STA 9701: Time Series Analysis

Final Project

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Analysis of the NY Region Housing Market Using ARMAX

Final Project 9701 – Time Series Fall 2010

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Introduction

Within the past two years the country has seen a dramatic failure of analysts to predict housing prices on a broad scale. As one of the largest segments of the US economy, with trillions of dollars of value in securities, effective analysis of the housing market is a goal of thousands of investors.

Our goals are two-fold. To learn about ARMAX and examine how it may be applied to this significant real world problem, and to evaluate the effectiveness of our model during the dramatic turnaround in prices of 2008.

In order to demonstrate the implementation of the ARMAX model, we first provide a short overview of the algorithmic process. Following the ARMAX overview, we use a replicated analysis on a data set taken from *Time Series Analysis and Its Applications* by Robert Shumway and David Stoffer. Lastly, we perform a similar analysis on the data set of interest, and develop a model for PPSF values in the New York MSA.

Data

For housing price data the team has selected the time series developed by Radar Logic LTD. Radar Logic produces a series of indices that track the price-per-square-foot value of housing prices in 25 different Metropolitan Statistical Areas, as well as a national index, which is a weighted composite of the MSA level indices.

We chose to use this data because it is not adjusted in retrospect and represents the raw spot price of real estate in the respective markets. Furthermore, the data was very transparent, in that it amalgamates all residential home closings, and publishes the median PPSF on a daily, weekly and monthly basis. Since the data was not created using any ‘black box’ methods, fluctuations and volatility in the market are evident.

In order to aid in the modeling of housing prices, we have selected two exogenous variables: the regional unemployment rate and the 30 year mortgage rate. Both time series are monthly and freely available from the Federal Reserve Bank of St Louis’ Federal Reserve Economic Data database (registration required) at research.stlouisfed.org.

Introduction To The ARMAX Model

Most people describe ARMAX as modeling time-series with an exogenous variable. In simpler language: modeling a time series when the researcher has data for more than one variable. However, a quick scan of the web will reveal that there are several conflicting interpretations of what that statement means. The most compelling of these is the exposition by Shumway and Stoffer[1]. They present several models of data analysis of time series when there are several time series in the process.

The first model Shumway and Stoffer present is very similar to a regression model for time series. The model equation takes the form:

$$Y_t = \beta_0 + \beta_1 t + \sum_{i=1}^{n} X_{it} + \varepsilon_t \qquad \{\varepsilon_t\} \sim WN(0, \sigma^2)$$

In this model, Y is the response variable, and there are n predictors, each with t measurement points. Finally, time is added as a component that is regressed against, as if time was just another linear prediction variable.

A second model that is more in keeping with ARMA modeling is Vector Autoregression. A working conceptual definition could be multivariate regression of time series. The equation to represent this model with i=1,…,t and j=1,...,k is:

$$\begin{bmatrix} y_{t1} \\ \vdots \\ y_{tk} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_k \end{bmatrix} + \sum_{i=1}^{n} \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_k \end{bmatrix} \begin{bmatrix} y_{t-i,1} \\ \vdots \\ y_{t-i,k} \end{bmatrix} + \begin{bmatrix} \omega_{t1} \\ \vdots \\ \omega_{tk} \end{bmatrix} \qquad \{\omega_{ij}\} \sim WN(0, \sigma_{tk}^2)$$

Or in matrix notation: $\mathbf{Y}_t = \boldsymbol{\alpha} + \Sigma\phi_i\mathbf{Y}_{ti} + \boldsymbol{\omega}_t$. In this model we assume $\mathrm{cov}(\omega_{si},\omega_{tj}) = \sigma_{ij}$ for all s=t or 0 otherwise. However, again this model is not quite what is desired, as the representation is of all three variables as if they were of equal importance when forecasting. Our goal is to have a single response variable and multiple predictors working in conjunction with the usual ARMA machinery.

The ARMAX model as defined by Shumway and Stoffer may also be referred to as VARMAX in the literature. For the full model there is a k dimensional set of response variables with an r dimensional vector of inputs represented as:

$$\mathbf{y}_t = \boldsymbol{\Gamma}\mathbf{u}_t + \sum_{j=1}^{p} \Phi_j \, \mathbf{y}_{t-j} + \sum_{j=1}^{q} \Theta_j \, \boldsymbol{\varepsilon}_{t-j} + \boldsymbol{\varepsilon}_t \qquad \text{where } \{\varepsilon_t\} \sim WN(0,\sigma^2)$$

Where $\mathbf{y}_t$ is a vector of the response variables at time t, Φ and Θ are ARMA coefficients for each of the response

variables, **u** is a vector made up of exogenous variables that are used to model and trend in the process and $\boldsymbol{\Gamma}$ is a k×r matrix representing the regression coefficients of the exogenous terms.

