Einstein Field Equations and Quantum Gravitational Approach to Contribution of Matter Density to the Fluctuating Metric and a Solution of Cosmological Constant

Suman Dey*
Department of Physics at Visva-Bharati University,
Santiniketan, West Bengal-731235
India

*Electronic address: ssuman7226@gmail.com

Abstract: We will first develop Einstein field equation with cosmological constant using Riemannian geometry. Now I considered that cosmological constant is associated with the vacuum energy density of particle physics model. Using Euclidean quantum gravity of path integral formalism and use of Robertson Walker metric(RW) I calculate the contribution of matter density to the fluctuating metric. This method rather leads to the observed cosmological constant without any need of fine tuning in the gravity and the particle physics model. this straightforward approach is very much useful and it is justified for all points of view.

1.1 Introduction to Riemannian Geometry:

As Euclidean is the study of flat space, between every pair of points there is a line segment which is unique which is the shortest distance between two points. All of these ideas in that Euclidean space is implemented in flat space. Pythagoras theorem and other sine and cosine that are in flat Euclidean space. Now instead of having flat space time we are going to introduce a curved space time, which is called Riemannian Space and the geometry associated with that called Riemannian Geometry.

Space-time coordinates are denoted by x^0, x^1, x^2, x^3rather $\sum_{i=0}^{3} x^i$. where x^0=ct, x^1 =x, x^2 =y, x^3=z. as the first coordinate represents the time component and others are the spatial components. As per Einstein summation convention, any time an upper index repeated as a lower index we are automatically summing over the index.

$$\sum_\mu V^\mu V_\mu = V^\mu V_\mu \ , \ \sum_\mu A^{\mu\nu}_{\gamma\mu} \ = \ A^{\mu\nu}_{\gamma\mu}$$

so, we are dropping $\sum$ notation all that above cases.

V^μ is called Contravariant component because of x^μ coordinates. Under space-time transformation the vector transforms like,

$$V'^\mu = \frac{\partial x'^\mu}{\partial x^\nu} V^\nu$$

For a particle moving space then spacetime coordinates will written as $x^\mu = x^\mu(\tau)$, where τ represents the Proper time.

Let a particle is moving in space, the proper time is τ. So we can write $x^\mu = x^\mu(\tau)$. Now the proper velocity is defined as $\frac{dx'^\mu}{d\tau} = \frac{dx'^\mu}{dx^\nu}\frac{dx^\nu}{d\tau}$

As we see that velocity transforms like a contravariant vector.

Now a Tensor field have two or more than components. The number of indices are the rank of Tensor. A Scalar is a ranked 0 tensor and a vector is ranked 1 tensor and $A^{\mu\nu}{}_{\gamma}$ is ranked 3 tensor.

Minkowski space is a space with a notation of distance between neighbouring points, in fact the square of a distance defined by Riemannian Geometry,

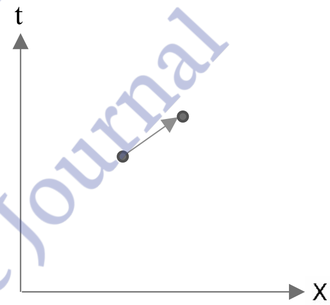
$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

The proper time between points $d\tau^2$

Given two points in space-time co-ordinate.

Proper time = $d\tau^2$, so we can write

$$dt^2 - \sum_i dx_i^2 = -ds^2, \quad where\ i = 1,2,3$$
$$= -g_{\mu\nu} dx^\mu dx^\nu$$



Where $g_{\mu\nu}$ is the Metric or g-metric of space-time.

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

As $g_{\mu\nu} \equiv g_{\mu\nu}(x,t)$ i.e. g-metric is a function of space and time also metric in space-time has one negative eigenvalue & three positive eigenvalues. So flat space means co-ordinate system where the g-metric look like this.

1.2 Minkowski Space-time:

Dealing with flat space we have introducing the Minkowski diagram to describe the uniformly accelerated frame as a point moving in hyperbolic nature.

