Data and Applications

Assignment 3

Functional Dependencies and Normalization

TEAM : Up To Data

- **Khushi Agarwal - 2020101092**
- **Soveet Kumar Nayak - 2020101086**
- **Naimeesh Narayan Tiwari - 2020101074**

2.1 Question 1

Consider $R(A_1, A_2, ..., A_n)$ to be a relation with functional dependencies defined as follows:

$$A_1 \rightarrow A_2A_3...A_n(i = 1)$$
$$A_2A_3 \rightarrow A_4A_5...A_nA_1(i = 2)$$
$$A_4A_5A_6 \rightarrow A_7A_8...A_nA_1A_2A_3(i = 3)$$

Functional dependencies of the sequence,

$$A_{\frac{(i-1)(i)}{2}+1}A_{\frac{(i-1)(i)}{2}+2}....A_{\frac{(i-1)(i)}{2}+i} \rightarrow A_{\frac{(i-1)(i)}{2}+i+1}....A_nA_1....A_{\frac{i(i-1)}{2}}$$

For i>3 and till $\frac{(i-1)(i)}{2} + i = n$.

Question 1.1 For what values of n is the above set of functional dependencies possible?

Solution:

Since,

$$A_1 \rightarrow A_2A_3...A_n(i = 1)$$
$$A_2A_3 \rightarrow A_4A_5...A_nA_1(i = 2)$$
$$A_4A_5A_6 \rightarrow A_7A_8...A_nA_1A_2A_3(i = 3)$$

and so on,

we should have n = 1 + 2 + 3 + ...

hence n should be of the form m(m+1)/2 , where m > 2

where m = no. of functional dependencies

for m ≤ 2 , the FD set is not possible as atleast one of the FDs do not exist.

Question 1.2 How many keys does the relation R have and what are they?

Solution:

The keys are :$A_1, A_2A_3, A_4A_5A_6, ...$

Hence, there are m keys where m(m+1)/2 = n [m being the number of functional dependencies]

as all attributes are prime attributes and the LHS of every functional dependency is a key.

Question 1.3 State the normal form of the above relation and normalize it to BCNF (if valid) using decomposition rules.

Solution:

→ Assuming all attributes are simple and indivisible in R, the relation is in 1NF.

→ Since the relation is 1NF and all attributes are prime attributes, hence the relation is in 2NF.

→ Since the relation is in 1NF and 2NF and all attributes are prime attributes, hence there is no non-prime attribute which is transitively dependent on the primary key , the relation R is in 3NF.

→ Since every functional dependency has a key in it hence in any functional dependency X → Y , X is a superkey , therefore, the relation R is in BCNF.

Question 1.4 Find the minimal cover of the above relation and use it to normalize it to BCNF (if valid).

Solution:

To get minimal cover,

Step 1: First reduce the RHS of all functional dependencies to a single attribute

$A_1 \rightarrow A_i \forall 2 \leq i \leq n$
$A_2 A_3 \rightarrow A_1$
$A_2 A_3 \rightarrow A_i \forall 4 \leq i \leq n, ...$

$A_{((i-1)(i)/2)+1}\ A_{((i-1)(i)/2)+2} \ldots A_{((i-1)(i)/2)+i} \ \rightarrow A_i \ \forall 1 \leq i \leq (i-1)(i)/2$

Step 2: remove the redundant functional dependencies

$A_1 \rightarrow A_i \forall 2 \leq i \leq n$
$A_2 A_3 \rightarrow A_1$
$A_4 A_5 A_6 \rightarrow A_1$

$A_{((i-1)(i)/2)+1}\ A_{((i-1)(i)/2)+2} \ldots A_{((i-1)(i)/2)+i} \ \rightarrow A_1$

2.2 Question 2

Consider $R(A_1, A_2, ...A_n)$ be a relation R with functional dependencies as follows:

$$A_i \rightarrow A_j \; \forall \; 1 \leq i < j \leq n$$

and,

$$A_i \rightarrow A_j \; \forall \; 1 \leq i > j \leq n$$

Given the premise, answer the following questions.

Question 2.1 How many keys does the relation R have and what are they?

here, $A_i \rightarrow A_j \forall 1 \leq i < j \leq n$

that implies that,

[after applying the Union property, that states that if X → Y and X → Z then X → YZ]

$A_1 \rightarrow A_2 A_3 A_4 A_5 .....A_n$

$A_2 \rightarrow A_3 A_4 A_5 .....A_n$

$A_3 \rightarrow A_4 A_5 ....A_n$

and so on..

now, $A_i \rightarrow A_j \forall 1 \leq i > j \leq n$, implies, similarly,

$A_n \rightarrow A_1 A_2 A_3 .....A_{n-1}$

$A_n - 1 \rightarrow A_1 A_2 A_3 ....A_{n-2}$

and so on,

this means that$A_i \rightarrow A_1 A_2 A_3 ....A_{i-1} A_{i+1} ......A_n$

hence, every attribute is a key in R.

Hence there are n keys.

Question 2.2 State the normal form of the above relation and normalize it to BCNF (if valid) using decomposition rules.

Solution:

→ Assuming all attributes are simple and indivisible in R, the relation is in 1NF.

→ Since the relation is in 1NF and all attributes are prime attributes, hence the relation is in 2NF.

→ Since the relation is in 1NF and 2NF and all attributes are prime attributes, hence there is no non-prime attribute which is transitively dependent on the primary key , the relation R is in 3NF.

→ Since every functional dependency has a key in it hence in any functional dependency $X \rightarrow Y$, X is a superkey, therefore, the relation R is in BCNF.

Question 2.3 Find the minimal cover of the above relation and use it to normalize it to BCNF (if valid).

Solution:

To get minimal cover,

Step 1: First reduce the RHS of all functional dependencies to a single attribute

But here, all the functional dependencies here have single attribute present.

Step 2: remove the redundant functional dependencies

here for a functional dependency to be redundant we have the condition :

X → Y is redundant X+ remains the same even after removing the functional dependency X → Y.

$A_i \rightarrow A_{i+1} \forall 1 \leq i \leq n-1$
$A_n \rightarrow A_1$

Hence, Any cyclic functional dependencies satisfy minimal cover here.

There are (n−1)! possible minimal covers. [permutations]

A_1 has n-1 options to determine

A_2 has n-2 options to determine

A_3 has n-3 options .....

and similarly we get (n-1)! minimal covers.