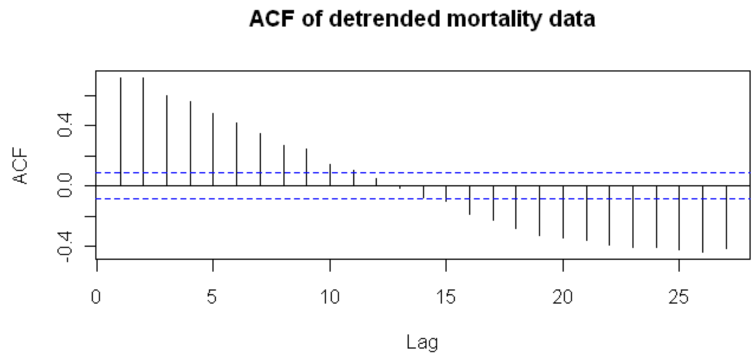
[1] Time Series Analysis and Its Applications With R Examples, Second Edition by Robert Shumway and David Stoffer http://www.stat.pitt.edu/stoffer/tsa2/

Replicating The Results of Shumway and Stoffer: An Example Of The Implementation Process

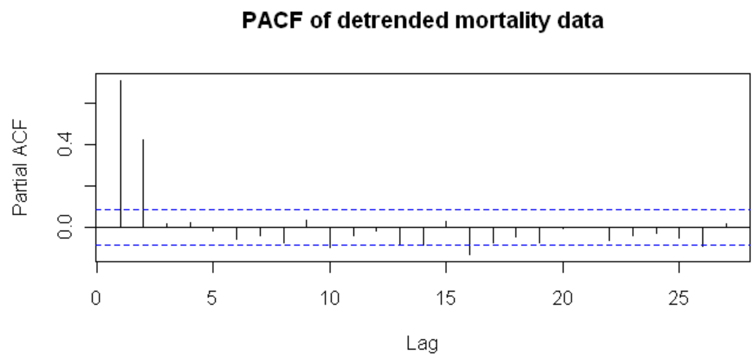
ARMAX is a complicated procedure for which there is no "push one button" function that runs through the whole analysis. Consequently, prior to analyzing new data sets the team attempted to replicate the results of Shumway and Stoffer. Unfortunately, the ARMAX example in the second edition is one of the examples that lack accompanying R code.

The following five steps are an overview of the process:

- Center and detrend the response variable.
- Fit an ARMA model to the response variable, this step is known as pre-whitening.
- Determine candidate exogenous variables that have leading lag correlation with the pre-whitened residuals.
- Fit a stepwise regression model against the pre-whitened data to determine included exogenous variables.
- Fit a regression model to the original data set determining values for the following parameters: the ARMA model ϕ_i and θ_i, as well as appropriately lagged series for all the significant lags of the exogenous variable. Notice that the estimates for ϕ_i and θ_i will change as a result of this step, generating estimates that accommodate the additional exogenous time series. Also, usual regression model fitting applies and some of the variables may be sources of the same information and consequently be dropped during the model evaluation stage.



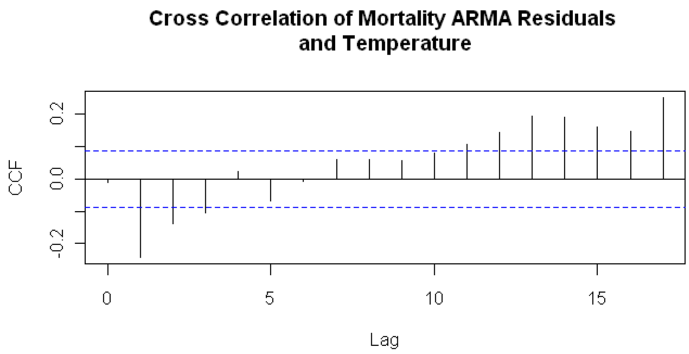
The first step is to load and detrend the data for the response variable. This is accomplished running a regression on the time series against time and storing the residuals.

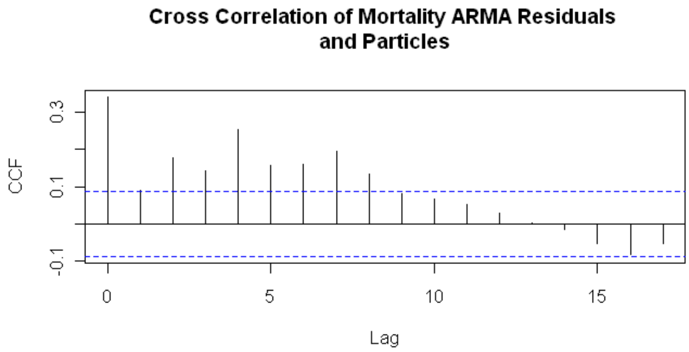


The second step is to fit an ARMA model to the resulting time series. As the ACF and PACF plots displayed to the right show the series is clearly an AR(2) process. This result is also confirmed by both BIC and EACF methods of order identification.