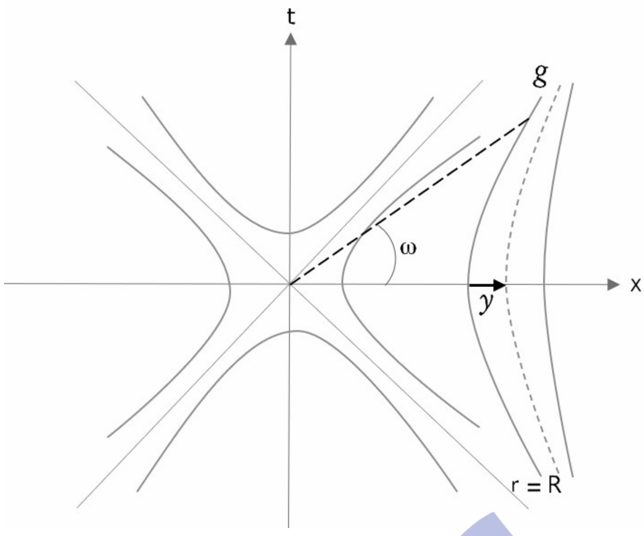


Fig.1.0. Minkowski diagram

As compared to the polar co-ordinate we introduced [1],

$$\text{Cos}\,\theta \xrightarrow{yields} \cosh\omega \qquad \text{, where } \omega = angle\ with\ spatial\ axis$$

$$\text{Sin}\,\theta \xrightarrow{yields} \sinh\omega$$

From co-ordinate transformation,

$$X = r\cosh\omega$$

$$T = r\sinh\omega$$

Here r is the Space-like co-ordinate and ω is the Time-like co-ordinate.

$$X^2 - T^2 = r^2$$

Now for the proper acceleration along such a trajectory of the particle, the acceleration will be, $A = \frac{c^2}{R}$, where c is the speed of light.

We are introducing some arbitrary set of co-ordinates and in that co-ordinate, we write the equation of motion of geodesic. We are going to see the equation of motion of the geodesic looks like a particle falling in a uniform gravitational field. In hyperbolic geometry, in analogue here (see Fig.1.0),

$$ds^2 = r^2 d\omega^2 - dr^2$$

Again we have, $A = \frac{c^2}{R} = g$

$$R = \frac{c^2}{g} \sim \frac{1}{g} \quad (c = 1)$$

making a new co-ordinate by, $r = R + y$ now,

$$\begin{aligned} d\tau^2 &= (R+y)^2 d\omega^2 - dy^2 \\ &= (R^2 + 2Ry + y^2)^2 d\omega^2 - dy^2 \\ &= \left(1 + \frac{2y}{R} + \frac{y^2}{R^2}\right) r^2 d\omega^2 - dy^2 \\ &= \left(1 + \frac{2y}{R}\right) R^2 d\omega^2 - dy^2 \\ &= \left(1 + \frac{2y}{R}\right) dt^2 - dy^2 \qquad where\ R\omega = t \\ &\cong (1 + 2gy)dt^2 - dy^2 \qquad \text{...........(1.a)} \end{aligned}$$

The $2gy$ term is the effective gravitational field.

Now in flat space,

I make that vector to be a unit vector,

Now the tangent vector or the unit Vector along the curve will be,

$$t^m = \frac{dX^m}{ds}$$

'ds' is the distance between two point.

We have, $ds^2 = g_{mn} dX^m dX^n$

Now,

$$D_m V^n = \frac{dV^n}{dX^m} + \Gamma^n_{mr} V^r$$

$$\Rightarrow dV^n + \Gamma_{mr}{}^n V^r dX^m = 0$$

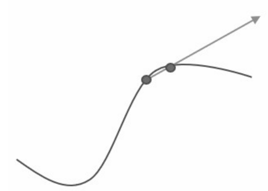


Fig.1.1.

