A VNS-Based Algorithm for the Mammography Unit Location Problem

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Abstract. This work deals with the mammography unit location problem in Brazil. In this problem, there is a set of mammography units to be installed in cities with hospital infrastructure and a set of cities, each with a demand for mammography screenings to be performed in women aged 40 to 69 years old. The goal is to decide where to install mammography units to maximize the total demand, satisfying the constraints that a woman can not travel more than 60 km to be attended and that not all cities are candidates to host a mammography unit. One mathematical programming formulation and a VNS-based algorithm are introduced. The methods were tested using data from Minas Gerais State, Brazil. We analyze the performance of the VNS algorithm considering several scenarios created from the base instance. The results show that the proposed algorithm is able to provide good quality solutions quickly. In addition, it has been shown that with the proposed allocation it is possible to increase the coverage of mammography screenings in the real instance.

Keywords: Mammography unit location · Maximal Covering Location Problem · Variable Neighborhood Search · Mathematical programming

1 Introduction

Among female population, cancer is the second leading cause of death worldwide, accounting for 14% of all deaths. Breast cancer is the most commonly diagnosed cancer among women in most countries of the world [17]. This situation is not different in Brazil [13].

According to [19], the reduction in the number of breast cancer-related deaths in the female population is directly related to the early diagnosis of this disease. On the other hand, the screening by mammography unit is the primary means of early detection of breast cancer.

The current recommendation of the Health Ministry of Brazil is that mammography screenings should be offered to women aged from 50 to 69 years old

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biennially, as this age group benefits more from the examination in terms of traceability [11,12]. Moreover, studies show an additional 8.9% screenings annually for a diagnostic indication to women in this age group. Thus, for women aged 50–69, the estimated demand is 58.9% of the female population per year [11]. Besides, according to these studies, annual screenings are required in 20% of women between the ages of 40 and 49, of which 10% are for diagnostic purposes and 10% for other indications.

Mammography screening is one of the diagnostic services offered by the Brazilian government health care, named Unified Health System (SUS, in Portuguese), through which a large part of the Brazilian population has its health care needs satisfied. According to the National Cancer Institute [14], each equipment is capable of performing 5,069 mammography screenings annually. Federal Government researches show that 70% of the population have SUS as a reference in health care and use the Health Care Network (RAS) to perform health services under the SUS, among them the diagnostic support services.

Regarding access to health services in RAS, Andrade et al. [4] emphasize that studies are essential to optimize mammography unit allocation. The woman travel distance to the place where the equipment is installed is one of the factors that most contribute to women failing the screening. In other words, many women do not perform the mammography screening simply because the equipment is installed far from their residences.

The SUS inefficiency in offering mammography screenings to the Brazilian female population is verified in several works, as in [3], [4] and [18]. These authors verified that considering only the demand for mammography screenings to be performed annually and the existing number of mammography units, the current number of equipment is sufficient. However, the distribution of this equipment is inadequate, since some regions are well covered and others are not. Besides, in many locations, there is a skilled labor shortage to operate the equipment.

This work deals with the Mammography Unit Location Problem (MULP) and contributes to the development of optimization models for a better distribution of mammography units, while at the same time doing a preliminary case study of Minas Gerais State, Brazil. A heuristic algorithm based on the Variable Neighborhood Search (VNS) method is presented to obtain approximate solutions to the Maximal Covering Location Problem (MCLP) [5]. This heuristic algorithm is proposed since the MULP is NP-hard [7].

The rest of this paper is organized as follows. In Sect. 2, a literature review is made, while in Sect. 3 the problem under study is described. Section 4 introduces a mathematical programming formulation for solving the MULP and Sect. 5 presents a VNS-based algorithm to obtain high quality solutions for it. In Sect. 6, the computational results are reported. Section 7 concludes the work and presents perspectives for future work.

