Homework # 7

Due Date: November 17 (Thursday), 18:00

Please upload your homework as an attachment through the website of the course on due time.

The file folder subject should be: 'matlab homework 7'. Please refer to "homework instructions.pdf" for detailed requirements on submission. If you have more than one files, please compress them in a single file.

If the questions ask you to plot or display something to the screen, please include the plot and screen output which your code generates. For example, for figures, save the figures, and insert them into a word ".doc" document. Submit the *.doc file together with your codes.

The present homework is designed to practice writing functions to solve problems. The problems in this homework are very common, and you meet similar ones in your other classes. The names of helpful functions are provided in bold where may be needed. Keep all your codes in scripts/functions. If a specific name is not mentioned in the problem statement, you can choose your own script/function names.

1. Fitting polynomials.

(a) Write a script to create a vector x with 100 equally distributed values in the range from -6 to 6. Then calculate $y = \sin(2x)e^{-\sqrt{2}x}$ with a perturbation (i.e., a random number in the range -0.5 to 0.5, by using **rand**) at each entry of the vector y. Now you have two vectors **x** and **y** with 100 entries.

(b) Then fit first, second, third, fourth, and fifth order polynomials to these points defined by the vectors x and y.

(c) Plot the data (points determined by **x** and **y**) as blue scatter dots (or others, e.g. stars) and a red solid line with a thickness 3 points on a figure, further plot all five polynomial fits using lines of different colors on the same axes with legends for all plots. Label the figure appropriately, including **xlabel** and **ylabel**. A suitable title should be given.

Hint: To get good fits, you may have to use the centering and scaling version of `polyfit` (the one that returns three arguments, see `help`) and its counterpart in `polyval` (the one that accepts the centering and scaling parameters).

2. Question-Interpolation and surface plots.

Write a script called randomSurface.m to do the following

(1). To make a random surface, make Z0 a 5x5 matrix of random values on the range [0, 2] (use **rand**).

(2). Make an X0 and Y0 using **meshgrid** and the vector 1:5 (use the same vector for both inputs into meshgrid). Now, X0, Y0, and Z0 define 100 points on a surface.

(3). We are going to interpolate intermediate values to make the surface seem smooth. Make X1 and Y1 using meshgrid and the vector 1:0.1:5 (again use the same vector for both inputs into meshgrid).

(4). Make Z1 by interpolating X0, Y0, and Z0 at the positions in X1 and Y1 using cubic interpolation (interp2, specify cubic as the interpolation method).

(5). Help of interp2.

(6). Plot a surface plot of Z1. Set the **colormap** to hsv and the shading property to interp (surf, colormap, shading).

(7). Hold on to the axes and plot the 15-line contour on the same axes (contour).

(8). Add a colorbar (colorbar).

(9). Set the color axis to be from 0 to 1 (caxis). The final figure should look something like this:

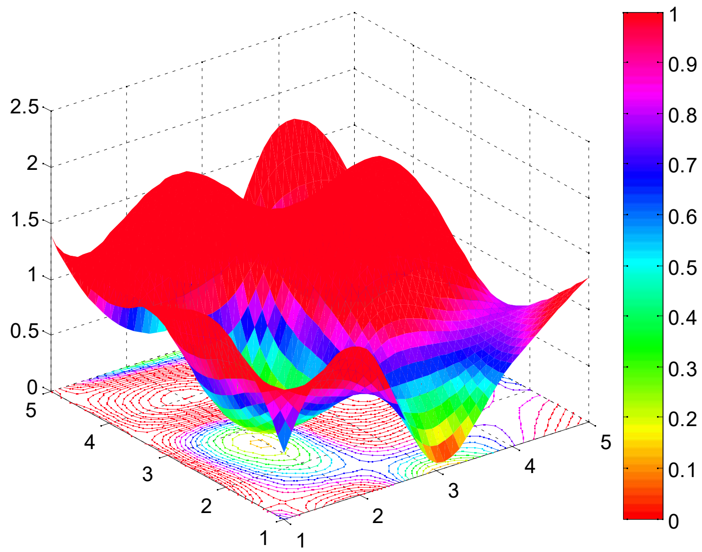


Figure 1

3. Optimization.

1） In this problem you use the fminbnd with variable lower and upper bounds in Matlab to find the minimum of the function $f(x) = 2x^2 \sin(4x)$.