Where, $\Gamma_{mr}{}^n$ *is called Christoffel Symbol*

$$\Gamma^n_{mr} = \frac{1}{2} g^{n\rho} \left(\frac{\partial g_{\rho m}}{\partial x^r} + \frac{\partial g_{\rho r}}{\partial x^m} - \frac{\partial g_{rm}}{\partial x^\rho} \right) \quad \ldots\ldots\ldots\ldots\ldots(1.2.a)$$

Plugging this into previous,

$$\Rightarrow dt^n + \Gamma_{mr}{}^n t^r dX^m = 0$$

$$\Rightarrow \frac{dt^n}{dS} = -\Gamma^n_{mr} t^r \frac{dX^m}{dS}$$

$$= -\Gamma_{mr}{}^n t^r t^m$$

So, as compared to the Fig.1.0.,

$$\frac{d^2y}{d\tau^2} = -\Gamma^y_{mr} \frac{dX^r}{dS} \frac{dX^m}{dS} \quad \ldots\ldots\ldots\ldots..(1.2.b)$$

1.3 Einstein field equation:

Consider a strong arbitrary gravitational field and X is an arbitrary point on that field. We can define a locally inertial coordinate system,

$$g_{\mu\nu}(X) = \eta_{\alpha\beta}$$

Such that, $\left(\frac{\partial g_{\alpha\beta}}{\partial x^\gamma}\right)_{x=X} = 0$, for x→X, $g_{\alpha\beta}$ *and* $\eta_{\alpha\beta}$ is different to each other. As Schwarzschild metric,

$$ds^2 = -\left(1 - \frac{2G\mu}{r}\right) dt^2 + \left(1 - \frac{2G\mu}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

$$\Longrightarrow ds^2 = -\left(1 - \frac{2G\mu}{r}\right) dt^2 + \left(1 - \frac{2G\mu}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\psi^2$$

$\varphi \equiv -\frac{G\mu}{r}$,the time-time component of metric tensor will be, $g_{00} \simeq -(1 + 2\varphi)$, φ is the Newtonian potential. From Poisson equation : $\nabla^2\varphi = 4\pi G\rho$,G is the gravitational constant.

The matter density $T_{00} \simeq$ *mass density* (ρ).

$$\nabla^2 g_{00} = -8\pi G T_{00} \quad \ldots\ldots\ldots\ldots(1.3.a)$$

The above equation is not Lorentz invariant yet, only supposed to hold for weak static fields generated by non-relativistic matter, for general distribution,

$$G_{\alpha\beta} = -8\pi G T_{\alpha\beta}$$

Now we have,

(I) $G_{\alpha\beta}$ is the liner combination of metric and from principle of equivalence for arbitrary strength must be in form of :

$$G_{\mu\nu} = -8\pi G T_{\mu\nu} \quad \ldots\ldots\ldots\ldots(1.3.b)$$

$G_{\mu\nu} \dashrightarrow G_{\alpha\beta}$ for weak fields. To remove the ambiguity in $G_{\mu\nu}$, we shall assume that the gravitational field equation are uniform in scale, so only up to second derivatives of is allowed.

(II) $T_{\mu\nu}$ is symmetric so, $G_{\mu\nu}$ is also symmetric.

(III) $G_{\mu\nu}$ only consists of terms with N=2 derivatives of metric, that is $G_{\mu\nu}$ contains only terms that are either liner in the second derivatives or quadratic in the first derivatives of the metric.