2 Literature Review

In [4], the authors analyzed the number of existing mammography units and the female population that requires mammography screenings in the State of Minas

Gerais in 2012. The authors concluded that distance and women displacement time to the equipment are important limiting factors for mammography screening. According to them, if the equipment is far from the women's residence, it is very likely that they will not travel to realize the mammography screenings. The authors emphasize the importance of studies to optimize these mammography unit allocation.

According to [3], several factors can create barriers to health services accessibility, such as educational level, socioeconomic status, transportation cost, health center location. The concept of accessibility is not only related to the availability of resources in a given period ([1] *apud* [3]); in fact, it is also related to the ability of individuals to appropriate the services offered. According to [8], it is not enough to offer the health service, it is also necessary that the patient can reach the center where it is offered at reasonable times and costs. In [3], the authors conclude that the availability of mammography units in Brazil is sufficient to cover the full demand of women for mammography screenings. However, when the maximum distance restriction is added in the context, the equipment distribution is inadequate since many of them do not cover all regions.

The demands of the public health medical specialties in the Minas Gerais State were studied in [16]. The object of research was the location of 51 Medical Specialty Centers (CEMs) in 853 cities of the State, and in five specialties: cardiology, pediatrics, mastology, gynecology, and endocrinology. They were chosen by the criterion of higher demand for medical attention in the State and medical care hours. The authors proposed a mixed integer programming model, based on the Maximal Covering Location Problem (MCLP), and considered three scenarios to define a set of candidate cities to receive a CEM. The first scenario considered 853 cities as candidates, the second 372, and the third 98. The maximum distance parameter varied in the values 400, 300, 200, and 100 kilometers (km), in order to identify the configuration that provides the highest coverage and the shortest average distance of displacement. The cities distance matrix was obtained by calculating the distance between two points according to the spherical law of the cosines, updated by a correction factor. The authors verified that the selected variations showed a better geographic distribution for the 51 CEMs with smaller distances of maximum coverage in all scenarios. Moreover, given the economic crisis in the Minas Gerais State, they suggested adopting the third scenario, considering the possibility of cost reduction as well as the number of CEMs to be installed, without coverage demand loss.

In [6], the authors analyzed the mammography units location in a set of 12 health regions of Minas Gerais State, involving 142 cities. The authors developed four mathematical programming formulations, all of them based on the p-median problem. In the first one, the goal is to minimize the total distance traveled by women when going to the mammography center. In the second formulation, the maximum displacement distance constraint is relaxed, and the distance exceeding the maximum distance is penalized in the objective function. The last two formulations differ from the previous two because they consider as objective function the distance and the women demand to be attended. More precisely, the objective function is given by the product between the traveled distance to the mammography unit and the number of women who travel. The objective of these last two formulations is to encourage the installation of equipment in cities with the highest mammography demand. As observed in [3], the authors concluded that there are more mammography units in the analyzed region than necessary and the current location is inadequate because it does not comply with the recommended Health Ministry rules.

A comprehensive review of models and solution methods for the healthcare facility location problem can be found in [2].

3 Problem Statement

The Mammography Unit Location Problem (MULP) addressed here has the following characteristics:

(a) There is a set S of n candidate cities to host p mammography units, with $p < n$;
(b) Each mammography unit has an annual capacity of realizing *cap* mammography screenings;
(c) Each city has an annual demand of mammography screenings for women in the age range indicated to do the screening, that is, 58.9% of women aged 50–69 and 20% of women between the ages of 40 and 49, according to the current recommendation of the Health Ministry of Brazil;
(d) A woman cannot travel more than R km to a city that hosts a mammography unit;
(e) Only cities with hospital infrastructure are candidates to host mammography units. In this paper we consider that a city is candidate to host mammography equipment if it has at least *demMin* women in the age range indicated for realizing the screening;
(f) Each city must be either fully covered by a mammography equipment or not covered. That is, we consider in this paper that a city cannot be partially covered. This restriction is imposed for administrative reasons, since this would require managing which women in a city should do the mammography screenings.

