Examples of supermodular functions. For $\{X, X'\} \subset \mathbb{R}^d$, label $\left\{\hat{X}, \tilde{X}\right\}$ their monotone rearrangement with $\hat{X}_i \leq \tilde{X}_i$. Let $\delta_{\text{call}}(t) = (t-K)^+$ and $\delta_{\text{put}}(t) = (K-t)^+$.

1. $c(X) = \delta_{\text{call}}(\min_i X_i)$. Take arbitrary $\{X, X'\} \subset \mathbb{R}^d$ and suppose wlog $\min_i X_i \leq \min_i X_i'$. By the monotone rearrangement,

$$\begin{aligned}\min_i \hat{X}_i &= \min_i \{\min\{X_i, X_i'\}\}\\ &= \min\left\{\min_i X_i, \min_i X_i'\right\}\\ &= \min_i X_i\end{aligned}$$

   and

$$\begin{aligned}\min \tilde{X}_i &= \min_i \{\max\{X_i, X_i'\}\}\\ &\geq \min_i X_i'\end{aligned}$$

   Then $c\left(\hat{X}\right) = c(X)$, and since δ_{call} is non-decreasing, $c\left(\tilde{X}\right) \geq c(X')$. We have the supermodular inequality $c\left(\hat{X}\right) + c\left(\tilde{X}\right) \geq c(X) + c(X')$.

2. $c(X) = \delta_{\text{put}}(\max X_i)$. Take arbitrary $\{X, X'\} \subset \mathbb{R}^d$ and suppose wlog $\max_i X_i \leq \max_i X_i'$. By an analogous reasoning as above, we have $\max \tilde{X}_i = \max X_i'$, $\max \hat{X}_i \leq \max X_i$, $c\left(\tilde{X}\right) = c(X')$ and $c\left(\hat{X}\right) \geq c(X)$ since δ_{put} is non-increasing. Again we have $c\left(\hat{X}\right) + c\left(\tilde{X}\right) \geq c(X) + c(X')$.