we get the average wall shear stress at statistically steady state as:

$$\tau_w = -\beta H. \quad (2)$$

This value of the average wall shear stress can be used to check when the bubbly flow has reached a steady state condition.

The numerical simulations are carried out by a finite volume/front-tracking method where one set of equations is used for the whole domain, including both the bubbles and the carrying liquid. It fully resolves the fluid flow around each bubble and accounts accurately for bubble-bubble interactions and bubble deformation. The "one-fluid" Navier-Stokes equation is

$$\begin{aligned} &\rho \frac{\partial \boldsymbol{u}}{\partial t} + \rho \nabla \cdot \boldsymbol{u}\boldsymbol{u} = -\nabla p + (\rho - \rho_{av})\boldsymbol{g} + \\ &\nabla \cdot \mu (\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T) + \sigma \int_F \kappa_f \boldsymbol{n}_f \delta(\boldsymbol{x} - \boldsymbol{x}_f) dA_f \end{aligned} \quad (3)$$

Here, **u** is the velocity vector, p is the pressure, ρ and μ are the discontinuous density and viscosity fields, respectively, and g is the gravity acceleration. σ is the constant surface tension, and δ is a three-dimensional delta function constructed by repeated multiplication of one-dimensional delta functions. $\kappa_f$ is twice the mean curvature, and $\mathbf{n}_f$ is a unit vector normal to the front. $\boldsymbol{x}$ is the point at which the equation is evaluated and $\boldsymbol{x}_f$ is the position of the front. The singular term ensures that the momentum equation implicitly contains the correct stress boundary conditions at the interface. Because of the incompressibility of the liquid and the bubble, the mass conservation equation reduces to

$$\nabla \cdot \boldsymbol{u} = 0 \quad (4)$$

for the whole flow field. We also take the density and viscosity of each fluid to be constants.

These equations are solved by a second-order accurate projection method on a fixed and staggered grid. The original method has been described in detail by Unverdi and Tryggvason (1992) and Tryggvason et al. (2001), and validation tests are described by Esmaeeli and Tryggvason (1998, 1999). For the simulations presented here we started with a fully parallel code written in Fortran 90/95 for the simulations described by Bunner and Tryggvason (2002,2003). Three major changes have been made for simulations of bubbles in a turbulent channel flow. First, the code is changed to accommodate non-uniform grids in the wall-normal direction. As the Reynolds number is increased, the resolution requirement increases, particularly near the wall. Non-uniform grids, clustered in the near wall region, are frequently used in simulations of wall bounded flows. Secondly, the demand on the advection solver also increases as the Reynolds number increases and we have implemented a third-order upwind scheme (QUICK) instead of the centered difference scheme used by Bunner and Tryggvason (2002) to allow us to accurately deal with such systems. Centered difference schemes often lead to unphysical oscillatory behavior or disastrous non-convergence in regions where advection strongly dominates diffusion. And thirdly, a nonconservative form of the governing equations is used in the new code. The original code used a conservative form of the governing equations, and it is found to cause increasing irregularities in the velocities near the front for high Reynolds number flows. The new code was tested extensively by comparing it with the original code (which has been thoroughly validated) and by grid refinement studies.

## LAMINAR BUBBLY CHANNEL FLOWS

In this section we review briefly recent results for the motion of several nearly spherical bubbles in laminar flow in a vertical channel, both for upflow and downflow, where all flow scales are fully resolved. The simulations showed that in both cases the flow consists of two well-defined regions: A thin wall-layer and a homogeneous core, occupying most of the channel. The formation of these regions is due to lift induced lateral motion of the bubbles. For a nearly spherical bubble rising due to buoyancy in a vertical shear, it is well known that the lift force pushes the bubble toward the side where the liquid is moving faster with respect to the bubble. Thus, in upflow a bubble near the wall is pushed toward the wall and in downflow the bubble is pushed away from the wall. The weight of the bubble/liquid mixture and the imposed pressure gradient must be balanced by a shear stress due to a velocity gradient. For upflow the mixture, on the average, must be sufficiently light so the imposed pressure gradient can push it upward. As bubbles are removed from the core, its average density decreases until the weight is balanced exactly by the pressure gradient. The shear is then zero and the migration of the bubbles to the wall stops. For downflow the opposite happens. Bubbles move into the core and make it more buoyant, until its weight is balanced by the pressure gradient and further lateral migration is stopped. Thus, in both cases the core is in