The objective is to decide where to install the mammography units in order to maximize the total demand for mammography screenings.

4 Mathematical Formulation

For applying the proposed formulation, we assume that the demand for mammography screenings of each city is smaller than the capacity for screenings of a mammography unit. When this does not happen, we allocate as many mammography units as necessary until the demand is less than the equipment capacity. In this way, the demand covered with this preprocessing is maximum.

In order to introduce the model, the input parameters and the decision variables are defined according to Table 1.

Table 1. Parameters and decision variables

Parameters	
N	Set of cities
d_{ij}	Distance from city i to city j
dem_j	Demand for mammography screenings in city j
cap	Annual screening capacity of an equipment
p	Amount of mammography units to be located
R	Maximum travel distance to be served
$demMin$	Minimum annual screening demand that a city must have to host an equipment
S_i	Set of cities whose distance from city i is less or equal to R km, that is, $S_i = \{j \in N \,\vert\, d_{ij} \leq R \text{ and } d_{ji} \leq R\}$
Decision variables	
x_{ij}	Binary variable that assumes value 1 if the women from city j are served by an equipment installed at city i and value 0, otherwise
y_i	Integer variable that represents the number of equipment installed at city i
z_i	Binary variable that assumes value 1 if the city i hosts some equipment and value 0, otherwise

Equations (1) to (11) represent the MULP:

$$\max \quad \sum_{i \in N} \sum_{j \in S_i} dem_j \cdot x_{ij} \tag{1}$$

$$\text{s. t.} \quad \sum_{i \in S_j} x_{ij} \leq 1 \quad \forall\, j \in N \tag{2}$$

$$\sum_{i \in N} y_i = p \tag{3}$$

$$\sum_{j \in S_i} dem_j \cdot x_{ij} \leq cap \cdot y_i \quad \forall\, i \in N \tag{4}$$

$$z_i \geq y_i/p \quad \forall\, i \in N \tag{5}$$

$$z_i \geq x_{ij} \quad \forall\, i, j \in N \tag{6}$$

$$x_{ii} = z_i \quad \forall\, i \in N \tag{7}$$

$$y_i = 0 \quad \forall\, i \in N \mid dem_i < demMin \tag{8}$$

$$x_{ij} \in \{0, 1\} \quad \forall\, i, j \in N \tag{9}$$

$$y_i \in \mathbb{Z}^+ \quad \forall\, i \in N \tag{10}$$

$$z_i \in \{0, 1\} \quad \forall\, i \in N \tag{11}$$

The objective function (1) aims to maximize the total demand for mammography screenings. Constraints (2) indicate that each city j must be served by a

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single mammography unit installed in city i, or not served. Constraint (3) determines that all available p equipment must be allocated, and a city may receive more than one equipment. Constraints (4) guarantee that the equipment' capacity must be respected. Constraints (5) ensure that if at least one equipment is installed in the city i then the variable z_i assumes the value 1. Constraints (6) ensure that a city j can only be served by a city i if an equipment is installed in this city. Constraints (7) ensure that the demand of city i has to be covered by the equipment installed in the city itself. Constraints (8) indicate that an equipment can only be installed in a city that has the demand for mammography screenings greater than or equal to *demMin*, to economically justify its installation. Finally, Constraints (9), (10) and (11) impose the domain of the decision variables.

5 The Proposed VNS Algorithm

As stated in Sect. 4, we assume that there is an initial preprocessing stage to allocate mammography units to each city with demand greater than the mammography unit's capacity. Furthermore, no modification of this allocation is made by any procedure (local search or shaking) during the search.

Subsection 5.1 describes the proposed VNS-based algorithm, and the following subsections describe its modules.

5.1 Variable Neighborhood Search

The Variable Neighborhood Search (VNS) algorithm [9] developed for the MULP is a basic VNS [10] and works according to Algorithm 1.