(IV) As $T_{\mu\nu}$ is conserved in equation (1.3.b) so, $G_{\mu\nu;\,\nu} = 0 \quad \Rightarrow g^{\mu\eta} G_{\mu\nu;\,\nu} = 0$

$$\Rightarrow G^\eta{}_{\nu;\,\eta} = 0$$

$$\Rightarrow G^\mu{}_{\nu;\,\mu} = 0$$

(V) By non-relativistic matter the 00 component of equation (1.3.b) must reduced to equation (1.3.a),

In that limit, , $G_{00} \simeq \nabla^2 g_{00}$ $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.3.c)$ Now the Riemann-Christoffel curvature tensor[4] is defined as,

$$R^{\lambda}_{\mu\nu\kappa} = \frac{\partial \Gamma^{\lambda}_{\mu\nu}}{\partial x^{\kappa}} - \frac{\partial \Gamma^{\lambda}_{\mu\kappa}}{\partial x^{\nu}} + \Gamma^{\eta}_{\mu\nu}\Gamma^{\lambda}_{\kappa\eta} - \Gamma^{\eta}_{\mu\kappa}\Gamma^{\lambda}_{\nu\eta}$$

This may be transformed as, $R_{\lambda\mu\nu\kappa} \equiv g_{\lambda\sigma} R^{\sigma}_{\mu\nu\kappa}$. $R_{\lambda\mu\nu\kappa}$ has symmetry and antisymmetric property both. Now I contracted $R_{\lambda\mu\nu\kappa}$ as,

$$R_{\mu\kappa} = g^{\lambda\nu} R_{\lambda\mu\nu\kappa}$$

$R_{\mu\kappa}$ is called Ricci tensor. Now Ricci tensor is always symmetric as, $R_{\mu\kappa} = R_{\kappa\mu}$. and the curvature scalar $R = R^{\mu}{}_{\mu}$.

A hidden assumption is, it is always possible to transfer co-ordinates into local cartesian co-ordinate system. (by equivalence principle)

After doing some calculations and using Bianchi identity[4] we can get this form,

$$R_{;\eta} - R^{\mu}{}_{\eta;\mu} + R^{\nu}{}_{\eta;\nu} = 0 \quad \text{.........(1.3.d)}$$

Or, $$(R^{\mu}{}_{\eta} - \tfrac{1}{2}\delta^{\mu}{}_{\mu} R)_{;\mu} = 0 \quad \text{.........(1.3.e)}$$

Or, $$(R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R)_{;\mu} = 0 \quad \text{.........(1.3.e.1)}$$

As by statement (III) $G_{\mu\nu}$ can be written as,

$$G_{\mu\nu} = C_1 R_{\mu\nu} + C_2 g_{\mu\nu} R \quad \text{............(1.3.f)}$$

From equation (1.3.f),

$$\Rightarrow g^{\mu\eta} G_{\mu\nu} = C_1 g^{\mu\eta} R_{\mu\nu} + C_2 g^{\mu\eta} g_{\mu\nu} R$$

$$\Rightarrow G^{\eta}{}_{\nu;\mu} = C_1 R^{\eta}{}_{\mu;\nu} + C_2 g^{\mu\eta} g_{\mu\nu} R_{;\mu}$$

$$\Rightarrow G^{\eta}{}_{\nu;\mu} = C_1 R^{\nu}{}_{\nu;\mu} + C_2 g^{\mu\nu} g_{\nu\mu} R_{;\nu}$$

$$\Rightarrow G^{\eta}{}_{\nu;\mu} = \frac{C_1}{2} \delta^{\nu}{}_{\mu} R_{;\nu} + C_2 \delta^{\nu}{}_{\mu} R_{;\nu} \text{ \{using eqn. (1.3.d)\& (1.3.e)\}}$$

$$\Rightarrow G^{\eta}{}_{\nu;\mu} = (\frac{C_1}{2} + C_2)\, R_{;\nu}$$

$$\Rightarrow \quad 0 = (\frac{C_1}{2} + C_2)\, R_{;\nu}$$

That is either $\frac{C_1}{2} = -C_2$ or, $R_{;\nu}$ vanishes.