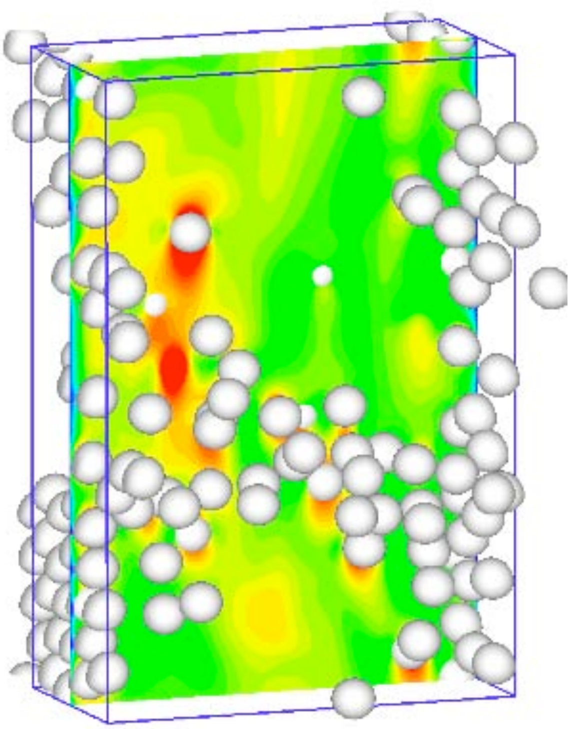

Figure 1. The bubble distribution at one time for laminar upflow. Isocontours of the vertical velocity are also shown for a plane going through the center of the channel.

hydrostatic equilibrium and it is only in the wall-layer where there is a non-zero velocity gradient. For upflow, where the weight of the mixture in the core is increased by pushing bubbles to the wall, the bubble rich mixture in the wall-layer is driven upward by the imposed pressure gradient. For downflow, on the other hand, bubbles must be drawn away from the wall to decrease the weight of the mixture in the core and the dense bubble-free wall-layer is driven downward by its weight and the imposed pressure gradient. This distribution is stable in the sense that if too many bubbles end up in the wall layer for upflow, the core slows down with respect to the wall layer, thus generating shear that will drive the bubbles out of the wall-layer. Similar if too many bubbles end up in the core for downflow, its velocity is reduced and bubbles are driven back to the wall. Figure 1 shows the bubble distribution and the vertical velocity at one time for upflow. For downflow see Lu, Biswas and Tryggvason (2006). The average void fraction is easily predicted given the considerations explained above and in figure 2 the average void fraction profile across the channel is plotted along with the model predictions. Obviously the agreement is excellent (the shape of the void fraction profile in the wall layer for upflow is different from the average, but the exact shape can be obtained also by assuming that the bubbles are nearly spherical).

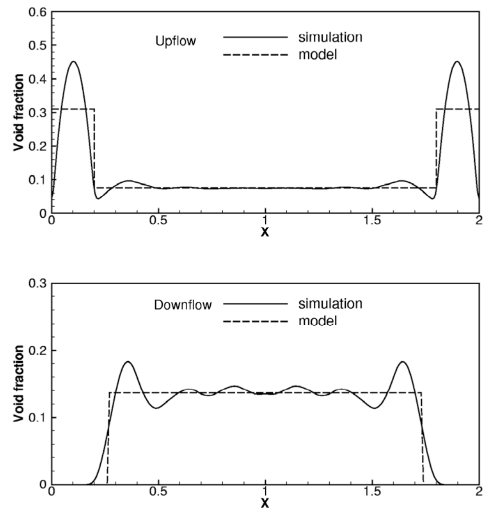


Figure 2. The average void fraction profiles across the channel for upflow (top frame) and downflow (bottom frame). The dashed line is the results of a simple analytical model. From Lu, Biswas and Tryggvason (2006).