Algorithm 1 : VNS

```
s0 ← InitialSolution()
s ← LocalSearch(s0)
while Stopping criterion is not satisfied do
    k ← 2;
    while  k ≤ r  do
        s' ← Shaking(s, k)
        s'' ← LocalSearch(s')
        if  ( f(s'') > f(s) )  then
            s ← s''
            k ← 2
        else
            k ← k + 1
        end if
    end while
end while
Return s;
```

Algorithm 1 begins in line 1 by constructing an initial solution according to Subsect. 5.4. Next, in line 2 it is refined by the local search procedure described in Subsect. 5.6. In order to avoid getting stuck in a local optimum, the algorithm goes into a loop that works as follows. Initially, all mammography units of k cities are removed, as well as all links associated with these k cities. In the next step, the solution is restored by a constructive mechanism described in Subsect. 5.4. Both steps (removal and construction) compose the Shaking procedure, which is described in Subsect. 5.7. In line 7, the current solution is refined. If this local optimum solution s'' is better than the current solution s (line 8), then s is updated and the level of perturbation returns to its minimum value ($k = 2$); otherwise, the perturbation is increased (line 12). The algorithm ends when the stopping criterion is satisfied.

5.2 Solution Representation

A solution s of the MULP is represented by a tuple $s = (u, v)$, where u and v are vectors, both of size n. Each position j of the vector u shows that the city j is covered by some equipment installed in the city u_j. If u_j assumes the value 0, it means that city j is not covered by any mammography unit. Each index j of the vector v indicates that the city j holds a total of v_j mammography units.

An example of a solution to the problem is shown in Table 2. In this case, $p = 2$ mammography units are available to cover up to $n = 8$ cities. The first line of the table shows the indexes of the cities; the second line, u_j, shows the links between cities, for example: cities 1, 2, and 3 are all covered by city 1, while cities 4 and 5 are covered by city 5. Cities 6, 7, and 8 are not covered by any city. Finally, the last line corresponds to the vector v that stores the number of equipment per city; in the case, one mammography unit was allocated to city 1 and another to city 5.

In Fig. 1(a), a map is displayed with the spatial distribution of 8 cities and 2 mammography units to be located. Each vertex in the map corresponds to a city and the number positioned inside the circle indicates the index of the respective city. A vertex in red color indicates that the respective city receives equipment. Thus, the mammography units were installed in cities 1 and 5. An edge connects the cities that will be served by these equipment. For example, edges $(1, 2)$ and $(1, 3)$ show that cities 2 and 3 are covered by city 1.

Table 2. Solution example

j	1	2	3	4	5	6	7	8
u_j	1	1	1	5	5	0	0	0
v_j	1	0	0	0	1	0	0	0

5.3 Evaluate Function

A solution $s = (u, v)$ is evaluated by the function f, given by Eq. (12), which must be maximized:

$$f(s) = \sum_{j \in N \mid u_j \neq 0} dem_j \tag{12}$$

where N is the set of cities, dem_j represents the demand for mammography screenings of the city j, and u_j is a variable that assumes a non-zero value if the city j is covered by some city and value 0, otherwise. The objective is to maximize the total number of screenings using p mammography units.

5.4 Initial Solution

We present below a constructive heuristic procedure to generate a solution for the MULP.

Step 1: Calculate, for each city i yet not covered, the demand of its coverage region, including its own demand for mammography screenings;
Step 2: Sort the cities, in decreasing order according to the demand from each region covered;
Step 3: Calculate the number of mammography units that are necessary to cover the city i that has the greatest demand. If this number is less than or equal to the number of mammography units available, cover the total demand of this city and update the available number of mammography units; otherwise, return to Step 1. In both cases, remove this city from the list;
Step 4: Calculate the amount of idle mammography screenings of the equipment installed in the city i and determine the cities in the region that are not covered;
Step 5: Solve the knapsack problem, considering the city i as a knapsack of capacity equal to the amount of idle mammography screenings and as items, the demands for mammography screenings of the cities still not covered that belong to its region;
Step 6: Assign the cities returned by Step 5 to the city i;
Step 7: If the remaining number of mammography units is still greater than zero, return to Step 1; otherwise, finalize the procedure and return the cities in which the mammography units will be installed, as well as the cities covered by them.