If, $R_{;\nu}$ is not vanishes then,

$$G^{\mu}{}_{\nu} = \left(\frac{C_1}{2} + C_2\right) R = -8\pi G T^{\mu}{}_{\nu}$$

As $\mu = \nu$:

$$G^{\mu}{}_{\mu} = \left(\frac{C_1}{2} + C_2\right) R = -8\pi G T^{\mu}{}_{\mu}$$

If, $R_{;\nu} \equiv \frac{\partial R}{\partial x^{\nu}}$ vanishes then we can write in right hand side $\frac{\partial T^{\mu}{}_{\mu}}{\partial x^{\nu}} = 0$, but this is not the case in the presence of inhomogeneous non relativistic matter.

So, $R_{;\nu} \neq 0$

As eqn. (1.3.f) continues, $\Rightarrow G_{\mu\nu} = C_1 (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R)$ ....(1.3.g)

Now from principle of equivalence for weak stationary field $G_{\mu\nu} \rightarrow G_{00}$ & $G_{00} \simeq \nabla^2 g_{00}$

Since we always talking about non relativistic case so $|T_{\alpha\beta}| \ll |T_{00}|$, $|G_{\alpha\beta}| \ll |G_{00}|$

And we can show that, $R \simeq 2R_{00}$ and yet we can find $G_{00} \simeq 2C_1 R_{00}$ .We take liner part of $R_{\lambda\mu\nu\kappa}$ [4] to calculate R_{00},

$$R_{\lambda\mu\nu\kappa} = \frac{1}{2}\left[\frac{\partial^2 g_{\lambda\nu}}{\partial x^{\kappa}\partial x^{\mu}} - \frac{\partial^2 g_{\mu\nu}}{\partial x^{\kappa}\partial x^{\lambda}} - \frac{\partial^2 g_{\lambda\kappa}}{\partial x^{\nu}\partial x^{\mu}} + \frac{\partial^2 g_{\mu\kappa}}{\partial x^{\nu}\partial x^{\lambda}}\right]$$

For static field all time derivatives vanish, so we will get $R_{0000} \simeq 0$ & $R_{i0j0} \simeq \frac{1}{2}\frac{\partial^2 g_{00}}{\partial x^i \partial x^j}$

Now, we can write by some calculations, $G_{00} \simeq 2C_1 \nabla^2 g_{00}$ as R_{i0i0} (at i=j) only contains second derivatives of g_{00} .

We can write, $C_1 = 1$

So, $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G T_{\mu\nu}$ ....(1.3.h)

Now as statement (III) tells that $G_{\mu\nu}$ contains terms with fewer then two derivatives of metric, I want to see if there will any possibility to add new terms to previous equation.

As first derivative of $G_{\mu\nu}$ are vanishes, $G_{\mu\nu;\nu} = 0$ as per statement (IV). So we can't use this.

Instead of that one can use metric tensor itself, then the possible new term will be $\Lambda g_{\mu\nu}$, Λ is a constant. As equation (1.3.e.1) tells us that, $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = constant$

$=\Lambda g_{\mu\nu}$

Rewriting equation (1.3.h), $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}$ ………..(1.3.i)

For empty space, $R_{\mu\nu} = 0$ and Λ is called Cosmological constant.

1.4 Quantum gravitational approach to contribution of matter density to the fluctuating metric & a solution to the Cosmological Constant:

Observation over the years in cosmology [2] it will be seen that Universe has a homogenous energy density that leads the expansion of universe or rather accelerating universe. This energy density is related to Cosmological constant. From some latest astrophysical data from type I supernovae [5]-[6], cosmic microwave background (CMB) [7], matter density [8], gravitational lensing [9], it is emulated that only small cosmological constant would be only possible as compatible to this experiment.

Now the problem will arise in association with the vacuum energy in Quantum field theory of the matter in the gravitational field. At any scale at which the quantum field theory considered the density is too large by enormous order of magnitude as compared to the practical value. As an example [10],[3]:

In Electroweak scale vacuum density in the order of $\rho_v^{EW} \approx (200\ GeV)^4$ as compared to experimental value $\rho_\Lambda \approx (10^{-12}\ GeV)^4$.