Once the order of the ARMA model has been identified, the next step is to determine parameter estimates for the variable. Running the arma function in R, and confirming with the PROC TIMESERIES in SAS, resulted in a pre-whitening model of: $\hat{Y}_t = .41097Y_{t-1} + .42310Y_{t-2} + \varepsilon_t$. The residuals of the model fit, ε_t, are the pre-whitened data used in the next step and $\hat{Y}_t$ is the fitted estimate for the centered and detrended mortality series at time t and $\{\varepsilon_t\}$ ~WN(0,σ^2). As mentioned above, these parameter estimates do not match the final $\hat{\phi}_1$ and $\hat{\phi}_2$ of the text book example. The purpose of this step is to generate the residuals against which the Cross Correlations will be determined.

The output of the next step, reviewing the cross correlation plots of the residuals from the ARMA model fitting, matches the text book results perfectly.





The technique to evaluate the CCF plots on the right is very similar to the analysis of the ACF and PACF. We are looking for correlations outside the bands, with more weight given to those correlations with a lag closer to zero. In this example lags of one and two were identified as candidate series for temperature and lags of zero, two, four, and seven were identified as candidates for level of particles in the air.

The penultimate step in the process is to use stepwise selection in R to make final selection of exogenous variables. A reduced model that matches the model fit in Shumway and Stoffer was selected with a lag one Temperature term and Particle terms of zero and four.

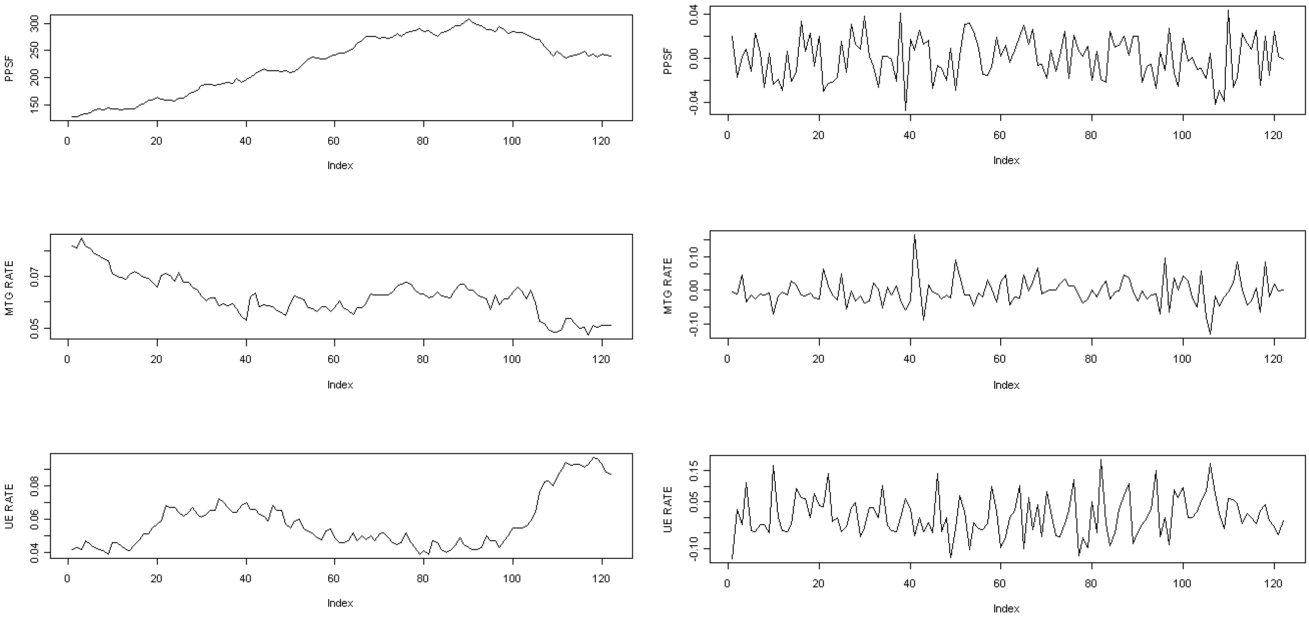
The final stage of ARMAX is to fit a regression using the selected exogenous variables along with the parameters of the pre-whitened ARMA model. This resulted in a mean function of:

$$\hat{M}_t = 42.9 - .01t - .18T_{t-1} + .11P_t + .05P_{t-4} + .31M_{t-1} + .30M_{t-2} + \hat{\omega}_t$$