Note that the cities in this constructive procedure are initially sorted in decreasing order of the total number of mammography screenings demanded by each city of the region that it can cover, i.e., the demand of the city itself is summed up with the demands of the cities that are in its coverage region. The coverage region of a city i is composed by all cities j which are at distance $d_{ij} \leq R$ km and $d_{ji} \leq R$ km from it.

In this sorting, only the cities that have hospital infrastructure are candidates, here considered those that have a high demand for mammography screenings, that is, greater than *demMin* ones. This value is adopted to economically justify the installation of mammography units in a city. Whenever a mammography unit is allocated to a city i, it is considered that this city will be fully covered. The amount of idle mammography screenings of this equipment is used to cover the demand of the cities that are in its coverage region.

The choice of which cities will be covered by the city i is done by solving the 0–1 Knapsack Problem (KP). The KP consists of filling a knapsack of capacity W with items of different weights and profits. The goal is to fill the knapsack with the highest possible profit so that it does not exceed its capacity. The following analogy is made in the construction of the initial solution for this problem: the uncovered cities of the region correspond to the items that can be inserted in the knapsack; each city or item has a profit and a weight and both match the demand for screenings from the respective city. Finally, the knapsack is represented by the city i, which has a mammography unit with idle capacity $W = cap - w_i$, where cap is the capacity of the equipment and w_i is the demand for mammography screenings of the city i. The method returns the list of cities that will be covered by the mammography unit installed in the city i.

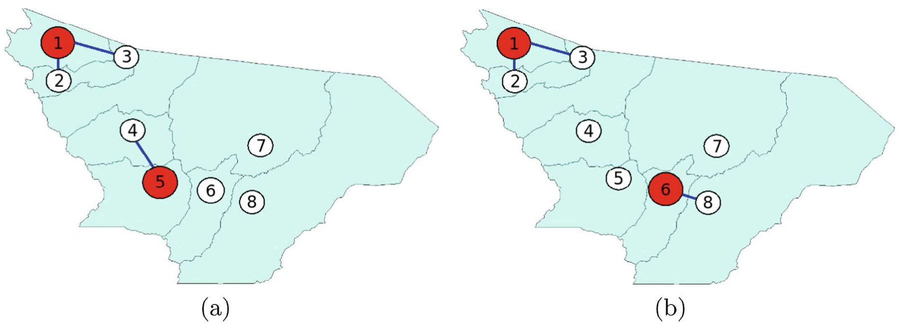

Fig. 1. Illustration of the exchange move (Color figure online)

We apply a dynamic programming algorithm[1] to solve the knapsack problem. Thus, the solution returned by it is exact. It has pseudo-polynomial time complexity, that is, $\mathcal{O}(mW)$, where m is the amount of items and W is the capacity of the knapsack. For the largest instance ($m = 310$ and $W < cap = 5069$), the problem is solved instantly.

5.5 Neighborhood Structure

The exploration of the solution space is made using a single type of move, called exchange function. This move consists of removing a mammography unit from a city and then allocating it to another city that has infrastructure to receive it but does not yet have that equipment.

[1] Its code was extracted from https://www.geeksforgeeks.org/0-1-knapsack-problem-dp-10/.

An application example of this move is shown in Figs. 1(a) and (b). In these figures, each vertex corresponds to a city, and the ones highlighted in red are the cities that have mammography units. The edges connecting two vertices indicate service dependency. In Fig. 1(a), for example, it is possible to verify that cities 1 and 5 have equipment and they cover the cities $\{1, 2, 3\}$ and $\{4, 5\}$, respectively.