Now I considered that cosmological constant is associated with the vacuum energy density of particle physics model. Using Euclidean quantum gravity of path integral formalism and use of Robertson Walker metric(RW) I calculate the contribution of matter density to the fluctuating metric. This method rather leads to the observed cosmological constant without any need of fine tuning in the gravity and the particle physics model.

The Einstein action is written as, $S = \int \sqrt{-g}[\frac{1}{2k}(R - 2\Lambda) + \mathcal{L}_M]d^4x$ where, $\mathcal{L}_M$= Lagrangian of matter. Now the Robertson Walker (RW) metric will be:

$$ds^2 = dt^2 - R^2(t)\left[\frac{1}{(1-kr^2)} + r^2(d\theta^2 + sin^2\theta)d\Phi^2\right]$$,where $R(t)$ is scalar factor

For k = 1,0,-1 means closed, open and flat geometrical spaces. $g \equiv detg_{\mu\nu}$ and $\kappa = 8\pi G c^{-4}$

Now the Hubble parameter is $H \equiv \frac{\dot{R}}{R}$ by looking at the Freidmann equation, we want to calculate the matter density parameter,

$$(\frac{\dot{R}}{R})^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2 = -\frac{kc^2}{R^2}$$
$$H^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2 = -\frac{kc^2}{R^2}$$

If the universe is flat then k =0 or if it has a critical density $\rho_{crit} = \frac{3H^2}{8\pi G}$ as Λ chosen to be zero. But this is not the density that the universe actually is, it would have to it if it was spatially flat by matter alone. I define the matter density parameter to describe the actual density of the universe relative to this critical density , $\Omega_M = \frac{\rho}{\rho_{crit}}$ for, $\Omega_M = 1$ the universe is spatially flat.

Expressing a similar kind of density parameter using Freidmann equation and take $\Omega_M = 0$, this gives

As for $\rho = 0, H^2 - \frac{1}{3}\Lambda_{cric}c^2 = 0 \Rightarrow \Lambda_{cric} = \frac{3H^2}{c^2}$

$\therefore$ dark energy density parameter, $\Omega_\Lambda = \frac{\Lambda}{\Lambda_{cric}}$

$$= \frac{\Lambda c^2}{3H^2}$$

Total, density parameter $\Omega = \Omega_c + \Omega_b + \Omega_\Lambda$, as $\Omega_c + \Omega_b \approx \Omega_M$

Now the rate of change of acceleration of universe expansion: $a = -R\frac{\ddot{R}}{\dot{R}} = \frac{\Omega_{\mathrm{M}}}{2} - \Omega_{\Lambda}$

Now as per astronomical data, the value of $\Omega_{\Lambda} = 0.694 \pm 0.007$ as k=0.

This corresponds to the value of $\Lambda \approx 4.33 \times 10^{-66} eV^2$.

	Planck TT+lowP+lensing	*Planck* TT+lowP+lensing+ext
$\Omega_{\mathrm{b}}h^2$	0.02226 ± 0.00023	0.02227 ± 0.00020
$\Omega_{\mathrm{c}}h^2$	0.1186 ± 0.0020	0.1184 ± 0.0012
$100\,\theta_{\mathrm{MC}}$	1.0410 ± 0.0005	1.0411 ± 0.0004
n_{s}	0.968 ± 0.006	0.968 ± 0.004
τ	0.066 ± 0.016	0.067 ± 0.013
$\ln(10^{10}\Delta^2_{\mathcal{R}})$	3.062 ± 0.029	3.064 ± 0.024
h	0.678 ± 0.009	0.679 ± 0.006
σ_8	0.815 ± 0.009	0.815 ± 0.009
Ω_{m}	0.308 ± 0.012	0.306 ± 0.007
Ω_{Λ}	0.692 ± 0.012	0.694 ± 0.007

Parameters constraints values taken from Review of Particle Physics group*[11] D 98, 030001(2018)