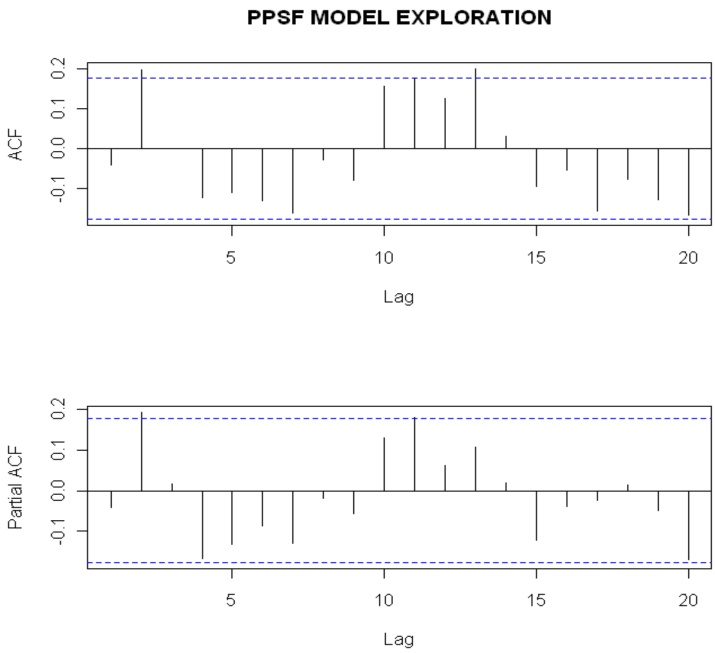
The adjusted R^2 for this function is 74.3%. These results perfectly match those of the text, and providing evidence that we are successfully replicating the ARMAX method as defined by Shumway and Stoffer.

Modeling Price-Per-Square-Foot of Real Estate in the New York MSA Using ARMAX

Time Series Plots: Pre-Transformation v. Transformation



In order to remove the trend from the time series, we first took the log difference of price-per-square-foot(PPSF) in order to break the correlations between observations and create a stable variance. Once that was complete, classical decomposition was employed to remove the linear trend.



Once PPSF, our dependent variable, was properly transformed to stationarity and the trend was removed, we transformed our independent variables, unemployment rate and the Freddie Mac northeastern average monthly mortgage rate, by taking the log difference. Improvement in the stability of the time series is evident by comparing the before and after plots above.

After pre-whitening our variables, we moved on to model exploration. When considering which model to begin with, it was evident from the ACF plots that an MA(2) model may be a sensible choice. In addition, the PACF reveals that an AR(2) could also be a good choice for this model. Clearly, additional exploration into the possibility of an ARMA model fit was necessary at this point.

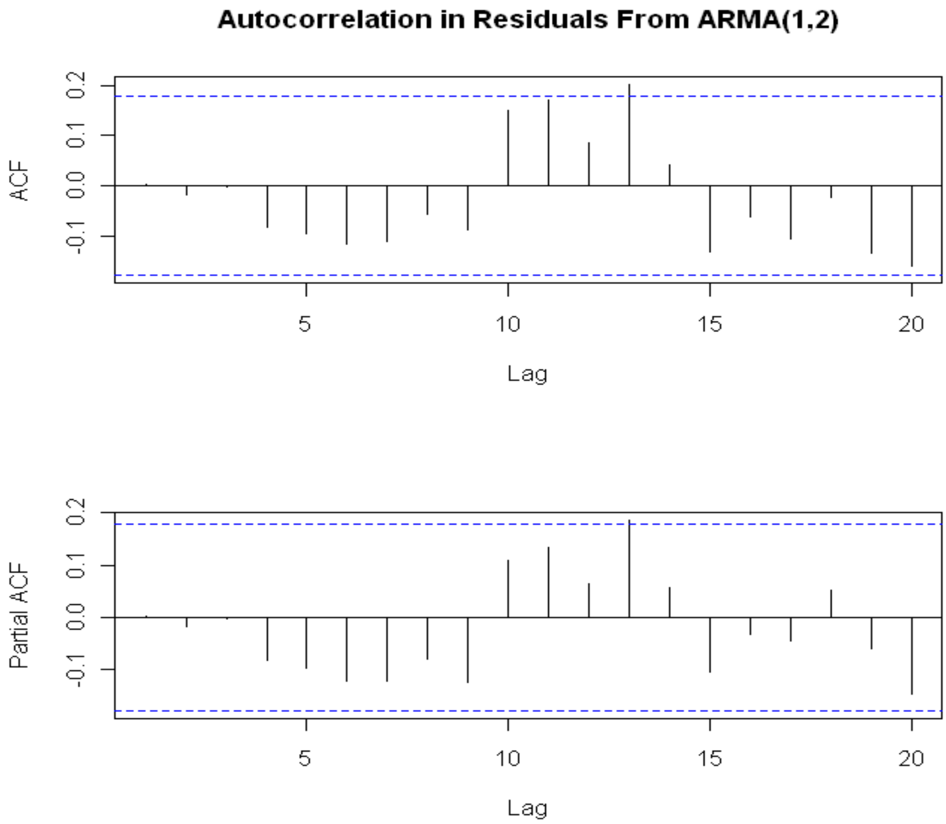
EACF Plot And AIC Criterion

AR \ MA	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	o	x	o	o	o	o	o	o	o	o	o	o	x	o
1	x	x	o	o	o	o	o	o	o	o	o	o	x	o
2	o	x	o	o	o	o	o	o	o	o	o	o	o	o
3	o	x	o	o	o	o	o	o	o	o	o	o	o	o
4	x	o	o	o	o	o	o	o	o	o	o	o	o	o
5	x	o	o	x	o	o	o	o	o	o	o	o	o	o
6	x	x	o	o	o	o	o	o	o	o	o	o	o	o
7	o	x	x	o	o	o	x	o	o	o	o	o	o	o

The EACF plot above shows that a variety of models may be appropriate for the PPSF time series. Specifically, we felt that ARMA(1,1), ARMA(2,2), ARMA(2,1) or ARMA(1,2) models warranted additional scrutiny.