In these figures, it can be seen that one mammography unit was removed from the city 5 and thus the city 4 is no longer covered by the city 5. Then it is necessary to choose another city that respects the restrictions to receive the equipment that is now available. In this example, city 6 was chosen to illustrate this move, as we can see in Fig. 1(b). Once these choices have been made, the next step is to determine which cities will be covered by city 5. Thus, the knapsack problem is solved. After applying the dynamic programming procedure mentioned before, city 8 was chosen to be served by city 6. Finally, in Fig. 1(b), we have the final solution resulting from the application of the exchange function move. The mammography unit of the city 5 is now in city 6 and this, in turn, only covers itself and the city 8.

5.6 Local Search

A solution s is refined by an Uphill Method with the First Improvement (FI) strategy using the exchange function move described in Subsect. 5.5.

Briefly, the method chooses a city from the current solution to remove its equipment and another city to receive it. This process is repeated as long as there are improvements. It is important to note that the mammography units allocated by preprocessing are not modified by the local search. They remain fixed throughout the procedure.

Initially, two sets of cities, S and V, are formed. The set S contains all cities that can host mammography units but do not yet have them, and the set V contains all cities that have at least one equipment. These sets are scrambled at each procedure call to prevent the method from always considering the same choice of cities in different runs.

Then, for each city $i \in V$, we remove its equipment. The next step is to choose a city $j \in S$ to receive such equipment. To fulfill this task, for each city $j \in S$ we solve a knapsack problem in order to choose the cities of the coverage region of the city j to be served. If the exchange move improves the current solution, it is accepted and the method is reset; otherwise, the method proceeds to the next city $j \in S$.

The method ends when all possible exchanges do not generate an improvement solution, thus ensuring that the returned solution is a local optimum in relation to the neighborhood used.

The uphill method described in Algorithm 2 takes as input a solution s. Initially it builds the sets S and V based on s (lines 5 and 6). In line 10, the RemoveEquipment(i) method is called. It is responsible for removing the mammography unit from city i and eliminating its dependencies, thus returning an incomplete solution s. In line 11, the loop that iterates on the cities that have infrastructure to receive an equipment starts. In line 12, the InsertEquipment(j)

method is responsible for inserting an equipment in the city j, returning a new solution s'. Then, the knapsack problem is performed in order to define which cities in the coverage region of city j will be covered. If the solution s'' returned by the knapsack improves the best solution found so far, then it is updated and the method restarts from line 3. Otherwise, the loop continues with the next city j. Finally, if the loop in line 3 ends without improvement, we terminate the method and return the solution s_{best} found with the assurance that we find a local optimum for this neighborhood.

5.7 Shaking Procedure

The shaking procedure works as follows. All mammography units and their dependencies of k cities ($k \leq r$) are removed from the current solution s (except those related to the preprocessing phase) and then this solution is restored by solving the knapsack problem according to Subsect. 5.4.

After the removal operation, the cities that will host the mammography units are selected in a partially greedy way as follows. For each city i that has not a mammography unit and that can host it, we calculate its potential of service, that is, the sum of the demands of mammography screenings of the coverage region of that city i. In other words, the service potential is calculated by summing the

Algorithm 2: First Improvement

Require: Solution s

1: $s_{best} \leftarrow s$
2: $hasImprovement \leftarrow true$
3: **while** $hasImprovement$ **do**
4: $\quad hasImprovement \leftarrow false$
5: $\quad S \leftarrow$ {Available cities in solution s that can host equipment}
6: $\quad V \leftarrow$ {Cities in solution s that have equipment}
7: $\quad$ Shuffle(S)
8: $\quad$ Shuffle(V)
9: $\quad$ **for** $i \in V$ **do**
10: $\qquad s \leftarrow$ RemoveEquipment(i)
11: $\qquad$ **for** $j \in S$ **do**
12: $\qquad\quad s' \leftarrow$ InsertEquipment(j, s)
13: $\qquad\quad s'' \leftarrow$ KnapsackProblem(j, s')
14: $\qquad\quad$ **if** $f(s'') > f(s_{best})$ **then**
15: $\qquad\qquad s \leftarrow s''$
16: $\qquad\qquad s_{best} \leftarrow s''$
17: $\qquad\qquad hasImprovement \leftarrow true$
18: $\qquad\qquad$ Goto line 3
19: $\qquad\quad$ **end if**
20: $\qquad$ **end for**
21: $\quad$ **end for**
22: **end while**
23: return s_{best}