Now, I consider Euclidean gravity in terms of RW metric $g_{\mu\nu} \to t_{\mu\nu} + \partial g_{\mu\nu}$, where $t_{\mu\nu}$is the RW metric and $\partial g_{\mu\nu}$ is the fluctuating metric. Expanding $\sqrt{-g}$ yields,

$$\sqrt{-\det g_{\mu\nu}} = \sqrt{-\det \partial g_{\mu\nu}}\,[1 + \tfrac{1}{2}\partial g^{\rho\sigma} t_{\sigma\rho} - \tfrac{1}{4}\partial g^{\rho\sigma} t_{\sigma\alpha}\partial g^{\alpha\beta} t_{\beta\rho} + \tfrac{1}{8}\partial g^{\rho\sigma} t_{\sigma\alpha}\partial g^{\alpha\beta} t_{\beta\rho} + \ldots]$$

Now the gravitational partition function as follows:

$$\mathcal{Z}_g = \int dt_{\mu\nu}\, \xi(t_{\mu\nu}) e^{[-\int \sqrt{-g}\frac{1}{2k}R(g)d^4x]} \times e^{[-\int \rho d^4x \sqrt{-\det \partial g_{\mu\nu}}[1+\frac{1}{2}\partial g^{\rho\sigma}t_{\sigma\rho} - \frac{1}{4}\partial g^{\rho\sigma}t_{\sigma\alpha}\partial g^{\alpha\beta}t_{\beta\rho} + \frac{1}{8}\partial g^{\rho\sigma}t_{\sigma\alpha}\partial g^{\alpha\beta}t_{\beta\rho} + \cdots]]}$$

$$\Rightarrow \mathcal{Z}_g = \int dt_{\mu\nu}\, \xi(t_{\mu\nu}) e^{[-\int \sqrt{-\partial g}\frac{1}{2k}R(\partial g)d^4x]} \times [\,correction\ terms\ of\ R(\partial g)] \times e^{[-\int \rho d^4x \sqrt{-\det \partial g_{\mu\nu}}[1+\frac{1}{2}\partial g^{\rho\sigma}t_{\sigma\rho} - \frac{1}{4}\partial g^{\rho\sigma}t_{\sigma\alpha}\partial g^{\alpha\beta}t_{\beta\rho} + \frac{1}{8}\partial g^{\rho\sigma}t_{\sigma\alpha}\partial g^{\alpha\beta}t_{\beta\rho} + \cdots]]}$$

$\xi(t_{\mu\nu})$ is a measure of path integral formalism.

For zeroth order contribution this term may be omitted.

$$\Rightarrow \mathcal{Z}_g = \int dt_{\mu\nu}\, e^{[-\int \sqrt{-\partial g}\frac{1}{2k}R(\partial g)d^4x]} \times [\,correction\ terms\ of\ R(\partial g)] \times e^{[-\int \rho d^4x \sqrt{-\det \partial g_{\mu\nu}}[1+\frac{1}{2}\partial g^{\rho\sigma}t_{\sigma\rho} - \frac{1}{4}\partial g^{\rho\sigma}t_{\sigma\alpha}\partial g^{\alpha\beta}t_{\beta\rho} + \frac{1}{8}\partial g^{\rho\sigma}t_{\sigma\alpha}\partial g^{\alpha\beta}t_{\beta\rho} + \cdots]]}$$

I integrate over the all degrees of freedom then we will get,

$$\mathcal{Z}_g \approx e^{[-\int \sqrt{-\partial g}\frac{1}{2k}R(\partial g)d^4x + \int \rho d^4x]} \times \det\left[\left\{-\frac{1}{4}\partial g^{\rho\sigma}t_{\sigma\alpha}\partial g^{\alpha\beta}t_{\beta\rho} + \frac{1}{8}\partial g^{\rho\sigma}t_{\sigma\alpha}\partial g^{\alpha\beta}t_{\beta\rho}\right\}(-\rho)\right]^{-\frac{1}{2}}$$