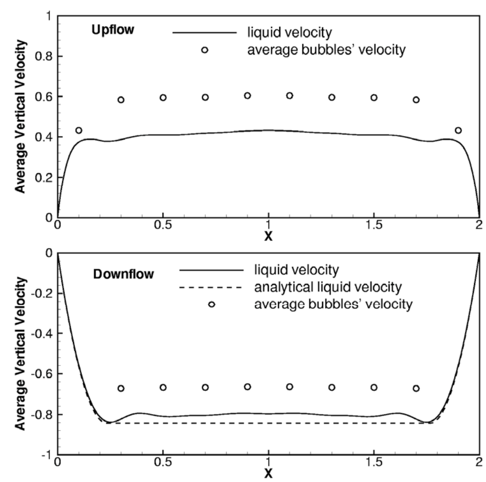


Figure 3. The average liquid velociy across the channel for upflow (top frame) and downflow (bottom frame). The open circles are the average bubble velocities. For the downflow, the dashed line shows the predictions of a simple analytical model. From Lu, Biswas and Tryggvason (2006).

For downflow, where the wall-layer is bubble free, the velocity profile is easily found by integrating the Navier-Stokes equations for steady laminar parallel flow and the flow rate can be predicted analytically, with a fair degree of accuracy. For upflow, on the other hand, the presence of the bubbles makes the situation more complex and the velocity profile is not as easily found. Figure 3 shows the average velocity profile across the channel for upflow and downflow. For downflow the model predictions are also included. Since the velocity increase across the wall-layer determines the liquid velocity in the core of the channel, it is critical for predicting the total flow rate. For the most part the bubbles in the wall layer interact only weakly with the bubbles in the core layer and as a first approximation it seems that they can be neglected, as long as the fluid is in hydrostatic equilibrium and the shear there is zero. We have therefore looked at the dynamics of a bubbly wall layer, neglecting the bubbles in the core region but eliminating any shear there by applying a body force adjusted to balance the pressure gradient there. For modest Reynolds numbers we have found that the contribution to the shear in the wall-layer from the Reynolds stress terms is insignificant, but the bubble deformation play a very significant role. While we do not fully understand yet how to predict the bubble deformation, we find that if we take information about the deformation from the computations and use them in a very simple model for flow, then we predict the velocity increase across the wall layer fairly accurately. There are slight differences that seem to be due to a small nonuniformity in the bubble concentration outside the wall layer and while we expect to explore how to account for those effects, their effect is small.

## TURBULENT CHANNEL FLOWS

Simulations of bubbly flows in turbulent channels suggest that the simplifications seen for laminar flow carry over, at least to some extend, to turbulent flows. We have started by looking at bubbles in a turbulent downflow in some detail. For the downflow case we expect the lift force to drive nearly spherical bubbles away from the walls, as for the laminar flow case. The velocity in the bubble free wall layer should therefore be given by the standard law of the wall. The main complication is if that if the wall layer is too thin, the presence of the bubbles may prevent the growth of turbulence structures near the wall and if the wall-layer is too thick, the core may meander in an unsteady way. Results obtained so far suggest that even for a very thin wall-layer (less than fifty wall units thick) the turbulence is sustained but that for thick wall layers the boundaries may vary in time due to meandering of the bubbly core. One frame from a simulation of bubbles in a turbulent downflow is shown in figure 4. For turbulent flow the velocity in the middle of the channel is relatively uniform in the absence of bubbles and since the main effect of adding the bubbles is to make the velocity there completely uniform, adding the bubbles causes surprisingly little change in the velocity. The main increase in velocity takes place in the bubble free wall layer where the velocity profile remains nearly the same and while the turbulent velocity profile without bubbles is not completely flat as it is after adding the bubbles, the differences are small. Since the flow in the core of the channel is uniform, the turbulent Reynolds stresses there are zero and in the buffer layer these are reduced. The slow growth of the velocity in the buffer layer and the wall region is also cut short at the outer edge of the wall layer and replaced by the uniform velocity characterizing the bubbly core.