demand of all cities that are less than 60 km from city i. The partially greedy choice is made so that different solutions are analyzed. We chose one of the four cities that have the greatest service potential.

Then, for each city i chosen to host an equipment, it is also necessary to define its service dependencies, that is, which cities in its coverage region will be served by the city i. For this decision we solve the knapsack problem (Subsect. 5.4), obtaining, thus, the greatest possible demand that the city i can cover.

After that, the city chosen to host the equipment and the cities it serves are included in the current solution, and the solution restoration method proceeds by recalculating the service potential of the remaining cities as previously presented.

6 Computational Experiments

The mathematical programming model presented in Sect. 4 was implemented in the Gurobi solver, academic version 8.0.0, with default settings, while the proposed VNS algorithm, presented in Sect. 5.1, was developed in C++ language. To test them was used an Intel Core i5 @ 2.5 GHz computer, with 8 GB of RAM under the Ubuntu 18.04 operating system.

Table 3. Characteristics of the instances

Instance	# Cities	# Cities with infrastructure	# Total equipment	# Preproc. equipment	# Remaining equipment
1	853	420	310	114	196
2	853	420	261	114	147
3	853	420	212	114	98
4	853	420	163	114	49
5	142	73	55	19	36
6	142	73	46	19	27
7	142	73	37	19	18
8	142	73	28	19	9

For testing the methods, 8 instances were used. These instances refer to female population data for the year 2010 of the Minas Gerais State, Brazil, and they are available at http://www.decom.ufop.br/prof/marcone/projects/MULP/instances-MG-2010.rar. This State has 853 cities and according to the sector of statistics of its State Secretary for Health (SES/MG), there were 310 mammography units in July of 2018 and the total demand was 1293968 mammography screenings. Considering the current equipment location and the problem characteristics described in Sect. 4, it is possible to perform 970103 mammography screenings, that is, only 75% of total demand. The distances between cities in instances 1 to 4 were obtained by applying the formula of Euclidean distance between cities, while in instances 5 to 8 these values refer to real distances obtained through Google Maps with travel by car. We consider that a city has hospital infrastructure if it has the demand for at least *demMin* mammography screenings. In our case, we set $demMin = 500$. The maximum travel distance to be served was set at $R = 60$ km, a value that is recommended by the Health Ministry of Brazil. The scenarios differ by the number of mammography units available for allocating and the number of cities considered. In the first four instances, the whole State is considered, while in the last four ones only cities that are 100 km from Ouro Preto city, except Belo Horizonte, are considered.

Table 3 summarizes the characteristics of these instances. Column 1 shows the instance number. The second column indicates the total number of cities and the third one shows the number of cities that have the infrastructure to receive equipment. The fourth column reports the number of equipment in the instance. The fifth column shows the number of equipment used by the preprocessing strategy established at the beginning of Sect. 5. Finally, the last column reports the amount of equipment available for applying the methods.

For calibrating the parameters of the VNS algorithm, we test empirically the following values: $r \in \{4, 6, 8, 10\}$ as the number of mammography units removed in the Shaking procedure according to Subsect. 5.7 and $iterMax \in \{100, 200, 300, 400\}$ as the maximum number of iterations without improvement. The best values found empirically were $r = 8$ and $iterMax = 100$.