Model	AIC Criterion
ARMA(1,1)	-610.2652
ARMA(1,2)	-613.155
ARMA(2,1)	-611.3828
ARMA(2,2)	-611.3895

In order to further evaluate the appropriateness of a specific ARMA model, we ran four different model fits and then evaluated the AIC for each of the models. In the end, the ARMA(1,2) model had the lowest AIC and was decided on as the most appropriate model. Although, given the closeness of values, any of the models would have been suitable as selections.

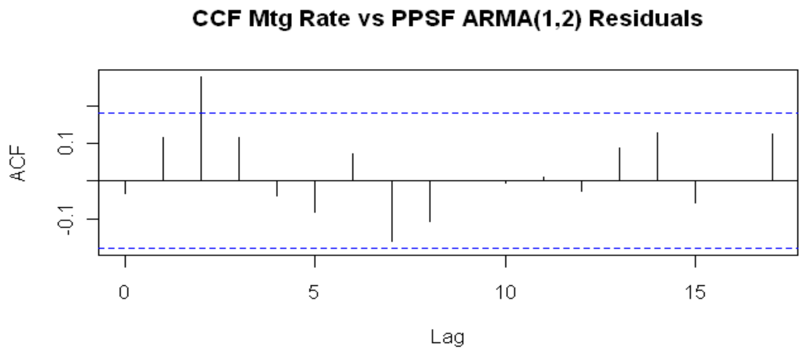


Plotting the ACF and PACF of the residuals of the fitted ARMA(1,2) model, we see that there is no significant autocorrelation in our residuals, which provides additional evidence to our model choice decision.

After the model for the dependent series had been validated, our next step was to look at the cross-correlation matrix of the independent variables to see if they would be effective in the ARMAX model.

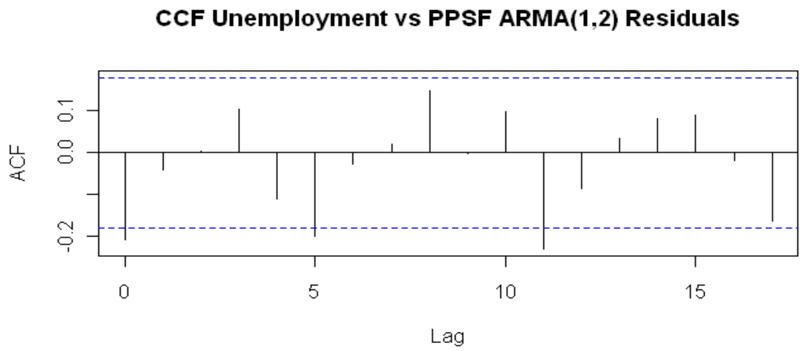
To do this, we looked at the cross-correlation plots of unemployment rate and mortgage rates vs. our PPSF dependent variable.

Evaluating our cross-correlation plot for mortgage rate vs PPSF we see that there is significant correlation at lag 2.



The cross-correlation plot of unemployment vs PPSF ends up having more correlations, which occur at lags 0, 5 and 11.

As an additional measure, we ran a stepwise selection, which chose very similar results of lag 2 for mortgage rate vs PPSF and lag 0 and 11 for unemployment vs PPSF. In response, we eliminated the lag 5 term for sake of simplicity.



Lastly, we introduced the appropriate lags for the two exogenous variables then moved on to the final model fitting procedure.

ARMA Model Summary

Coefficients:	ar1	ma1	ma2	intercept
	-0.0302	-0.0092	0.254	0.0003
s.e.	0.3220	0.3066	0.098	0.0022

sigma^2 estimated as 0.0003648: log likelihood = 281.78, aic = -555.55

ARMAX Model Summary

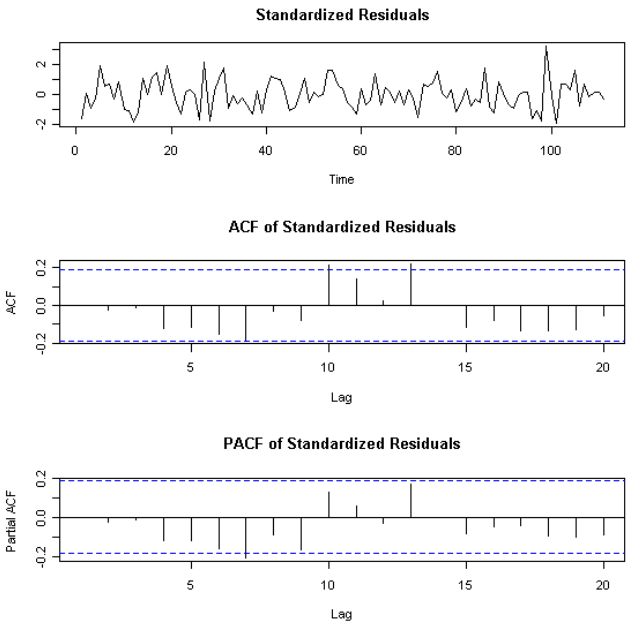
Coefficients:	ar1	ma1	ma2	intercept	NE_MTG_Rate.ld.lag2	UE_RATE.ld.lag0	UE_RATE.ld.lag11
	0.0093	-0.1286	0.307	0.0014	0.1433	-0.0605	-0.0373
s.e.	0.2823	0.2628	0.099	0.0020	0.0384	0.0241	0.0230