We have also conducted several simulations of downflow with bubbles of different sizes. The results show that the bubble size has relatively little effect on the velocity profile, although the rise velocity of smaller bubbles is smaller than for larger bubbles at the same void fraction.

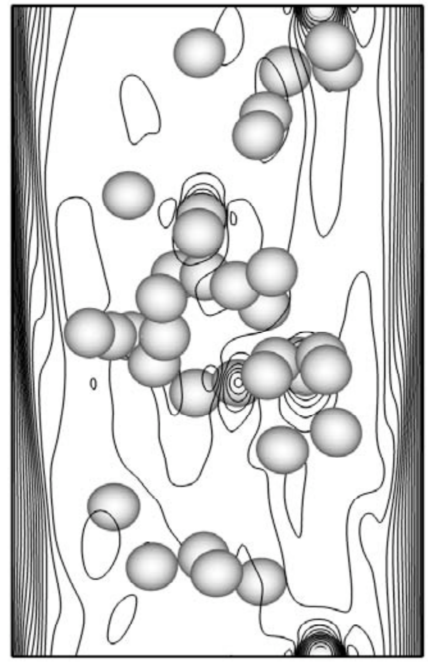

Figure 4. One frame from a simulation of turbulent bubbly downflow in a channel.

## CONCLUSIONS

While the basic structure of bubbly flows in a vertical channel has been observed experimentally before, the computational studies discussed here have allowed us to explore the dynamics in detail in a very well controlled and characterized situation. Results for both laminar and turbulent flows show that the structure of the flow is determined by the lift on the bubbles. At steady state the flow in the center of channel is in hydrostatic equilibrium for both upflow and downflow, and the dynamics is well described by results for homogeneous flows. The wall region is, however, very different for up-and downflow. For downflow, where there are no bubbles near the wall, the flow is particularly simple. In upflows, when the wall layer contains a large number of bubbles, the dynamics depends sensitively on both the number of bubbles and their deformability. Further studies of bubbly wall layers are in progress.

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## REFERENCES


Antal, S.P., Lahey, Jr., R.T., and Flaherty, J.E., 1991. Analysis of Phase Distribution in Fully Developed Laminar Bubbly Two-Phase Flow. Int'l. J. Multiphase Flow 17, 635-652.

Azpitarte, O.E., and Buscaglia, G.C., 2003. Analytical and numerical evaluation of two-fluid model solutions for laminar fully developed bubbly two-phase flows. Chemical Engineering Science 58, 3765-3776.

Biswas, S., Esmaeeli, A., and Tryggvason, G., 2005. Comparison of results from DNS of bubbly flows with a two-fluid model for two-dimensional laminar flows. Int'l. J. Multiphase Flow 31, 1036-1048.

Bunner, B. and Tryggvason, G. "Dynamics of homogeneous bubbly flows. Part 1. Rise velocity and microstructure of the bubbles," J. Fluid Mech. 466, 17 (2002).

Bunner, B. and Tryggvason, G. "Dynamics of homogeneous bubbly flows. Part 2. Velocity fluctuations," J. Fluid Mech. 466, 53 (2002).

Bunner, B. and Tryggvason, G. "Effect of bubble deformation on the properties of bubbly flows," J. Fluid Mech. 495, 77 (2003).

Delhay, J.M., Handbook of Multiphase Systems (editor Hetsroni, G.), Hemisphere-McGraw Hill, 1982

Drew, D.A., Passman, S.L., 1999. Theory of Multicomponent Fluids. Springer.

Esmaeeli, A. and Tryggvason, G. "Direct numerical simulations of bubbly flows. Part 1. Low Reynolds number arrays," J. Fluid Mech. 377, 313 (1998).

Esmaeeli, A. and Tryggvason, G. "Direct numerical simulations of bubbly flows. Part 2. Moderate Reynolds number arrays," J. Fluid Mech. 385, 325 (1999).

Guet, S. Ooms, G., Oliemans, R.V.A. and Mudde, R.F., 2004. Bubble size effect on low liquid input drift-flux parameters. Chem. Engr. Science 59, 3315-3329.

Guet, S. Ooms, G. and Oliemans, R.V.A., 2005. Simplified two-fluid model for gas-lift efficiency predictions AIChe Journal, 51, 1885-1896.