Table 4. Results Gurobi × VNS

Inst.	Demand preproc.	Gurobi			VNS		
		ub	Best	Time (s)	Best	Average	Time (s)
1	577866	1291621	**1291621**	3.37	**1291621**	**1291621**	1.25
2	577866	1290753	**1288076**	3600.00	1278820	1276812	2466.89
3	577866	1074628	1074466	3600.00	**1074563**	1074541	3600.00
4	577866	826247	826225	3600.00	**826247**	**826247**	19.80
5	96311	221140	**221140**	0.94	**221140**	220690	3.32
6	96311	221140	**221140**	102.17	**221140**	216462	71.53
7	96311	187553	**187544**	3600.00	187478	187386	326.37
8	96311	141932	**141932**	2.95	**141932**	141930	494.65

Table 4 reports the results considering that a mammography unit performs 5069 mammography screenings per year [14]. Column 1 shows the instance, and column 2 indicates the demand served by preprocessing (114 equipment were allocated after this phase for the instances 1 to 4 and 19 for the instances 5 to 8). Columns 3–5 show the upper bound and the value returned by the Gurobi solver with the remaining equipment, as well as the time consumed for solving the instance. Since the solver has been applied only for 3600 s of processing time, then this value returned by Gurobi is optimal if the time spent by it is less than 3600 s. The last three columns show the best result, the average result and the time spent by the VNS algorithm, respectively. The columns "Best" and

"Average" represent the total demand, including the preprocessing one. Values highlighted in bold indicate the best solution for the instance.

According to Table 4, in instance 1, which represents the real instance of Minas Gerais State, both the VNS algorithm and Gurobi Solver are able to find the optimal solution quickly. The total demand met, of 1291621 mammography screenings, is higher than the current allocation of 970103 ones. We can also observe that the VNS algorithm achieves the optimal solution in 5 instances and produces a better solution than Gurobi in two instances (instances 3 and 4). Only in two instances (2 and 7) the VNS algorithm did not overcome Gurobi.

Figure 2 illustrates a typical evolution of the best solution' value produced by the VNS algorithm over the time. As we can see, the VNS method is able to improve the value of the solution over the time.

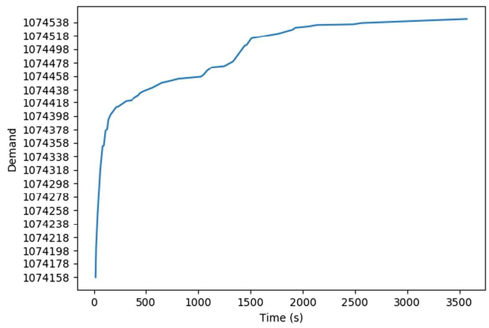

Fig. 2. Evolution of the best solution: instance 3

7 Conclusions and Future Work

This work addressed the Mammography Unit Location Problem (MULP). To solve it, a mathematical programming formulation and a VNS-based heuristic algorithm were developed. In order to test them, eight instances related to data from the State of Minas Gerais, Brazil, were used.

The proposed algorithm was able to produce good quality solutions and outperform the Gurobi solver in two instances. The variability of the final solutions is also low, except in instance 6. Besides it, the algorithm was able to improve the value of the solution over time. It is interesting to note that the total demand of the Minas Gerais State was almost fully covered with the existing mammography units. In fact, the demand served was 99.8% in instance 1. On the other hand, the demand covered by the two models (exact and heuristic) was much higher than the present one. This is due to the fact that the allocations of these equipment have much political influence.

The variability of the final solutions of the VNS algorithm can be reduced by adequately calibrating its parameters. This can be done, for example, by using the Irace package [15]. In addition, other neighborhood structures can be designed to improve its performance.

We also suggest as future work to consider: (1) more recent data of the female population in the age range indicated for mammography screenings; (2) the current grouping of cities in health regions; (3) the real distances between cities; (4) that a city can be partially covered by an equipment and (5) the proposition of itineraries for mobile mammography units to cover cities not covered by the current location of the mammography units.

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