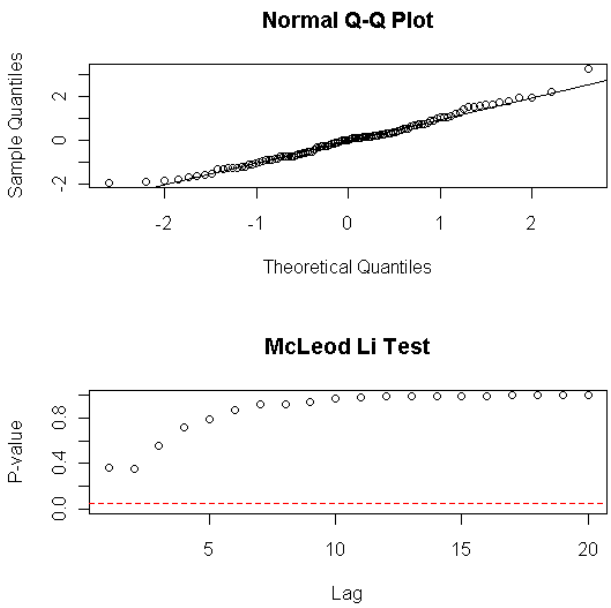
sigma^2 estimated as 0.0003010: log likelihood = 292.42, aic = -570.84

We can now compare our ARMA model to our ARMAX model. We see that the ARMAX model has a smaller AIC Criterion than the ARMA model as well as a smaller variance function. We also notice a smaller standard error around the ARMA process terms within the ARMAX model, when compared to the ARMA model fit.

From the above tables the mortgage rate at lag 2 has a positive association with price per square foot and unemployment at lags 0 and 11 have a negative association.

As a final check several diagnostic checks were run on the residuals. As the diagrams below show the acf and pacf appear to be white noise with an acceptable number of auto-correlations more than two standard deviations from 0. The QQ plot is not perfectly normal, however, the Shapiro-Wilk test for normality did not reject the null hypothesis that the residuals are normal. Similarly the McLeod-Li test for conditional heteroskedasticity did not reject the null. However, the Box-Ljung test did reject the null hypothesis when testing against a lag greater than 12. This suggests that further analysis is warranted.





In conclusion, the ARMAX method is a viable alternative to a standard ARMA model, and its implementation is worth exploring, especially when analyzing data sets that are influenced by exogenous variables.