a. You may try different lower bounds, e.g. -10, -8, -6, …, 8, or some other more values;
b. Try different upper bounds, e.g. -8, -6, …, 8, 10, or some other values;
c. You can try different ranges [-10,-8], [-10, -6],…, [-10,8], [-10, 10], [-8, -6], …,[8,10] to find the minimum of the function.
d. Please run fminbnd with these different boundaries, and plot the distribution of the minimum with the x axis representing the left boundary and the y axis representing the right boundary. You may find some similar example via the link
http://pundit.pratt.duke.edu/wiki/MATLAB:Fminbnd_and_fminsearch

2）In this problem you use the optimization toolbox in Matlab to find the minimum of Banana function. You may find more about Banana functions by searching 'Rosenbrock function'.

a. Create a function for the famous Rosenbrock banana function as follows

$$f(x, y) = (x-2)^2 + 4(x^2 - y)^2$$

b. Create a figure to show the graphic representation of *f*, (using: **Surf, Contour**, or others).

C. Find the minimum of the Banana function *f* using the Matlab optimization algorithm **fminunc, fmincon, fminsearch**. (Hint: you should create a function for calculating the banana function, and call the function when doing the optimization using **fminunc, fmincon, fminsearch**)

d. Compare the algorithms (**fminunc, fmincon, fminsearch**) on the following measures:
- Robustness: ability to find a global optimum and dependence of performance on initial guess.
- Efficiency: how many function evaluations were required?

3) In this problem you use the fminbnd and fminsearch in Matlab to find the maximum of the function

$f(x) = 2x^2 \sin(2x) + 4\mathrm{x}$.

(a) For fminbnd, the range is -10 to 10.
(b) For fminsearch, you may try different starting guesses.
(c) Please compare the maximum obtained from **fminbnd** and **fminsearch.** Please plot the function $f(x)$ and indicate the maximum found in the figure.
Hint: you may use anonymous function or a separate function file.

4. From the previous optimization, you may find one minimum value of the banana function. Now, please try the so-called Branin function, which has three global minima. The Branin function is written as

$$f(x,y) = a\left(x_2 - bx_1^2 + cx_1 - r\right)^2 + s(1-t)\cos(x_1) + s$$

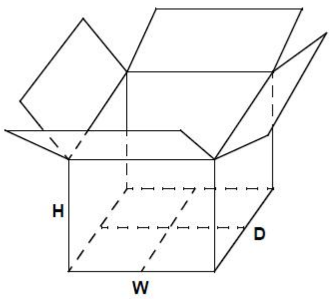
where $a = 1$, $b = 5.1/\left(4\pi^2\right)$, $c = 5/\pi$, r=6, s =10, $t = 1/(8\pi)$.

In this problem you use the optimization toolbox (optimtool) in Matlab to find the three minima of Branin function by trying different starting guess.

(a) Create a figure to show the graphic representation of *f*, on the square $x_1 \in [-5, 10]$, $x_2 \in [0, 15]$ (using: **Surf, Contour**, or others).

(b) Find the three minima of the Banana function *f* using the Matlab optimization algorithm **fminunc, fmincon, or fminsearch**. (e.g. one of the global minima $x^* = (-\pi, 12.275)$, $f\left(x^*\right) = 0.397887$)

5. Box Design. This is a simple box design problem. The goal is to determine dimensions H, W, and D to minimize the surface area required to enclose a volume of at least 2 cubic meter.



The problem is presented in the form as

Minimize surface area S, where S = 2.0*(W*H +D*H+2.0*W*D)

Subject to

2.0 –H*D*W <=0.0

H, W, D>=0.0

You may use the initial value for H, W and D as 1.0. Please try to solve such a problem using fmincon function. You are asked to present the optimum value of H, W, and D, and the optimum value of the objective. The iteration history may be accessed from Matlab and presented too.

Hint: here it is a constrained optimization problem.