Kuo, T.C., Pan, C. and Chieng, C.C., 1997. Eulerian-Lagrangian Computations on phase distribution of two-phase bubbly flows. Int. J. Num. Meth. Fluids, 24, 579-593.

Lopez-De-Bertodano, M., Lahey, R.T. Jr, and Jones O.C. 1987. Development of a k-ε model for bubbly two-phase flow. J. Fluids Engineering 13, 327-343.

M. Lopez de Bertodano, R. T. J. Lahey, and O. C. Jones, "Phase distribution in bubbly two-phase flow in vertical ducts," Int. J. Multiphase Flow 20, 805 (1994).

Liu, T.-J. 1997. Investigation of the wall shear stress in vertical bubbly flow under different bubble size conditions, Int. J. Multiphase Flow, Volume 23, 1085-1109.

Liu, T.-J. and Bankoff, S. G. "Structure of air-water bubbly flow in a vertical pipe. Part 2, Void fraction, bubble velocity and bubble size distribution," Int. J. Heat Mass Transfer 36, 1061 (1993).

Liu, T.-J. and Bankoff, S. G. ""Structure of Air Water Bubbly Flow in A Vertical Pipe. Part I, Liquid Mean Velocity and Turbulence Measurements," Int. J. Heat Mass Transfer 36, 1049 (1993).

Luo, R., Pan, X. H. and Yang, X. Y. 2003. Laminar light particle and liquid two-phase flows in a vertical pipe. Int. J. Multiphase Flow, 29, 603-620.

Lu, J. Biswas, S. and Tryggvason, G. "A DNS study of laminar bubbly flows in a vertical channel," Int. J. Multiphase Flow 32, To be published (2006).

Nakoryakov, V. E., Kashinsky, O. N., Randin, V. V. and Timkin, L. S. "Gas-liquid bubbly flow in vertical pipes," J. Fluids Eng.-T. ASME 118, 377 (1996).

Nakoryakov, V. E., Kashinsky, O. N., Burdukov, A. P. and Odnoral, V. P. "Local characteristics of upward gas-liquid flows," Int. J. Multiphase Flow 7, 63 (1981).

Matos, A. de, Rosa, E. S. and Franca, F. A., 2004. The phase distribution of upward co-current bubbly flows in a vertical square channel. J. Braz. Soc. Mech. Sci. & Eng., 26, 308-316.

Serizawa, A., Kataoka, I. and Michiyoshi, I. "Turbulence structure of air-water bubbly flow. II. Local properties," Int. J. Multiphase Flow 2, 235 (1975).

Serizawa, A., Kataoka, I. and Michiyoshi, I. "Turbulence structure of air-water bubbly flow. III. Transport properties," Int. J. Multiphase Flow 2, 247 (1975).

So, S., Morikita, H., Takagi, S., and Matsumoto, Y. 2002. Laser Doppler velocimetry measurement of turbulent bubbly channel flow. Experiments in Fluids 33 (2002), 135-142.

Song, Q., Luo, R., Yang, X.Y., and Wang, Z. 2001. Phase distributions for upward laminar dilute bubbly flows with non-uniform bubble sizes in a vertical pipe. Int. J. Multiphase Flow, 27, 379-390

Tryggvason, G., Bunner, B., Esmaeeli, A., Juric, D., Al-Rawahi, N., Tauber, W., Han, J., Nas, S. and Jan, Y. J. "A front-tracking method for the computations of multiphase flow," J. Comput. Phys. 169, 708 (2001).

Unverdi S. O. and Tryggvason, G. "A front-tracking method for viscous incompressible multi-fluid flows," J. Comput. Phys. 100, 25 (1992).

Wang, S. K., Lee, S. J., Jones, O. C. J. and Lahey, R. T. J. "3-D Turbulence struture and phase distribution measurements in bubbly two-phase flows," Int. J. Multiphase Flow 13, 327 (1987).

Zhang, D.Z. and Prosperetti, A., 1994. Averaged equations for inviscid disperse two-phase flow. J. Fluid Mech. 267, 185-219.