Appendix R Code To Replicate Shumway and Stoffer Example 5.12

```
# lm result matches textbook page 319
require(TSA)
require(tseries)

options(width=128)

#######
## Change the filedir!!!
#######
filedir <- "c:/data/sta9701/"
filedir <- "E:/Data/"

mort = scan(paste(filedir,"cmort.dat",sep=""))
temp = scan(paste(filedir,"temp.dat",sep=""))
part = scan(paste(filedir,"part.dat",sep=""))

# create detrend series
dmort = resid(lm(mort~time(mort)))
dtemp = resid(lm(temp~time(temp)))
dpart = resid(lm(part~time(part)))
data <- data.frame(t=1:NROW(mort),mort,temp,part,dmort,dtemp,dpart)

# look at the original data series
par(mfrow=c(3,1))
plot(data$mort,type="l")
plot(data$temp,type="l")
plot(data$part,type="l")

# look at the detrended data series
plot(data$dmort,type="l")
plot(data$dtemp,type="l")
plot(data$dpart,type="l")

# preliminary ARMA order exploration
par(mfrow=c(2,1))
acf(data$dmort,na.action=na.remove, main="ACF of detrended mortality data")
pacf(data$dmort,na.action=na.remove,main="PACF of detrended mortality data")

# model#3 - ARMA(2,0) has lowest AIC
m0 <- eacf(data$dmort)

print(m3 <- arma(data$dmort,c(2,0)))

# no significant autocorrelation in residuals
data <- transform(data,dmort.pw=residuals(m3))
acf(data$dmort.pw,na.action=na.remove)
pacf(data$dmort.pw,na.action=na.remove)

# dtemp is significant at a lag of 1
# dpart is significant at a lag of 0,2,4,7
```

```
ccftmp <- ccf(data$dmort.pw,data$dtemp,na.action=na.remove)
ccfpar <- ccf(data$dmort.pw,data$dpart,na.action=na.remove)

plot(ccftmp[0:17],main="Cross Correlation of Mortality ARMA Residuals \nand Temperature", ylab="CCF")
plot(ccfpar[0:17],main="Cross Correlation of Mortality ARMA Residuals \nand Particles", ylab="CCF")

## CCF plots match the book! :)

# add raw lags to the data set
data <- transform(data,mort.lag1=c(rep(NA,1),embed(mort,2)[,2]))
data <- transform(data,mort.lag2=c(rep(NA,2),embed(mort,3)[,3]))
data <- transform(data,temp.lag1=c(rep(NA,1),embed(temp,2)[,2]))
data <- transform(data,temp2.lag1=temp.lag1^2)
data <- transform(data,part.lag0=part)
data <- transform(data,part.lag2=c(rep(NA,2),embed(part,3)[,3]))
data <- transform(data,part.lag4=c(rep(NA,4),embed(part,5)[,5]))
data <- transform(data,part.lag7=c(rep(NA,7),embed(part,8)[,8]))

# add diff lags to the data set
data <- transform(data,dmort.lag1=c(rep(NA,1),embed(dmort,2)[,2]))
data <- transform(data,dmort.lag2=c(rep(NA,2),embed(dmort,3)[,3]))
data <- transform(data,dtemp.lag1=c(rep(NA,1),embed(dtemp,2)[,2]))
data <- transform(data,dtemp2.lag1=dtemp.lag1^2)
data <- transform(data,dpart.lag0=dpart)
data <- transform(data,dpart.lag2=c(rep(NA,2),embed(dpart,3)[,3]))
data <- transform(data,dpart.lag4=c(rep(NA,4),embed(dpart,5)[,5]))
data <- transform(data,dpart.lag7=c(rep(NA,7),embed(dpart,8)[,8]))

# stepwise selection chooses dtemp lag l, dpart 0,2,4
# we'll use dtemp lag l, dpart 0,4 to replicate example
scope <- as.formula(dmort.pw~t+dtemp.lag1+dtemp2.lag1+dpart.lag0+dpart.lag2+dpart.lag4+dpart.lag7)
print(summary(pw.m0 <- step(lm(dmort.pw~1,data=data[complete.cases(data),]),scope=scope)))

# stepwise selection with raw predictors chooses different subset
scope <- as.formula(dmort.pw~t+temp.lag1+temp2.lag1+part.lag0+part.lag2+part.lag4+part.lag7)
print(summary(pw.m1 <- step(lm(dmort.pw~1,data=data[complete.cases(data),]),scope=scope)))

# remove incomplete cases and adjust trend
datasub <- data[5:508,]
datasub$t <- 1:NROW(datasub)

# fit arimax model using lm - matches example results exactly
# arimax model suggests negative association between mortaltiy and lag1 temperature
# suggests positive association between mortaltiy and particulate count at lag0 and lag4
print(lm0 <- lm(mort~t+temp.lag1+part.lag0+part.lag4+mort.lag1+mort.lag2,data=datasub))

# fit equivalent model using arima with xreg - parameter scale seems different, mabey issue with R
print(ax0 <- arima(datasub$mort,order=c(2,0,0),xreg=datasub[,c("t","temp.lag1","part.lag0","part.lag4")]))

# model residuals, acf, and pacf look good
par(mfrow=c(3,1))
stdres <- rstandard(lm0)
```

```
plot(stdres,type="l")
acf(stdres)
pacf(stdres)

# residuals have a fat right tail
par(mfrow=c(1,1))
qqnorm(stdres)
qqline(stdres)

# residuals fail tests of normality but pass Ljung-Box test suggesting white noise
# also fail McLeod Li test suggesting conditional variance

# residuals fail Shapiro and Jarque-Bera tests of normality
# however pass Ljung-Box test suggesting whire noise
# McLeod-Li test suggest conditional heteroscedacity

# ho:random residuals ha:non-random residuals
print(Box.test(stdres,lag=20,type="Ljung-Box"))

# ho:normal residuals ha:non-normal residuals
print(shapiro.test(stdres))

# ho:normal residuals ha:non-normal residuals
print(jarque.bera.test(stdres))

# ho:random heteroscedascity ha:conditional heteroscedacity
McLeod.Li.test(y=stdres,gof.lag=20)
```