I did not extract the factor $\sqrt{-\det \partial g_{\mu\nu}}$ as this is the property of the curved lattice one may use for this space. The contribution of ρ in the right side of the equation,

Consider only the determinant part containing ρ,

$$[\det \rho]^{-1/2} = e^{[Trace[-\frac{1}{2}ln\rho]]} = e^{[-\sqrt{-\partial g}\frac{16}{2}\int \frac{ln\rho}{(2\pi)^4}d^4p]}$$

As the two indices tensor has 16 components and trace is considered over curved space also all there are 16 degrees of freedom of the tensor $t_{\mu\nu}$. p suggests for pressure in rest frame. Combining previous two equations we get,

$$\mathcal{Z}_g = e^{[-d^4x\sqrt{-\partial g}[\frac{1}{2k}R(\partial g) + \rho + 8\int \frac{ln\rho}{(2\pi)^4}d^4p]}$$

Now the Einstein field equations are now,

$$R_{\mu\nu} - \frac{1}{2}\partial g_{\mu\nu}R + \Lambda \partial g_{\mu\nu} = -8\pi G T_{\mu\nu}$$

So, $\Lambda = \frac{8\pi G}{c^4}\left[\rho + 8\int \frac{ln\rho}{(2\pi)^4} d^4p\right]$

At take $\mathcal{M}_P^2 = \frac{hc}{2\pi G} \simeq 1.488 \times 10^{38}(GeV)^2$

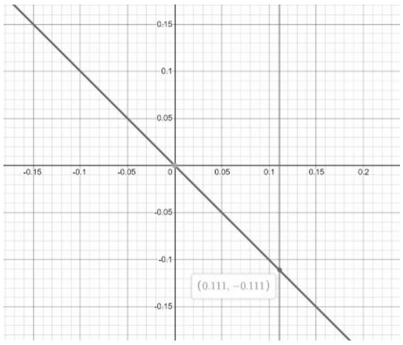
I introduce $\mathcal{M}$ as a ultraviolet scale of QFT theoretical approach. taking $\rho \approx \omega\mathcal{M}$ where ω is a dimensionless constant. ω should not be fine-tuned.

This leads to, $\Lambda = \frac{8\pi\mathcal{M}^4}{\mathcal{M}_P{}^2}\left[\omega + \frac{ln\omega}{2\pi^2}\right]$

Take, Planck scale as $\mathcal{M} = \mathcal{M}_P = \sqrt{1.488 \times 10^{38}}\, GeV$

And $\Lambda \approx 4.33 \times 10^{-66} eV^2$, $\omega^{1/4} \approx 0.577$.

Now again take, $\mathcal{M} = 200$ GeV and we get $\omega \approx 0.1112$ by solving graphically. This shows that the value is not tuned so, it does not matter what scale either its ultraviolet scale or the actual value of the quantum field theory vacuum this kind of approach leads to correct value of cosmological constant. The approach may also be done in different scale.



[1] Lecture on Gravitation and Cosmology by L. Susskind, Stanford Univ. (2012) [2] P.J. Peebles, Astrophysics, J.**284,** 439 (1984).
[3] S.M. Carroll, Living Rev. rel. 4,1 (2001), arXiv:astro-ph/0004075.
[4] Gravitation and Cosmology: Principle and application to the general relativity, Weinberg.S,1972 pub.
[5] A.G. Riess et al.[Hi-Z Supernova Team collaboration] Astrophysics journal.**426**, 57 (1994).
[6] P.M. Garnavich et al. .[Hi-Z Supernova Team collaboration] Astrophysics journal.**509**, L74 (1998).
[7] W.Hu and M.White, astro-ph/9606140 (1996).
[8] S. Burles et al., Phys. Rev. Lett. **82,** 4176 (1999).
[9] M.S. Turner, astro-ph/9703161 (1997).
[10] R. Jora arXiv:9779(2020).
[11] *Review of Particle Physics group[11] D 98, 030001(2018)