Appendix R Code To Replicate Model Fit Of New York Real Estate ARMAX Model

```
# fit an arimax model to price per sqft data
require(TSA)
require(tseries)
options(width=128)
filedir <- "E:/Data/"
#filedir <- "c:/data/sta9701/"
load(paste(filedir,"sta9701_data.rda",sep=""))

# look at the original data series
par(mfrow=c(3,1))
plot(data$PPSF,type="l")
plot(data$NE_MTG_Rate,type="l")
plot(data$UE_RATE,type="l")

# transform to stationarity
data <- transform(data,PPSF.ld=c(NA,diff(log(PPSF))))
data <- transform(data,PPSF.ld.dt=c(NA,resid(lm(PPSF.ld~time(PPSF.ld)))))
data <- transform(data,NE_MTG_Rate.ld=c(NA,diff(log(NE_MTG_Rate))))
data <- transform(data,UE_RATE.ld=c(NA,diff(log(UE_RATE))))
data <- data[complete.cases(data),]

# look at the transformed data series
plot(data$PPSF.ld.dt,type="l")
plot(data$NE_MTG_Rate.ld,type="l")
plot(data$UE_RATE.ld,type="l")

# preliminary ARMA order exploration
par(mfrow=c(2,1))
acf(data$PPSF.ld.dt,na.action=na.remove)
pacf(data$PPSF.ld.dt,na.action=na.remove)

# model#2 - ARMA(1,2) has lowest AIC
m0 <- eacf(data$PPSF.ld.dt)
summary(m1 <- arma(data$PPSF.ld.dt,order=c(1,1)))$aic
summary(m2 <- arma(data$PPSF.ld.dt,order=c(1,2)))$aic
summary(m3 <- arma(data$PPSF.ld.dt,order=c(2,1)))$aic
summary(m4 <- arma(data$PPSF.ld.dt,order=c(2,2)))$aic

# no significant autocorrelation in residuals
data <- transform(data,PPSF.ld.dt.pw=residuals(m2))
acf(data$PPSF.ld.dt.pw,na.action=na.remove)
pacf(data$PPSF.ld.dt.pw,na.action=na.remove)

# mtg_rate is significant at a lag of 2.
# une_rate is significant at a lag of 0,5,11.
ccfmtg <- ccf(data$PPSF.ld.dt.pw,data$NE_MTG_Rate.ld,na.action=na.remove)
ccfune  <- ccf(data$PPSF.ld.dt.pw,data$UE_RATE.ld,na.action=na.remove)
plot(ccfmtg[0:17],main="CCF Mtg Rate vs PPSF ARMA(1,2) Residuals")
plot(ccfune[0:17],main="CCF Unemployment vs PPSF ARMA(1,2) Residuals")
```

```
# add lagged series to data set
data <- transform(data,NE_MTG_Rate.ld.lag2=c(rep(NA,2),embed(NE_MTG_Rate.ld,3)[,3]))
data <- transform(data,UE_RATE.ld.lag0=UE_RATE.ld)
data <- transform(data,UE_RATE.ld.lag5=c(rep(NA,5),embed(UE_RATE.ld,6)[,6]))
data <- transform(data,UE_RATE.ld.lag11=c(rep(NA,11),embed(UE_RATE.ld,12)[,12]))

# stepwise selection chooses mtg_rate lag 2, une_rate lags 0,11
data  <- data[complete.cases(data),]
scope <- as.formula(PPSF.ld.dt.pw~NE_MTG_Rate.ld.lag2+UE_RATE.ld.lag0+UE_RATE.ld.lag5+UE_RATE.ld.lag11)
print(summary(pw.m0 <- step(lm(PPSF.ld.dt.pw~1,data=data),scope=scope)))

# need to use arima with xreg to fit because we have MA terms
# compare orignal arima model to new arimax model
# arimax is supperior, has more degrees of freedom but better AIC
# arimax model suggests possitive association between ppsf and lag2 mortgage rate
# suggests negative association between ppsf and unemployment at lag0 and lag11

print(mx0 <- arima(data$PPSF.ld.dt,order=c(1,0,2)))
print(mx1 <-
arima(data$PPSF.ld.dt,order=c(1,0,2),xreg=data[,c("NE_MTG_Rate.ld.lag2","UE_RATE.ld.lag0","UE_RATE.ld.lag11")]))

# model residuals, acf, and pacf look good at lower lags
par(mfrow=c(3,1))
stdres <- rstandard(mx1)
plot(stdres,type="l", main="Standardized Residuals",ylab="")
acf(stdres, main="ACF of Standardized Residuals")
pacf(stdres, main="PACF of Standardized Residuals")

# residuals have a narrow left tail and fat right tail, but for the most part fairly normal
par(mfrow=c(2,1))
qqnorm(stdres)
qqline(stdres)

# residuals pass Shapiro and Jarque-Bera tests of normality
# however Ljung-Box test suggests additonal autocorrelation
# McLeod-Li test suggest random heteroscedacity

# ho:random residuals ha:non-random residuals
print(Box.test(stdres,lag=20,type="Ljung-Box"))

# ho:normal residuals ha:non-normal residuals
print(shapiro.test(stdres))

# ho:normal residuals ha:non-normal residuals
print(jarque.bera.test(stdres))

# ho:random heteroscedascity ha:conditional heteroscedacity
McLeod.Li.test(y=stdres,gof.lag=20,main="McLeod Li Test")
```

Works Cited

Radar Logic LTD. (n.d.). Retrieved December 01, 2010, from Radar Logic Data: www.radarlogic.com

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